Competition and Collusion Among Strategic Traders Who Face Uncertainty

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Abstract

Conventional wisdom suggests that informed investors benefit from colluding in their trading. However, we show that this may not hold when investors face uncertainty about other traders' behavior. In a Kyle (1985) framework, we compare trading profits under collusive and competitive equilibria when informed investors face uncertainty about liquidity trading volatility. While low uncertainty favors collusion, we show that the expected profit of an *individual* investor under competition can be higher than the *total* profits for all investors under collusion when uncertainty is sufficiently high. Allowing traders to endogenously choose whether or not to collude gives rise to novel predictions: an small increase in uncertainty about liquidity trading can lead to jumps in trading volume and expected price informativeness.

JEL: D82, D84, G12, G14

Keywords: collusive trading, strategic behavior, uncertainty about liquidity

1   Introduction

Antitrust enforcement and the identification of collusive behavior are some of the primary objectives of regulatory agencies worldwide. Recently such issues have come to the forefront as the implementation of machine learning, artificial intelligence, and big data in pricing and trading mechanisms has sparked concerns regarding the potential for such algorithms to learn to collude tacitly. A series of policy reports and proposed rules (e.g., OECD (2017), SEC (2023)), legal studies (e.g., Azzutti, Ringe, and Stiehl (2022)), and experimental works (e.g., Calvano, Calzolari, Denicolò, and Pastorello (2020), Colliard, Foucault, and Lovo (2022), Dou, Goldstein, and Ji (2024)) have brought attention to these issues. A common theme of the latter is that tacit collusion should result in, and be identifiable via, excess profits relative to a competitive benchmark.

Such logic has merit. For example, antitrust enforcement in financial markets is often based on the notion that informed investors are better off if they are able to collude and coordinate their trading behavior. This common wisdom is derived from analysis of strategic trading models which show that when investors with correlated information compete, they trade aggressively on their signals and drive down trading profits (e.g., Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), Back, Cao, and Willard (2000)).

We show this may no longer be the case if traders face uncertainty about the volatility of liquidity trading. In a standard strategic trading model, we show that when uncertainty about the volatility of liquidity trading is sufficiently low, traders are better off by pooling their information and trading in a coordinated manner i.e., by colluding. However, when uncertainty is high, expected profits from collusion may be *lower* than from competing against others. In fact, as the uncertainty becomes sufficiently high, the expected profit for any individual trader in a competitive equilibrium exceeds the total profits for all investors in the collusive equilibrium. When traders endogenously choose whether or not to pay a cost to collude, they will choose to do so only when uncertainty is sufficiently low. We show that this can lead to novel predictions: small increases in uncertainty about liquidity trading can lead to jumps in trading volume and expected price informativeness.

Model and intuition. Specifically, we consider a two-trader extension of Kyle (1985), where traders observe conditionally independent signals about the terminal value of the risky security and submit market orders. A risk neutral competitive market maker sets the price as the conditional expected value of the security, given the total order flow from informed investors and liquidity (noise) traders. Following Hong and Rady (2002), we assume that while the market maker knows the distribution of noise trading, the informed investors do not — instead, they face uncertainty about whether noise trading volatility is low or high. This reflects a realistic feature of many market settings: while investors may be better informed about security fundamentals than the market maker, they are likely to be less well-informed about market conditions, and, in particular, the trading behavior of other participants.[1] Moreover, such uncertainty is particularly important

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[1]For example, funds may outsource the evaluation of trade execution costs to third parties such as Investment Technology Group, recently acquired by Virtu Financial.

for machine-learning based algorithms, which are designed under the premise of uncertain trading environments and payoffs.

We characterize the linear equilibrium in this setting under collusion and competition between investors. In the collusive equilibrium, we assume investors can perfectly collude by pooling their signals when choosing how to trade. In the competitive equilibrium, each investor conditions only on her own signal and best responds to her conjectures about the other investor's and the market maker's strategy. In either equilibrium, the intensity with which the investors trade on their information depends on the precision of their signals and their expectation of the price impact they will face, based on whether noise trading volatility is high or low. In contrast, the market maker conditions on both the order flow and the volatility of noise trading when setting the price impact: when noise trading volatility is high (low), price impact is low (high, respectively).

We show that, holding fixed the average level of noise trading volatility, an increase in uncertainty about noise trading volatility leads investors to trade less aggressively on their private signals. Intuitively, this is because the *average* price impact they face is increasing in the uncertainty about noise trading volatility.[2] As a result, trading profits also decrease with uncertainty about noise trading volatility.

The key difference across the collusive and competitive equilibria is how quickly trading intensity and profits decrease with uncertainty. When uncertainty is low, the standard intuition applies. Trading intensity in the collusive equilibrium is lower than in the competitive equilibrium, and as a result, trading profits are higher. However, higher uncertainty about noise trading volatility (holding fixed the mean) implies that, in some states, noise trading volatility (and consequently liquidity) will be very low and price impact will be very high. In the collusive equilibrium, investors' strategies are perfectly coordinated and equivalent to that of a *single* strategic investor who observes both signals. As a result, each investor internalizes the impact of their trading not only on the order flow, but also on the trading intensity of the other investor, and so cuts back their trading intensity sharply in response to their uncertainty. In fact, as uncertainty increases, the equilibrium trading intensity and expected profits approach zero.

In contrast, each investor does not fully internalize the impact of their trades in the competitive equilibrium, because they take the trading behavior of the other investor as given. As a result, trading intensity and expected profits decrease more slowly as uncertainty increases. This implies that while trading profits are higher under the collusive equilibrium when uncertainty is low, they are eventually higher under the competitive equilibrium as uncertainty increases. Moreover, we show that the greater trading intensity and liquidity in the competitive equilibrium translate to greater expected price informativeness.

Implications Our main results imply a surprising conclusion. To the extent that speculators face uncertainty about the behavior of liquidity traders, realized profits of competitive speculators may exceed the profits of speculators colluding perfectly. Importantly, this implies that the focus on

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[2]For fixed trading strategies, market liquidity (i.e., $\frac{1}{\lambda}$) is linear in the variance of noise trading (σ_z^2), which implies that price impact (λ) is convex in it. This implies average price impact is increasing in the uncertainty about σ_z^2.

speculative profits along when trying to identify collusive behavior in financial markets might be misleading.

Our notion of collusion so far is somewhat limited, since we assume traders either compete or coordinate for all possible levels of uncertainty. In an extension, we allow traders to pay a cost to coordinate on trading after they learn the level of uncertainty about noise trading (but before they observe signals about payoffs). In this case, traders endogenously choose to collude only when uncertainty about noise trading volatility is below an (endogenous) threshold. As such, one should expect that collusion is more prevalent during times of low market uncertainty and competition is more prevalent during times of high uncertainty.

This extension also generates a number of novel predictions about how market uncertainty affects observables. We show that both return volatility and expected price informativeness can increase with uncertainty about noise trading volatility. Moreover, while expected volume tends to decline with uncertainty in general, there can be regions in which small increases in uncertainty lead to jumps in volume.

To see why, note that conditional on an equilibrium, return volatility and expected price informativeness do not depend on uncertainty about noise trading and expected trading volume decreases with it.[3] However, we show that price volatility, expected price informativeness and expected volume are all higher in the competitive equilibrium than in the collusive one because traders trade more aggressively on their private information under competition. With endogenous collusion, this implies that a small increase in uncertainty around the threshold can lead to a switch from the collusive to the competitive equilibria, which in turn leads to a jump in return volatility, expected price informativeness and trading volume.

Related literature. Our analysis extends the model in Hong and Rady (2002) by allowing investors to have conditionally independent signals. As in this earlier work, we show that profits decrease with uncertainty about the volatility of liquidity trading. However, our focus is on how uncertainty affects the relative profits of investors under competition and under collusion. In Section 5, we show that our results also obtain when investors are perfectly informed about asset values and when the number of investors is arbitrarily high (but fixed): in either case, expected profits are higher under competition when uncertainty about noise trading volatility is sufficiently high.

More generally, our paper is related to the broader literature in which market participants face uncertainty about other participants in the market. These include models in which traders are unsure about the presence or number of other informed traders, or the precision of their information.[4] Collin-Dufresne and Fos (2016) consider a related model in which the volatility of noise trading is stochastic — however, importantly, because the model is in continuous time, traders face no uncertainty about it. In contrast to this literature, which focuses on the impact of uncertainty on

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[3]Since the market maker knows the volatility of noise trading, he is able to condition on the order flow efficiently, and so volatility and expected price informativeness are independent of the noise trading volatility (as in Kyle (1985). However, higher uncertainty decreases the trading intensity and so leads to lower trading volume.

[4]For instance, see Altı, Kaniel, and Yoeli (2012), Li (2013), Banerjee and Green (2015), Back, Crotty, and Li (2018), Dai, Wang, and Yang (2021), and Banerjee and Breon-Drish (2020)).

individual trader profits, our analysis is concerned with how this uncertainty affects the incentives for traders to collude versus compete.

Our analysis also speaks to the recent literature on artificial intelligence and machine learning that studies the potential for tacit collusion among algorithms in agent-based settings. A growing number of papers study such effects not only in the financial market settings (e.g., Dou et al. (2024), Colliard et al. (2022)) but also in the more traditional product-pricing sectors (e.g., Cho and Williams (2024), Calvano et al. (2020)). These papers argue that, in simulations, algorithms appear to implicitly converge to strategies that are consistent with collusive behavior, even though they are unable to explicitly communicate or coordinate with each other.[5] Our analysis suggests a possible confound in such settings. In the presence of parameter uncertainty, as is likely faced by such algorithms, profits may be high even in the absence of tacit collusion. As such, our analysis suggests that one should account for the *direct* impact of parameter uncertainty when interpreting the evidence from such simulations as being evidence of collusive behavior.

It is worth noting that our economic mechanism does not rely on ambiguity aversion (e.g., Caballero and Krishnamurthy (2008), Condie and Ganguli (2011), Easley and O'Hara (2009)) or overconfidence (e.g., Kyle and Wang (1997), Benos (1998)). Instead, ours is a setting of two-sided private information: investors are better informed about asset values while the market maker is better informed about liquidity trading volatility.

Overview. The rest of the paper is organized as follows. Section 2 presents the description of the model and a discussion of important assumptions. Section 3 presents the main analysis of the paper, and Section 5 explores how our results change under different assumptions. Section 6 discusses implications of our analysis for delegated portfolio management and antitrust regulatory policy and presents concluding remarks. All proofs are in the Appendix.

2 Model

Our model is a single-period variant of the multi-trader Kyle model in Hong and Rady (2002), where strategic traders face uncertainty about noise trading volatility. There are two assets: a risky asset and a risk-free asset with interest rate normalized to zero. The risky asset pays off a terminal value of $v \sim N\left(0, \sigma_v^2\right)$ at the end of the period.

We extend the setting in Hong and Rady (2002) by assuming that traders have conditionally independent, private signals about the value of the risky asset. Specifically, there are two strategic traders, indexed by $i \in \{1, 2\}$. Trader i observes a private signal of the form:

$$s_i = v + \varepsilon_i, \quad \text{where } \varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right), \tag{1}$$

and submits market order x_i. The aggregate trade from noise traders is denoted by $z \sim N\left(0, \sigma_z^2\right)$,

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[5]Empirically identifying such effects is inherently challenging, although Assad, Clark, Ershov, and Xu (2024) conjecture that collusive effects may be present in the German retail gasoline markets.

where the variance of noise trading is distributed according to:

$$\sigma_z^2 \in \left\{ \bar{\sigma}^2 - \delta, \bar{\sigma}^2 + \delta \right\}, \text{ where } \Pr\left(\sigma_z^2 = \bar{\sigma}^2 - \delta\right) = \frac{1}{2}, \tag{2}$$

and $\bar{\sigma}^2 > \delta \geq 0$. Finally, there is a risk neutral, competitive market maker, who is privately informed about the realization of σ_z^2, and sets the price P of the risky asset conditional on this information and the total order flow, which we denote by $y = x_1 + x_2 + z$. We assume that v, ε_1, ε_2, and z, as governed by the realization of σ_z^2, are all mutually independent.

