# Heterogenous Beliefs and Disagreement in Financial Markets

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# Why do people disagree?

Standard approach to model beliefs is **rational expectations (RE)**

- Investors have common priors over payoffs and signals
- The common prior is the objective one (shared by nature and econometrician)
- Only disagree if they have private, asymmetric information
- Very useful in understanding learning, information aggregation

  (see yesterday's sessions!)

Empirically, Rational Expectations is difficult to reconcile with:

- Public, persistent disagreement should converge to agreement
  - Even though most information is public macro forecasts
- Magnitude and dynamics of trading volume and volatility
- Return predictability and over-valuation

# Why do people disagree?

Alt. approach: **heterogeneous priors** or **difference of opinions (DO)**

- Investors have heterogenous priors over payoffs and signals
- Intuitively, they have different models of the world
- Can "agree to disagree" even after observing the same signals
  - Interpret the same information / news differently

Useful for understanding empirical regularities, difficult to reconcile with RE

- Persistent, public disagreement
- Over-valuation and speculative bubbles
- Trading volume magnitudes and dynamics
- Return predictability

# Overview of Talk

- Background Results – No trade theorem
- Basic Framework: RE vs. DO implications
- Applications of the DO approach:
  - Over-valuation
  - Speculative Bubbles
  - Trading volume dynamics
  - Predictability in returns
- Choosing to disagree

Caveat: Notation and models differ from papers to facilitate exposition

# Background: Agree to Disagree and No Trade Theorems

# General Setup

Two assets:

- Risk-free asset normalized to numeraire
- Risky asset has terminal payoff $V \sim f_V(v)$ and equilibrium price $P$

Investors are indexed by $i \in \mathcal{I}$

- Investor $i$ privately observes signal $s_i$, where $s_i|V \sim g_{s|V}(s;v)$
- Investor $i$ submits optimal demand $x_i(s_i, P)$ for the risky assets, and aggregate supply of the asset is $Q$. Market clearing implies:

$$\int_{i \in \mathcal{I}} x_i(\cdot) di = Q$$

# Types of Beliefs

- Investors have **common prior beliefs** if they agree on the joint distribution of payoffs and signals i.e., agree on joint distribution of $(V, \{s_i\}_i, s_p)$

- Investors have **concordant beliefs** if they agree on the conditional distribution of signals given the payoffs
  - They agree on the *interpretation* of signals i.e., $g_{s|V}(s; v)$
  - They can disagree on the prior marginal distribution of payoffs i.e., $f_V(v)$

- Investors have **mutually absolute continuous beliefs** if they assign zero probabilities to the same events
  - Always assume this!
  - Otherwise, investors want to sell Arrow Debreu securities that pay for a zero probability event

# Agree to Disagree and No Trade Theorems

**Aumann 1976:** If two people have **common priors** and their posteriors about a given event are **common knowledge**, then these posteriors are equal i.e., we cannot "agree to disagree"

**Tirole 1982:** Strictly risk-averse investors with **common priors** do not engage in **purely speculative trade**; risk-neutral investors are indifferent to purely speculative trade

**Milgrom and Stokey 1982:** Suppose investors are **strictly risk-averse** with **Pareto optimal** allocations, **concordant** beliefs and **private information**. Then, it cannot be **common knowledge** that all investors strictly prefer the same (non-zero) trade.

**(Tirole 1982):** Risk-averse investors with **common priors** do not engage in **purely speculative trade.**

**Proof by Contradiction:** Suppose **strictly** risk-averse investors are willing to engage in purely speculative trade (i.e., payoff $V$ is independent of endowments)

- Investor $i$ observes private signal $s_i$ and transacts $x_i$ : Gain $G_i = x_i(V - P)$
- Investors have **common priors** about $V, \{s_i\}_i$
- Market clearing condition: $\sum_i x_i = 0$ determines price $P(\{s_i\}_i)$

Note that the total monetary gain must be zero i.e., $\sum_i G_i = 0$

Since each $i$ is strictly risk averse, she must expect a strictly non-negative gain i.e.,

$$\mathbb{E}[G_i|\, s_i, P(\{s_i\}_i)] > 0 \quad \text{for each } i$$

$$\Rightarrow \mathbb{E}[G_i|P(\{s_i\}_i)] > 0 \quad \text{for each } i \ \text{(by the law of iterated expectations)}$$

$$\Rightarrow \mathbb{E}[\textstyle\sum_i G_i|P(\{s_i\}_i)] > 0, \quad \text{which is a contradiction!}$$

There is **no price** in a rational expectations equilibrium at which $V$ can be traded between strictly risk-averse investors!

