Competition and Collusion Among Strategic Traders Who Face Uncertainty

Snehal Banerjee and George Malikov*

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Abstract

Conventional wisdom suggests that informed investors benefit from colluding in their trading. However, we show that this may not hold when investors face uncertainty about other traders' behavior. In a Kyle (1985) framework, we compare trading profits under collusive and competitive equilibria when informed investors face uncertainty about liquidity trading volatility. While low uncertainty favors collusion, we show that the expected profit of an *individual* investor under competition can be higher than the *total* profits for all investors under collusion when uncertainty is sufficiently high. This finding cautions against relying solely on profits to detect collusive behavior.

JEL: D82, D84, G12, G14

Keywords: collusive trading, strategic behavior, uncertainty about liquidity

*Banerjee (snehalb@umich.edu) is at the University of Michigan, Ann Arbor, and Malikov (gmalikov@ivey.ca) is at the University of Western Ontario. We thank Sugato Bhattacharyya, Bradyn Breon-Drish, Paolo Pasquariello, Filipp Prokopev, Kevin Smith, and Liyan Yang for useful discussions. All errors are our own.

1 Introduction

Antitrust enforcement and the identification of collusive behavior are some of the primary objectives of regulatory agencies worldwide. Recently such issues have come to the forefront as the implementation of machine learning, artificial intelligence, and big data in pricing and trading mechanisms has sparked concerns regarding the potential for such algorithms to learn to collude tacitly. A series of policy reports and proposed rules (e.g., OECD (2017), SEC (2023)), legal studies (e.g., Azzutti, Ringe, and Stiehl (2022)), and experimental works (e.g., Calvano, Calzolari, Denicolò, and Pastorello (2020), Colliard, Foucault, and Lovo (2022), Dou, Goldstein, and Ji (2024)) have brought attention to these issues. A common theme of the latter is that tacit collusion should result in, and be identifiable via, excess profits relative to a competitive benchmark.

Such logic has merit. For example, antitrust enforcement in financial markets is often based on the notion that informed investors are better off if they are able to collude and coordinate their trading behavior. This common wisdom is derived from analysis of strategic trading models which show that when investors with correlated information compete, they trade aggressively on their signals and drive down trading profits (e.g., Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), Back, Cao, and Willard (2000)).

We show this may no longer be the case if traders face uncertainty about the volatility of liquidity trading. Specifically, we consider a two-trader extension of Kyle (1985), where traders observe conditionally independent signals about the terminal value of the risky security and submit market orders. A risk neutral competitive market maker sets the price as the conditional expected value of the security, given the total order flow from informed investors and liquidity (noise) traders. Following Hong and Rady (2002), we assume that while the market maker knows the distribution of noise trading, the informed investors do not — instead, they face uncertainty about whether noise trading volatility is low or high. This reflects a realistic feature of many market settings: while investors may be better informed about security fundamentals than the market maker, they are likely to be less well-informed about market conditions, and, in particular, the trading behavior of other participants.[1] Moreover, such uncertainty is particularly important for machine-learning based algorithms, which are designed under the premise of uncertain trading environments and payoffs.

We characterize the linear equilibrium in this setting under collusion and competition between investors. In the collusive equilibrium, we assume investors can perfectly collude by pooling their signals when choosing how to trade. In the competitive equilibrium, each investor conditions only on her own signal and best responds to her conjectures about the other investor's and the market maker's strategy. In either equilibrium, the intensity with which the investors trade on their information depends on the precision of their signals and their expectation of the price impact they will face, based on whether noise trading volatility is high or low. In contrast, the market maker conditions on both the order flow and the volatility of noise trading when setting the price impact:

[1] For example, funds may outsource the evaluation of trade execution costs to third parties such as Investment Technology Group, recently acquired by Virtu Financial.

when noise trading volatility is high (low), price impact is low (high, respectively).

We show that, holding fixed the average level of noise trading volatility, an increase in uncertainty about noise trading volatility leads investors to trade less aggressively on their private signals. Intuitively, this is because the *average* price impact they face is increasing in the uncertainty about noise trading volatility.[2] As a result, trading profits also decrease with uncertainty about noise trading volatility.

The key difference across the collusive and competitive equilibria is how quickly trading intensity and profits decrease with uncertainty. When uncertainty is low, the standard intuition applies. Trading intensity in the collusive equilibrium is lower than in the competitive equilibrium, and as a result, trading profits are higher. However, higher uncertainty about noise trading volatility (holding fixed the mean) implies that, in some states, noise trading volatility (and consequently liquidity) will be very low and price impact will be very high. In the collusive equilibrium, investors' strategies are perfectly coordinated and equivalent to that of a *single* strategic investor who observes both signals. As a result, each investor internalizes the impact of their trading not only on the order flow, but also on the trading intensity of the other investor, and so cuts back their trading intensity sharply in response to their uncertainty. In fact, as uncertainty increases, the equilibrium trading intensity and expected profits approach zero.

In contrast, each investor does not fully internalize the impact of their trades in the competitive equilibrium, because they take the trading behavior of the other investor as given. As a result, trading intensity and expected profits decrease more slowly as uncertainty increases. This implies that while trading profits are higher under the collusive equilibrium when uncertainty is low, they are eventually higher under the competitive equilibrium as uncertainty increases. Moreover, we show that the greater trading intensity and liquidity in the competitive equilibrium translate to greater expected price informativeness.

As such, our results point to a potential complication in identifying collusive behavior in financial markets: the focus on speculative profits alone may be misleading. To the extent that speculators face uncertainty about the behavior of liquidity traders, realized profits of competitive speculators may exceed the profits of speculators colluding perfectly. Therefore, caution is warranted when relying on profits as the primary measure of collusive behavior for regulatory purposes.

Our analysis also speaks to the recent literature on artificial intelligence and machine learning that studies the potential for tacit collusion among algorithms in agent-based settings. A growing number of papers study such effects not only in the financial market settings (e.g., Dou et al. (2024), Colliard et al. (2022)) but also in the more traditional product-pricing sectors (e.g., Cho and Williams (2024), Calvano et al. (2020)). These papers argue that, in simulations, algorithms appear to implicitly converge to strategies that are consistent with collusive behavior, even though they are unable to explicitly communicate or coordinate with each other.[3] Our analysis suggests a

[2]For fixed trading strategies, market liquidity (i.e., $\frac{1}{\lambda}$) is linear in the variance of noise trading (σ_z^2), which implies that price impact (λ) is convex in it. This implies average price impact is increasing in the uncertainty about σ_z^2.

[3]Empirically identifying such effects is inherently challenging, although Assad, Clark, Ershov, and Xu (2024) conjecture that collusive effects may be present in the German retail gasoline markets.

possible confound in such settings. In the presence of parameter uncertainty, as is likely faced by such algorithms, profits may be high even in the absence of tacit collusion. As such, our analysis suggests that one should account for the *direct* impact of parameter uncertainty when interpreting the evidence from such simulations as being evidence of collusive behavior.

Our analysis extends the model in Hong and Rady (2002) by allowing investors to have conditionally independent signals. As in this earlier work, we show that profits decrease with uncertainty about the volatility of liquidity trading. However, our focus is on how uncertainty affects the relative profits of investors under competition and under collusion. In Section 4, we show that our results also obtain when investors are perfectly informed about asset values and when the number of investors is arbitrarily high (but fixed): in either case, expected profits are higher under competition when uncertainty about noise trading volatility is sufficiently high.

It is worth noting that our economic mechanism does not rely on ambiguity aversion (e.g., Caballero and Krishnamurthy (2008), Condie and Ganguli (2011), Easley and O'Hara (2009)) or overconfidence (e.g., Kyle and Wang (1997), Benos (1998)). Instead, ours is a setting of two-sided private information: investors are better informed about asset values while the market maker is better informed about liquidity trading volatility.

The rest of the paper is organized as follows. Section 2 presents the description of the model and a discussion of important assumptions. Section 3 presents the main analysis of the paper, and Section 4 explores how our results change under different assumptions. Section 5 discusses implications of our analysis for delegated portfolio management and antitrust regulatory policy and presents concluding remarks. All proofs are in the Appendix.

2 Model

Our model is a single-period variant of the multi-trader Kyle model in Hong and Rady (2002), where strategic traders face uncertainty about noise trading volatility. There are two assets: a risky asset and a risk-free asset with interest rate normalized to zero. The risky asset pays off a terminal value of $v \sim N\left(0, \sigma_v^2\right)$ at the end of the period.

We extend the setting in Hong and Rady (2002) by assuming that traders have conditionally independent, private signals about the value of the risky asset. Specifically, there are two strategic traders, indexed by $i \in \{1, 2\}$. Trader i observes a private signal of the form:

$$s_i = v + \varepsilon_i, \quad \text{where } \varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right), \tag{1}$$

and submits market order x_i. The aggregate trade from noise traders is denoted by $z \sim N\left(0, \sigma_z^2\right)$, where the variance of noise trading is distributed according to:

$$\sigma_z^2 \in \left\{\bar{\sigma}^2 - \delta, \bar{\sigma}^2 + \delta\right\}, \text{ where } \Pr\left(\sigma_z^2 = \bar{\sigma}^2 - \delta\right) = \frac{1}{2}, \tag{2}$$

and $\bar{\sigma}^2 > \delta \geq 0$. Finally, there is a risk neutral, competitive market maker, who is privately informed about the realization of σ_z^2, and sets the price P of the risky asset conditional on this information and the total order flow, which we denote by $y = x_1 + x_2 + z$. We assume that v, ε_1, ε_2, and z, as governed by the realization of σ_z^2, are all mutually independent.

As in Hong and Rady (2002), we assume that the market maker knows the realization of σ_z^2 while the strategic traders do not. The above specification implies that the volatility of noise trading variance is given by δ, since

$$\mathbb{V}\left[\sigma_z^2\right] = \frac{1}{4}(2\delta)^2 = \delta^2. \tag{3}$$

For ease of exposition, and with some abuse of notation, we will refer to the parameter δ as the **uncertainty** about noise trading volatility in what follows.

We restrict attention to symmetric, linear equilibria in which (i) the equilibrium trade by investor i is given by $x_i = \beta s_i$, and (ii) the market maker's pricing rule is (conditionally) linear in the order flow $y = x_1 + x_2 + z$. Denote the pricing rule by:

$$P\left(y; \sigma_z^2\right) = \begin{cases} \mathbb{E}\left[v|\sigma_z^2 = \bar{\sigma}^2 - \delta\right] = \lambda_h y & \text{if } \sigma_z^2 = \bar{\sigma}^2 - \delta \\ \mathbb{E}\left[v|\sigma_z^2 = \bar{\sigma}^2 + \delta\right] = \lambda_l y & \text{if } \sigma_z^2 = \bar{\sigma}^2 + \delta \end{cases}. \tag{4}$$

2.1 Discussion of assumptions

Our goal is to explore the impact of uncertainty about noise trading volatility in a setting that deviates minimally from the traditional Kyle (1985) framework. For instance, one could allow for more general distributions over noise trading volatility, but this would make the analysis less tractable and the intuition less transparent. It is worth distinguishing our setting from one in which investors face ambiguity about the distribution of noise trading. Importantly, investors in our setting know the true distribution of noise trading volatility (given by (2)) and are risk-neutral. As such, our results are not driven by ambiguity or ambiguity aversion.