As in Hong and Rady (2002), we assume that the market maker knows the realization of σ_z^2 while the strategic traders do not. The above specification implies that the volatility of noise trading variance is given by δ, since

$$\mathbb{V}\left[\sigma_z^2\right] = \frac{1}{4}\left(2\delta\right)^2 = \delta^2. \tag{3}$$

For ease of exposition, and with some abuse of notation, we will refer to the parameter δ as the **uncertainty** about noise trading volatility in what follows.

We restrict attention to symmetric, linear equilibria in which (i) the equilibrium trade by investor i is given by $x_i = \beta s_i$, and (ii) the market maker's pricing rule is (conditionally) linear in the order flow $y = x_1 + x_2 + z$. Denote the pricing rule by:

$$P\left(y; \sigma_z^2\right) = \begin{cases} \mathbb{E}\left[v|y, \sigma_z^2 = \bar{\sigma}^2 - \delta\right] = \lambda_h y & \text{if } \sigma_z^2 = \bar{\sigma}^2 - \delta \\ \mathbb{E}\left[v|y, \sigma_z^2 = \bar{\sigma}^2 + \delta\right] = \lambda_l y & \text{if } \sigma_z^2 = \bar{\sigma}^2 + \delta \end{cases}. \tag{4}$$

2.1 Discussion of assumptions

Our goal is to explore the impact of uncertainty about noise trading volatility in a setting that deviates minimally from the traditional Kyle (1985) framework. For instance, one could allow for more general distributions over noise trading volatility, but this would make the analysis less tractable and the intuition less transparent. It is worth distinguishing our setting from one in which investors face ambiguity about the distribution of noise trading. Importantly, investors in our setting know the true distribution of noise trading volatility (given by (2)) and are risk-neutral. As such, our results are not driven by ambiguity or ambiguity aversion.

The key assumption in our analysis is that investors are less informed about the distribution of noise trading than the market maker. The stronger assumption that the market maker knows the volatility of noise trading perfectly is made for analytical tractability. In a setting where the market maker faces uncertainty about noise trading volatility, he would update not only on the value of the asset, but also on the volatility of noise trading from the order flow. However, (generically) this would imply that the price would no longer be linear in the order flow, which in turn would (generically) imply that the investors' strategies are no longer linear in their signals, making the analysis less tractable.

3 Analysis

In what follows, we compare the equilibrium when traders can collude to the equilibrium in which they compete.

3.1 Collusion

We begin by considering a benchmark in which the two traders collude perfectly by pooling their information when choosing how to trade. This is equivalent to a single strategic trader who can observe both signals $\{s_1, s_2\}$, and trades x_M to maximize the following objective

$$\mathbb{E}\left[x_M \left(v - \lambda \left(x_M + z\right)\right) | s_1, s_2\right] = x_M \mathbb{E}\left[v | s_1, s_2\right] - \left(\frac{\lambda_h + \lambda_l}{2}\right) x_M^2, \tag{5}$$

where the equality follows from the above conjecture for the market maker's pricing rule. This implies that the optimal trade is given by

$$x_M = \frac{1}{\lambda_h + \lambda_l} \mathbb{E}\left[v | s_1, s_2\right] = \frac{1}{\lambda_h + \lambda_l} \left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{2}{\sigma_\varepsilon^2}}\right) (s_1 + s_2) \equiv \beta_M (s_1 + s_2), \tag{6}$$

since $\mathbb{E}\left[v | s_1, s_2\right] = \frac{\frac{s_1}{\sigma_\varepsilon^2} + \frac{s_2}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{2}{\sigma_\varepsilon^2}}$. Notably, this implies that the collusive trading strategy puts equal weight on the two traders' signals.

Since the market maker can condition on the order flow and the noise trading volatility, σ_z^2, her problem implies that

$$\lambda\left(\sigma_z^2\right) = \frac{\mathbb{C}\left(v, \beta_M \left(s_1 + s_2\right) + z\right)}{\mathbb{V}\left(\beta_M \left(s_1 + s_2\right) + z\right)} = \frac{2\beta_M \sigma_v^2}{4\beta_M^2 \sigma_v^2 + 2\beta_M^2 \sigma_\varepsilon^2 + \sigma_z^2} \tag{7}$$

Solving the above system of equations for $\{\beta_M, \lambda_{M,h}, \lambda_{M,l}\}$ gives us the following result.

Proposition 1. *When traders collude perfectly, there exists a unique, linear equilibrium with $x_M = \beta_M (s_1 + s_2)$ and*

$$P\left(y; \sigma_z^2\right) = \begin{cases} \lambda_{M,h} y & \text{if } \sigma_z^2 = \bar{\sigma}^2 - \delta \\ \lambda_{M,l} y & \text{if } \sigma_z^2 = \bar{\sigma}^2 + \delta \end{cases}, \tag{8}$$

where

$$\beta_M = \frac{\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}, \quad \lambda_{M,h} = \frac{2\beta_M \sigma_v^2}{4\beta_M^2 \sigma_v^2 + 2\beta_M^2 \sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta}, \quad \text{and} \quad \lambda_{M,l} = \frac{2\beta_M \sigma_v^2}{4\beta_M^2 \sigma_v^2 + 2\beta_M^2 \sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta}. \tag{9}$$

Moreover, β_M is decreasing in δ, $\lambda_{M,h}$ is increasing in δ, $\lambda_{M,l}$ is decreasing in δ, but $\frac{\lambda_{M,h} + \lambda_{M,l}}{2}$ is increasing in δ. The investors' combined expected trading profits are given by

$$\pi_M = \mathbb{E}\left[x_M \left(v - P\right)\right] = \beta_M \sigma_v^2 = \frac{\sigma_v^2 \sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}, \tag{10}$$

Figure 1: Equilibrium β, λ and π versus uncertainty δ with collusion versus competition.

The equilibrium is characterized by $\{\beta_M, \lambda_{l,M}, \lambda_{h,M}, \pi_M\}$ for the collusive equilibrium and $\{\beta_C, \lambda_{l,C}, \lambda_{h,C}, \pi_C\}$ for the competitive equilibrium. Other parameters are set to $\sigma_v = 2$, $\sigma_\varepsilon = 1$ and $\bar{\sigma} = 2$.

(a) β versus δ

(b) λ versus δ

(c) π versus δ
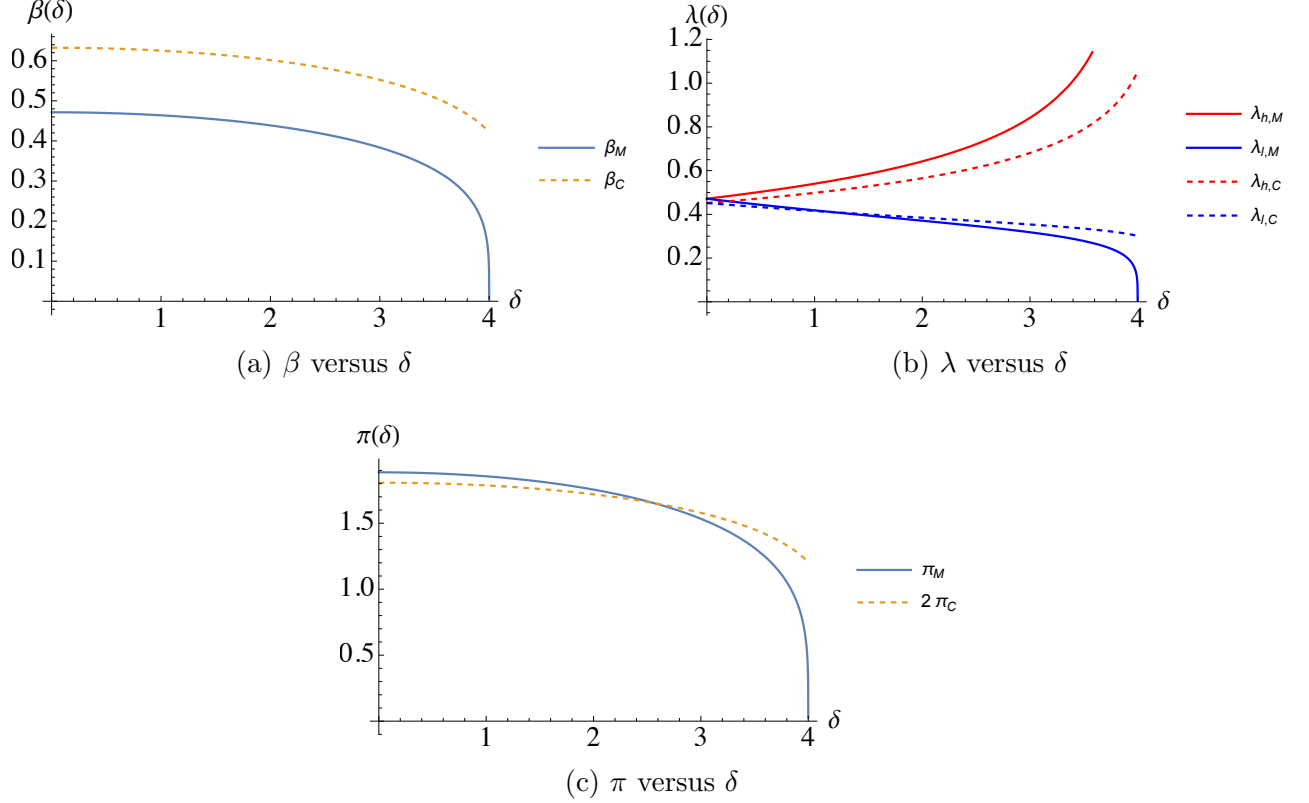
which is decreasing in δ.

The above result highlights that the collusive trading intensity β_M and trading profits π_M are decreasing in uncertainty about noise trading volatility, δ — Figure 1 provides an illustration of these results. The key to this intuition is that, for a fixed β_M, the price impact $\lambda(\sigma_z^2)$ is a convex function of noise trade volatility σ_z^2, as highlighted by (7).[6] This implies that holding the mean $\bar{\sigma}^2$ fixed, an increase in the variation of noise trading volatility increases the *average* price impact the (collusive) strategic trader faces, i.e., $\frac{\lambda_h + \lambda_l}{2}$ increases with δ. As a result, her trading intensity, β, decreases with δ. Finally, note that expected profits are proportional to trading intensity and, therefore, decreasing in δ, since

$$\pi_M = \mathbb{E}\left[x\left(v - \tfrac{\lambda_h + \lambda_l}{2}x\right)\right] = \beta_M \sigma_v^2, \tag{11}$$

as we show in the proof of the proposition.

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[6]It is worth noting that β_M does not depend on the realized σ_z^2, but $\lambda(\sigma_z^2)$ does.

3.2 Competition

Now consider the case where each trader is trading individually on her own signal, taking the strategy of the other participants as given. Specifically, investor i chooses to maximize:

$$\mathbb{E}\left[x_i\left(v - \lambda x_i - \lambda\left(\beta_j s_j + z\right)\right)|s_i\right] = x_i\left(1 - \frac{\lambda_h + \lambda_l}{2}\beta_j\right)\mathbb{E}\left[v|s_i\right] - \frac{\lambda_h + \lambda_l}{2}x_i^2. \tag{12}$$

This implies that the optimal trade is given by

$$x_i = \frac{\left(1 - \frac{\lambda_h + \lambda_l}{2}\beta_j\right)}{\lambda_h + \lambda_l}\mathbb{E}\left[v|s_i\right] = \frac{\left(1 - \frac{\lambda_h + \lambda_l}{2}\beta_j\right)}{\lambda_h + \lambda_l}\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2}}s_i \equiv \beta_i s_i, \tag{13}$$

since $\mathbb{E}\left[v|s_i\right] = \frac{\frac{s_i}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2}}$. Symmetry implies

$$x_j = \frac{\left(1 - \frac{\lambda_h + \lambda_l}{2}\beta_i\right)}{\lambda_h + \lambda_l}\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2}}s_j \equiv \beta_j s_j \tag{14}$$

and in a symmetric equilibrium, we have:

$$\beta_1 = \beta_2 \equiv \beta_C = \frac{1}{\lambda_h + \lambda_l}\frac{2\sigma_v^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}. \tag{15}$$

Since the market maker can condition on the order flow and the noise trading volatility, σ_z^2, her problem implies that

$$\lambda\left(\sigma_z^2\right) = \frac{\mathbb{C}\left(v, \beta_1 s_1 + \beta_2 s_2 + z\right)}{\mathbb{V}\left(\beta_1 s_1 + \beta_2 s_2 + z\right)} = \frac{\left(\beta_i + \beta_j\right)\sigma_v^2}{\left(\beta_i + \beta_j\right)^2\sigma_v^2 + \left(\beta_i^2 + \beta_j^2\right)\sigma_\varepsilon^2 + \sigma_z^2}. \tag{16}$$

Solving the above system of equations for $\{\beta_C, \lambda_{C,h}, \lambda_{C,l}\}$ after imposing $\beta_1 = \beta_2 \equiv \beta_C$ gives us the following result.