# RE versus DO benchmarks

# Setup: Payoffs

Two assets:

- Risk-free asset normalized to numeraire
- Risky asset has terminal payoff $V$ is normally distributed with mean $\mu$ and precision $\tau$ i.e.,

$$V \sim \mathcal{N}(\mu, 1/\tau)$$

  - WLOG normalize prior mean $\mu = 0$

- Aggregate supply of the risky asset is normalized to $Q = 0$
  - Papers consider non-zero supply, which has implications for expected returns and risk-premia

# Setup: Preferences and Signals

Investors are indexed by $i \in \mathcal{I}$

- Investor $i$ has negative exponential utility (CARA) over terminal wealth with constant absolute risk aversion $\gamma$. Optimal demand in date 1 is given by:

$$x_{i,1} = \arg\max_{x_1} \mathbb{E}_i\left[-e^{-\gamma\left(W_1 + x_1(V - P_1)\right)} \mid s_i, P_1\right]$$

- Investor $i$ observes **private** signal $s_i$ and **public** signal $s_\eta$

$$s_i = V + \varepsilon_i, \text{ where } \varepsilon_i \sim \mathcal{N}(0\,, 1/\tau_e)$$

$$s_\eta = V + \eta, \text{ where } \eta \sim \mathcal{N}\left(0\,, 1/\tau_\eta\right)$$

# Setup: Timing

Three dates: $t = \{0,1,2\}$

- $\boldsymbol{t = 0}$**:** investors trade with no information, $x_{i,0} = 0$ ; $P_0 = \mathbb{E}[V] = 0$,
- $\boldsymbol{t = 1}$**:** investors observe signals and submit demand $x_{i,1}$ ; market clearing price $P_1$
- $\boldsymbol{t = 2}$**:** risky asset pays off $V$

Denote (dollar) returns by: $R_1 = P_1 - P_0$ and $R_2 = V - P_1$

# Note: Normal + CARA is tractable!

1. The projection theorem for normal distributions implies:

$$V|s_i, s_\eta \sim \mathcal{N}\left(\frac{\tau\,\mu + \tau_e\,s_i + \tau_\eta\,s_\eta}{\tau + \tau_e + \tau_\eta}, \frac{1}{\tau + \tau_e + \tau_\eta}\right)$$

2. For $z \sim \mathcal{N}(\mu, 1/\tau)$, we know $\mathbb{E}[e^{tz}] = e^{t\mu + \frac{t^2}{2\tau}}$.

This implies, optimal demand for investor $i$ is given by:

$$x_{i,1} = \frac{\mathbb{E}_i[V|s_i, P_1] - P_1}{\gamma\ var_i[V|s_i, P_1]}$$

where $\mathbb{E}_i[\cdot]$ and $var_i[\cdot]$ denote the expectation and variance under investor $i$'s subjective beliefs.

# Benchmark: Fully revealing RE equilibrium

**Benchmark setting:** Suppose there is no public signal – investors only observe private signals and the price

Solve for the equilibrium using "guess and check":

1. Conjecture that the price reveals a signal $s_p = \frac{1}{n}\sum_i s_i \equiv \bar{s}$
2. Characterize optimal demand for investor $i$ given private signal and price
3. Solve for the equilibrium price and verify the above conjecture.