The key assumption in our analysis is that investors are less informed about the distribution of noise trading than the market maker. The stronger assumption that the market maker knows the volatility of noise trading perfectly is made for analytical tractability. In a setting where the market maker faces uncertainty about noise trading volatility, he would update not only on the value of the asset, but also on the volatility of noise trading from the order flow. However, (generically) this would imply that the price would no longer be linear in the order flow, which in turn would (generically) imply that the investors' strategies are no longer linear in their signals, making the analysis less tractable.

3 Analysis

In what follows, we compare the equilibrium when traders can collude to the equilibrium in which they compete.

3.1 Collusion

We begin by considering a benchmark in which the two traders collude perfectly by pooling their information when choosing how to trade. This is equivalent to a single strategic trader who can observe both signals $\{s_1, s_2\}$, and trades x_M to maximize the following objective

$$\mathbb{E}\left[x_M\left(v-\lambda\left(x_M+z\right)\right)|s_1,s_2\right]=x_M\mathbb{E}\left[v|s_1,s_2\right]-\left(\tfrac{\lambda_h+\lambda_l}{2}\right)x_M^2, \tag{5}$$

where the equality follows from the above conjecture for the market maker's pricing rule. This implies that the optimal trade is given by

$$x_M=\frac{1}{\lambda_h+\lambda_l}\mathbb{E}\left[v|s_1,s_2\right]=\frac{1}{\lambda_h+\lambda_l}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2}+\frac{2}{\sigma_\varepsilon^2}}\right)\left(s_1+s_2\right)\equiv\beta_M\left(s_1+s_2\right), \tag{6}$$

since $\mathbb{E}\left[v|s_1,s_2\right]=\frac{\frac{s_1}{\sigma_\varepsilon^2}+\frac{s_2}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2}+\frac{2}{\sigma_\varepsilon^2}}$. Notably, this implies that the collusive trading strategy puts equal weight on the two traders' signals.

Since the market maker can condition on the order flow and the noise trading volatility, σ_z^2, her problem implies that

$$\lambda\left(\sigma_z^2\right)=\frac{\mathbb{C}\left(v,\beta_M\left(s_1+s_2\right)+z\right)}{\mathbb{V}\left(\beta_M\left(s_1+s_2\right)+z\right)}=\frac{2\beta_M\sigma_v^2}{4\beta_M^2\sigma_v^2+2\beta_M^2\sigma_\varepsilon^2+\sigma_z^2} \tag{7}$$

Solving the above system of equations for $\{\beta_M, \lambda_{M,h}, \lambda_{M,l}\}$ gives us the following result.

Proposition 1. *When traders collude perfectly, there exists a unique, linear equilibrium with* $x_M=\beta_M\left(s_1+s_2\right)$ *and*

$$P\left(y;\sigma_z^2\right)=\begin{cases}\lambda_{M,h}y & \text{if } \sigma_z^2=\bar{\sigma}^2-\delta\\ \lambda_{M,l}y & \text{if } \sigma_z^2=\bar{\sigma}^2+\delta\end{cases}, \tag{8}$$

where

$$\beta_M=\frac{\sqrt[4]{\bar{\sigma}^4-\delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2+2\sigma_v^2}},\quad \lambda_{M,h}=\frac{2\beta_M\sigma_v^2}{4\beta_M^2\sigma_v^2+2\beta_M^2\sigma_\varepsilon^2+\sigma_z^2-\delta},\ \text{and}\quad \lambda_{M,l}=\frac{2\beta_M\sigma_v^2}{4\beta_M^2\sigma_v^2+2\beta_M^2\sigma_\varepsilon^2+\sigma_z^2+\delta}. \tag{9}$$

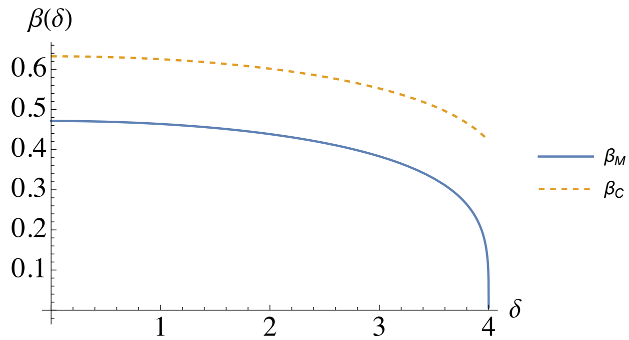
Moreover, β_M *is decreasing in* δ, $\lambda_{M,h}$ *is increasing in* δ, $\lambda_{M,l}$ *is decreasing in* δ, *but* $\frac{\lambda_{M,h}+\lambda_{M,l}}{2}$ *is increasing in* δ. *The investor's expected trading profits are given by*

$$\pi_M=\mathbb{E}\left[x_M\left(v-P\right)\right]=\beta_M\sigma_v^2=\frac{\sigma_v^2\sqrt[4]{\bar{\sigma}^4-\delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2+2\sigma_v^2}}, \tag{10}$$

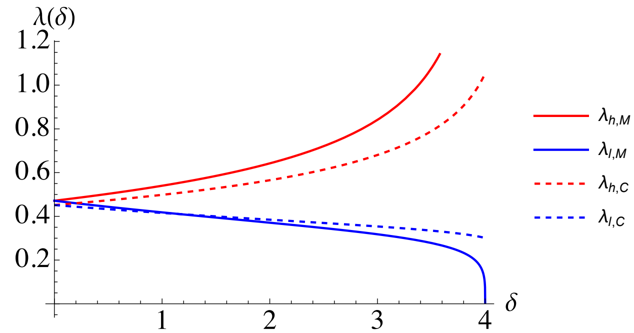
which is decreasing in δ.

The above result highlights that the collusive trading intensity β_M and trading profits π_M are decreasing in uncertainty about noise trading volatility, δ — Figure 1 provides an illustration of

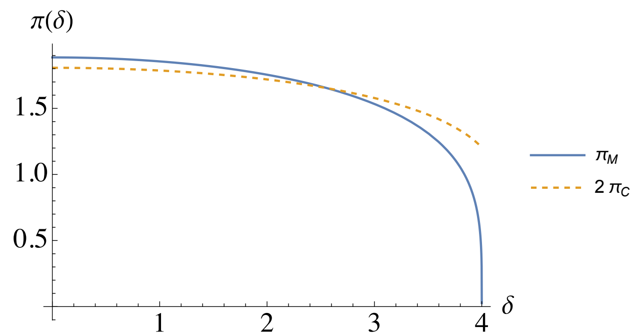
Figure 1: Equilibrium β, λ and π versus uncertainty δ with collusion versus competition. The equilibrium is characterized by $\{\beta_M, \lambda_{l,M}, \lambda_{h,M}, \pi_M\}$ for the collusive equilibrium and $\{\beta_C, \lambda_{l,C}, \lambda_{h,C}, \pi_C\}$ for the competitive equilibrium. Other parameters are set to $\sigma_v = 2$, $\sigma_\varepsilon = 1$ and $\bar{\sigma} = 2$.



(a) β versus δ



(b) λ versus δ



(c) π versus δ

these results. The key to this intuition is that, for a fixed β_M, the price impact $\lambda(\sigma_z^2)$ is a convex function of noise trade volatility σ_z^2, as highlighted by (7).[4] This implies that holding the mean $\bar{\sigma}^2$ fixed, an increase in the variation of noise trading volatility increases the *average* price impact the (collusive) strategic trader faces, i.e., $\frac{\lambda_h+\lambda_l}{2}$ increases with δ. As a result, her trading intensity, β, decreases with δ. Finally, note that expected profits are proportional to trading intensity and, therefore, decreasing in δ, since

$$\pi_M = \mathbb{E}\left[x\left(v - \frac{\lambda_h+\lambda_l}{2}x\right)\right] = \beta_M \sigma_v^2, \tag{11}$$

as we show in the proof of the proposition.

[4]It is worth noting that β_M does not depend on the realized σ_z^2, but $\lambda(\sigma_z^2)$ does.

3.2 Competition

Now consider the case where each trader is trading individually on her own signal, taking the strategy of the other participants as given. Specifically, investor i chooses to maximize:

$$\mathbb{E}\left[x_i\left(v-\lambda x_i-\lambda\left(\beta_j s_j+z\right)\right)|s_i\right]=x_i\left(1-\tfrac{\lambda_h+\lambda_l}{2}\beta_j\right)\mathbb{E}\left[v|s_i\right]-\tfrac{\lambda_h+\lambda_l}{2}x_i^2. \tag{12}$$

This implies that the optimal trade is given by

$$x_i=\frac{\left(1-\frac{\lambda_h+\lambda_l}{2}\beta_j\right)}{\lambda_h+\lambda_l}\mathbb{E}\left[v|s_i\right]=\frac{\left(1-\frac{\lambda_h+\lambda_l}{2}\beta_j\right)}{\lambda_h+\lambda_l}\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\varepsilon^2}}s_i\equiv\beta_i s_i, \tag{13}$$

since $\mathbb{E}\left[v|s_i\right]=\frac{\frac{s_i}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\varepsilon^2}}$. Symmetry implies

$$x_j=\frac{\left(1-\frac{\lambda_h+\lambda_l}{2}\beta_i\right)}{\lambda_h+\lambda_l}\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\varepsilon^2}}s_j\equiv\beta_j s_j \tag{14}$$

and in a symmetric equilibrium, we have:

$$\beta_1=\beta_2\equiv\beta_C=\frac{1}{\lambda_h+\lambda_l}\frac{2\sigma_v^2}{2\sigma_\varepsilon^2+3\sigma_v^2}. \tag{15}$$

Since the market maker can condition on the order flow and the noise trading volatility, σ_z^2, her problem implies that

$$\lambda\left(\sigma_z^2\right)=\frac{\mathbb{C}\left(v,\beta_1 s_1+\beta_2 s_2+z\right)}{\mathbb{V}\left(\beta_1 s_1+\beta_2 s_2+z\right)}=\frac{\left(\beta_i+\beta_j\right)\sigma_v^2}{\left(\beta_i+\beta_j\right)^2\sigma_v^2+\left(\beta_i^2+\beta_j^2\right)\sigma_\varepsilon^2+\sigma_z^2}. \tag{16}$$

Solving the above system of equations for $\{\beta_C,\lambda_{C,h},\lambda_{C,l}\}$ after imposing $\beta_1=\beta_2\equiv\beta_C$ gives us the following result.

Proposition 2. *When traders compete, there exists a unique, symmetric, linear equilibrium with $x_i=\beta_C s_i$ and*

$$P\left(y;\sigma_z^2\right)=\begin{cases}\lambda_{C,h}y & \text{if } \sigma_z^2=\bar{\sigma}^2-\delta\\ \lambda_{C,l}y & \text{if } \sigma_z^2=\bar{\sigma}^2+\delta\end{cases}, \tag{17}$$

where $\beta_C=\frac{1}{2}\sqrt{\frac{\sqrt{\bar{\sigma}^4(2\sigma_\varepsilon^2+3\sigma_v^2)^2-4\delta^2(\sigma_\varepsilon^4+3\sigma_\varepsilon^2\sigma_v^2+2\sigma_v^4)}+\bar{\sigma}^2\sigma_v^2}{\sigma_\varepsilon^4+3\sigma_\varepsilon^2\sigma_v^2+2\sigma_v^4}}$,

$$\lambda_{C,h}=\frac{2\beta_C\sigma_v^2}{4\beta_C^2\sigma_v^2+2\beta_C^2\sigma_\varepsilon^2+\bar{\sigma}^2-\delta},\quad \text{and } \lambda_{C,l}=\frac{2\beta_C\sigma_v^2}{4\beta_C^2\sigma_v^2+2\beta_C^2\sigma_\varepsilon^2+\bar{\sigma}^2+\delta}. \tag{18}$$

Moreover, β_C is decreasing in δ, $\lambda_{C,h}$ is increasing in δ, $\lambda_{C,l}$ is decreasing in δ, but $\frac{\lambda_{C,h}+\lambda_{C,l}}{2}$ is *increasing in δ. The investor's expected trading profits are given by*

$$\pi_C = \mathbb{E}\left[x_C\left(v-P\right)\right] = \beta_C \sigma_v^2 \tfrac{\sigma_v^2+\sigma_\varepsilon^2}{2\sigma_\varepsilon^2+3\sigma_v^2}, \tag{19}$$

which is decreasing in δ.