Proposition 2. *When traders compete, there exists a unique, symmetric, linear equilibrium with* $x_i = \beta_C s_i$ *and*

$$P\left(y; \sigma_z^2\right) = \begin{cases} \lambda_{C,h}y & \text{if } \sigma_z^2 = \bar{\sigma}^2 - \delta \\ \lambda_{C,l}y & \text{if } \sigma_z^2 = \bar{\sigma}^2 + \delta \end{cases}, \tag{17}$$

where $\beta_C = \frac{1}{2}\sqrt{\frac{\sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4\right)} + \bar{\sigma}^2\sigma_v^2}{\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4}}$,

$$\lambda_{C,h} = \frac{2\beta_C\sigma_v^2}{4\beta_C^2\sigma_v^2 + 2\beta_C^2\sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta}, \quad \text{and } \lambda_{C,l} = \frac{2\beta_C\sigma_v^2}{4\beta_C^2\sigma_v^2 + 2\beta_C^2\sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta}. \tag{18}$$

Moreover, β_C *is decreasing in* δ, $\lambda_{C,h}$ *is increasing in* δ, $\lambda_{C,l}$ *is decreasing in* δ, *but* $\frac{\lambda_{C,h} + \lambda_{C,l}}{2}$ *is*

increasing in δ. The investor's expected trading profits are given by

$$\pi_C = \mathbb{E}\left[x_C\left(v-P\right)\right] = \beta_C \sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}, \tag{19}$$

which is decreasing in δ.

Figure 1 provides an illustration of these results. As in the collusive equilibrium, trading intensity β_C and profits are decreasing in uncertainty about noise trading volatility, because the average price impact that traders face (i.e., $\frac{\lambda_{C,h} + \lambda_{C,l}}{2}$) is increasing in δ. The next section characterizes how these equilibria differ in their response to uncertainty.

3.3 Relative benefits of collusion

Given the characterization of equilibria in the previous subsections, we now characterize how uncertainty affects the relative benefits of collusion.

Proposition 3. *There exist $0 < \underline{\delta} < \bar{\delta} < \bar{\sigma}^2$ such that:*
  (i) *if $\delta < \underline{\delta}$, collusion generates higher total profits than competition, i.e., $\pi_M > 2\pi_C$;*
  (ii) *if $\underline{\delta} < \delta < \bar{\delta}$, total profits are higher under competition than collusion, but individual profits are not, i.e., $2\pi_C > \pi_M > \pi_C$;*
  (iii) *if $\bar{\delta} < \delta$, individual profits are higher under competition than total profits under collusion, i.e., $\pi_C > \pi_M$.*

The above result highlights that the relative benefit of collusion among strategic traders depends on the uncertainty they face about the distribution of noise trading. Specifically, we show that when uncertainty about noise trading volatility is sufficiently high, expected profits for an *individual* investor under competition can be higher than *total* profits for all investors under collusion.

The key difference between the competitive and collusive equilibria is how aggressively investors trade on their private signals, and how this varies with uncertainty. Note that when $\delta \to 0$, we have

$$\beta_M = \frac{\bar{\sigma}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}} \quad \text{and} \quad \beta_C = \frac{\bar{\sigma}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + \sigma_v^2}} > \beta_M,$$

which reflects the fact that, in the absence of uncertainty, investors trade less aggressively on their private information in the collusive equilibrium. This is consistent with the intuition from the existing literature (e.g., Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), Back et al. (2000)), and implies that (total) expected profits are higher under collusion than under competition i.e.,

$$\pi_M = \frac{\bar{\sigma}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}} \times \sigma_v^2 > 2\pi_C = \frac{\bar{\sigma}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + \sigma_v^2}} \frac{2(\sigma_v^2 + \sigma_\varepsilon^2)}{2\sigma_\varepsilon^2 + 3\sigma_v^2} \times \sigma_v^2.$$

As δ increases, investors trade less aggressively under both equilibria i.e., both β_C and β_M fall. However, the response is steeper in the collusive equilibrium, because each investor internalizes not only the impact of their trading on the price, but also on the trading of the other investor. To see

Figure 2: Best response functions

The figure plots the average price impact $\frac{\lambda_h + \lambda_l}{2}$ (solid line) as a best response to the intensity β chosen by traders, and the trading intensities β_M (dotted) and β_C (dashed) as best responses to the average price impact in the collusive and competitive equilibrium respectively. Panel (a) considers a setting with no uncertainty (i.e., $\delta = 0$), while panel (b) considers a setting with high uncertainty (i.e., $\delta = 3.5$). Other parameters are set to $\sigma_v = 2$, $\sigma_\varepsilon = 1$ and $\bar{\sigma} = 2$.

(a) Best responses for $\delta = 0$ (b) Best responses for $\delta = 3.5$
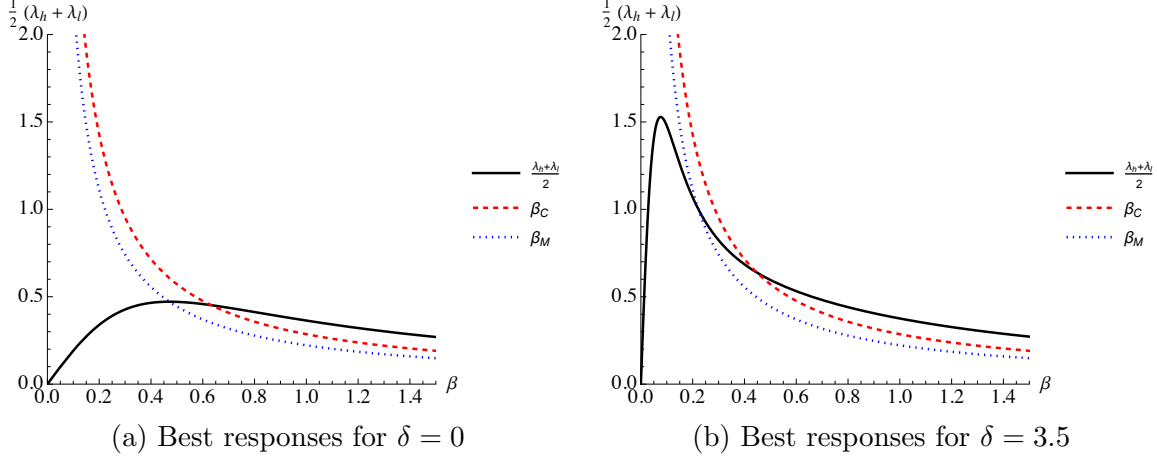
why, it is useful to compare the best response functions of the market participants in each setting. Figure 2 provides an illustration.

First, note that equations (7) and (16) immediately imply that given intensity β chosen by traders, the best response choice of $\lambda(\sigma_z^2)$ for the market maker in either case is the same. In Figure 2, this is plotted as a solid line. Comparing panels (a) and (b), we note that for any fixed β, an increase in uncertainty increases the average price impact i.e., $\frac{\partial}{\partial \delta} \frac{\lambda_h + \lambda_l}{2} > 0$.

Second, note that in either equilibrium, the traders' best response β is a decreasing function of the average price impact $\frac{\lambda_h + \lambda_l}{2}$, as illustrated by the dotted and dashed lines in Figure 2. Moreover, for a given average price impact $\frac{\lambda_h + \lambda_l}{2}$, the traders' best response β is always higher under the competitive equilibrium than under the collusive equilibrium (i.e., $\beta_C > \beta_M$). Intuitively, this is because in the collusive equilibrium, each investor internalizes not only the impact of their trading on the price, but also on the trading of the other investor.

The equilibrium trading intensities in the collusive and competitive equilibria, β_M and β_C, are characterized by the intersection of best response functions. In Figure 2, these correspond to the points at which the solid line intersects the dotted and dashed lines, respectively. As the panels in Figure 2 illustrate, when uncertainty increases, the drop in the equilibrium trading intensity (β_M) in the collusive equilibrium is larger than the corresponding drop in equilibrium intensity (β_C) in the competitive equilibrium. Moreover, the difference between the equilibrium average price impact faced by the competitive and collusive investors increases with greater uncertainty.

Intuitively, in the collusive equilibrium, one can think of the investors as acting as a single, "monopolistic" investor. Holding the mean noise trading volatility ($\bar{\sigma}^2$) fixed, an increase in uncertainty

implies that half the time, noise trading volatility is very low and the order flow is very informative (i.e., $\lambda_{M,h}$ is very high). The "monopolistic" trader responds to this by decreasing trading intensity aggressively. In fact, in the limit, as $\delta \to \bar{\sigma}^2$, the trading intensity under the collusive equilibrium drops to zero i.e., $\beta_M \to 0$. Moreover, in equilibrium, this behavior is reinforced by the market maker's response since, in the limit, $\lambda_{M,h} \to \infty$. As a result, the expected profit for investors in the collusive equilibrium also drops to zero.

In the competitive equilibrium, each investor takes the other investor's trading strategy as given, and so does not fully internalize the impact of their own trading on the trading intensity of the other investor. As a result, the trading intensity β_C responds more slowly to changes in δ. In contrast to the collusive equilibrium, as $\delta \to \bar{\sigma}^2$, the equilibrium trading intensity remains strictly positive in the limit i.e.,

$$\lim_{\delta \to \bar{\sigma}^2} \beta_C = \frac{\bar{\sigma}\sigma_v}{\sqrt{2}\sqrt{(\sigma_\varepsilon^2 + \sigma_v^2)(\sigma_\varepsilon^2 + 2\sigma_v^2)}},$$

and as a result, so does the expected profit for each trader.

3.4   Endogenous Collusion

So far, our notion of collusion has been limited: investors cannot optimally choose to collude or compete depending on the level of uncertainty regarding noise trading volatility. In this section, we assume that before trading, investors can choose to pay a cost c to choose whether or not pool their information and coordinate their trading. This cost reflects not only the direct costs of monitoring and enforcing coordination, sharing information and resources, but also indirect costs of litigation and regulatory risk.

The following result characterizes the endogenous choice of collusion in this setting.

Corollary 1. *There exists a $\bar{c} > 0$, such that:*
   (i) if $c > \bar{c}$, the investors never choose to coordinate; and
   (ii) if $c \leq \bar{c}$, there exists a threshold $\bar{\delta}(c)$ such that the investors choose to coordinate if and only if $\delta < \bar{\delta}(c)$. Furthermore, $\bar{\delta}(c)$ is decreasing in c.

Note that investors should choose to collude if and only if their net benefit from doing so exceeds the cost i.e., if

$$\Pi(\delta) \equiv \pi_M(\delta) - 2\pi_C(\delta) \geq c.$$

The proof of Proposition 3 establishes that the difference between the expected profits $\Pi(\delta)$ in the two cases is maximized for $\delta = 0$ and decreasing in δ. This implies that if $c > \bar{c}$, where

$$\bar{c} \equiv \Pi(0) = \frac{\bar{\sigma}\sigma_v^2}{\sqrt{2}} \left(\frac{1}{\sqrt{2\sigma_v^2 + \sigma_\varepsilon^2}} - \frac{2\sqrt{\sigma_v^2 + \sigma_\varepsilon^2}}{3\sigma_v^2 + 2\sigma_\varepsilon^2} \right),$$

then the investors will never choose to collude. Moreover, for $c < \bar{c}$, investors are indifferent between investing and not when $\delta = \bar{\delta}(c)$, where $\Pi(\bar{\delta}(c)) = c$. Since $\Pi(\cdot)$ is a decreasing function, the above implies $\bar{\delta}(c)$ is decreasing in c.

Intuitively, investors choose to collude when both the cost of collusion and uncertainty about noise trading is sufficiently low. As we discuss further in Section 6, this has implications for the delegated portfolio management sector, in which groups of investors (funds) can chose to compete or collude depending on the uncertainty that they face.