Standard approach for RE equilibria

**Step 1:** Suppose the price reveals the average signal i.e.,

$$\bar{s} \equiv \frac{1}{n}\sum_{i} s_i \middle| V \sim \mathcal{N}\left(V, \frac{1}{n\tau_e}\right)$$

Note that the above signal subsumes the private signal $s_i$, so that:

$$V|s_i, P_1 \sim \mathcal{N}\left(\frac{\tau\, 0 + n\tau_e\, \bar{s}}{\tau + n\,\tau_e}, \frac{1}{\tau + n\,\tau_e}\right)$$

**Step 2:** The optimal demand is the same across investors:

$$x_{i,1} = \frac{\mathbb{E}[V|s_i, P_1] - P_1}{\gamma \times var[V|s_i, P_1]} = \frac{\frac{\tau\, 0 + n\tau_e\, \bar{s}}{\tau + n\,\tau_e} - P_1}{\gamma \times \left(\frac{1}{\tau + n\,\tau_e}\right)}$$

**Step 3:** Market clearing implies $\sum_i x_{i,1} = Q = 0$, which yields

$$P_1 = \frac{n\tau_e}{\tau + n\,\tau_e}\ \bar{s}$$

But this verifies the conjecture about the information revealed by the price.

Note: $x_{i,1} = 0$!

# RE equilibrium: Features and Paradoxes

There is **zero trading volume** i.e.,

$$\mathcal{V} \equiv \frac{1}{n}\sum_i \left|x_{i,1} - x_{i,0}\right| = \frac{1}{n}\sum_i |0-0| = 0$$

Returns are **unpredictable** - since $\mathbb{E}[V|P_1] = P_1$, we have:

$$\mathbb{E}[R_2|R_1] = \mathbb{E}[V - P_1|P_1 - P_0] = 0$$

Some problematic features:

- **Grossman Paradox:** Investor $i$ does not need to use private signal $s_i$ to form optimal demand, so how does the price reflect it?
- **Grossman-Stiglitz Paradox:** But then why acquire information $s_i$ ?
- **Beja Paradox:** How does the information in $s_i$ get impounded into prices, since optimal equilibrium demand is $x_{i,1} = Q\ /\ n = 0$ for each $i$

# Resolution: Noise

The RE approach has relied on **noise** to work around problems:

- Noise traders / Liquidity traders: (Unmodeled) price inelastic demand for the asset, usually distributed normally
- Endowment / Labor Income / Alt. Investments: generate non-informational trading demand for CARA investors

**Benefits:** Can avoid the above paradoxes, rich set of models with well defined volume, return predictability

**Cost:** Often, assumptions about the noise drive the results, so

- Difficult to separate impact of information vs. noise
- Noise is often "reduced form," so difficult to test / detect predictions

# Difference of Opinions

Suppose each investor $i$ believes their own signal $s_i$ is informative i.e.,

$$s_i =_i V + \varepsilon_i, \text{ where } \varepsilon_i \sim \mathcal{N}(0\,,1\,/\,\tau_e)$$

but that others' signals are not informative i.e.,

$$s_j =_i \theta + \varepsilon_j, \text{ where } \theta \text{ and } \varepsilon_j \text{ are independent of } V$$

Note: Beliefs are **not** concordant!

- This implies that each investor (incorrectly) believes the price is uninformative so that optimal demand is given by:

$$x_i = \frac{\mathbb{E}_i[V|s_i, P_1] - P_1}{\gamma \times var_i[V|s_i, P_1]} = \frac{\frac{\tau_e}{\tau + \tau_e} s_i - P_1}{\gamma \times \frac{1}{\tau + \tau_e}}$$

- Market clearing implies:

$$P_1 = \frac{\tau_e}{\tau + \tau_e}\ \bar{s}$$

# DO equilibrium: Features

**Trading volume** is positive:

$$\mathcal{V} \equiv \frac{1}{n}\sum_i \left|x_{i,1} - x_{i,0}\right| = \frac{1}{n}\sum_i \left|\tau_e(s_i - \bar{s})\right| > 0$$

Returns are **predictable:** Since $P_1$ reveals $\bar{s}$, we know

$$\mathbb{E}[V|P_1] = \mathbb{E}[V|\bar{s}] = \frac{n\tau_e}{\tau + n\,\tau_e}\;\bar{s}$$

But since $R_1 = P_1 - P_0 = \frac{\tau_e}{\tau+\tau_e}\;\bar{s}$, this implies:

$$\mathbb{E}[R_2|R_1] = \mathbb{E}[V - P_1|R_1] = \left(\frac{n\tau_e}{\tau + n\,\tau_e} - \frac{\tau_e}{\tau + \tau_e}\right)\mathbb{E}[\,\bar{s}|R_1] = \frac{(n-1)\tau}{\tau + n\tau_e}\,R_1$$

so that returns are positively correlated (when $n > 1$)

**Intuition:** Investors under-react to the information in prices

# DO equilibrium: Implications

The **Grossman, Grossman-Stiglitz** and **Beja** paradoxes do not arise: Investor $i$'s optimal demand depends on their private signal $s_i$, so private information is valuable and impounded into prices through trade

Variants of DO models:

- Have different priors about $V$, but agree on interpretation of signals i.e., $s|V$
- Dismiss / Under-estimate the information that others have
- Interpret the same signal differently

Offer flexibility, but must be disciplined by data!