Figure 1 provides an illustration of these results. As in the collusive equilibrium, trading intensity β_C and profits are decreasing in uncertainty about noise trading volatility, because the average price impact that traders face (i.e., $\frac{\lambda_{C,h}+\lambda_{C,l}}{2}$) is increasing in δ. The next section characterizes how these equilibria differ in their response to uncertainty.

3.3 Relative benefits of collusion

Given the characterization of equilibria in the previous subsections, we now characterize how uncertainty affects the relative benefits of collusion.

Proposition 3. *There exist $0 < \underline{\delta} < \bar{\delta} < \bar{\sigma}^2$ such that:*

(i) if $\delta < \underline{\delta}$, collusion generates higher total profits than competition, i.e., $\pi_M > 2\pi_C$;

(ii) if $\underline{\delta} < \delta < \bar{\delta}$, total profits are higher under competition than collusion, but individual profits are not, i.e., $2\pi_C > \pi_M > \pi_C$;

(iii) if $\bar{\delta} < \delta$, individual profits are higher under competition than total profits under collusion, i.e., $\pi_C > \pi_M$.

The above result highlights that the relative benefit of collusion among strategic traders depends on the uncertainty they face about the distribution of noise trading. Specifically, we show that when uncertainty about noise trading volatility is sufficiently high, expected profits for an *individual* investor under competition can be higher than *total* profits for all investors under collusion.

The key difference between the competitive and collusive equilibria is how aggressively investors trade on their private signals, and how this varies with uncertainty. Note that when $\delta \to 0$, we have

$$\beta_M = \frac{\bar{\sigma}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}} \quad \text{and} \quad \beta_C = \frac{\bar{\sigma}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + \sigma_v^2}} > \beta_M,$$

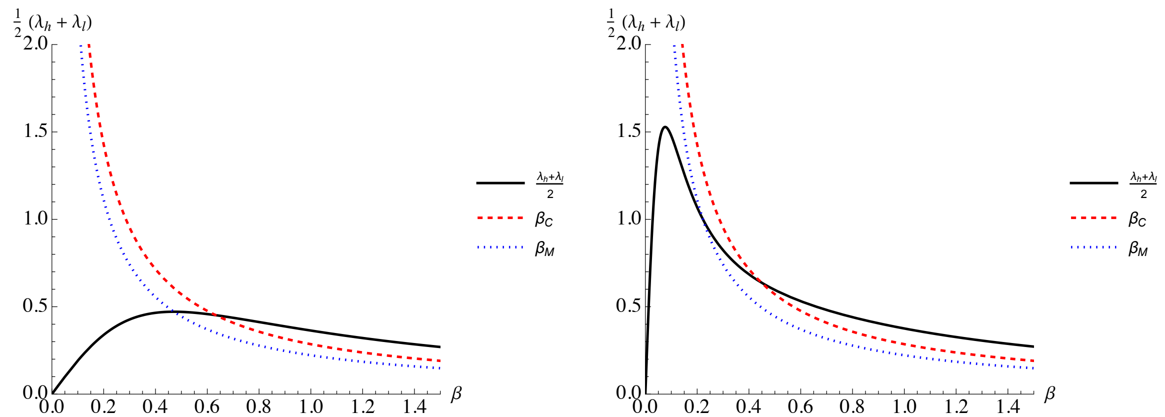
which reflects the fact that, in the absence of uncertainty, investors trade less aggressively on their private information in the collusive equilibrium. This is consistent with the intuition from the existing literature (e.g., Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), Back et al. (2000)), and implies that (total) expected profits are higher under collusion than under competition i.e.,

$$\pi_M = \tfrac{\bar{\sigma}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2+2\sigma_v^2}} \times \sigma_v^2 > 2\pi_C = \tfrac{\bar{\sigma}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2+\sigma_v^2}} \tfrac{2(\sigma_v^2+\sigma_\varepsilon^2)}{2\sigma_\varepsilon^2+3\sigma_v^2} \times \sigma_v^2.$$

As δ increases, investors trade less aggressively under both equilibria i.e., both β_C and β_M fall. However, the response is steeper in the collusive equilibrium, because each investor internalizes not only the impact of their trading on the price, but also on the trading of the other investor. To see

Figure 2: Best response functions

The figure plots the average price impact $\frac{\lambda_h+\lambda_l}{2}$ (solid line) as a best response to the intensity β chosen by traders, and the trading intensities β_M (dotted) and β_C (dashed) as best responses to the average price impact in the collusive and competitive equilibrium respectively. Panel (a) considers a setting with no uncertainty (i.e., $\delta = 0$), while panel (b) considers a setting with high uncertainty (i.e., $\delta = 3.5$). Other parameters are set to $\sigma_v = 2$, $\sigma_\varepsilon = 1$ and $\bar{\sigma} = 2$.



(a) Best responses for $\delta = 0$ (b) Best responses for $\delta = 3.5$

why, it is useful to compare the best response functions of the market participants in each setting. Figure 2 provides an illustration.

First, note that equations (7) and (16) immediately imply that given intensity β chosen by traders, the best response choice of $\lambda(\sigma_z^2)$ for the market maker in either case is the same. In Figure 2, this is plotted as a solid line. Comparing panels (a) and (b), we note that for any fixed β, an increase in uncertainty increases the average price impact i.e., $\frac{\partial}{\partial \delta}\frac{\lambda_h+\lambda_l}{2} > 0$.

Second, note that in either equilibrium, the traders' best response β is a decreasing function of the average price impact $\frac{\lambda_h+\lambda_l}{2}$, as illustrated by the dotted and dashed lines in Figure 2. Moreover, for a given average price impact $\frac{\lambda_h+\lambda_l}{2}$, the traders' best response β is always higher under the competitive equilibrium than under the collusive equilibrium (i.e., $\beta_C > \beta_M$). Intuitively, this is because in the collusive equilibrium, each investor internalizes not only the impact of their trading on the price, but also on the trading of the other investor.

The equilibrium trading intensities in the collusive and competitive equilibria, β_M and β_C, are characterized by the intersection of best response functions. In Figure 2, these correspond to the points at which the solid line intersects the dotted and dashed lines, respectively. As the panels in Figure 2 illustrate, when uncertainty increases, the drop in the equilibrium trading intensity (β_M) in the collusive equilibrium is larger than the corresponding drop in equilibrium intensity (β_C) in the competitive equilibrium. Moreover, the difference between the equilibrium average price impact faced by the competitive and collusive investors increases with greater uncertainty.

Intuitively, in the collusive equilibrium, one can think of the investors as acting as a single, "monopolistic" investor. Holding the mean noise trading volatility ($\bar{\sigma}^2$) fixed, an increase in uncertainty implies that half the time, noise trading volatility is very low and the order flow is very informative (i.e., $\lambda_{M,h}$ is very high). The "monopolistic" trader responds to this by decreasing trading intensity aggressively. In fact, in the limit, as $\delta \to \bar{\sigma}^2$, the trading intensity under the collusive equilibrium drops to zero i.e., $\beta_M \to 0$. Moreover, in equilibrium, this behavior is reinforced by the market maker's response since, in the limit, $\lambda_{M,h} \to \infty$. As a result, the expected profit for investors in the collusive equilibrium also drops to zero.

In the competitive equilibrium, each investor takes the other investor's trading strategy as given, and so does not fully internalize the impact of their own trading on the trading intensity of the other investor. As a result, the trading intensity β_C responds more slowly to changes in δ. In contrast to the collusive equilibrium, as $\delta \to \bar{\sigma}^2$, the equilibrium trading intensity remains strictly positive in the limit i.e.,

$$\lim_{\delta \to \bar{\sigma}^2} \beta_C = \frac{\bar{\sigma}\sigma_v}{\sqrt{2}\sqrt{(\sigma_\varepsilon^2 + \sigma_v^2)(\sigma_\varepsilon^2 + 2\sigma_v^2)}},$$

and as a result, so does the expected profit for each trader.

3.4 Implications for price informativeness

Let $PI(P) = -\mathbb{V}[v|P]$ denote the price informativeness in a given equilibrium. The following lemma establishes that, as in Hong and Rady (2002), price informativeness in either equilibrium depends on the realization of the price when investors face uncertainty about the volatility of noise trading.

Lemma 1. *Price informativeness can be expressed as*

$$PI(P) \equiv -\mathbb{V}[v|P] = -\left(\phi(P) \times \sigma_v^2 (1 - 2\beta\lambda_h) + (1 - \phi(P)) \times \sigma_v^2 (1 - 2\beta\lambda_l)\right),$$

where

$$\phi(P) \equiv \Pr(\sigma_z^2 = \bar{\sigma}^2 - \delta | P) = \frac{1}{1 + \sqrt{\frac{\lambda_h}{\lambda_l}} e^{-\frac{P^2}{2\sigma_v^2}\frac{\lambda_h - \lambda_l}{2\beta\lambda_h\lambda_l}}}, \tag{20}$$

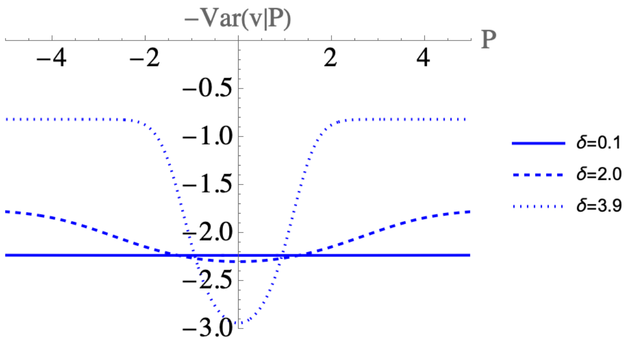
and $\beta = \beta_M, \lambda_h = \lambda_{M,h}$ and $\lambda_l = \lambda_{M,l}$ for the collusive equilibrium, and $\beta = \beta_C, \lambda_h = \lambda_{C,h}$ and $\lambda_l = \lambda_{C,l}$ for the competitive equilibrium.

Figure 3 provides an illustration of this dependence for the collusive and competitive equilibrium. As emphasized by Hong and Rady (2002), price informativeness is higher for larger absolute realizations of P (larger $|P|$), since these realizations allow one to better distinguish the high noise trading volatility state from the low noise trading volatility state. Specifically, one can show that $\lim_{P^2 \to \infty} \phi(P) = 1$, i.e., for sufficiently large realizations of P, one becomes arbitrarily certain that $\sigma_z^2 = \bar{\sigma}^2 - \delta$.