4 Volatility, Volume and Price Informativeness

In this section, we characterize how uncertainty about noise trading affects observables like price volatility, trading volume and price informativeness. Note that since the market maker is risk-neutral, the expected return on the security is always zero.

4.1 Volatility

We can compute the volatility of the price as the square root of the unconditional variance $\mathbb{V}(P)$, which by the law of total variance can be expressed as

$$\mathbb{V}(P) = \mathbb{E}[\mathbb{V}(P|\sigma_z^2)] + \mathbb{V}(\mathbb{E}[P|\sigma_z^2]).$$

Since $\mathbb{E}[P|\sigma_z^2] = 0$, the above simplifies to:

$$\mathbb{V}(P) = \frac{1}{2}\left(\lambda_h^2 \mathbb{V}(y|\sigma_z^2 = \bar{\sigma}^2 - \delta) + \lambda_l^2 \mathbb{V}(y|\sigma_z^2 = \bar{\sigma}^2 + \delta)\right)$$
$$= (\lambda_h + \lambda_l)\beta\sigma_v^2,$$

where the second equality follows from the definition of λ in each equilibrium. This implies the following result.

Proposition 4. *(i) Under the collusive equilibrium, the unconditional variance of price is given by:*

$$\mathbb{V}(P) = \sigma_v^2 \times \frac{\sigma_v^2}{2\sigma_v^2 + \sigma_\varepsilon^2} \equiv V_M,$$

while under the competitive equilibrium, the unconditional variance of the price is given by:

$$\mathbb{V}(P) = \sigma_v^2 \times \frac{2\sigma_v^2}{3\sigma_v^2 + 2\sigma_\varepsilon^2} \equiv V_C.$$

Moreover, $V_M < V_C$.
(ii) If collusion is endogenous and $c < \bar{c}$, then the volatility of price is weakly increasing in the uncertainty about noise trading.

The result is reminiscent of the traditional Kyle (1985) setting without uncertainty about noise trading. Intuitively, because the market maker knows the realized value of σ_z^2, he is able to set price impact λ to efficiently extract the information in order flow in both the low and high volatility regimes. The strategic investors anticipate this when choosing their optimal trades: even

though they do not know the realized volatility of noise trading, their strategy is a best response to the average price impact. As a result, unconditional price volatility is completely unaffected by the volatility of noise trading σ_z^2, and in particular, δ.

The variance in the price is affected by the degree of collusion, however. Since investors trade less aggressively on their information in the collusive equilibrium, price volatility is always lower in this case than in the competitive equilibrium. This implies that if the choice of collusion is endogenous, an increase in uncertainty about noise trader volatility makes it more likely that investors choose the collusive equilibrium, which in turn implies that the volatility in prices is higher. While the result might appear to be immediate, the mechanism that gives rise to it is novel to our setting, and relies importantly on investors ability to endogenously choose whether or not to collude in response to changes in uncertainty.

4.2   Volume

Following Admati and Pfleiderer (1988), we define expected volume as the sum of the expectation of the absolute value of trades by the informed investors, the noise traders and the market maker i.e.,

$$\mathcal{V} = \mathbb{E}[|\sum_i x_i|] + \mathbb{E}[|z|] + \mathbb{E}[|y|].$$

Moreover, note that conditional on σ_z^2, each of these trades is normally distributed with mean zero. This implies we can characterize expected volume as in the following result.

Proposition 5. *(i) Total expected volume is given by:*

$$\mathcal{V}(\delta) = \left(\sigma_x + \tfrac{1}{2}\left(\sqrt{\bar{\sigma}^2 + \delta} + \sqrt{\bar{\sigma}^2 - \delta}\right) + \tfrac{1}{2}\left(\sqrt{\bar{\sigma}^2 + \delta + \sigma_x^2} + \sqrt{\bar{\sigma}^2 - \delta + \sigma_x^2}\right)\right)\frac{2\sqrt{2}}{\sqrt{\pi}}$$

where $\sigma_x \equiv \sqrt{\mathbb{V}\left(\sum_i x_i\right)} = \beta\sqrt{4\sigma_v^2 + 2\sigma_\varepsilon^2}$. Moreover, $\mathcal{V}(\delta)$ is decreasing in δ.
(ii) All else equal, total expected volume is higher under the competitive equilibrium than under the collusive equilibrium i.e., $\mathcal{V}_C(\delta) > \mathcal{V}_M(\delta)$.

The above result highlights that total expected volume decreases with uncertainty about noise trading volatility, δ. Intuitively, the trading intensity of informed investors, β, is decreasing in uncertainty, as illustrated in Figure 1. Moreover, expected trading volume of noise traders is also decreasing in δ, which in turn implies that the trading volume induced by the market maker is also decreasing in δ.[7]

Part (ii) of the result implies the competitive equilibrium is characterized by a higher expected volume than the collusive one. This comes from a higher trading intensity under the competitive equilibrium: $\beta_M < \beta_C$. The above results also imply that if collusion is endogenous and $c < \bar{c}$,

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[7]Even though the expected *variance* of noise trading σ_z^2 is unaffected by δ, the expected *volatility* (or standard deviation) of noise trading, which determines expected volume, is a decreasing function. This is because $\mathbb{E}[\sqrt{x}] \leq \sqrt{\mathbb{E}[x]}$.

then expected volume is strictly decreasing in δ, interrupted by a discontinuous upward jump at $\bar{\delta}(c)$, after which it resumes its decreasing trend.

4.3   Price Informativeness

Let $PI(P) = -\mathbb{V}[v|P]$ denote the price informativeness in a given equilibrium. The following lemma establishes that, as in Hong and Rady (2002), price informativeness in either equilibrium depends on the realization of the price when investors face uncertainty about the volatility of noise trading.

Lemma 1. *Price informativeness can be expressed as*

$$PI(P) \equiv -\mathbb{V}[v|P] = -\left(\phi\left(P\right) \times \sigma_v^2 \left(1 - 2\beta\lambda_h\right) + \left(1 - \phi\left(P\right)\right) \times \sigma_v^2 \left(1 - 2\beta\lambda_l\right)\right),$$

where

$$\phi(P) \equiv \Pr(\sigma_z^2 = \bar{\sigma}^2 - \delta|P) = \frac{1}{1 + \sqrt{\frac{\lambda_h}{\lambda_l}} e^{-\frac{P^2}{2\sigma_v^2} \frac{\lambda_h - \lambda_l}{2\beta\lambda_h\lambda_l}}}, \tag{20}$$

and $\beta = \beta_M, \lambda_h = \lambda_{M,h}$ and $\lambda_l = \lambda_{M,l}$ for the collusive equilibrium, and $\beta = \beta_C, \lambda_h = \lambda_{C,h}$ and $\lambda_l = \lambda_{C,l}$ for the competitive equilibrium.

Figure 3 provides an illustration of this dependence for the collusive and competitive equilibrium. As emphasized by Hong and Rady (2002), price informativeness is higher for larger absolute realizations of P (larger $|P|$), since these realizations allow one to better distinguish the high noise trading volatility state from the low noise trading volatility state. Specifically, one can show that $\lim_{P^2 \to \infty} \phi(P) = 1$, i.e., for sufficiently large realizations of P, one becomes arbitrarily certain that $\sigma_z^2 = \bar{\sigma}^2 - \delta$.

Figure 3: Price informativeness $PI(P)$ under the collusive and competitive equilibria

The equilibrium is characterized by $\{\beta_M, \lambda_{l,M}, \lambda_{h,M}\}$ for the collusive equilibrium and $\{\beta_C, \lambda_{l,C}, \lambda_{h,C}\}$ for the competitive equilibrium. Other parameters are set to $\sigma_v = 2$, $\sigma_\varepsilon = 1$ and $\bar{\sigma} = 2$.

(a) $PI(P)$ for collusive equilibrium   (b) $PI(P)$ for competitive equilibrium

15
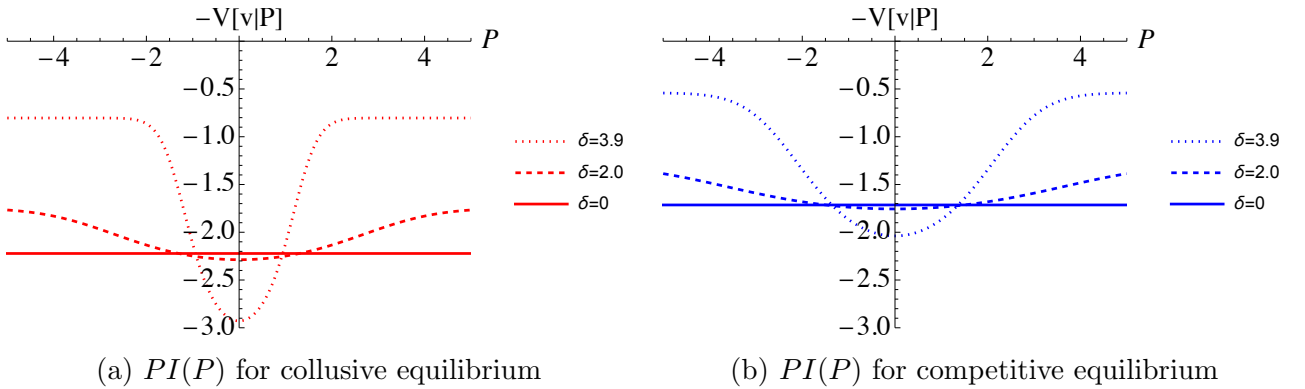
While price informativeness for a given realization of P varies with uncertainty δ, as illustrated by Figure 2, expected price informativeness is independent of δ. The following result establishes how this varies across equilibria in our setting.

Proposition 6. *(i) Under the collusive equilibrium, expected price informativeness is given by:*

$$\mathbb{E}[PI(P)] = -\sigma_v^2 \left(\frac{\sigma_\varepsilon^2 + \sigma_v^2}{2\sigma_v^2 + \sigma_\varepsilon^2} \right) \equiv PI_M,$$

while under the competitive equilibrium, expected price informativeness is given by:

$$\mathbb{E}[PI(P)] = -\sigma_v^2 \left(\frac{2\sigma_\varepsilon^2 + \sigma_v^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2} \right) \equiv PI_C.$$

Moreover, $PI_C > PI_M$.
(ii) If collusion is endogenous and $c < \bar{c}$, then the expected price informativeness is weakly increasing in the uncertainty about noise trading.

Part (i) of the above result establishes that expected price informativeness is higher under the competitive equilibrium, even when investors are uncertain about noise trading volatility. While this might initially appear to be at odds with our earlier results, note that one can express expected price informativeness as

$$\mathbb{E}[PI(P)] = \mathbb{E}[-\mathbb{V}[v|P]] = -\sigma_v^2(1 - \beta(\lambda_l + \lambda_h)),$$

as we verify in the proof of the above result. This implies that one can express expected profits under collusion and competition as:

$$\pi_M = \beta_M \sigma_v^2 = \frac{1}{2}(\sigma_v^2 + PI_M) \times \left(\frac{\lambda_{M,h} + \lambda_{M,l}}{2} \right)^{-1}, \quad \text{and} \tag{21}$$

$$\pi_C = \beta_C \sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2} = \frac{1}{2}(\sigma_v^2 + PI_C) \times \left(\frac{\lambda_{C,h} + \lambda_{C,l}}{2} \right)^{-1} \times \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}, \tag{22}$$

respectively, which implies that profits are positively related to expected price informativeness and negatively related to expected market impact. Therefore, greater competitive profits are generated in tandem with more informative prices and more liquid markets.

Part (ii) implies that when investors can endogenously choose to collude, price informativeness can actually *increase* with uncertainty about noise trading volatility. It is worth noting that contrary to existing models of "crowding in" of information, this is not driven by endogenous information acquisition. Instead, it arises even when investors' information is fixed, purely due to a change in the trading strategies that firms employ. This distinguishes our analysis from existing work, and suggests novel implications for the impact of regulatory policy (which affects the cost c of collusion) for price informativeness.

5   Robustness

Our benchmark analysis restricts attention to a simple setting with noisy signals and two investors to facilitate exposition. In this section, we discuss how our results change when we modify these assumptions.