# Applications of the DO approach

The DO approach generates a number of empirically relevant predictions:

- Overvaluation in the presence of short sales: Miller 1977
- Speculative Bubbles: Harrison and Kreps 1979, Scheinkman and Xiong 2003
- Trading Volume Dynamics: Kandel and Pearson 1995, Banerjee and Kremer 2010
- Return Predictability: Banerjee, Kaniel and Kremer 2011

# Application 1: Overvaluation

# Miller (1977): DO + Short Sales Constraints

In the basic framework, assume:

- Continuum of investors indexed by $i$
- Given signals, investor $i$'s (posterior) beliefs about payoffs are given by

$$V \sim_i \mathcal{N}(\mu_i, 1 / \hat{\tau})$$

where $\mu_i \sim \mathcal{U}[\mu - \Delta, \mu + \Delta]$ . The optimal demand for investor $i$ is

$$x_i = \frac{\hat{\tau}}{\gamma}(\mu_i - P_1)$$

- Aggregate supply of risky asset is $Q$
- **Important:** Assume short-sales are **banned**.

# DO + Short Sales Constraints ⇒ Overvaluation

Market clearing implies that the aggregate demand is equal to $Q$ i.e.,

$$\frac{1}{2\Delta}\int_{P}^{\mu+\Delta}\frac{\hat{\tau}(\mu_i - P_1)}{\gamma}\,di = \frac{\hat{\tau}(\mu+\Delta-P)^2}{4\gamma\Delta} = Q$$

which implies

$$P_1 = \mu + \Delta - 2\sqrt{\frac{\gamma\Delta Q}{\hat{\tau}}}$$

- Without short-sales ban, the price would be $P_1' = \mu - \frac{\gamma}{\hat{\tau}}Q$

When disagreement is sufficiently high (i.e., $\Delta > \gamma Q/\hat{\tau}$),

$$P_1 - P_1' = \left(\sqrt{\Delta} - \sqrt{\frac{\gamma}{\hat{\tau}}Q}\right)^2 > 0$$

i.e., the price with the short-sales ban is **higher** than without the ban.

# DO + Short Sales Constraints ⇒ Overvaluation

- Possible explanation for over-valuation of IPO stocks
- Consistent with Diether, Malloy and Scherbina (2002), who document a **negative** cross-sectional relation between analyst forecast dispersion and expected returns higher $P_1 \Leftrightarrow$ lower $R_2$
- DO assumption is important: With RE, Diamond and Verrecchia (1987) show that short-sales do not bias prices on average
- Short-sales ban can be relaxed:
  - Intuitively works if short-sales are costly, not banned
  - In a dynamic setting, Banerjee (2011) shows that in a DO equilibrium, disagreement and expected returns are negatively related

# Application 2: Speculative Bubbles

# Speculative Bubbles

A **speculative bubble** is when the price exceeds the valuation of the most optimistic investor

- Agents pay a higher price than their own valuation for the asset today because they believe they can sell it to others tomorrow for an even higher price "resale option value"

- **Need:** Heterogeneous beliefs, short sales constraints and **change** in the identity of optimist

**Harrison and Kreps (1979)** - Risk neutral setting

**Scheinkman and Xiong (2003)** - Risk averse setting

# Simple Overlapping Generations Setup

Consider an infinite horizon, OLG model with single consumption good

- Generation $t$ is born with endowment at date $t$, consumes at $t+1$
- Risk-free asset with gross return $1+r$
- Risky asset with aggregate supply 1 has dividend of $d_t$ in period $t$, where

$$d_{t+1} = d_t \, G_{t+1} \text{ where } \mathbb{E}_t[G_{t+1}] = \mathbb{E}_t[G_{t+s}] = 1 \text{ for all } s \geq 1$$