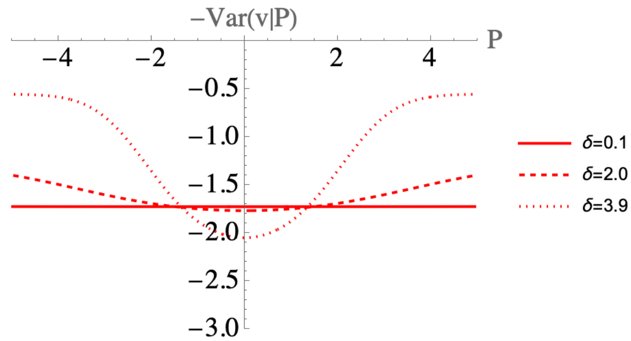
While price informativeness for a given realization of P varies with uncertainty δ, as illustrated by Figure 2, expected price informativeness is independent of δ. The following result establishes how this varies across equilibria in our setting.

Figure 3: Price informativeness $PI(P)$ under the collusive and competitive equilibria

The equilibrium is characterized by $\{\beta_M, \lambda_{l,M}, \lambda_{h,M}\}$ for the collusive equilibrium and $\{\beta_C, \lambda_{l,C}, \lambda_{h,C}\}$ for the competitive equilibrium. Other parameters are set to $\sigma_v = 2$, $\sigma_\varepsilon = 1$ and $\bar{\sigma} = 2$.



(a) $PI(P)$ for collusive equilibrium



(b) $PI(P)$ for competitive equilibrium

Proposition 4. *Under the collusive equilibrium, expected price informativeness is given by:*

$$\mathbb{E}[PI(P)] = -\sigma_v^2 \left(\frac{\sigma_\varepsilon^2 + \sigma_v^2}{2\sigma_v^2 + \sigma_\varepsilon^2} \right) \equiv PI_M,$$

while under the competitive equilibrium, expected price informativeness is given by:

$$\mathbb{E}[PI(P)] = -\sigma_v^2 \left(\frac{2\sigma_\varepsilon^2 + \sigma_v^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2} \right) \equiv PI_C.$$

Moreover, $PI_C > PI_M$.

The above result establishes that expected price informativeness is higher under the competitive equilibrium, even when investors are uncertain about noise trading volatility. While this might initially appear to be at odds with our earlier results, note that one can express expected price informativeness as

$$\mathbb{E}[PI(P)] = \mathbb{E}[-\mathbb{V}[v|P]] = -\sigma_v^2(1 - \beta(\lambda_l + \lambda_h)),$$

as we verify in the proof of the above result. This implies that one can express expected profits under collusion and competition as:

$$\pi_M = \beta_M \sigma_v^2 = \frac{1}{2}(\sigma_v^2 + PI_M) \times \left(\frac{\lambda_{M,h} + \lambda_{M,l}}{2} \right)^{-1}, \quad \text{and} \tag{21}$$

$$\pi_C = \beta_C \sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2} = \frac{1}{2}(\sigma_v^2 + PI_C) \times \left(\frac{\lambda_{C,h} + \lambda_{C,l}}{2} \right)^{-1} \times \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}, \tag{22}$$

respectively, which implies that profits are positively related to expected price informativeness and negatively related to expected market impact. Therefore, greater competitive profits are generated in tandem with more informative prices and more liquid markets.

4 Robustness

Our benchmark analysis restricts attention to a simple setting with noisy signals and two investors to facilitate exposition. In this section, we discuss how our results change when we modify these assumptions.

4.1 Perfect information benchmark

One might wonder whether the result that expected profits are higher under competition when uncertainty is sufficiently high is driven by the assumption that investors have noisy information about payoffs. To explore the robustness of our results along this dimension, in this section, we assume that the investors have access to perfect information, i.e., $s_i = v$. This corresponds to the limit of the equilibria as $\sigma_\varepsilon \to 0$. In the limit, the price coefficients are given by:

$$\beta_M = \frac{\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{2\sigma_v} \quad \beta_C = \frac{\sqrt{\sqrt{9\bar{\sigma}^4 - 8\delta^2} + \bar{\sigma}^2}}{2\sqrt{2}\sigma_v}. \tag{23}$$

Notably, when there is no uncertainty, we have

$$\beta_M = \frac{\bar{\sigma}}{2\sigma_v} < \frac{\bar{\sigma}}{\sqrt{2}\sigma_v} = \beta_C,$$

while in the limit, as $\delta \to \bar{\sigma}^2$, $\beta_M \to 0$, while $\beta_C > 0$.

Moreover, in this case, the ratio of profits is given by:

$$\rho = \frac{\pi_M}{2\pi_C} = \frac{3\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sqrt{9\bar{\sigma}^4 - 8\delta^2} + \bar{\sigma}^2}}.$$

Note that this implies:

$$\lim_{\delta \to 0} \rho = \frac{3}{2\sqrt{2}} > 1 \tag{24}$$

$$\lim_{\delta \to \bar{\sigma}^2} \rho = 0 \tag{25}$$

$$\frac{\partial \rho}{\partial \delta} = -\frac{3\delta\bar{\sigma}^2}{2\sqrt{2}\left(\bar{\sigma}^4 - \delta^2\right)^{3/4}\sqrt{\left(9\bar{\sigma}^4 - 8\delta^2\right)\left(\sqrt{9\bar{\sigma}^4 - 8\delta^2} + \bar{\sigma}^2\right)}} < 0. \tag{26}$$

This implies that, as in the benchmark analysis, there exists a $\underline{\delta} \in \left(0, \bar{\sigma}^2\right)$ such that $\rho\left(\underline{\delta}\right) = 1$, and another $\bar{\delta} \in \left(0, \bar{\sigma}^2\right)$ such that $\rho\left(\bar{\delta}\right) = \frac{1}{2}$.

As such, we find that our main conclusion remains unchanged, i.e., while total expected profits are higher under collusion when investors face no uncertainty, they are higher under competition when uncertainty is sufficiently high.

4.2 Multiple informed traders

Another feature of our benchmark analysis is that we assume there are only two informed investors. In this section, we explore how our results change when we instead assume there are an arbitrary number of investors.

Suppose there are $N > 1$ traders who each observe $s_i = v + \varepsilon_i$. The following result provides a characterization of the equilibrium and the relative profits under the collusive and competitive equilibrium.

Proposition 5. *In either equilibrium, investor i submits a trade $x_i = \beta s_i$ and the market maker sets the price as*

$$P\left(y;\sigma_z^2\right) = \begin{cases} \lambda_h y & \text{if } \sigma_z^2 = \bar{\sigma}^2 - \delta \\ \lambda_l y & \text{if } \sigma_z^2 = \bar{\sigma}^2 + \delta \end{cases} \tag{27}$$

where

$$\lambda_h = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta} \quad \lambda_l = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta}. \tag{28}$$

In the collusive equilibrium, the trading intensity is given by

$$\beta = \beta_M = \sqrt[4]{\frac{\bar{\sigma}^4 - \delta^2}{N^2\left(\sigma_\varepsilon^2 + N\sigma_v^2\right)^2}} \tag{29}$$

and the total expected profits are given by

$$\pi_M = \frac{1}{2}\beta_M N\sigma_v^2. \tag{30}$$

In the competitive equilibrium, the trading intensity is given by

$$\beta = \beta_C = \sqrt{\frac{\sqrt{\bar{\sigma}^4(2\sigma_\varepsilon^2+(N+1)\sigma_v^2)^2-4\delta^2(\sigma_\varepsilon^2+\sigma_v^2)(\sigma_\varepsilon^2+N\sigma_v^2)}+(N-1)\bar{\sigma}^2\sigma_v^2}{2N(\sigma_\varepsilon^2+\sigma_v^2)(\sigma_\varepsilon^2+N\sigma_v^2)}} \tag{31}$$

and the total expected profits are given by

$$N\pi_C = \frac{\left(\sigma_v^2 + \sigma_\varepsilon^2\right)}{\left(\left(N+1\right)\sigma_v^2 + 2\sigma_\varepsilon^2\right)}\beta_C N\sigma_v^2. \tag{32}$$

The above result generalizes the analysis in our main model. In fact, one can verify that the expressions coincide if we set $N = 2$. The next result establishes how the relative benefit of collusion depends on uncertainty and the number of investors.

Proposition 6. *Let $\rho\left(\delta, N\right) \equiv \frac{\pi_M}{N\pi_C}$ denote the ratio of total expected profits under the collusive and competitive equilibria. Then,* *(i) for a fixed δ,*

$$\lim_{N\to\infty} \pi_M = \frac{1}{2}\sigma_v \sqrt[4]{\bar{\sigma}^4 - \delta^2} \text{ and } \lim_{N\to\infty} N\pi_C = 0, \tag{33}$$

so that $\lim_{N\to\infty} \rho(\delta, N) = \infty$,

(ii) for a fixed $N > 1$, $\rho(0, N) > 1$, $\lim_{\delta\to\bar{\sigma}^2} \rho(\delta, N) = 0$, *and* $\frac{\partial\rho}{\partial\delta} < 0$.

For a fixed level of uncertainty, we show that expected profits under collusion are higher when the number of investors is sufficiently high. Intuitively, for a fixed δ, as N increases, investors compete more aggressively in the competitive equilibrium — in the limit, as $N \to \infty$, *total* expected profits reduce to zero in this case. In contrast, total expected profits stay strictly positive under the collusive equilibrium. This result is consistent with earlier work (e.g., Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), Back et al. (2000)).

However, we also show that for a fixed number N of investors, investors have lower expected profits under the collusive equilibrium when uncertainty about noise trading volatility is sufficiently high. As such, the effects of uncertainty about noise trading volatility, which we highlight in our benchmark analysis, obtain even if the number of investors in the economy is large.

5 Implications and Concluding Remarks

We consider a multi-investor extension of the Kyle (1985) model in which investors face uncertainty about the volatility of liquidity trading. We compare expected trading profits under a competitive equilibrium to those under a perfectly collusive equilibrium, in which all investors combine their information and coordinate perfectly. We show that when uncertainty is low, expected profits are higher under the collusive equilibrium, consistent with existing analysis. However, we find that when uncertainty increases, this may no longer be the case. In fact, when uncertainty about liquidity trading volatility is sufficiently high, we show that the expected profit for an *individual* investor in the competitive equilibrium can be higher than the *total* profit for all investors under the collusive equilibrium. As such, uncertainty about liquidity trading can have a substantive impact on the relative benefits of collusion among strategic investors.

As we outline below, our analysis has implications for the delegated portfolio management sector and antitrust regulation.

Delegated portfolio management. Mutual fund family structure has been shown to provide for strategic benefits for the top-performers within a family at the expense of the under-performers (e.g., Bhattacharya, Lee, and Pool (2013), Gaspar, Massa, and Matos (2006), Eisele, Nefedova, Parise, and Peijnenburg (2020)). Our model can be viewed through the lens of competition vs. cooperation within the fund family (e.g., Evans, Prado, and Zambrana (2020)), whereby a family may choose to have the same portfolio manager run multiple funds or assign the investment decisions in these funds to different managers. Our analysis suggests that in periods of greater uncertainty, or for investment strategies where liquidity regimes are highly variable and difficult to predict, fund families would benefit from inducing competition among its portfolio managers, even if these managers have correlated ideas and strategies. Conversely, when traders face low uncertainty about liquidity, the family should encourage cooperation among portfolio managers running different strategies (i.e., via the sharing of ideas as in Kacperczyk and Seru (2012) and Cici, Jaspersen, and Kempf (2017)) or have the same portfolio manager responsible for multiple funds (thereby fully internalizing the effect of her trading across the different strategies). A natural starting point for testing this implication of the model is to consider various measures of team-managed vs. solo-managed funds (e.g., Patel and Sarkissian (2017)) and interact these with the variation in noise trading volatility (e.g., using the approach in Peress and Schmidt (2021)).