5.1   Perfect information benchmark

One might wonder whether the result that expected profits are higher under competition when uncertainty is sufficiently high is driven by the assumption that investors have noisy information about payoffs. To explore the robustness of our results along this dimension, in this section, we assume that the investors have access to perfect information, i.e., $s_i = v$. This corresponds to the limit of the equilibria as $\sigma_\varepsilon \to 0$. In the limit, the price coefficients are given by:

$$\beta_M = \frac{\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{2\sigma_v} \quad \beta_C = \frac{\sqrt{\sqrt{9\bar{\sigma}^4 - 8\delta^2} + \bar{\sigma}^2}}{2\sqrt{2}\sigma_v}. \tag{23}$$

Notably, when there is no uncertainty, we have

$$\beta_M = \frac{\bar{\sigma}}{2\sigma_v} < \frac{\bar{\sigma}}{\sqrt{2}\sigma_v} = \beta_C,$$

while in the limit, as $\delta \to \bar{\sigma}^2$, $\beta_M \to 0$, while $\beta_C > 0$.

Moreover, in this case, the ratio of profits is given by:

$$\rho = \frac{\pi_M}{2\pi_C} = \frac{3\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sqrt{9\bar{\sigma}^4 - 8\delta^2} + \bar{\sigma}^2}}.$$

Note that this implies:

$$\lim_{\delta \to 0} \rho = \frac{3}{2\sqrt{2}} > 1 \tag{24}$$

$$\lim_{\delta \to \bar{\sigma}^2} \rho = 0 \tag{25}$$

$$\frac{\partial \rho}{\partial \delta} = -\frac{3\delta\bar{\sigma}^2}{2\sqrt{2}\left(\bar{\sigma}^4 - \delta^2\right)^{3/4}\sqrt{\left(9\bar{\sigma}^4 - 8\delta^2\right)\left(\sqrt{9\bar{\sigma}^4 - 8\delta^2} + \bar{\sigma}^2\right)}} < 0. \tag{26}$$

This implies that, as in the benchmark analysis, there exists a $\underline{\delta} \in \left(0, \bar{\sigma}^2\right)$ such that $\rho\left(\underline{\delta}\right) = 1$, and another $\bar{\delta} \in \left(0, \bar{\sigma}^2\right)$ such that $\rho\left(\bar{\delta}\right) = \frac{1}{2}$.

As such, we find that our main conclusion remains unchanged, i.e., while total expected profits are higher under collusion when investors face no uncertainty, they are higher under competition when uncertainty is sufficiently high.

5.2 Multiple informed traders

Another feature of our benchmark analysis is that we assume there are only two informed investors. In this section, we explore how our results change when we instead assume there are an arbitrary number of investors.

Suppose there are $N > 1$ traders who each observe $s_i = v + \varepsilon_i$. The following result provides a characterization of the equilibrium and the relative profits under the collusive and competitive equilibrium.

Proposition 7. *In either equilibrium, investor i submits a trade $x_i = \beta s_i$ and the market maker sets the price as*

$$P\left(y; \sigma_z^2\right) = \begin{cases} \lambda_h y & \text{if } \sigma_z^2 = \bar{\sigma}^2 - \delta \\ \lambda_l y & \text{if } \sigma_z^2 = \bar{\sigma}^2 + \delta \end{cases} \tag{27}$$

where

$$\lambda_h = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta} \quad \lambda_l = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta}. \tag{28}$$

In the collusive equilibrium, the trading intensity is given by

$$\beta = \beta_M = \sqrt[4]{\frac{\bar{\sigma}^4 - \delta^2}{N^2 \left(\sigma_\varepsilon^2 + N\sigma_v^2\right)^2}} \tag{29}$$

and the total expected profits are given by

$$\pi_M = \frac{1}{2}\beta_M N\sigma_v^2. \tag{30}$$

In the competitive equilibrium, the trading intensity is given by

$$\beta = \beta_C = \sqrt{\frac{\sqrt{\bar{\sigma}^4(2\sigma_\varepsilon^2+(N+1)\sigma_v^2)^2 - 4\delta^2(\sigma_\varepsilon^2+\sigma_v^2)(\sigma_\varepsilon^2+N\sigma_v^2)} + (N-1)\bar{\sigma}^2\sigma_v^2}{2N(\sigma_\varepsilon^2+\sigma_v^2)(\sigma_\varepsilon^2+N\sigma_v^2)}} \tag{31}$$

and the total expected profits are given by

$$N\pi_C = \frac{\left(\sigma_v^2 + \sigma_\varepsilon^2\right)}{\left((N+1)\sigma_v^2 + 2\sigma_\varepsilon^2\right)}\beta_C N\sigma_v^2. \tag{32}$$

The above result generalizes the analysis in our main model. In fact, one can verify that the expressions coincide if we set $N = 2$. The next result establishes how the relative benefit of collusion depends on uncertainty and the number of investors.

Proposition 8. *Let $\rho\left(\delta, N\right) \equiv \frac{\pi_M}{N\pi_C}$ denote the ratio of total expected profits under the collusive and competitive equilibria. Then,*

(i) for a fixed δ,

$$\lim_{N \to \infty} \pi_M = \frac{1}{2}\sigma_v \sqrt[4]{\bar{\sigma}^4 - \delta^2} \ \text{ and } \ \lim_{N \to \infty} N\pi_C = 0, \tag{33}$$

so that $\lim_{N \to \infty} \rho(\delta, N) = \infty$,

(ii) for a fixed $N > 1$, $\rho(0, N) > 1$, $\lim_{\delta \to \bar{\sigma}^2} \rho(\delta, N) = 0$, *and* $\frac{\partial \rho}{\partial \delta} < 0$.

For a fixed level of uncertainty, we show that expected profits under collusion are higher when the number of investors is sufficiently high. Intuitively, for a fixed δ, as N increases, investors compete more aggressively in the competitive equilibrium — in the limit, as $N \to \infty$, *total* expected profits reduce to zero in this case. In contrast, total expected profits stay strictly positive under the collusive equilibrium. This result is consistent with earlier work (e.g., Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), Back et al. (2000)).

However, we also show that for a fixed number N of investors, investors have lower expected profits under the collusive equilibrium when uncertainty about noise trading volatility is sufficiently high. As such, the effects of uncertainty about noise trading volatility, which we highlight in our benchmark analysis, obtain even if the number of investors in the economy is large.

6   Implications and Concluding Remarks

We consider a multi-investor extension of the Kyle (1985) model in which investors face uncertainty about the volatility of liquidity trading. We compare expected trading profits under a competitive equilibrium to those under a perfectly collusive equilibrium, in which all investors combine their information and coordinate perfectly. We show that when uncertainty is low, expected profits are higher under the collusive equilibrium, consistent with existing analysis. However, we find that when uncertainty increases, this may no longer be the case. In fact, when uncertainty about liquidity trading volatility is sufficiently high, we show that the expected profit for an *individual* investor in the competitive equilibrium can be higher than the *total* profit for all investors under the collusive equilibrium. As such, uncertainty about liquidity trading can have a substantive impact on the relative benefits of collusion among strategic investors.

As we outline below, our analysis has implications for the delegated portfolio management sector and antitrust regulation.

Delegated portfolio management. Mutual fund family structure has been shown to provide for strategic benefits for the top-performers within a family at the expense of the under-performers (e.g., Bhattacharya, Lee, and Pool (2013), Gaspar, Massa, and Matos (2006), Eisele, Nefedova, Parise, and Peijnenburg (2020)). Our model can be viewed through the lens of competition vs. cooperation within the fund family (e.g., Evans, Prado, and Zambrana (2020)), whereby a family may choose to have the same portfolio manager run multiple funds or assign the investment decisions in these funds to different managers. Our analysis suggests that in periods of greater uncertainty, or for investment strategies where liquidity regimes are highly variable and difficult to predict,

fund families would benefit from inducing competition among its portfolio managers, even if these managers have correlated ideas and strategies. Conversely, when traders face low uncertainty about liquidity, the family should encourage cooperation among portfolio managers running different strategies (i.e., via the sharing of ideas as in Kacperczyk and Seru (2012) and Cici, Jaspersen, and Kempf (2017)) or have the same portfolio manager responsible for multiple funds (thereby fully internalizing the effect of her trading across the different strategies). A natural starting point for testing this implication of the model is to consider various measures of team-managed vs. solo-managed funds (e.g., Patel and Sarkissian (2017)) and interact these with the variation in noise trading volatility (e.g., using the approach in Peress and Schmidt (2021)).

Antitrust policy. Several academic and policy papers have raised concerns regarding the potential for AI/ML-based algorithms to learn to collude tacitly (e.g., Azzutti et al. (2022)). A common theme is that tacit collusion may result in either supra-competitive speculative profits (e.g., Dou et al. (2024)) or market maker mark-ups (e.g., Colliard et al. (2022)). Our analysis highlights a confound in relying solely on measures of profitability as an indicator for tacit collusion. Because trading algorithms inherently face parameter uncertainty, the less-aggressive trading may be an optimal response to such uncertainty as opposed to an intent to collude tacitly. Regulations aimed at making order flow more transparent have the potential to resolve this confounding effect: if traders are endowed with a reliable estimate of noise trading volatility, one may argue with greater confidence that the supra-competitive profits are indeed outcomes of collusive behavior. However, as highlighted by Azzutti et al. (2022), greater transparency might also facilitate collusion because deviations from optimal behavior are more immediate and punishments for deviations are easier to implement.

On a related note, policies aimed at reducing the uncertainty regarding noise trading volatility for informed speculators (i.e., reducing δ) have the potential to improve speculative profits in both competitive and collusive settings – an immediate consequence of Propositions 1 and 2. Importantly, however, our analysis uncovers a potential drawback of such policies. Note that as uncertainty about liquidity falls, collusion becomes more attractive to traders (Proposition 3). Moreover, Proposition 6 establishes that average price informativeness is lower in the collusive equilibrium than under competition. As such, an increase in transparency about liquidity trading can inadvertently lead to more collusive behavior among sophisticated traders and, consequently, lower price informativeness.

Future work. Our model is stylized for tractability and expositional clarity, but may be extended along a number of dimensions. It would be interesting to compare the impact of competition versus collusion in a dynamic version of our model in which noise trading volatility evolves stochastically, and strategic traders learn about this over time. It would also be informative to consider the impact of investor uncertainty along other dimensions (e.g., the number of other investors in the market or their risk aversion). Finally, allowing for heterogeneous information quality and endogenous information acquisition would further test the robustness of our main result. We leave these extensions for future work.

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A   Proofs

A.1   Proof of Proposition 1

The solutions $\{\beta_M, \lambda_{M,h}, \lambda_{M,l}\}$ follow from solving the system of equations:

$$\beta = \frac{1}{\lambda_h + \lambda_l}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{2}{\sigma_\varepsilon^2}}\right) \tag{34}$$

$$\lambda_h = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \bar\sigma^2 - \delta} \tag{35}$$

$$\lambda_l = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \bar\sigma^2 + \delta}. \tag{36}$$

There are four sets of solutions, but we select the set with $\lambda_h, \lambda_l > 0$ to ensure that the trader's second order condition is satisfied. This yields $\beta = \frac{\sqrt[4]{\bar\sigma^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}$,

$$\lambda_h = \frac{\sigma_v^2\sqrt[4]{\bar\sigma^4 - \delta^2}\left(\sqrt{\bar\sigma^4 - \delta^2} - \bar\sigma^2 + \delta\right)}{\sqrt{2}\delta\left(\bar\sigma^2 - \delta\right)\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}, \text{ and } \lambda_l = \frac{\sigma_v^2\sqrt[4]{\bar\sigma^4 - \delta^2}\left(\bar\sigma^2 + \delta - \sqrt{\bar\sigma^4 - \delta^2}\right)}{\sqrt{2}\delta\left(\bar\sigma^2 + \delta\right)\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}. \tag{37}$$