- Two groups of risk-neutral investors $i \in \{A, B\}$, who only **disagree** about next period's growth $G_{t+1}$

$$\mathbb{E}_{A,t}[G_{t+1}] = 1 \quad \text{and} \quad \mathbb{E}_{B,t}[G_{t+1}] = 1 + g$$

- But, have **correct** beliefs about subsequent growth i.e., for $s \geq 2$

$$\mathbb{E}_{A,t}[G_{t+s}] = \mathbb{E}_{B,t}[G_{t+s}] = 1$$

- Short selling is **banned**.

# Buy and hold valuations

Conditional on dividend $d_t$, beliefs about future dividends are:

- For $A$ investors, $\mathbb{E}_{A,t}[d_{t+1}] = \mathbb{E}_{A,t}[d_{t+s}] = d_t$ for all $s \geq 1$
- For $B$ investors, $\mathbb{E}_{B,t}[d_{t+1}] = \mathbb{E}_{B,t}[d_{t+s}] = d_t(1+g)$ for all $s \geq 1$

So the "buy and hold" valuation for each type of investor is:

$$P_A(d_t) = \frac{d_t}{r} \text{ and } P_B(d_t) = \frac{d_t(1+g)}{r}$$

This is the valuation for the asset if trading was not allowed.

**Note:** $B$-investors' valuation is always more optimistic.

# Speculative premium: Price > Most optimistic valuation

Consider a stationary, linear equilibrium where $P_t(d_t) = A\, d_t$

- The price can be defined recursively as:

$$A\, d_t \equiv P_t = \max_{i \in \{A,B\}} \frac{\mathbb{E}_{i,t}[d_{t+1} + P_{t+1}]}{1+r} = \max_{i \in \{A,B\}} \frac{\mathbb{E}_{i,t}[d_{t+1}(1+A)\,]}{1+r}$$

- But this implies:

$$A\, d_t = \frac{d_t(1+g)(1+A)}{1+r} \Rightarrow A = \frac{1+g}{r-g}$$

So the price

$$P_t(d_t) = \left(\frac{1+g}{r-g}\right) d_t > P_B(d_t)$$

Price features a resale option: Optimists pay more today because they expect to sell to future generations who will be even more optimistic than them…

**Key:** Identity of the most optimistic investor switches

In the OLG example, new generations guarantee this

In Harrison and Kreps (1979) and Scheinkman and Xiong (2003):

- investors are **long-lived**, but
- beliefs are such that the **groups alternate between** being **optimistic** and **pessimistic**
  - (e.g., investors in HK disagree on the persistence of the dividend process)

# Application 3: Trading Volume Dynamics

# Models of Volume

Standard models of trade are driven by the following possible motives:

- Hedging risks
- Asymmetric information – but we need liquidity traders / noise traders (usually unmodeled)
- Difference of opinions

Difference of opinions are a useful approach in modeling volume since they can match a number of empirical patterns.

# Trading volume around earnings announcements

A number of papers that empirically study trading volume (and price dynamics) around earnings announcements

- **Kandel and Pearson (1995):** significant volume, often unrelated to price changes
  - They also document that analyst forecasts **flip / diverge after announcement**
- **Chae (2005):** volume is low before a public announcement, spikes at the announcement and then dies down gradually after the announcement

Difficult to reconcile with RE models

- Usually, public signals lead to more **agreement** (exception Kondor (2012))
- In RE models, volume is always accompanied by price changes
- Dynamic, noisy RE models (e.g., He and Wang, 1995) predict high volume leading up to announcement, and then low volume afterwards

# DO and Volume dynamics

**Banerjee and Kremer (2010)** build on intuition from Kandel and Pearson (1995) and Harris and Raviv (1993)

- Finite horizon, dynamic model (also, have infinite horizon OLG version)
- Two groups of investors $i \in \{A, B\}$ maximize CARA utility over final wealth
- Risky asset has zero net supply and pays terminal dividend $\boldsymbol{D}$

$$D = V + d, \text{ where } d \sim \mathcal{N}(0, 1/\eta) \text{ and } V \sim \mathcal{N}(0, 1/\tau_0)$$