Antitrust policy. Several academic and policy papers have raised concerns regarding the potential for AI/ML-based algorithms to learn to collude tacitly (e.g., Azzutti et al. (2022)). A common theme is that tacit collusion may result in either supra-competitive speculative profits (e.g., Dou et al. (2024)) or market maker mark-ups (e.g., Colliard et al. (2022)). Our analysis highlights a confound in relying solely on measures of profitability as an indicator for tacit collusion. Because trading algorithms inherently face parameter uncertainty, the less-aggressive trading may be an optimal response to such uncertainty as opposed to an intent to collude tacitly. Regulations aimed at making order flow more transparent have the potential to resolve this confounding effect: if traders are endowed with a reliable estimate of noise trading volatility, one may argue with greater confidence that the supra-competitive profits are indeed outcomes of collusive behavior. However, as highlighted by Azzutti et al. (2022), greater transparency might also facilitate collusion because deviations from optimal behavior are more immediate and punishments for deviations are easier to implement.

On a related note, policies aimed at reducing the uncertainty regarding noise trading volatility for informed speculators (i.e., reducing δ) have the potential to improve speculative profits in both competitive and collusive settings – an immediate consequence of Propositions 1 and 2. Importantly, however, our analysis uncovers a potential drawback of such policies. Note that as uncertainty about liquidity falls, collusion becomes more attractive to traders (Proposition 3). Moreover, Proposition 4 establishes that average price informativeness is lower in the collusive equilibrium than under competition. As such, an increase in transparency about liquidity trading can inadvertently lead to more collusive behavior among sophisticated traders and, consequently, lower price informativeness.

Future work. Our model is stylized for tractability and expositional clarity, but may be extended along a number of dimensions. It would be interesting to compare the impact of competition versus collusion in a dynamic version of our model in which noise trading volatility evolves stochastically, and strategic traders learn about this over time. It would also be informative to consider the impact of investor uncertainty along other dimensions (e.g., the number of other investors in the market or their risk aversion). Finally, allowing for heterogeneous information quality and endogenous information acquisition would further test the robustness of our main result. We leave these extensions for future work.

References

Assad, S., R. Clark, D. Ershov, and L. Xu (2024). Algorithmic pricing and competition: empirical evidence from the german retail gasoline market. *Journal of Political Economy 132*(3), 723–771. 3

Azzutti, A., W.-G. Ringe, and S. Stiehl (2022). Machine learning, market manipulation, and collusion: why the "black box" matters'. *Working paper*. 1, 5

Back, K., C. H. Cao, and G. A. Willard (2000). Imperfect competition among informed traders. *The Journal of Finance 55*(5), 2117–2155. 1, 3.3, 4.2

Benos, A. V. (1998). Aggressiveness and survival of overconfident traders. *Journal of Financial Markets 1*(3-4), 353–383. 1

Bhattacharya, U., J. H. Lee, and V. K. Pool (2013). Conflicting family values in mutual fund families. *The Journal of Finance 68*(1), 173–200. 5

Caballero, R. and A. Krishnamurthy (2008). Collective risk management in a flight to quality episode. *Journal of Finance 63*(5), 821–845. 1

Calvano, E., G. Calzolari, V. Denicolò, and S. Pastorello (2020). Artificial intelligence, algorithmic pricing, and collusion. *American Economic Review 110*(10), 3267–3297. 1

Cho, I.-K. and N. Williams (2024). Collusive outcomes without collusion: algorithmic pricing in a duopoly model. *Working paper*. 1

Cici, G., S. Jaspersen, and A. Kempf (2017). Speed of information diffusion within fund families. *Review of Asset Pricing Studies 7*(1), 144–170. 5

Colliard, J.-E., T. Foucault, and S. Lovo (2022). Algorithmic pricing and liquidity in securities markets. *HEC Paris Research Paper*. 1, 5

Condie, S. and J. Ganguli (2011). Ambiguity and rational expectations equilibria. *Review of Economic Studies 78*(3), 821–845. 1

Dou, W. W., I. Goldstein, and Y. Ji (2024). Ai-powered trading, algorithmic collusion, and price efficiency. *Working paper*. 1, 5

Easley, D. and M. O'Hara (2009). Ambiguity and nonparticipation: the role of regulation. *Review of Financial Studies 22*(5), 1817–1843. 1

Eisele, A., T. Nefedova, G. Parise, and K. Peijnenburg (2020). Trading out of sight: An analysis of cross-trading in mutual fund families. *Journal of Financial Economics 135*(2), 359–378. 5

Evans, R. B., M. P. Prado, and R. Zambrana (2020). Competition and cooperation in mutual fund families. *Journal of Financial Economics 136*(1), 168–188. 5

Foster, F. D. and S. Viswanathan (1996). Strategic trading when agents forecast the forecasts of others. *The Journal of Finance 51*(4), 1437–1478. 1, 3.3, 4.2

Gaspar, J.-M., M. Massa, and P. Matos (2006). Favoritism in mutual fund families? Evidence on strategic cross-fund subsidization. *The Journal of Finance 61*(1), 73–104. 5

Holden, C. W. and A. Subrahmanyam (1992). Long-lived private information and imperfect competition. *The Journal of Finance 47*(1), 247–270. 1, 3.3, 4.2

Hong, H. and S. Rady (2002). Strategic trading and learning about liquidity. *Journal of Financial Markets 5*(5), 419–450. 1, 2, 2, 3.4, 3.4

Kacperczyk, M. T. and A. Seru (2012). Does firm organization matter? Evidence from centralized and decentralized mutual funds. *Working Paper*. 5

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica 53*(6), 1315–1335. 1, 2.1, 5

Kyle, A. S. and F. A. Wang (1997). Speculation duopoly with agreement to disagree: Can overconfidence survive the market test? *The Journal of Finance 52*(5), 2073–2090. 1

OECD (2017). Algorithms and collusion: competition policy in the digital age. 1

Patel, S. and S. Sarkissian (2017). To group or not to group? Evidence from mutual fund databases. *Journal of Financial and Quantitative Analysis 52*(5), 1989–2021. 5

Peress, J. and D. Schmidt (2021). Noise traders incarnate: Describing a realistic noise trading process. *Journal of Financial Markets 54*. 5

SEC (2023). Conflicts of interest associated with the use of predictive data analytics by broker-dealers and investment advisers. 1

A Proofs

A.1 Proof of Proposition 1

The solutions $\{\beta_M, \lambda_{M,h}, \lambda_{M,l}\}$ follow from solving the system of equations:

$$\beta = \frac{1}{\lambda_h + \lambda_l}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{2}{\sigma_\varepsilon^2}}\right) \tag{34}$$

$$\lambda_h = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta} \tag{35}$$

$$\lambda_l = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta}. \tag{36}$$

There are four sets of solutions, but we select the set with $\lambda_h, \lambda_l > 0$ to ensure that the trader's second order condition is satisfied. This yields $\beta = \frac{\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}$,

$$\lambda_h = \frac{\sigma_v^2 \sqrt[4]{\bar{\sigma}^4 - \delta^2}\left(\sqrt{\bar{\sigma}^4 - \delta^2} - \bar{\sigma}^2 + \delta\right)}{\sqrt{2}\delta\left(\bar{\sigma}^2 - \delta\right)\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}, \text{ and } \lambda_l = \frac{\sigma_v^2 \sqrt[4]{\bar{\sigma}^4 - \delta^2}\left(\bar{\sigma}^2 + \delta - \sqrt{\bar{\sigma}^4 - \delta^2}\right)}{\sqrt{2}\delta\left(\bar{\sigma}^2 + \delta\right)\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}. \tag{37}$$

Note that

$$\frac{\partial\beta}{\partial\delta} = -\frac{\delta}{2\sqrt{2}\left(\bar{\sigma}^4 - \delta^2\right)^{3/4}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}} = -\frac{\delta}{2\bar{\sigma}^4 - 2\delta^2}\beta < 0, \tag{38}$$

which together with

$$\beta = \frac{1}{\lambda_h + \lambda_l}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{2}{\sigma_\varepsilon^2}}\right) \tag{39}$$

$$= \frac{2}{\lambda_h + \lambda_l}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{2}{\sigma_v^2} + \frac{4}{\sigma_\varepsilon^2}}\right), \tag{40}$$

implies that

$$\frac{\partial}{\partial\delta}\frac{\lambda_h + \lambda_l}{2} > 0. \tag{41}$$

Moreover, note that

$$\frac{\partial\lambda_h}{\partial\delta} = \frac{2\sigma_v^2\left(\frac{\partial\beta}{\partial\delta}\left(\bar{\sigma}^2 - \delta\right) + \beta - 2\beta^2\frac{\partial\beta}{\partial\delta}\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)}{\left(\bar{\sigma}^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{42}$$

$$= \frac{\beta\sigma_v^2\left(2\bar{\sigma}^4 - \delta^2 - \delta\left(\bar{\sigma}^2 - 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)\right)}{\left(\bar{\sigma}^4 - \delta^2\right)\left(\bar{\sigma}^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{43}$$

$$= \frac{\beta\sigma_v^2\left(2\bar{\sigma}^4 - \delta^2 - \delta\left(\bar{\sigma}^2 - 2\left(\frac{\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}\right)^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)\right)}{\left(\bar{\sigma}^4 - \delta^2\right)\left(\bar{\sigma}^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{44}$$

$$= \frac{\beta\sigma_v^2 \left(2\bar{\sigma}^4 - \delta^2 - \delta\left(\bar{\sigma}^2 - \sqrt{\bar{\sigma}^4 - \delta^2}\right)\right)}{\left(\bar{\sigma}^4 - \delta^2\right)\left(\bar{\sigma}^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{45}$$

$$> \frac{\beta\sigma_v^2 \left(2\bar{\sigma}^4 - \delta^2 - \delta\bar{\sigma}^2\right)}{\left(\bar{\sigma}^4 - \delta^2\right)\left(\bar{\sigma}^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} > 0, \tag{46}$$

since $\bar{\sigma}^2 > \delta$. Similarly,

$$\frac{\partial\lambda_l}{\partial\delta} = -\frac{2\sigma_v^2\left(-\frac{\partial\beta}{\partial\delta}\left(\bar{\sigma}^2 + \delta\right) + \beta + 2\beta^2\frac{\partial\beta}{\partial\delta}\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)}{\left(\bar{\sigma}^2 + \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{47}$$

$$= -\frac{\beta\sigma_v^2\left(-\delta\bar{\sigma}^2 - 2\bar{\sigma}^4 + \delta^2 + 2\beta^2\delta\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)}{\left(\delta^2 - \bar{\sigma}^4\right)\left(\bar{\sigma}^2 + \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{48}$$

$$= -\frac{\beta\sigma_v^2\left(-\delta\bar{\sigma}^2 - 2\bar{\sigma}^4 + \delta^2 + 2\delta\left(\frac{\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}\right)^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)}{\left(\delta^2 - \bar{\sigma}^4\right)\left(\bar{\sigma}^2 + \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{49}$$

$$= -\frac{\beta\sigma_v^2\left(2\bar{\sigma}^4 - \delta^2 + \delta\left(\bar{\sigma}^2 - \sqrt{\bar{\sigma}^4 - \delta^2}\right)\right)}{\left(\bar{\sigma}^4 - \delta^2\right)\left(\bar{\sigma}^2 + \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} < 0 \tag{50}$$