Note that

$$\frac{\partial\beta}{\partial\delta} = -\frac{\delta}{2\sqrt{2}\left(\bar\sigma^4 - \delta^2\right)^{3/4}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}} = -\frac{\delta}{2\bar\sigma^4 - 2\delta^2}\beta < 0, \tag{38}$$

which together with

$$\beta = \frac{1}{\lambda_h + \lambda_l}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{2}{\sigma_\varepsilon^2}}\right) \tag{39}$$

$$= \frac{2}{\lambda_h + \lambda_l}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{2}{\sigma_v^2} + \frac{4}{\sigma_\varepsilon^2}}\right), \tag{40}$$

implies that

$$\frac{\partial}{\partial\delta}\frac{\lambda_h + \lambda_l}{2} > 0. \tag{41}$$

Moreover, note that

$$\frac{\partial\lambda_h}{\partial\delta} = \frac{2\sigma_v^2\left(\frac{\partial\beta}{\partial\delta}\left(\bar\sigma^2 - \delta\right) + \beta - 2\beta^2\frac{\partial\beta}{\partial\delta}\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)}{\left(\bar\sigma^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{42}$$

$$= \frac{\beta\sigma_v^2\left(2\bar\sigma^4 - \delta^2 - \delta\left(\bar\sigma^2 - 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)\right)}{\left(\bar\sigma^4 - \delta^2\right)\left(\bar\sigma^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{43}$$

$$= \frac{\beta\sigma_v^2\left(2\bar\sigma^4 - \delta^2 - \delta\left(\bar\sigma^2 - 2\left(\frac{\sqrt[4]{\bar\sigma^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}\right)^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)\right)}{\left(\bar\sigma^4 - \delta^2\right)\left(\bar\sigma^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{44}$$

$$= \frac{\beta \sigma_v^2 \left(2\bar{\sigma}^4 - \delta^2 - \delta \left(\bar{\sigma}^2 - \sqrt{\bar{\sigma}^4 - \delta^2} \right) \right)}{\left(\bar{\sigma}^4 - \delta^2 \right) \left(\bar{\sigma}^2 - \delta + 2\beta^2 \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \right)^2} \tag{45}$$

$$> \frac{\beta \sigma_v^2 \left(2\bar{\sigma}^4 - \delta^2 - \delta \bar{\sigma}^2 \right)}{\left(\bar{\sigma}^4 - \delta^2 \right) \left(\bar{\sigma}^2 - \delta + 2\beta^2 \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \right)^2} > 0, \tag{46}$$

since $\bar{\sigma}^2 > \delta$. Similarly,

$$\frac{\partial \lambda_l}{\partial \delta} = -\frac{2\sigma_v^2 \left(-\frac{\partial \beta}{\partial \delta} \left(\bar{\sigma}^2 + \delta \right) + \beta + 2\beta^2 \frac{\partial \beta}{\partial \delta} \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \right)}{\left(\bar{\sigma}^2 + \delta + 2\beta^2 \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \right)^2} \tag{47}$$

$$= -\frac{\beta \sigma_v^2 \left(-\delta \bar{\sigma}^2 - 2\bar{\sigma}^4 + \delta^2 + 2\beta^2 \delta \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \right)}{\left(\delta^2 - \bar{\sigma}^4 \right) \left(\bar{\sigma}^2 + \delta + 2\beta^2 \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \right)^2} \tag{48}$$

$$= -\frac{\beta \sigma_v^2 \left(-\delta \bar{\sigma}^2 - 2\bar{\sigma}^4 + \delta^2 + 2\delta \left(\frac{\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}} \right)^2 \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \right)}{\left(\delta^2 - \bar{\sigma}^4 \right) \left(\bar{\sigma}^2 + \delta + 2\beta^2 \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \right)^2} \tag{49}$$

$$= -\frac{\beta \sigma_v^2 \left(2\bar{\sigma}^4 - \delta^2 + \delta \left(\bar{\sigma}^2 - \sqrt{\bar{\sigma}^4 - \delta^2} \right) \right)}{\left(\bar{\sigma}^4 - \delta^2 \right) \left(\bar{\sigma}^2 + \delta + 2\beta^2 \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \right)^2} < 0 \tag{50}$$

Finally, note that expected trading profits are given by:

$$\pi_M = \mathbb{E} \left[x_M \left(v - \frac{\lambda_h + \lambda_l}{2} x_M \right) \right] \tag{51}$$

$$= \mathbb{E} \left[x_M \left(\mathbb{E} \left[v | s_1, s_2 \right] - \frac{\lambda_h + \lambda_l}{2} x_M \right) \right] \tag{52}$$

$$= \frac{\lambda_h + \lambda_l}{2} \mathbb{E} \left[x_M^2 \right] \tag{53}$$

$$= \frac{\lambda_h + \lambda_l}{2} \beta^2 \mathbb{E} \left[\left(s_1 + s_2 \right)^2 \right] \tag{54}$$

$$= \frac{\beta_M}{2} \left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{2}{\sigma_\varepsilon^2}} \right) \left(4\sigma_v^2 + 2\sigma_\varepsilon^2 \right) \tag{55}$$

$$= \beta_M \sigma_v^2 \tag{56}$$

which implies profits are decreasing in δ. $\qquad\square$

A.2 Proof of Proposition 2

The equilibrium is characterized by the system of equation:

$$\beta = \frac{2\sigma_v^2}{\left(\lambda_h + \lambda_l \right) \left(2\sigma_\varepsilon^2 + 3\sigma_v^2 \right)} \tag{57}$$

$$\lambda_h = \frac{2\beta \sigma_v^2}{4\beta^2 \sigma_v^2 + 2\beta^2 \sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta} \tag{58}$$

$$\lambda_l = \frac{2\beta \sigma_v^2}{4\beta^2 \sigma_v^2 + 2\beta^2 \sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta}. \tag{59}$$

As before, there are four sets of solutions, but we select the set with $\lambda_h, \lambda_l > 0$ to ensure that the trader's second order condition is satisfied. This yields:

$$\beta = \frac{1}{2} \sqrt{\frac{\sqrt{\bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2 \left(\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4\right)} + \bar{\sigma}^2\sigma_v^2}{\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4}}. \tag{60}$$

Let $\Gamma \equiv \bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2 \left(\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4\right)$. Note that

$$\Gamma > \bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4 \left(\bar{\sigma}^4\right) \left(\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4\right) = \bar{\sigma}^4 \sigma_v^4 > 0 \tag{61}$$

and

$$\frac{\partial \Gamma}{\partial \delta} = -8\delta \left(\sigma_\varepsilon^2 + \sigma_v^2\right) \left(\sigma_\varepsilon^2 + 2\sigma_v^2\right) < 0. \tag{62}$$

and that $\beta = \frac{1}{2}\sqrt{\frac{\sqrt{\Gamma} + \bar{\sigma}^2\sigma_v^2}{\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4}}$, which implies

$$\frac{\partial \beta}{\partial \delta} = -\frac{\delta}{2\sqrt{\Gamma}} \times \frac{1}{\beta} < 0. \tag{63}$$

This, together with

$$\beta = \frac{2\sigma_v^2}{\left(\lambda_h + \lambda_l\right) \left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)} \tag{64}$$

$$= \frac{2}{\lambda_h + \lambda_l} \times \frac{\sigma_v^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}, \tag{65}$$

implies that

$$\frac{\partial}{\partial \delta} \frac{\lambda_h + \lambda_l}{2} > 0. \tag{66}$$

Moreover, note that

$$\lim_{\delta \to 0} \beta = \frac{\bar{\sigma}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + \sigma_v^2}} \equiv \bar{\beta} \tag{67}$$

and

$$\lim_{\delta \to \bar{\sigma}^2} \beta = \frac{\bar{\sigma}\sigma_v}{\sqrt{2}\sqrt{\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)}} \equiv \underline{\beta} \tag{68}$$

Next, note that

$$\lambda_l^{-1} = \frac{\bar{\sigma}^2 + \delta}{2\beta\sigma_v^2} + \beta\left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right) > 0 \tag{69}$$

$$\lambda_h^{-1} = \frac{\bar{\sigma}^2 - \delta}{2\beta\sigma_v^2} + \beta\left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right) > 0 \tag{70}$$

This implies

$$\frac{\partial \lambda_l^{-1}}{\partial \delta} = \left(2 - \frac{\bar{\sigma}^2 + \delta}{2\beta^2\sigma_v^2} + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right) \frac{\partial \beta}{\partial \delta} + \frac{1}{2\beta\sigma_v^2} \tag{71}$$

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$$= -\frac{\delta}{2\sqrt{\Gamma}} \times \frac{1}{\beta} \left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2} - \frac{\bar{\sigma}^2 + \delta}{2\beta^2\sigma_v^2} \right) + \frac{1}{2\beta\sigma_v^2} \tag{72}$$

$$= \frac{1}{2\sqrt{\Gamma}\beta} \left(-\delta \left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2} - \frac{\bar{\sigma}^2 + \delta}{2\beta^2\sigma_v^2} \right) + \frac{\sqrt{\Gamma}}{\sigma_v^2} \right) \tag{73}$$

$$> \frac{1}{2\sqrt{\Gamma}\beta} \left(-\delta \left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2} - \frac{\bar{\sigma}^2 + \delta}{2\beta^2\sigma_v^2} \right) + \frac{\sqrt{\bar{\sigma}^4\sigma_v^4}}{\sigma_v^2} \right) \tag{74}$$

$$= \frac{1}{2\sqrt{\Gamma}\beta} \left(\bar{\sigma}^2 + \delta\frac{\bar{\sigma}^2 + \delta}{2\beta^2\sigma_v^2} - \delta \left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2} \right) \right) \tag{75}$$

$$> \frac{1}{2\sqrt{\Gamma}\beta} \left(\bar{\sigma}^2 + \delta\frac{\bar{\sigma}^2 + \delta}{2\bar{\beta}^2\sigma_v^2} - \delta \left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2} \right) \right) \tag{76}$$

$$= \frac{1}{2\sqrt{\Gamma}\beta} \left(\bar{\sigma}^2 + \frac{\delta^2 \left(\sigma_\varepsilon^2 + \sigma_v^2 \right)}{\bar{\sigma}^2\sigma_v^2} - \delta \right) > 0, \tag{77}$$

which implies $\frac{\partial \lambda_l}{\partial \delta} < 0$.

Similarly, note that:

$$\frac{\partial \lambda_h^{-1}}{\partial \delta} = \left(2 - \frac{\bar{\sigma}^2 - \delta}{2\beta^2\sigma_v^2} + \frac{\sigma_\varepsilon^2}{\sigma_v^2} \right) \frac{\partial \beta}{\partial \delta} - \frac{1}{2\beta\sigma_v^2} \tag{78}$$

$$= -\frac{\delta}{2\sqrt{\Gamma}} \times \frac{1}{\beta} \left(2 - \frac{\bar{\sigma}^2 - \delta}{2\beta^2\sigma_v^2} + \frac{\sigma_\varepsilon^2}{\sigma_v^2} \right) - \frac{1}{2\beta\sigma_v^2} \tag{79}$$

$$= -\frac{1}{2\sqrt{\Gamma}\beta} \left(\delta \left(2 - \frac{\bar{\sigma}^2 - \delta}{2\beta^2\sigma_v^2} + \frac{\sigma_\varepsilon^2}{\sigma_v^2} \right) + \frac{\sqrt{\Gamma}}{\sigma_v^2} \right) \tag{80}$$

$$= -\frac{1}{2\sqrt{\Gamma}\beta\sigma_v^2} \left(\delta \left(2\sigma_v^2 + \sigma_\varepsilon^2 - \frac{\bar{\sigma}^2 - \delta}{2\beta^2} \right) + \sqrt{\Gamma} \right) \tag{81}$$

Let

$$f(\delta) \equiv \delta \left(2\sigma_v^2 + \sigma_\varepsilon^2 - \frac{\bar{\sigma}^2 - \delta}{2\beta^2} \right) + \sqrt{\Gamma} \tag{82}$$

$$= \frac{\sqrt{\Gamma} \left(\sigma_v^2 \left(\bar{\sigma}^2 + 2\delta \right) + \delta\sigma_\varepsilon^2 \right) + \delta \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \left(2\delta \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) - \bar{\sigma}^2 \left(2\sigma_\varepsilon^2 + \sigma_v^2 \right) \right) + \Gamma}{\bar{\sigma}^2\sigma_v^2 + \sqrt{\Gamma}} \tag{83}$$

Note that $f(\delta) > 0$ if and only if

$$\sqrt{\Gamma} \left(\sigma_v^2 \left(\bar{\sigma}^2 + 2\delta \right) + \delta\sigma_\varepsilon^2 \right) > -\delta \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \left(2\delta \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) - \bar{\sigma}^2 \left(2\sigma_\varepsilon^2 + \sigma_v^2 \right) \right) - \Gamma \tag{84}$$

$$= \delta\bar{\sigma}^2 \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \left(2\sigma_\varepsilon^2 + \sigma_v^2 \right) + 2\delta^2 \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) - \bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2 \right)^2 \tag{85}$$

Next, note that RHS of (85) is given by

$$\delta\bar{\sigma}^2 \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \left(2\sigma_\varepsilon^2 + \sigma_v^2 \right) + 2\delta^2 \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) - \bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2 \right)^2 \tag{86}$$

$$< \bar{\sigma}^4 \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \left(2\sigma_\varepsilon^2 + \sigma_v^2 \right) + 2\bar{\sigma}^4 \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) - \bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2 \right)^2 \tag{87}$$

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$$= -\bar{\sigma}^4 \sigma_v^2 \left(\sigma_\varepsilon^2 + 3\sigma_v^2 \right) < 0 \tag{88}$$

and so $f(\delta) > 0$ always. This implies $\frac{\partial \lambda_h}{\partial \delta} > 0$.