- At each date $t$, investors observe a **public** signal

$$s_t = V + \varepsilon_t \text{ where } \varepsilon_t \sim \mathcal{N}(0,1/q_t)$$

which they interpret differently:

$$\varepsilon_t \sim_i \mathcal{N}(e_{i,t}, 1/q_t), \text{ where } e_{i,t} \sim \mathcal{N}(0, \lambda_t)$$

**Intuition:** Same earnings number "beats expectations" for some, "misses" for others

- The variance $\boldsymbol{\lambda_t}$ captures the extent of disagreement about date $t$ information

# Beliefs and optimal demand

Conditional on date $t$ information, investor $i$'s beliefs can be recursively defined as: $V \sim_i \mathcal{N}(\mu_{i,t}, 1/\tau_t)$, where

$$\boldsymbol{\mu_{i,t+1}} = (1-\pi_t)\boldsymbol{\mu_{i,t}} + \pi_t\left(s_{t+1} - \boldsymbol{e_{i,t+1}}\right), \text{ and } \tau_{t+1} = \tau_t + q_{t+1}$$

where $\pi_t = \frac{q_{t+1}}{\tau_t + q_{t+1}}$

Working back from the final date, we can show that optimal demand is:

$$x_{i,t} = \omega_t \frac{\mathbb{E}_{i,t}[P_{t+1}] - P_t}{var_{i,t}[P_{t+1}]} + (1-\omega_t)\frac{\mathbb{E}_{i,t}[V] - P_t}{var_{i,t}[V]}$$

for some weight $\omega_t > 0$.

There is a speculative component and a fundamental component of demand

# Equilibrium prices and demand

Given assumptions about symmetry and conditional independence, can show:

$$\mathbb{E}_{i,t}[P_{t+1} - P_t] \propto \mathbb{E}_{i,t}[V - P_t]$$

so that optimal demand is linear in beliefs about fundamentals!

**Equilibrium price** reflects the **average** valuation:

$$P_t = \frac{1}{2}\left(\mathbb{E}_{A,t}[V] + \mathbb{E}_{B,t}[V]\right) \equiv \bar{\mu}_t$$

**Optimal demand** reflects difference in valuations, or **disagreement**:

$$x_{A,t} = \phi_t\left(\mathbb{E}_{A,t}[V - P_t]\right) = \frac{\phi_t\left(\mathbb{E}_{A,t}[V] - \mathbb{E}_{B,t}[V]\right)}{2} \equiv \phi_t \Delta_{\mu,t}$$

# Trading volume

Trading **volume** is driven by **changes in disagreement**

$$\mathcal{V}_{t,t+1} \propto \left|x_{i,t+1} - x_{i,t}\right| = \left|(\phi_{t+1}(1-\pi_t) - \phi_t)\boldsymbol{\Delta_{\mu,t}} - \phi_{t+1}\pi_t\boldsymbol{\Delta_{e,t+1}}\right|$$

– Moreover, $(\phi_{t+1}(1-\pi_t) - \phi_t) \leq 0$ for reasonable parameters

**Idiosyncratic term:** $\Delta_{e,t+1} = e_{A,t+1} - e_{B,t+1}$

- Effect of disagreement about date $t+1$ information
- Induces divergence in beliefs and trade – assumed i.i.d.

**Belief convergence term:** $\Delta_{\mu,t} = \mu_{A,t} - \mu_{B,t}$

- If investors interpret date $t+1$ information identically ($\Delta_{e,t+1} = 0$), beliefs will converge on average since $(\phi_{t+1}(1-\pi_t) - \phi_t) \leq 0$
- Induces persistence in trade

# Implications

Forecasts can diverge / flip after public announcement

Volume can exhibit clustering and persistence, even if news is i.i.d.

- Informative, high disagreement signal (e.g., earnings announcement) followed by less informative / low disagreement signals generate a spike in trading volume (idiosyncratic term) followed by more volume due to belief convergence
- Can generate volume without price changes ($\Delta_{e,t+1}$ and $\bar{e}_{t+1}$ are uncorrel.)