Finally, note that expected trading profits are given by:

$$\pi_M = \mathbb{E}\left[x_M\left(v - \tfrac{\lambda_h + \lambda_l}{2}x_M\right)\right] \tag{51}$$

$$= \mathbb{E}\left[x_M\left(\mathbb{E}\left[v|s_1, s_2\right] - \tfrac{\lambda_h + \lambda_l}{2}x_M\right)\right] \tag{52}$$

$$= \tfrac{\lambda_h + \lambda_l}{2}\mathbb{E}\left[x_M^2\right] \tag{53}$$

$$= \tfrac{\lambda_h + \lambda_l}{2}\beta^2\mathbb{E}\left[\left(s_1 + s_2\right)^2\right] \tag{54}$$

$$= \frac{\beta_M}{2}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{2}{\sigma_\varepsilon^2}}\right)\left(4\sigma_v^2 + 2\sigma_\varepsilon^2\right) \tag{55}$$

$$= \beta_M\sigma_v^2 \tag{56}$$

which implies profits are decreasing in δ. □

A.2 Proof of Proposition 2

The equilibrium is characterized by the system of equation:

$$\beta = \frac{2\sigma_v^2}{\left(\lambda_h + \lambda_l\right)\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)} \tag{57}$$

$$\lambda_h = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta} \tag{58}$$

$$\lambda_l = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta}. \tag{59}$$

As before, there are four sets of solutions, but we select the set with $\lambda_h, \lambda_l > 0$ to ensure that the trader's second order condition is satisfied. This yields:

$$\beta = \frac{1}{2}\sqrt{\frac{\sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4\right)} + \bar{\sigma}^2\sigma_v^2}{\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4}}. \tag{60}$$

Let $\Gamma \equiv \bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4\right)$. Note that

$$\Gamma > \bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\left(\bar{\sigma}^4\right)\left(\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4\right) = \bar{\sigma}^4\sigma_v^4 > 0 \tag{61}$$

and

$$\frac{\partial\Gamma}{\partial\delta} = -8\delta\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right) < 0. \tag{62}$$

and that $\beta = \frac{1}{2}\sqrt{\frac{\sqrt{\Gamma} + \bar{\sigma}^2\sigma_v^2}{\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4}}$, which implies

$$\frac{\partial\beta}{\partial\delta} = -\frac{\delta}{2\sqrt{\Gamma}} \times \frac{1}{\beta} < 0. \tag{63}$$

This, together with

$$\beta = \frac{2\sigma_v^2}{\left(\lambda_h + \lambda_l\right)\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)} \tag{64}$$

$$= \frac{2}{\lambda_h + \lambda_l} \times \frac{\sigma_v^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}, \tag{65}$$

implies that

$$\frac{\partial}{\partial\delta}\frac{\lambda_h + \lambda_l}{2} > 0. \tag{66}$$

Moreover, note that

$$\lim_{\delta\to 0}\beta = \frac{\bar{\sigma}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + \sigma_v^2}} \equiv \bar{\beta} \tag{67}$$

and

$$\lim_{\delta\to\bar{\sigma}^2}\beta = \frac{\bar{\sigma}\sigma_v}{\sqrt{2}\sqrt{\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)}} \equiv \underline{\beta} \tag{68}$$

Next, note that

$$\lambda_l^{-1} = \frac{\bar{\sigma}^2 + \delta}{2\beta\sigma_v^2} + \beta\left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right) > 0 \tag{69}$$

$$\lambda_h^{-1} = \frac{\bar{\sigma}^2 - \delta}{2\beta\sigma_v^2} + \beta\left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right) > 0 \tag{70}$$

This implies

$$\frac{\partial\lambda_l^{-1}}{\partial\delta} = \left(2 - \frac{\bar{\sigma}^2 + \delta}{2\beta^2\sigma_v^2} + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right)\frac{\partial\beta}{\partial\delta} + \frac{1}{2\beta\sigma_v^2} \tag{71}$$

$$= -\frac{\delta}{2\sqrt{\Gamma}} \times \frac{1}{\beta}\left(2+\frac{\sigma_\varepsilon^2}{\sigma_v^2}-\frac{\bar{\sigma}^2+\delta}{2\beta^2\sigma_v^2}\right)+\frac{1}{2\beta\sigma_v^2} \tag{72}$$

$$= \frac{1}{2\sqrt{\Gamma}\beta}\left(-\delta\left(2+\frac{\sigma_\varepsilon^2}{\sigma_v^2}-\frac{\bar{\sigma}^2+\delta}{2\beta^2\sigma_v^2}\right)+\frac{\sqrt{\Gamma}}{\sigma_v^2}\right) \tag{73}$$

$$> \frac{1}{2\sqrt{\Gamma}\beta}\left(-\delta\left(2+\frac{\sigma_\varepsilon^2}{\sigma_v^2}-\frac{\bar{\sigma}^2+\delta}{2\beta^2\sigma_v^2}\right)+\frac{\sqrt{\bar{\sigma}^4\sigma_v^4}}{\sigma_v^2}\right) \tag{74}$$

$$= \frac{1}{2\sqrt{\Gamma}\beta}\left(\bar{\sigma}^2+\delta\frac{\bar{\sigma}^2+\delta}{2\beta^2\sigma_v^2}-\delta\left(2+\frac{\sigma_\varepsilon^2}{\sigma_v^2}\right)\right) \tag{75}$$

$$> \frac{1}{2\sqrt{\Gamma}\beta}\left(\bar{\sigma}^2+\delta\frac{\bar{\sigma}^2+\delta}{2\bar{\beta}^2\sigma_v^2}-\delta\left(2+\frac{\sigma_\varepsilon^2}{\sigma_v^2}\right)\right) \tag{76}$$

$$= \frac{1}{2\sqrt{\Gamma}\beta}\left(\bar{\sigma}^2+\frac{\delta^2\left(\sigma_\varepsilon^2+\sigma_v^2\right)}{\bar{\sigma}^2\sigma_v^2}-\delta\right)>0, \tag{77}$$

which implies $\frac{\partial\lambda_l}{\partial\delta}<0$.

Similarly, note that:

$$\frac{\partial\lambda_h^{-1}}{\partial\delta} = \left(2-\frac{\bar{\sigma}^2-\delta}{2\beta^2\sigma_v^2}+\frac{\sigma_\varepsilon^2}{\sigma_v^2}\right)\frac{\partial\beta}{\partial\delta}-\frac{1}{2\beta\sigma_v^2} \tag{78}$$

$$= -\frac{\delta}{2\sqrt{\Gamma}} \times \frac{1}{\beta}\left(2-\frac{\bar{\sigma}^2-\delta}{2\beta^2\sigma_v^2}+\frac{\sigma_\varepsilon^2}{\sigma_v^2}\right)-\frac{1}{2\beta\sigma_v^2} \tag{79}$$

$$= -\frac{1}{2\sqrt{\Gamma}\beta}\left(\delta\left(2-\frac{\bar{\sigma}^2-\delta}{2\beta^2\sigma_v^2}+\frac{\sigma_\varepsilon^2}{\sigma_v^2}\right)+\frac{\sqrt{\Gamma}}{\sigma_v^2}\right) \tag{80}$$

$$= -\frac{1}{2\sqrt{\Gamma}\beta\sigma_v^2}\left(\delta\left(2\sigma_v^2+\sigma_\varepsilon^2-\frac{\bar{\sigma}^2-\delta}{2\beta^2}\right)+\sqrt{\Gamma}\right) \tag{81}$$

Let

$$f(\delta) \equiv \delta\left(2\sigma_v^2+\sigma_\varepsilon^2-\frac{\bar{\sigma}^2-\delta}{2\beta^2}\right)+\sqrt{\Gamma} \tag{82}$$

$$= \frac{\sqrt{\Gamma}\left(\sigma_v^2\left(\bar{\sigma}^2+2\delta\right)+\delta\sigma_\varepsilon^2\right)+\delta\left(\sigma_\varepsilon^2+2\sigma_v^2\right)\left(2\delta\left(\sigma_\varepsilon^2+\sigma_v^2\right)-\bar{\sigma}^2\left(2\sigma_\varepsilon^2+\sigma_v^2\right)\right)+\Gamma}{\bar{\sigma}^2\sigma_v^2+\sqrt{\Gamma}} \tag{83}$$

Note that $f(\delta)>0$ if and only if

$$\sqrt{\Gamma}\left(\sigma_v^2\left(\bar{\sigma}^2+2\delta\right)+\delta\sigma_\varepsilon^2\right) > -\delta\left(\sigma_\varepsilon^2+2\sigma_v^2\right)\left(2\delta\left(\sigma_\varepsilon^2+\sigma_v^2\right)-\bar{\sigma}^2\left(2\sigma_\varepsilon^2+\sigma_v^2\right)\right)-\Gamma \tag{84}$$

$$= \delta\bar{\sigma}^2\left(\sigma_\varepsilon^2+2\sigma_v^2\right)\left(2\sigma_\varepsilon^2+\sigma_v^2\right)+2\delta^2\left(\sigma_\varepsilon^2+\sigma_v^2\right)\left(\sigma_\varepsilon^2+2\sigma_v^2\right)-\bar{\sigma}^4\left(2\sigma_\varepsilon^2+3\sigma_v^2\right)^2 \tag{85}$$

Next, note that RHS of (85) is given by

$$\delta\bar{\sigma}^2\left(\sigma_\varepsilon^2+2\sigma_v^2\right)\left(2\sigma_\varepsilon^2+\sigma_v^2\right)+2\delta^2\left(\sigma_\varepsilon^2+\sigma_v^2\right)\left(\sigma_\varepsilon^2+2\sigma_v^2\right)-\bar{\sigma}^4\left(2\sigma_\varepsilon^2+3\sigma_v^2\right)^2 \tag{86}$$

$$<\bar{\sigma}^4\left(\sigma_\varepsilon^2+2\sigma_v^2\right)\left(2\sigma_\varepsilon^2+\sigma_v^2\right)+2\bar{\sigma}^4\left(\sigma_\varepsilon^2+\sigma_v^2\right)\left(\sigma_\varepsilon^2+2\sigma_v^2\right)-\bar{\sigma}^4\left(2\sigma_\varepsilon^2+3\sigma_v^2\right)^2 \tag{87}$$

$$= -\bar{\sigma}^4\sigma_v^2\left(\sigma_\varepsilon^2+3\sigma_v^2\right)<0 \tag{88}$$

and so $f(\delta)>0$ always. This implies $\frac{\partial\lambda_h}{\partial\delta}>0$.