Finally, note that expected profits for a single trader are given by:

$$\pi_{C,i} = \mathbb{E}\left[x_i \left(1 - \frac{\lambda_h + \lambda_l}{2} \beta_j \right) \mathbb{E}\left[v | s_i \right] - \frac{\lambda_h + \lambda_l}{2} x_i^2 \right] \tag{89}$$

$$= \mathbb{E}\left[\beta_C \left(1 - \frac{\lambda_h + \lambda_l}{2} \beta_C \right) \frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2}} s_i^2 - \frac{\lambda_h + \lambda_l}{2} \beta_C^2 s_i^2 \right] \tag{90}$$

$$= \beta_C \left[\left(1 - \frac{\lambda_h + \lambda_l}{2} \beta_C \right) \frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2}} - \frac{\lambda_h + \lambda_l}{2} \beta_C \right] \mathbb{E}\left[s_i^2 \right] \tag{91}$$

$$= \beta_C \left[(\lambda_h + \lambda_l) \beta_C - \frac{\lambda_h + \lambda_l}{2} \beta_C \right] \mathbb{E}\left[s_i^2 \right] \tag{92}$$

$$= \frac{\beta_C}{2} \frac{2\sigma_v^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2} \mathbb{E}\left[s_i^2 \right] \tag{93}$$

$$= \frac{\beta_C}{2} \frac{2\sigma_v^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2} \left(\sigma_v^2 + \sigma_\varepsilon^2 \right) \tag{94}$$

$$= \beta_C \sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2} \tag{95}$$

which completes the proof. $\qquad\square$

The following Lemma is useful for proving Proposition 3.

Lemma 2. *The trading intensity under collusion is lower than the trading intensity under competition i.e., $\beta_C > \beta_M$. Moreover, $\lim_{\delta \to 0} \beta_M / \beta_C = \sqrt{\frac{\sigma_\varepsilon^2 + \sigma_v^2}{\sigma_\varepsilon^2 + 2\sigma_v^2}}$, $\lim_{\delta \to \bar{\sigma}^2} \beta_M / \beta_C = 0$, and β_M / β_C is decreasing in δ.*

Proof. Let $B \equiv \frac{\beta_M}{\beta_C}$. Then, given the above expressions

$$B^2 = \frac{2\sqrt{\bar{\sigma}^4 - \delta^2} \left(\sigma_\varepsilon^2 + \sigma_v^2 \right)}{\sqrt{\bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2 \right)^2 - 4\delta^2 \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right)} + \bar{\sigma}^2 \sigma_v^2}. \tag{96}$$

Now,

$$B^2 \le 1 \tag{97}$$

$$\Leftrightarrow \sqrt{\bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2 \right)^2 - 4\delta^2 \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right)} + \bar{\sigma}^2 \sigma_v^2 \ge 2\sqrt{\bar{\sigma}^4 - \delta^2} \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) \tag{98}$$

$$\Leftrightarrow \sqrt{\bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2 \right)^2 - 4\delta^2 \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right)} \ge 2\sqrt{\bar{\sigma}^4 - \delta^2} \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) - \bar{\sigma}^2 \sigma_v^2 \tag{99}$$

$$\Leftrightarrow \bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2 \right)^2 - 4\delta^2 \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) \left(\sigma_\varepsilon^2 + 2\sigma_v^2 \right) \ge \begin{array}{c} 4 \left(\bar{\sigma}^4 - \delta^2 \right) \left(\sigma_\varepsilon^2 + \sigma_v^2 \right)^2 + \bar{\sigma}^4 \sigma_v^4 \\ -4\sqrt{\bar{\sigma}^4 - \delta^2} \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) \bar{\sigma}^2 \sigma_v^2 \end{array} \tag{100}$$

$$4\sigma_v^2 \left(\bar{\sigma}^4 - \delta^2 \right) \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) \ge -4\sqrt{\bar{\sigma}^4 - \delta^2} \left(\sigma_\varepsilon^2 + \sigma_v^2 \right) \bar{\sigma}^2 \sigma_v^2 \tag{101}$$

which always holds since $\bar{\sigma}^2 > \delta$. This implies that $\beta_C > \beta_M$. Moreover,

$$\lim_{\delta \to 0} B^2 = \frac{1}{2 - \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_v^2}} = \frac{\sigma_\varepsilon^2 + \sigma_v^2}{\sigma_\varepsilon^2 + 2\sigma_v^2}, \tag{102}$$

$$\lim_{\delta \to \bar{\sigma}^2} B^2 = \lim_{\delta \to \bar{\sigma}^2} \frac{2\sqrt{\bar{\sigma}^4 - \delta^2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)}{\sqrt{\bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2 \left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right) + \bar{\sigma}^2 \sigma_v^2}} \tag{103}$$

$$= \lim_{\delta \to \bar{\sigma}^2} \frac{2\sqrt{\bar{\sigma}^4 - \delta^2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)}{\sqrt{\bar{\sigma}^4 \sigma_v^4 + \bar{\sigma}^2 \sigma_v^2}} = 0 \tag{104}$$

and

$$\frac{\partial B^2}{\partial \delta} = -\frac{\delta \bar{\sigma}^2 \sigma_v^2}{\left(\bar{\sigma}^4 - \delta^2\right)\sqrt{\bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2 \left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)}} B^2 < 0, \tag{105}$$

which completes the proof. $\qquad\square$

A.3 Proof of Proposition 3

Recall that in the monopolist case, we can express profits as $\pi_M = \beta_M \sigma_v^2$ and in the competitive case we have $\pi_C = \beta_C \sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}$. This implies

$$\rho \equiv \frac{\pi_M}{2\pi_C} = \frac{\beta_M \sigma_v^2}{2\beta_C \sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}} = B \frac{2\sigma_\varepsilon^2 + 3\sigma_v^2}{2\sigma_v^2 + 2\sigma_\varepsilon^2}. \tag{106}$$

Lemma 2 implies

$$\lim_{\delta \to 0} \rho = \sqrt{\frac{\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2}{4\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)}} > 1 \tag{107}$$

and

$$\lim_{\delta \to \bar{\sigma}^2} \rho = 0 \tag{108}$$

and

$$\frac{\partial \rho}{\partial \delta} = \frac{\partial B}{\partial \delta} \frac{2\sigma_\varepsilon^2 + 3\sigma_v^2}{2\sigma_v^2 + 2\sigma_\varepsilon^2} < 0. \tag{109}$$

This implies there exists a $\underline{\delta} \in \left(0, \bar{\sigma}^2\right)$ such that $\rho\left(\underline{\delta}\right) = 1$, and another $\bar{\delta} \in \left(0, \bar{\sigma}^2\right)$ such that $\rho\left(\bar{\delta}\right) = \frac{1}{2}$. Additionally,

$$\frac{\partial \rho}{\partial \delta} < 0 \iff 2\pi_C \frac{\partial \pi_M}{\partial \delta} - \pi_M \frac{\partial 2\pi_C}{\partial \delta} < 0 \iff \frac{\partial \pi_M}{\partial \delta}\left(\frac{\partial 2\pi_C}{\partial \delta}\right)^{-1} > \frac{\pi_M}{2\pi_C}, \tag{110}$$

which implies that if $\pi_M > 2\pi_C$ (i.e., if $\delta \in (0, \underline{\delta})$) then $-\frac{\partial \pi_M}{\partial \delta} > -\frac{\partial 2\pi_C}{\partial \delta}$. $\qquad\square$

A.4 Proof of Proposition 5

(i) Note that conditional on σ_z, the aggregate informed trading, $\sum_i x_i$, noise trading, z, and aggregate order flow, y, are normally distributed with mean zero.

Next, note that for $\mathcal{Z} \sim N(0, \sigma^2)$, it holds that $\mathbb{E}[|\mathcal{Z}|] = 2\mathbb{E}[\mathcal{Z}|\mathcal{Z} > 0] = 2\left(0 + \sigma \frac{\phi(0)}{1-\Phi(0)}\right) = \frac{2\sqrt{2}}{\sqrt{\pi}}\sigma$, where $\phi()$ and $\Phi()$ are the density and CDF of a standard normal random variable.

Next, the volatility of the flow from informed investors is

$$\mathcal{V}_I = \tfrac{2\sqrt{2}}{\sqrt{\pi}}\sigma_x = \tfrac{2\sqrt{2}}{\sqrt{\pi}}\beta\left(4\sigma_v^2 + 2\sigma_\varepsilon^2\right)$$

From Propositions 1 and 2, informed investors trade less aggressively when uncertainty is higher, i.e., $\beta_\delta < 0$. Thus, $(\mathcal{V}_I)_\delta < 0$.

Analogously, the volatility of the flow from noise traders, given by

$$\mathcal{V}_N = \tfrac{2\sqrt{2}}{\sqrt{\pi}}\left(\tfrac{1}{2}\sqrt{\bar{\sigma}^2 + \delta} + \tfrac{1}{2}\sqrt{\bar{\sigma}^2 - \delta}\right)$$

and the volatility of the aggregate flow

$$\mathcal{V}_M = \tfrac{2\sqrt{2}}{\sqrt{\pi}}\left(\tfrac{1}{2}\sqrt{\bar{\sigma}^2 + \delta + \sigma_x^2} + \tfrac{1}{2}\sqrt{\bar{\sigma}^2 - \delta + \sigma_x^2}\right)$$

both decrease with δ by Jensen's inequality.

Therefore, $\mathcal{V} = \mathcal{V}_I + \mathcal{V}_N + \mathcal{V}_M$ is decreasing in δ.