Abnormal volume and volatility are positive correlated over time

- Variation in disagreement in interpretations (i.e., $\lambda_t$) can drive correlation
- Novel predictions on how persistence in volume depends on volatility
  - Some empirical evidence for these in cross section of stocks

# Application 4: Return Predictability

# Predictability, Disagreement and Higher Order Beliefs

Empirically, one observes that momentum and other predictability "anomalies" are stronger for firms with greater disagreement (e.g., Zhang 2006, Verardo 2006)

In dynamic settings, disagreement generates "beauty contest" effects

- Investors buy an asset not only if they consider it attractive, but also if they believe others do (see Keynes, 1936)
- If investors care about future prices, they must form beliefs about the valuations of other investors
- "Higher order beliefs" i.e., beliefs about beliefs of others

# Higher Order Beliefs and Predictability

**Allen, Morris and Shin (2006):** higher-order beliefs may generate persistence

Consider a finite horizon model with **myopic** investors

- Asset pays off $V \sim \mathcal{N}(0, 1\,/\,\tau)$ at terminal date $T$ i.e., $P_T = V$
- At date $t$, investors observe $s_t = V + e_t$, where $e_t \sim \mathcal{N}(0,1/\tau_e)$, and optimally demands: $x_t = \frac{\mathbb{E}_t[P_{t+1} - P_t]}{\gamma\, var_t[P_{t+1}]}$
- Aggregate supply of the asset is $z_t \sim N(0,1/\tau_z)$ i.i.d. over time so that

$$P_{T-1} = \bar{\mathbb{E}}_{T-1}[P_T] - \gamma var_{T-1}[P_T] z_{T-1} = \frac{\tau_e}{\tau + \tau_e} s_{T-1} - \gamma var_{T-1}[P_T] z_{T-1}$$

$$P_{T-2} = \bar{\mathbb{E}}_{T-2}[P_{T-1}] - \gamma var_{T-2}[P_{T-1}] z_{T-2} = \frac{\tau_e}{\tau + \tau_e}\left(\frac{\boldsymbol{\tau_e}}{\boldsymbol{\tau} + \boldsymbol{\tau_e}} \boldsymbol{s_{T-2}}\right) - \gamma var_{T-2}\,[P_{T-1}] z_{T-2}$$

which implies

$$\mathbb{E}[P_{t+1} - P_t | V] = \underbrace{\left(\frac{\tau_e}{\tau + \tau_e}\right)^{T-t-1}\left(1 - \frac{\tau_e}{\tau + \tau_e}\right)}_{\geq 0} V$$

# Not quite there…

Predictability should be conditional on **past** information, i.e., look at

$$\mathbb{E}[P_{t+1} - P_t | \boldsymbol{P_t}], \text{ not } \mathbb{E}[P_{t+1} - P_t | \boldsymbol{V}]$$

In a **rational expectations** model, investors condition on prices **correctly**!

i.e., for each $i$,

$$\mathbb{E}_{i,t}[P_{t+1}] = \mathbb{E}\left[P_{t+1} | S_{i,t}, P_t\right]$$

$$\Rightarrow \quad \forall i, 0 = \mathbb{E}\left[P_{t+1} - \mathbb{E}_{i,t}[P_{t+1}] | P_t\right]$$

But this implies:

$$\mathbb{E}[P_{t+1} - P_t | P_t] = \mathbb{E}[P_{t+1} - (\overline{\mathbb{E}}_t[P_{t+1}] - \gamma var_t[P_{t+1}] z_t) | P_t]$$

$$= \gamma\, var_t[P_{t+1}] \;\; \mathbb{E}[\boldsymbol{z_t} | \boldsymbol{P_t}]$$

so **predictability** is driven by assumptions about (persistence of ) $\boldsymbol{z_t}$

- i.i.d. aggregate supply ⇒ reversals, Persistent aggregate supply ⇒ drift

# Difference of opinions ⇒ Predictability

**Banerjee, Kaniel and Kremer (2009)** show that:

- Higher order beliefs **do not** generate drift in RE models
- Drift can arise in DO models because investors under-react to price

  e.g., with no aggregate noise (like AMS suggest!)

$$P_t = \overline{\mathbb{E}}_t[P_{t+1}] = \overline{\mathbb{E}}_t[V] = \left(\frac{\tau_e}{\tau + \tau_e}\right)^{T-t} V$$

$$\Rightarrow \ \mathbb{E}_t \ [P_{t+1} - P_t | P_t] = \frac{\tau}{\tau_e} \ P_t$$

- With aggregate noise and fully dynamic investors, need "**higher order disagreement**" to generate predictability

  Investors must agree to disagree not only about fundamentals, but also about beliefs about the average valuation and so on.