Finally, note that expected profits for a single trader are given by:

$$\begin{aligned}
\pi_{C,i} &= \mathbb{E}\left[x_i\left(1-\frac{\lambda_h+\lambda_l}{2}\beta_j\right)\mathbb{E}\left[v|s_i\right]-\frac{\lambda_h+\lambda_l}{2}x_i^2\right] && (89)\\
&= \mathbb{E}\left[\beta_C\left(1-\frac{\lambda_h+\lambda_l}{2}\beta_C\right)\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\varepsilon^2}}s_i^2-\frac{\lambda_h+\lambda_l}{2}\beta_C^2s_i^2\right] && (90)\\
&= \beta_C\left[\left(1-\frac{\lambda_h+\lambda_l}{2}\beta_C\right)\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\varepsilon^2}}-\frac{\lambda_h+\lambda_l}{2}\beta_C\right]\mathbb{E}\left[s_i^2\right] && (91)\\
&= \beta_C\left[\left(\lambda_h+\lambda_l\right)\beta_C-\frac{\lambda_h+\lambda_l}{2}\beta_C\right]\mathbb{E}\left[s_i^2\right] && (92)\\
&= \frac{\beta_C}{2}\frac{2\sigma_v^2}{2\sigma_\varepsilon^2+3\sigma_v^2}\mathbb{E}\left[s_i^2\right] && (93)\\
&= \frac{\beta_C}{2}\frac{2\sigma_v^2}{2\sigma_\varepsilon^2+3\sigma_v^2}\left(\sigma_v^2+\sigma_\varepsilon^2\right) && (94)\\
&= \beta_C\sigma_v^2\tfrac{\sigma_v^2+\sigma_\varepsilon^2}{2\sigma_\varepsilon^2+3\sigma_v^2} && (95)
\end{aligned}$$

which completes the proof. □

The following Lemma is useful for proving Proposition 3.

Lemma 2. *The trading intensity under collusion is lower than the trading intensity under competition i.e.,* $\beta_C>\beta_M$. *Moreover,* $\lim_{\delta\to 0}\beta_M/\beta_C=\sqrt{\frac{\sigma_\varepsilon^2+\sigma_v^2}{\sigma_\varepsilon^2+2\sigma_v^2}}$, $\lim_{\delta\to\bar{\sigma}^2}\beta_M/\beta_C=0$, *and* β_M/β_C *is decreasing in* δ.

Proof. Let $B\equiv\frac{\beta_M}{\beta_C}$. Then, given the above expressions

$$B^2=\frac{2\sqrt{\bar{\sigma}^4-\delta^2}\left(\sigma_\varepsilon^2+\sigma_v^2\right)}{\sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2+3\sigma_v^2\right)^2-4\delta^2\left(\sigma_\varepsilon^2+\sigma_v^2\right)\left(\sigma_\varepsilon^2+2\sigma_v^2\right)}+\bar{\sigma}^2\sigma_v^2}. \tag{96}$$

Now,

$$\begin{aligned}
& B^2\le 1 && (97)\\
\Leftrightarrow\ & \sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2+3\sigma_v^2\right)^2-4\delta^2\left(\sigma_\varepsilon^2+\sigma_v^2\right)\left(\sigma_\varepsilon^2+2\sigma_v^2\right)}+\bar{\sigma}^2\sigma_v^2\ge 2\sqrt{\bar{\sigma}^4-\delta^2}\left(\sigma_\varepsilon^2+\sigma_v^2\right) && (98)\\
\Leftrightarrow\ & \sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2+3\sigma_v^2\right)^2-4\delta^2\left(\sigma_\varepsilon^2+\sigma_v^2\right)\left(\sigma_\varepsilon^2+2\sigma_v^2\right)}\ge 2\sqrt{\bar{\sigma}^4-\delta^2}\left(\sigma_\varepsilon^2+\sigma_v^2\right)-\bar{\sigma}^2\sigma_v^2 && (99)\\
\Leftrightarrow\ & \bar{\sigma}^4\left(2\sigma_\varepsilon^2+3\sigma_v^2\right)^2-4\delta^2\left(\sigma_\varepsilon^2+\sigma_v^2\right)\left(\sigma_\varepsilon^2+2\sigma_v^2\right)\ge \begin{array}{c}4\left(\bar{\sigma}^4-\delta^2\right)\left(\sigma_\varepsilon^2+\sigma_v^2\right)^2+\bar{\sigma}^4\sigma_v^4\\ -4\sqrt{\bar{\sigma}^4-\delta^2}\left(\sigma_\varepsilon^2+\sigma_v^2\right)\bar{\sigma}^2\sigma_v^2\end{array} && (100)\\
& 4\sigma_v^2\left(\bar{\sigma}^4-\delta^2\right)\left(\sigma_\varepsilon^2+\sigma_v^2\right)\ge -4\sqrt{\bar{\sigma}^4-\delta^2}\left(\sigma_\varepsilon^2+\sigma_v^2\right)\bar{\sigma}^2\sigma_v^2 && (101)
\end{aligned}$$

which always holds since $\bar{\sigma}^2 > \delta$. This implies that $\beta_C > \beta_M$. Moreover,

$$\lim_{\delta \to 0} B^2 = \frac{1}{2 - \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_v^2}} = \frac{\sigma_\varepsilon^2 + \sigma_v^2}{\sigma_\varepsilon^2 + 2\sigma_v^2}, \tag{102}$$

$$\lim_{\delta \to \bar{\sigma}^2} B^2 = \lim_{\delta \to \bar{\sigma}^2} \frac{2\sqrt{\bar{\sigma}^4 - \delta^2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)}{\sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)} + \bar{\sigma}^2\sigma_v^2} \tag{103}$$

$$= \lim_{\delta \to \bar{\sigma}^2} \frac{2\sqrt{\bar{\sigma}^4 - \delta^2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)}{\sqrt{\bar{\sigma}^4\sigma_v^4} + \bar{\sigma}^2\sigma_v^2} = 0 \tag{104}$$

and

$$\frac{\partial B^2}{\partial \delta} = -\frac{\delta\bar{\sigma}^2\sigma_v^2}{\left(\bar{\sigma}^4 - \delta^2\right)\sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)}} B^2 < 0, \tag{105}$$

which completes the proof. □

A.3 Proof of Proposition 3

Recall that in the monopolist case, we can express profits as $\pi_M = \beta_M \sigma_v^2$ and in the competitive case we have $\pi_C = \beta_C \sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}$. This implies

$$\rho \equiv \frac{\pi_M}{2\pi_C} = \frac{\beta_M \sigma_v^2}{2\beta_C \sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}} = B \frac{2\sigma_\varepsilon^2 + 3\sigma_v^2}{2\sigma_v^2 + 2\sigma_\varepsilon^2}. \tag{106}$$

Lemma 2 implies

$$\lim_{\delta \to 0} \rho = \sqrt{\frac{\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2}{4\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)}} > 1 \tag{107}$$

and

$$\lim_{\delta \to \bar{\sigma}^2} \rho = 0 \tag{108}$$

and

$$\frac{\partial \rho}{\partial \delta} = \frac{\partial B}{\partial \delta} \frac{2\sigma_\varepsilon^2 + 3\sigma_v^2}{2\sigma_v^2 + 2\sigma_\varepsilon^2} < 0. \tag{109}$$

This implies there exists a $\underline{\delta} \in \left(0, \bar{\sigma}^2\right)$ such that $\rho\left(\underline{\delta}\right) = 1$, and another $\bar{\delta} \in \left(0, \bar{\sigma}^2\right)$ such that $\rho\left(\bar{\delta}\right) = \frac{1}{2}$. Additionally,

$$\frac{\partial \rho}{\partial \delta} < 0 \iff 2\pi_C \frac{\partial \pi_M}{\partial \delta} - \pi_M \frac{\partial 2\pi_C}{\partial \delta} < 0 \iff \frac{\partial \pi_M}{\partial \delta}\left(\frac{\partial 2\pi_C}{\partial \delta}\right)^{-1} > \frac{\pi_M}{2\pi_C}, \tag{110}$$

which implies that if $\pi_M > 2\pi_C$ (i.e., if $\delta \in (0, \underline{\delta})$) then $-\frac{\partial \pi_M}{\partial \delta} > -\frac{\partial 2\pi_C}{\partial \delta}$. □

A.4 Proof of Lemma 1

In either equilibrium, $P = \lambda(x+z)$ where $x = \beta(s_1 + s_2)$ and $s_i = v + \varepsilon_i$. This implies:

$$\lambda\left(\sigma_z^2\right) = \frac{\mathbb{C}\left(v, x+z|\sigma_z^2\right)}{\mathbb{V}\left(x+z|\sigma_z^2\right)} = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \sigma_z^2} \tag{111}$$

$$\mathbb{V}\left[v|P,\sigma_z^2\right] = \sigma_v^2 - \frac{\mathbb{C}\left(v, x+z|\sigma_z^2\right)^2}{\mathbb{V}\left(x+z|\sigma_z^2\right)} \tag{112}$$

$$= \sigma_v^2\left(1 - \frac{4\beta^2\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \sigma_z^2}\right) \tag{113}$$

$$= \sigma_v^2\left(1 - 2\beta\lambda\right) \tag{114}$$

Moreover, this implies that conditional on σ_z^2, P is normally distributed with mean zero and variance:

$$\mathbb{V}\left[P|\sigma_z^2\right] = \lambda^2\left(4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \sigma_z^2\right) = 2\beta\lambda\sigma_v^2 \tag{115}$$

Note that

$$\phi(P) \equiv \Pr\left(\sigma_z^2 = \bar{\sigma}^2 - \delta|P\right) \tag{116}$$

$$= \frac{\Pr\left(\sigma_z^2 = \bar{\sigma}^2 - \delta, P = p\right)}{\Pr\left(P = p\right)} \tag{117}$$

$$= \frac{\Pr(P = p|\sigma_z^2 = \bar{\sigma}^2 - \delta)\Pr(\sigma_z^2 = \bar{\sigma}^2 - \delta)}{\Pr(P = p|\sigma_z^2 = \bar{\sigma}^2 - \delta)\Pr(\sigma_z^2 = \bar{\sigma}^2 - \delta) + \Pr(P = p|\sigma_z^2 = \bar{\sigma}^2 + \delta)\Pr(\sigma_z^2 = \bar{\sigma}^2 + \delta)} \tag{118}$$

$$= \frac{\frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_h\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h\sigma_v^2}}\right)}{\frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_h\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h\sigma_v^2}}\right) + \frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_l\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_l\sigma_v^2}}\right)} \tag{119}$$

$$= \frac{1}{1 + \sqrt{\frac{\lambda_h}{\lambda_l}} e^{-\frac{P^2}{2\sigma_v^2}\frac{\lambda_h - \lambda_l}{2\beta\lambda_h\lambda_l}}}, \tag{120}$$

where $f(\cdot)$ is the pdf of the standard normal distribution. The result follows from noting that:

$$\mathbb{V}\left[v|P\right] = \mathbb{E}\left[\mathbb{V}\left[v|P,\sigma_z^2\right]|P\right] + \mathbb{V}\left[\mathbb{E}\left[v|P,\sigma_z^2\right]|P\right] \tag{121}$$

$$= \mathbb{E}\left[\mathbb{V}\left[v|P,\sigma_z^2\right]|P\right] + \mathbb{V}[P|P] \tag{122}$$

$$= \phi(P) \times \sigma_v^2\left(1 - 2\beta\lambda_h\right) + \left(1 - \phi(P)\right) \times \sigma_v^2\left(1 - 2\beta\lambda_l\right). \tag{123}$$

$\square$

A.5 Proof of Proposition 4

Note that

$$\mathbb{E}\left[\mathbb{V}\left[v|P\right]\right] = \int_{-\infty}^{\infty} \left(\sigma_v^2\left(1-2\beta\lambda_l\right) + \phi\left(P\right) \times 2\sigma_v^2\beta\left(\lambda_l - \lambda_h\right)\right) F\left(P\right) dP \tag{124}$$

where

$$F\left(P\right) = \frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_h\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h\sigma_v^2}}\right) + \frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_l\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_l\sigma_v^2}}\right) \tag{125}$$

is the unconditional distribution of P. This implies:

$$\begin{aligned}
\mathbb{E}\left[\mathbb{V}\left[v|P\right]\right] &= \sigma_v^2\left(1-2\beta\lambda_l\right) + \int_{-\infty}^{\infty} \sigma_v^2\beta\left(\lambda_l-\lambda_h\right) \times \frac{1}{\sqrt{2\beta\lambda_h\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h\sigma_v^2}}\right) dP &(126)\\
&= \sigma_v^2\left(1-2\beta\lambda_l\right) + \sigma_v^2\beta\left(\lambda_l-\lambda_h\right) &(127)\\
&= \sigma_v^2\left(1-\beta\left(\lambda_l+\lambda_h\right)\right) &(128)
\end{aligned}$$

The result follows from plugging in the expressions for $\{\beta, \lambda_h, \lambda_l\}$ for each equilibrium and simplifying. $\square$

A.6 Proof of Proposition 5

The proof follows the same steps as the two investor benchmark analysis, so we present only a sketch.

