Leaks, disclosures and internal communication*

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Abstract

We study how leaks affect a firm's communication decisions and real efficiency. A privately informed manager strategically chooses both public disclosure and internal communication to employees. Public disclosure is noisy, but in the absence of leaks, internal communication is perfectly informative because it maximizes employee coordination and efficiency. The possibility of public leaks distorts these choices: we show that more leakage worsens internal communication and can reduce real efficiency, despite increasing public disclosure. We discuss the implications of our results for recent regulations that protect and encourage whistleblowers in financial markets.

Keywords: leaks, whistleblower, disclosure, internal communication, efficiency

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1 Introduction

Publicity is justly commended as a remedy for social and industrial diseases. Sunlight is said to be the best of disinfectants; electric light the most efficient policeman.

— Louis Brandeis, Other People's Money – and How Bankers Use It (1914)

By encouraging public leaks of internal information, regulatory policy strives to deter wrong-doing by firms. For example, the SEC's Dodd-Frank Whistleblower Program is designed to improve protection and provide incentives to whistleblowers in financial markets.[1] Since inception, the program has awarded more than \$111 million to 34 whistleblowers, and has led to SEC enforcement actions in which over \$584 million in financial sanctions were ordered. In fiscal year 2016 alone, the SEC's Office of the Whistleblower received over 4,200 tips and awarded over \$57 million as part of the program.

While these policies are arguably effective at detecting unlawful behavior by market participants, they also have unintended consequences. By increasing the likelihood that internal communication is leaked publicly, these policies can distort how a firm's employees communicate with each other within the firm and to market participants. Specifically, we show that increasing the likelihood of leaks leads to more public disclosure, but coarser internal communication. Moreover, the overall effect of more leakage on real (allocative) efficiency is not monotonic, and depends on the ex-ante informativeness of internal communication.

We consider a setting where a manager, who is privately informed about the firm's fundamentals, strategically chooses both public disclosure and internal communication. Because public disclosures affect the short-term stock price, the manager has an incentive to withhold negative news about fundamentals from the public. However, she prefers her internal communication to be as informative as possible, since this improves the extent to which her employees can coordinate on the appropriate action for the firm. We assume that the manager's public disclosures are verifiable, but that internal communication is cheap talk.

We show that the possibility of leakage introduces a tradeoff between internal and external communication. On the one hand, the benefit from withholding negative news from the public is lower since it may be leaked anyways. On the other hand, the possibility of leaks creates an incentive for the manager to distort her message to employees, which makes it more difficult to sustain informative communication internally. As a result, more leakage

[1] The SEC Whistleblower Program is a recent addition to a number of other, similar financial market regulations (e.g., US Whistleblower Protection Act (1989), Section 806 of Sarbanes-Oxley Act (2002), Whistleblower Protection Enhancement Act (2007)). See the U.S. Securities and Exchange Commission, 2016 Annual Report to Congress on The Dodd-Frank Whistleblower Program (https://www.sec.gov/files/owb-annual-report-2016.pdf) for more details.

leads to more informative public disclosure, but less informative internal communication.

In our model, real efficiency is driven by the degree of coordination among employees. As a benchmark, we first consider the case when the manager's incentives are aligned with those of the employees. In this case, we show efficiency is maximized when there are no leaks: internal communication is perfectly informative, and as a result, coordination is maximized. An increase in leakage has two offsetting effects on efficiency. On the one hand, internal communication is coarser, which reduces the information available to employees and, consequently, makes coordination more difficult. On the other hand, there is more public disclosure, which provides more information to employees and, thereby, improves coordination. Initially, the first effect dominates and so efficiency decreases with leakage. However, when the probability of leaks is sufficiently high, internal communication is uninformative and the second effect takes over. As a result, when the likelihood of leakage is very high, real efficiency increases with leakage.

We then extend the analysis to the case in which there are conflicts of interest between the manager and the other stakeholders of the firm. We show that in this case, leakage can have a disciplinary effect: efficiency can be higher in the presence of leaks than in the absence of leaks. Because the manager's incentives are not aligned with the employees', internal communication is coarse even in the absence of leakage. When the probability of leakage is sufficiently high, the impact of the increased public disclosure offsets the decrease due to worse internal communication, and overall efficiency can be higher as a result.