# Choosing to Disagree

# Choosing Subjective Beliefs

Both RE and DO approaches **assume** investors hold certain beliefs:

- RE: assume subjective beliefs are consistent with objective beliefs
  - Theoretically motivated: Objective beliefs are "accurate"
- DO: assume a specific deviation, disciplined by empirics
  - Overwhelming (direct and indirect) empirical evidence for biases like overconfidence, dismissiveness, extrapolative expectations…
  - DO / Behavioral approach is silent on when / why such biases arise

If given the freedom to choose beliefs, what do investors prefer to do?

# Choosing to Disagree

**Banerjee, Davis and Gondhi (2019)** allow for subjective belief choice

- **Anticipation** of future outcomes affects subjective **utility** and **actions**
  e.g., anxiety before an exam, fear of getting bad news about health
- Investors can **choose** how to interpret the information available to them, subject to a **cost** of distorting beliefs away from RE, to **maximize** anticipated utility

The paper shows that

- Investors always **choose** to deviate from RE
- In any symmetric equilibrium, investors "agree to disagree": overweight private signals, but underweight the information in prices
- When aggregate risk tolerance is sufficiently high, investors endogenously separate into two types: fundamental (who ignore price info) and technical trades (who overweight price info)

# Intuition: Anticipated Utility and Signal Precision

$$AU = \mathbb{E}_i\left[\mathbb{E}_i\left[-e^{-\gamma\left(W_1 + x_i^*(V-P_1)\right)}\middle|\, s_i, P_1\right]\right] = -\sqrt{\frac{var_i[V|s_i, P]}{var_i[V-P]}}$$

Higher perceived precision of signals affects AU through two channels:

- **Information Channel** ($var_i[V|s_i, P]$) Investor faces less uncertainty about payoff ⇒ Trades more aggressively ⇒ higher anticipated utility
- **Speculative Channel** ($var_i[V-P]$) If investor believes prices track fundamentals more closely ⇒ Less opportunity for speculation ⇒ lower anticipated utility

# Intuition: Private Signals vs. Price Signals

$$AU = -\sqrt{\frac{var_i[V|s_i, P]}{var_i[V - P]}}$$

Higher perceived precision of signals affects AU through two channels:

- Beliefs about **private signal** only affects information channel:

  Higher perceived precision of private signal (higher $\delta_{e,i}$) **increases** AU

- Beliefs about **price** affects both information and speculative channels:

  Higher perceived precision of price (higher $\delta_{z,i}$) can **increase or decrease** AU, depending on others' beliefs

# Implications

- Preference for price information depends on what others are doing:
  - If others are using prices, speculative channel dominates
  - If others are **not** using prices, information channel can dominate

- Same underlying motives can lead to

  (i) multiple "biases" – over-confidence, dismissiveness

  (ii) endogenous heterogeneity: – when risk tolerance is high,

  most investors ignore prices, but others overweight it

- Approach allows us to better understand under what conditions behavioral biases arise

# Conclusions

# Disagreement in Financial Markets

Rational expectations is the traditional approach

- Very useful benchmark
- Difficult to reconcile some empirical patterns

Difference of opinions is a useful alternative

- Natural explanation for why people continue to disagree, especially after observing same information
- Delivers simple mechanisms for a number of empirical regularities
- Must be disciplined by empirical / psychological evidence: avoid the “anything goes” criticism

# Future areas of research

Develop richer, "quantitative" models that can be calibrated / estimated

- Challenge: Beliefs / Information not usually observable
- Possible approaches:
  - Derive distinguishing predictions on observables (e.g., Banerjee, 2011)
  - Use data from forecasts / surveys

Subjective belief choice is a promising approach

- Gives insight into what type of biases / deviations from RE arise in different environments

Study impact of policy / regulatory changes to information environment when investors do not exhibit RE (e.g., Banerjee, Davis and Gondhi, 2018)