Collusive equilibrium Under collusion, the "representative" investor can combine all the signals to $\bar{s} = \sum_i s_i = Nv + \sum \varepsilon_i \sim N\left(Nv, N\sigma_\varepsilon^2\right)$. In this case, the optimal trading strategy maximizes:

$$\begin{aligned}
\pi_M &= \max_x \mathbb{E}\left[x\left(v-\lambda x\right)|\bar{s}\right] &(129)\\
&= \max_x x\mathbb{E}\left[v|\bar{s}\right] - x^2\frac{\lambda_h+\lambda_l}{2} &(130)
\end{aligned}$$

and so

$$x_M = \frac{1}{\lambda_h+\lambda_l}\frac{N\sigma_v^2}{N^2\sigma_v^2+N\sigma_\varepsilon^2}\bar{s} \equiv \beta_M\bar{s}, \tag{131}$$

where

$$\beta_M = \frac{1}{\lambda_h+\lambda_l}\frac{N\sigma_v^2}{N^2\sigma_v^2+N\sigma_\varepsilon^2}. \tag{132}$$

The market maker sets

$$\lambda = \frac{\mathbb{C}\left(v, \beta\sum_i s_i + z\right)}{\mathbb{V}\left(\beta\sum_i s_i + z\right)} = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \sigma_z^2} \tag{133}$$

which implies:

$$\lambda_h = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta} \tag{134}$$

$$\lambda_l = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta} \tag{135}$$

Solving for the equilibrium gives us:

$$\beta_M = \frac{1}{\sqrt[4]{\frac{N^2(\sigma_\varepsilon^2 + N\sigma_v^2)^2}{\bar{\sigma}^4 - \delta^2}}} \tag{136}$$

and expected profits of

$$\begin{aligned}
\pi_M &= \mathbb{E}\left[x\mathbb{E}\left[v|\bar{s}\right] - x^2\frac{\lambda_h + \lambda_l}{2}\right] &(137)\\
&= \frac{1}{2}\mathbb{E}\left[\left(\lambda_h + \lambda_l\right)x^2\right] &(138)\\
&= \frac{1}{2}\beta_M\frac{N\sigma_v^2}{N^2\sigma_v^2 + N\sigma_\varepsilon^2}\mathbb{E}\left[\bar{s}^2\right] &(139)\\
&= \frac{1}{2}\beta_M N\sigma_v^2 &(140)\\
&= \frac{N\sigma_v^2}{2\sqrt[4]{\frac{N^2(\sigma_\varepsilon^2 + N\sigma_v^2)^2}{\bar{\sigma}^4 - \delta^2}}} &(141)
\end{aligned}$$

Competitive Equilibrium Each trader chooses her strategy given the behavior of others. Specifically,

$$\begin{aligned}
\pi_i &= \max_x \mathbb{E}\left[x_i\left(v - \lambda x_i - \lambda\left(\sum_{j\neq i}\beta_j s_j + z\right)\right)\right] &(142)\\
&= \max_x \mathbb{E}\left[x_i\left(1 - \lambda\beta\left(N-1\right)\right)v - \lambda x_i^2|s_i\right] &(143)\\
&= \max_x x_i\left(1 - \frac{\lambda_h + \lambda_l}{2}\beta\left(N-1\right)\right)\mathbb{E}\left[v|s_i\right] - \frac{\lambda_h + \lambda_l}{2}x_i^2 &(144)
\end{aligned}$$

The FOC implies:

$$x_i = \frac{1 - \frac{\lambda_h + \lambda_l}{2}\beta\left(N-1\right)}{\lambda_h + \lambda_l}\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}s_i \equiv \beta_i s_i \tag{145}$$

In equilibrium, $\beta_i = \beta$ and so

$$\beta = \frac{2}{\left(\lambda_h + \lambda_l\right)\left(N + 1 + \frac{2\sigma_\varepsilon^2}{\sigma_v^2}\right)} \tag{146}$$

The market maker sets

$$\lambda = \frac{\mathbb{C}\left(v, \beta \sum_i s_i + z\right)}{\mathbb{V}\left(\beta \sum_i s_i + z\right)} = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \sigma_z^2} \tag{147}$$

This implies:

$$\beta = \frac{\sqrt{\frac{\sqrt{\bar{\sigma}^4(2\sigma_\varepsilon^2+(N+1)\sigma_v^2)^2-4\delta^2(\sigma_\varepsilon^2+\sigma_v^2)(\sigma_\varepsilon^2+N\sigma_v^2)}+(N-1)\bar{\sigma}^2\sigma_v^2}{N(\sigma_\varepsilon^2+\sigma_v^2)(\sigma_\varepsilon^2+N\sigma_v^2)}}}{\sqrt{2}} \tag{148}$$

Insider profits in this case are given by

$$\begin{aligned}
\pi_C &= \mathbb{E}\left[x_i\left(1 - \frac{\lambda_h + \lambda_l}{2}\beta\left(N-1\right)\right)\mathbb{E}\left[v|s_i\right] - \frac{\lambda_h + \lambda_l}{2}x_i^2\right] && (149)\\
&= \mathbb{E}\left[x_i^2\left(\lambda_h + \lambda_l\right) - \frac{\lambda_h + \lambda_l}{2}x_i^2\right] && (150)\\
&= \beta^2\frac{\lambda_h + \lambda_l}{2}\mathbb{E}\left[s_i^2\right] && (151)\\
&= \beta_C\sigma_v^2\frac{\sigma_v^2 + \sigma_\varepsilon^2}{\left(\left(N+1\right)\sigma_v^2 + 2\sigma_\varepsilon^2\right)} && (152)
\end{aligned}$$

This completes the characterization of the equilibria. □

A.7 Proof of Proposition 6

Let

$$\begin{aligned}
\rho &= \frac{\pi_M}{N\pi_C} && (153)\\
&= \frac{2\sigma_\varepsilon^2 + (N+1)\sigma_v^2}{\sqrt{2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\sqrt[4]{\frac{(\sigma_\varepsilon^2+N\sigma_v^2)^2}{\bar{\sigma}^4-\delta^2}}\sqrt{\frac{\sqrt{\bar{\sigma}^4(2\sigma_\varepsilon^2+(N+1)\sigma_v^2)^2-4\delta^2(\sigma_\varepsilon^2+\sigma_v^2)(\sigma_\varepsilon^2+N\sigma_v^2)}+(N-1)\bar{\sigma}^2\sigma_v^2}{(\sigma_\varepsilon^2+\sigma_v^2)(\sigma_\varepsilon^2+N\sigma_v^2)}}} && (154)
\end{aligned}$$

(i) Note that

$$\lim_{N\to\infty} \pi_M = \frac{1}{2}\sigma_v\sqrt[4]{\bar{\sigma}^4 - \delta^2} \tag{155}$$

but for

$$\lim_{N\to\infty} N\pi_C = \lim_{N\to\infty} \beta_C N\sigma_v^2\frac{\sigma_v^2 + \sigma_\varepsilon^2}{\left(\left(N+1\right)\sigma_v^2 + 2\sigma_\varepsilon^2\right)} \tag{156}$$

Now,

$$\lim_{N\to\infty} \beta_C = \lim_{N\to\infty} \frac{\sqrt{\frac{\sqrt{\bar{\sigma}^4(2\sigma_\varepsilon^2+(N+1)\sigma_v^2)^2-4\delta^2(\sigma_\varepsilon^2+\sigma_v^2)(\sigma_\varepsilon^2+N\sigma_v^2)}+(N-1)\bar{\sigma}^2\sigma_v^2}{N(\sigma_\varepsilon^2+\sigma_v^2)(\sigma_\varepsilon^2+N\sigma_v^2)}}}{\sqrt{2}} = 0 \tag{157}$$

and

$$\lim_{N\to\infty} N\sigma_v^2\frac{\sigma_v^2 + \sigma_\varepsilon^2}{\left(\left(N+1\right)\sigma_v^2 + 2\sigma_\varepsilon^2\right)} = \sigma_v^2 \tag{158}$$

so that

$$\lim_{N\to\infty} N\pi_C = 0. \tag{159}$$

(ii) Note that

$$\lim_{\delta\to 0} \rho = \frac{2\sigma_\varepsilon^2 + (N+1)\sigma_v^2}{2\sqrt{(\sigma_\varepsilon^2+\sigma_v^2)\,(\sigma_\varepsilon^2+N\sigma_v^2)}} \equiv \rho_0 \tag{160}$$

$$\lim_{\delta\to\bar\sigma^2} \rho = 0 \tag{161}$$

$$\frac{\partial\rho}{\partial\delta} = -\frac{\delta(N-1)\bar\sigma^2\sigma_v^2}{2\left(\bar\sigma^4-\delta^2\right)\sqrt{\bar\sigma^4\left(2\sigma_\varepsilon^2+(N+1)\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^2+\sigma_v^2\right)\left(\sigma_\varepsilon^2+N\sigma_v^2\right)}} \times \rho \tag{162}$$

$$< 0 \tag{163}$$

Moreover, note

$$\rho_0 > 1 \tag{164}$$

$$\Leftrightarrow \left(2\sigma_\varepsilon^2+(N+1)\sigma_v^2\right)^2 > 4\left(\sigma_\varepsilon^2+\sigma_v^2\right)\left(\sigma_\varepsilon^2+N\sigma_v^2\right) \tag{165}$$

$$\Leftrightarrow 4\sigma_\varepsilon^4 + (N+1)^2\sigma_v^4 + 4(N+1)\,\sigma_\varepsilon^2\sigma_v^2 > 4\left(\sigma_\varepsilon^4 + N\sigma_v^4 + (N+1)\,\sigma_\varepsilon^2\sigma_v^2\right) \tag{166}$$

$$\Leftrightarrow (N+1)^2 > 4N \tag{167}$$

which is true for $N \geq 2$. This completes the proof. $\square$