Our analysis suggests a novel channel through which regulatory policy towards whistleblowers can affect firm behavior, and offers a number of implications. A higher likelihood of leakage should be associated with more public disclosure, but less informative internal communication. These effects should be especially pronounced in firms where compensation contracts over-weight short-term performance (e.g., through higher stock / option grants). Policies that increase leakage are more likely to be beneficial for firms with severe conflicts of interest and poor internal communication, but counter-productive for firms with good governance and internal communication. Accounting for this cross-sectional variation when evaluating the impact of regulatory policies is important in accurately measuring their effectiveness.

Our model builds on the seminal work by Crawford and Sobel (1982) on cheap talk and by Dye (1985) and Jung and Kwon (1988) on strategic verifiable disclosure. Our paper is part of a larger literature that studies communication in settings with a single sender and multiple receivers (e.g., Farrell and Gibbons (1989), Newman and Sansing (1993) and Goltsman and Pavlov (2011)). However, much of this literature has restricted attention to cheap talk communication. In contrast, the sender engages in different types of communication in our model: public disclosures are verifiable but internal communication is via cheap talk. Moreover, the incentives for the two types of communication are interdependent due to leakage. The manager's incentive to distort her cheap-talk message to employees arises endogenously because her internal communication can be leaked publicly, and such leaks affect the short-term stock price.

The rest of the paper is as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium disclosure and internal communication by the manager, and Section 4 presents the comparative statics and efficiency results for the benchmark model with no conflicts of interest. Section 5 extends the analysis to the case where the manager's incentives are misaligned with the other stakeholders. Section 6 discusses some implications of the model and concludes. Unless noted otherwise, proofs are in the Appendix.

2 Model

There are three dates $t \in \{1, 2, 3\}$, and three types of players: a manager M, a continuum of shareholders $s \in S$, and a continuum of employees $i \in I$. The value of the firm, V, depends on its fundamentals θ, and the action x_i of its employees. We assume that the value is increasing in fundamentals and increasing in the degree of coordination across its employees. Specifically, suppose the firm's value is given by:

$$V = \theta - \beta \int_{i \in I} (x_i - \theta)^2 di, \tag{1}$$

where $\beta > 0$ measures the relative importance of coordination within the firm. For tractability, suppose that θ is uniformly distributed between l and $h > l$ i.e., $f(\theta) = \frac{1}{h-l}$.[2]

Date $t = 1$. At date $t = 1$, the manager observes fundamentals θ with probability π, and denote the indicator variable for this event by $\xi \in \{0, 1\}$ (so that $\Pr(\xi = 1) = \pi$).[3] An informed manager chooses whether to publicly disclose her information (i.e., $d = \theta$) or not (i.e., $d = \emptyset$). This public disclosure is observable by both shareholders and employees. Moreover, while the manager can choose not to publicly disclose her information, we assume she cannot publicly lie, i.e., such external disclosures are verifiable. The manager can also internally communicate to her employees via a cheap-talk message m. This internal communication is not verifiable (i.e., the manager can lie) and not immediately observable to

[2]The linear-quadratic specification for V and the distributional assumption for θ are made for tractability. We expect the qualitative nature of our results to survive in more general settings, but a formal analysis is not immediately tractable.

[3]Since $\pi < 1$, voluntary disclosure is not fully unraveling. This approach has been extensively adopted in the literature starting with Dye (1985) and Jung and Kwon (1988).

shareholders.

Date $t = 2$**.** At date $t = 2$, employees optimally choose their action to maximize the value of the firm. In particular, employee i chooses action x_i to maximize:

$$x_i \equiv \arg\max_x \mathbb{E}\left[V \middle| m, d\right] \tag{2}$$

$$\Rightarrow x_i = \mathbb{E}\left[\theta \middle| m, d\right]. \tag{3}$$

Importantly, the message m to employees may be leaked to shareholders with probability ρ, and denote the indicator variable for this event by $\lambda \in \{0, 1\}$. Given their information, shareholders set the price P of the firm equal to its conditional expectation i.e.,

$$P = \mathbb{E}\left[V \middle| d,\ \lambda \times m \times \xi\right]. \tag{4}$$

Date $t = 3$**.** At the final date, the value of the firm is publicly revealed. The manager receives a payoff U which depends on a weighted average of the date 2 price and the final value V of the firm:

$$U = (1 - \delta) P + \delta V, \tag{5}$$

where $\delta \in [0, 1)$. We assume that $\delta < 1$ to ensure that the manager's payoff depends on the price P, and so her disclosure policy is non-trivial. Specifically, when $\delta = 1$, the manager can only affect her payoff through the information she conveys to her employees. As a result, she is indifferent between a large class of disclosure / messaging strategies, as long as they ensure employees are perfectly informed about θ.[4]

Moreover, to ensure that an informed manager has an incentive to withhold information for sufficiently low fundamentals, we make the following assumption.