(ii) This follows immediately from $\beta_M < \beta_C$

$\square$

A.5 Proof of Lemma 1

In either equilibrium, $P = \lambda(x + z)$ where $x = \beta(s_1 + s_2)$ and $s_i = v + \varepsilon_i$. This implies:

$$\lambda\left(\sigma_z^2\right) = \frac{\mathbb{C}\left(v, x + z | \sigma_z^2\right)}{\mathbb{V}\left(x + z | \sigma_z^2\right)} = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \sigma_z^2} \tag{111}$$

$$\mathbb{V}\left[v | P, \sigma_z^2\right] = \sigma_v^2 - \frac{\mathbb{C}\left(v, x + z | \sigma_z^2\right)^2}{\mathbb{V}\left(x + z | \sigma_z^2\right)} \tag{112}$$

$$= \sigma_v^2 \left(1 - \frac{4\beta^2\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \sigma_z^2}\right) \tag{113}$$

$$= \sigma_v^2 (1 - 2\beta\lambda) \tag{114}$$

Moreover, this implies that conditional on σ_z^2, P is normally distributed with mean zero and variance:

$$\mathbb{V}\left[P | \sigma_z^2\right] = \lambda^2\left(4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \sigma_z^2\right) = 2\beta\lambda\sigma_v^2 \tag{115}$$

Note that

$$\phi(P) \equiv \Pr\left(\sigma_z^2 = \bar{\sigma}^2 - \delta | P\right) \tag{116}$$

$$= \frac{\Pr\left(\sigma_z^2 = \bar{\sigma}^2 - \delta, P = p\right)}{\Pr\left(P = p\right)} \tag{117}$$

$$= \frac{\Pr(P = p|\sigma_z^2 = \bar{\sigma}^2 - \delta)\Pr(\sigma_z^2 = \bar{\sigma}^2 - \delta)}{\Pr(P = p|\sigma_z^2 = \bar{\sigma}^2 - \delta)\Pr(\sigma_z^2 = \bar{\sigma}^2 - \delta) + \Pr(P = p|\sigma_z^2 = \bar{\sigma}^2 + \delta)\Pr(\sigma_z^2 = \bar{\sigma}^2 + \delta)} \tag{118}$$

$$= \frac{\frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_h\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h\sigma_v^2}}\right)}{\frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_h\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h\sigma_v^2}}\right) + \frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_l\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_l\sigma_v^2}}\right)} \tag{119}$$

$$= \frac{1}{1 + \sqrt{\frac{\lambda_h}{\lambda_l}} e^{-\frac{P^2}{2\sigma_v^2}\frac{\lambda_h - \lambda_l}{2\beta\lambda_h\lambda_l}}}, \tag{120}$$

where $f(\cdot)$ is the pdf of the standard normal distribution. The result follows from noting that:

$$\mathbb{V}\left[v|P\right] = \mathbb{E}\left[\mathbb{V}\left[v|P, \sigma_z^2\right]|P\right] + \mathbb{V}\left[\mathbb{E}\left[v|P, \sigma_z^2\right]|P\right] \tag{121}$$

$$= \mathbb{E}\left[\mathbb{V}\left[v|P, \sigma_z^2\right]|P\right] + \mathbb{V}[P|P] \tag{122}$$

$$= \phi\left(P\right) \times \sigma_v^2\left(1 - 2\beta\lambda_h\right) + \left(1 - \phi\left(P\right)\right) \times \sigma_v^2\left(1 - 2\beta\lambda_l\right). \tag{123}$$

$$\square$$

A.6 Proof of Proposition 6

Note that

$$\mathbb{E}\left[\mathbb{V}\left[v|P\right]\right] = \int_{-\infty}^{\infty} \left(\sigma_v^2\left(1 - 2\beta\lambda_l\right) + \phi\left(P\right) \times 2\sigma_v^2\beta\left(\lambda_l - \lambda_h\right)\right) F\left(P\right) dP \tag{124}$$

where

$$F\left(P\right) = \frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_h\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h\sigma_v^2}}\right) + \frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_l\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_l\sigma_v^2}}\right) \tag{125}$$

is the unconditional distribution of P. This implies:

$$\mathbb{E}\left[\mathbb{V}\left(v|P\right)\right] = \sigma_v^2\left(1 - 2\beta\lambda_l\right) + \int_{-\infty}^{\infty} \sigma_v^2\beta\left(\lambda_l - \lambda_h\right) \times \frac{1}{\sqrt{2\beta\lambda_h\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h\sigma_v^2}}\right) dP \tag{126}$$

$$= \sigma_v^2\left(1 - 2\beta\lambda_l\right) + \sigma_v^2\beta\left(\lambda_l - \lambda_h\right) \tag{127}$$

$$= \sigma_v^2\left(1 - \beta\left(\lambda_l + \lambda_h\right)\right) \tag{128}$$

The result follows from plugging in the expressions for $\{\beta, \lambda_h, \lambda_l\}$ for each equilibrium and simplifying. $\square$

A.7 Proof of Proposition 7

The proof follows the same steps as the two investor benchmark analysis, so we present only a sketch.

Collusive equilibrium Under collusion, the "representative" investor can combine all the signals to $\bar{s} = \sum_i s_i = Nv + \sum \varepsilon_i \sim N\left(Nv, N\sigma_\varepsilon^2\right)$. In this case, the optimal trading strategy maximizes:

$$\pi_M = \max_x \mathbb{E}\left[x\left(v - \lambda x\right)|\bar{s}\right] \tag{129}$$

$$= \max_x x\mathbb{E}\left[v|\bar{s}\right] - x^2\frac{\lambda_h + \lambda_l}{2} \tag{130}$$

and so

$$x_M = \frac{1}{\lambda_h + \lambda_l}\frac{N\sigma_v^2}{N^2\sigma_v^2 + N\sigma_\varepsilon^2}\bar{s} \equiv \beta_M\bar{s}, \tag{131}$$

where

$$\beta_M = \frac{1}{\lambda_h + \lambda_l}\frac{N\sigma_v^2}{N^2\sigma_v^2 + N\sigma_\varepsilon^2}. \tag{132}$$

The market maker sets

$$\lambda = \frac{\mathbb{C}\left(v, \beta\sum_i s_i + z\right)}{\mathbb{V}\left(\beta\sum_i s_i + z\right)} = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \sigma_z^2} \tag{133}$$

which implies:

$$\lambda_h = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta} \tag{134}$$

$$\lambda_l = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta} \tag{135}$$

Solving for the equilibrium gives us:

$$\beta_M = \frac{1}{\sqrt[4]{\frac{N^2(\sigma_\varepsilon^2 + N\sigma_v^2)^2}{\bar{\sigma}^4 - \delta^2}}} \tag{136}$$

and expected profits of

$$\pi_M = \mathbb{E}\left[x\mathbb{E}\left[v|\bar{s}\right] - x^2\frac{\lambda_h + \lambda_l}{2}\right] \tag{137}$$

$$= \frac{1}{2}\mathbb{E}\left[\left(\lambda_h + \lambda_l\right)x^2\right] \tag{138}$$

$$= \frac{1}{2}\beta_M\frac{N\sigma_v^2}{N^2\sigma_v^2 + N\sigma_\varepsilon^2}\mathbb{E}\left[\bar{s}^2\right] \tag{139}$$

$$= \frac{1}{2}\beta_M N\sigma_v^2 \tag{140}$$

$$= \frac{N\sigma_v^2}{2\sqrt[4]{\frac{N^2(\sigma_\varepsilon^2 + N\sigma_v^2)^2}{\bar{\sigma}^4 - \delta^2}}} \tag{141}$$

Competitive Equilibrium Each trader chooses her strategy given the behavior of others. Specif-

ically,

$$\pi_i = \max_x \mathbb{E}\left[x_i\left(v - \lambda x_i - \lambda\left(\sum_{j\neq i}\beta_j s_j + z\right)\right)\right] \tag{142}$$

$$= \max_x \mathbb{E}\left[x_i\left(1 - \lambda\beta\left(N-1\right)\right)v - \lambda x_i^2 | s_i\right] \tag{143}$$

$$= \max_x x_i\left(1 - \frac{\lambda_h + \lambda_l}{2}\beta\left(N-1\right)\right)\mathbb{E}\left[v|s_i\right] - \frac{\lambda_h + \lambda_l}{2}x_i^2 \tag{144}$$

The FOC implies:

$$x_i = \frac{1 - \frac{\lambda_h + \lambda_l}{2}\beta\left(N-1\right)}{\lambda_h + \lambda_l}\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}s_i \equiv \beta_i s_i \tag{145}$$

In equilibrium, $\beta_i = \beta$ and so

$$\beta = \frac{2}{\left(\lambda_h + \lambda_l\right)\left(N + 1 + \frac{2\sigma_\varepsilon^2}{\sigma_v^2}\right)} \tag{146}$$

The market maker sets

$$\lambda = \frac{\mathbb{C}\left(v, \beta\sum_i s_i + z\right)}{\mathbb{V}\left(\beta\sum_i s_i + z\right)} = \frac{N\beta\sigma_v^2}{N^2\beta^2\sigma_v^2 + N\beta^2\sigma_\varepsilon^2 + \sigma_z^2} \tag{147}$$

This implies:

$$\beta = \frac{\sqrt{\frac{\sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2 + (N+1)\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + N\sigma_v^2\right)} + (N-1)\bar{\sigma}^2\sigma_v^2}{N\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + N\sigma_v^2\right)}}}{\sqrt{2}} \tag{148}$$

Insider profits in this case are given by

$$\pi_C = \mathbb{E}\left[x_i\left(1 - \frac{\lambda_h + \lambda_l}{2}\beta\left(N-1\right)\right)\mathbb{E}\left[v|s_i\right] - \frac{\lambda_h + \lambda_l}{2}x_i^2\right] \tag{149}$$

$$= \mathbb{E}\left[x_i^2\left(\lambda_h + \lambda_l\right) - \frac{\lambda_h + \lambda_l}{2}x_i^2\right] \tag{150}$$

$$= \beta^2\frac{\lambda_h + \lambda_l}{2}\mathbb{E}\left[s_i^2\right] \tag{151}$$

$$= \beta_C\sigma_v^2\frac{\sigma_v^2 + \sigma_\varepsilon^2}{\left((N+1)\sigma_v^2 + 2\sigma_\varepsilon^2\right)} \tag{152}$$

This completes the characterization of the equilibria. $\qquad\square$

A.8   Proof of Proposition 8

Let

$$\rho = \frac{\pi_M}{N\pi_C} \tag{153}$$

$$= \frac{2\sigma_\varepsilon^2 + (N+1)\sigma_v^2}{\sqrt{2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\sqrt[4]{\frac{(\sigma_\varepsilon^2 + N\sigma_v^2)^2}{\bar{\sigma}^4 - \delta^2}}\sqrt{\frac{\sqrt{\bar{\sigma}^4(2\sigma_\varepsilon^2 + (N+1)\sigma_v^2)^2 - 4\delta^2(\sigma_\varepsilon^2 + \sigma_v^2)(\sigma_\varepsilon^2 + N\sigma_v^2) + (N-1)\bar{\sigma}^2\sigma_v^2}}{(\sigma_\varepsilon^2 + \sigma_v^2)(\sigma_\varepsilon^2 + N\sigma_v^2)}}} \tag{154}$$

(i) Note that

$$\lim_{N\to\infty} \pi_M = \frac{1}{2}\sigma_v\sqrt[4]{\bar{\sigma}^4 - \delta^2} \tag{155}$$

but for

$$\lim_{N\to\infty} N\pi_C = \lim_{N\to\infty} \beta_C N\sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{\left((N+1)\sigma_v^2 + 2\sigma_\varepsilon^2\right)} \tag{156}$$

Now,

$$\lim_{N\to\infty} \beta_C = \lim_{N\to\infty} \frac{\sqrt{\frac{\sqrt{\bar{\sigma}^4(2\sigma_\varepsilon^2 + (N+1)\sigma_v^2)^2 - 4\delta^2(\sigma_\varepsilon^2 + \sigma_v^2)(\sigma_\varepsilon^2 + N\sigma_v^2) + (N-1)\bar{\sigma}^2\sigma_v^2}}{N(\sigma_\varepsilon^2 + \sigma_v^2)(\sigma_\varepsilon^2 + N\sigma_v^2)}}}{\sqrt{2}} = 0 \tag{157}$$

and

$$\lim_{N\to\infty} N\sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{\left((N+1)\sigma_v^2 + 2\sigma_\varepsilon^2\right)} = \sigma_v^2 \tag{158}$$

so that

$$\lim_{N\to\infty} N\pi_C = 0. \tag{159}$$

(ii) Note that

$$\lim_{\delta\to 0} \rho = \frac{2\sigma_\varepsilon^2 + (N+1)\sigma_v^2}{2\sqrt{\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + N\sigma_v^2\right)}} \equiv \rho_0 \tag{160}$$

$$\lim_{\delta\to\bar{\sigma}^2} \rho = 0 \tag{161}$$

$$\frac{\partial\rho}{\partial\delta} = -\frac{\delta(N-1)\bar{\sigma}^2\sigma_v^2}{2\left(\bar{\sigma}^4 - \delta^2\right)\sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2 + (N+1)\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + N\sigma_v^2\right)}} \times \rho \tag{162}$$

$$< 0 \tag{163}$$

Moreover, note

$$\rho_0 > 1 \tag{164}$$

$$\Leftrightarrow \left(2\sigma_\varepsilon^2 + (N+1)\sigma_v^2\right)^2 > 4\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + N\sigma_v^2\right) \tag{165}$$

$$\Leftrightarrow 4\sigma_\varepsilon^4 + (N+1)^2\sigma_v^4 + 4(N+1)\sigma_\varepsilon^2\sigma_v^2 > 4\left(\sigma_\varepsilon^4 + N\sigma_v^4 + (N+1)\sigma_\varepsilon^2\sigma_v^2\right) \tag{166}$$

$$\Leftrightarrow (N+1)^2 > 4N \tag{167}$$

which is true for $N \geq 2$. This completes the proof. $\qquad\square$