Assumption 1. *Assume that* $\frac{h+l}{2} - \beta\frac{(h-l)^2}{12} > l$.

When the manager is uninformed about fundamentals, the unconditional expected value of the firm is given by

$$\mathbb{E}[V] = \mathbb{E}[\theta] - \beta\mathbb{E}\left[(\theta - \mathbb{E}[\theta])^2\right] = \frac{h+l}{2} - \frac{\beta}{12}(h-l)^2. \tag{6}$$

In contrast, by revealing her information perfectly to her employees (and thereby enabling perfect coordination), a manager of type $\theta = l$ can ensure that the firm has value of $V = l$. Assumption 1 ensures that a manager of the lowest type (i.e., $\theta = l$) would strictly prefer to withhold her type and pool with the uninformed managers.

[4] For instance, she is indifferent between disclosing θ publicly, or not disclosing θ to shareholders, but disclosing it perfectly to shareholders.

We focus on pure strategy, Perfect Bayesian equilibrium. In particular, an equilibrium is characterized by: (i) communication strategies $\{d,\ m\}$ that maximize the manager's expected utility U at date 1, (ii) optimal employee actions x_i that maximize the conditional expected value of the firm, (iii) a pricing rule given by (4), and (iv) participants' beliefs that satisfy Bayes' rule wherever it is well-defined. Since the manager communicates with her employees using cheap talk, there always exist equilibria that feature babbling. We shall instead focus on characterizing equilibria that feature informative cheap-talk communication between the manager and employees.

Remark 1. The manager's payoff specification in (5) implies that there is no explicit conflict of interest between manager and the shareholders or employees. This serves as a useful benchmark in which to study the effects of leakage on the manager's external and internal communication. In Section 5, we relax this assumption by allowing for biases in the manager's payoffs that explicitly distort her incentives relative to those of the other stakeholders in the firm.

Remark 2. In practice, leaks often reveal negative information about the firm. Our model captures this feature, since in equilibrium, the manager withholds relatively negative information, and this is the information that may be leaked to the market. However, a limitation of our model is that the probability of leakage ρ is constant, and does not depend on either the fundamentals θ of the firm, or the message m sent by the manager. This is primarily for tractability. As a first step to exploring the robustness of our results to this restriction, we consider an extension in Appendix B where the probability of leakage ρ is a decreasing function of θ. While this assumption reflects the notion that worse news about fundamentals is more likely to be leaked, it significantly limits our ability to characterize the equilibrium. However, our numerical analysis suggests that the main conclusions of our benchmark analysis persist: increasing the likelihood of leaks improves public disclosure but worsens internal communication. We hope to extend this analysis and explore an endogenous model of what information is leaked in future work.

3 Equilibrium Communication and Leakage

In this section, we characterize the equilibrium communication by the manager. We begin with the benchmark case where there is no possibility of leakage (i.e., $\rho = 0$), and show that in this case, an informed manager optimally chooses to perfectly reveal her information to her employees. We then consider the case with leakage (i.e., $\rho > 0$). The possibility that her internal communication is leaked to shareholders creates an endogenous incentive for the manager to mislead employees — all else equal, she would prefer to report that fundamentals are higher than they actually are. In equilibrium, this leads to noisy internal communication: the manager's cheap-talk message is only partially informative.

3.1 Preliminaries

We first characterize an uninformed manager's strategy. Since external disclosures to shareholders are verifiable, an uninformed manager must publicly report that she is uninformed i.e., $d = \emptyset$. Moreover, conditional on her beliefs, the value of the firm is maximized when

$$\max_{x_i} \mathbb{E}\left[V\left(\theta, \{x_i\}_i\right)\right] = \max_{x_i} \mathbb{E}\left[\theta\right] - \beta \mathbb{E}\left[\int_i \left(\theta - x_i\right)^2 di\right] \tag{7}$$

$$\Rightarrow x_i^* = \mathbb{E}\left[\theta\right] = \tfrac{h+l}{2}. \tag{8}$$

But the manager can induce this action by revealing to her employees that she is uninformed i.e., $m = \{\theta \in [l, h]\}$. This gives us the following result.

Lemma 1. *In any equilibrium, an uninformed manager discloses that she is uninformed to shareholders and employees i.e., $d = \emptyset$ and $m = \{\theta \in [l, h]\}$.*

The above result implies that in the following analysis, we can characterize the equilibrium by specifying the strategy of the informed manager, since an uninformed manager has the same communication strategy in equilibrium.

Next, consider an informed manager who observes a realization θ of fundamentals. Since external disclosures are verifiable and observable by both shareholders and employees, a disclosure of θ implies that $P = \theta$ and

$$V = \theta - \beta \int_{i \in I} (\mathbb{E}\left[\theta|\theta\right] - \theta)^2 di = \theta, \tag{9}$$

so that the manager's payoff from disclosing θ is $U_D\left(\theta\right) = \theta$, irrespective of whether or not there is a leak. Moreover, this implies that the manager's optimal disclosure policy must follow a cutoff rule i.e., she should only disclosure θ when $\theta \geq q$ for some disclosure threshold q. This implies the following observation.

Lemma 2. *In any equilibrium, an informed manager's optimal disclosure decision follows a cutoff strategy.*

Next, we turn to the benchmark case without leakage.

3.2 Equilibrium with no leakage

Suppose there is no possibility of leakage i.e., $\rho = 0$. Since the message to her employees will not be revealed to the shareholders, an informed manager has no incentive to distort her internal communication. In this case, we have the following result.

Proposition 1. *Suppose Assumption 1 holds and there is no leakage i.e, $\rho = 0$. Let*

$$q^{NoLeak} = h - (h-l)\,\tfrac{6-\sqrt{6(1-\pi)(6-\pi\beta(h-l))}}{6\pi}. \tag{10}$$

Then, there exists an equilibrium in which an informed manager (i) discloses θ to shareholders if and only if $\theta \geq q^{NoLeak}$, and (ii) perfectly reveals θ to her employees.

In the equilibrium described above, the manager perfectly reveals her information to her employees, and so the value of the firm (conditional on the manager being informed) is θ. The overall payoff to the manager depends on whether she discloses θ to shareholders. If she does, then her payoff is given by

$$U_D(\theta) = (1-\delta)\,P + \delta\theta = \theta, \tag{11}$$

since $P = \mathbb{E}[V|\theta] = \theta$. On the other hand, by not disclosing her information publicly, she can pool with the uninformed managers, which yields a price

$$P_{ND}\left(q^{NoLeak}\right) = \frac{(1-\pi)\,\mathbb{E}\left[V\middle|\xi=0\right] + \pi\Pr\left(\theta < q^{NoLeak}\right)\mathbb{E}\left[V\middle|\xi=1, d=\emptyset\right]}{1-\pi+\pi\Pr\left(\theta < q^{NoLeak}\right)} \tag{12}$$

and an overall payoff of

$$U_{ND}(\theta) = (1-\delta)\,P_{ND} + \delta\theta. \tag{13}$$

The equilibrium disclosure cutoff q^{NoLeak} is pinned down by the natural indifference condition: a manager with fundamentals $\theta = q^{NoLeak}$ is indifferent between disclosing her information to shareholders and not i.e.,

$$U_{ND}\left(q^{NoLeak}\right) = U_D\left(q^{NoLeak}\right). \tag{14}$$

As we show in the proof of Proposition 1, Assumption 1 ensures the existence of such a cutoff $q^{NoLeak} \in [l, h]$.

The cutoff threshold q^{NoLeak} is decreasing in β and π. All else equal, a higher β reduces the value of an "uninformed" firm (i.e., the expected value of the firm conditional on the manager being uninformed), and consequently decreases the benefit of pooling with the uninformed managers. This leads an informed manager to disclose more often. Similarly, when π is higher, the benefit from not disclosing θ is lower since conditional on no disclosure, shareholders attribute a higher likelihood of the manager being informed. In the limit, when $\pi = 1$, shareholders assume that the manager is informed irrespective of whether she discloses or not, and as a result, there is no benefit from withholding information i.e., $q^{NoLeak} = l$.

Proposition 1 characterizes the equilibrium with the most informative internal communication. Since the manager uses cheap talk to communicate with her employees, there exist other equilibria in which internal communication is coarser. However the above result establishes that, in the absence of leaks, perfectly informative cheap talk can be sustained in equilibrium. Next, we show that this is no longer the case when there is leakage.

3.3 Equilibrium with leakage

The possibility that messages to her employees may be leaked to shareholders distorts an informed manager's external and internal communication decisions. On the one hand, the likelihood of a leak reduces the manager's incentives from withholding information from shareholders. On the other hand, the possibility of a leakage creates an incentive for the manager to distort her message to employees: because the message will be leaked to shareholders with positive probability, she has an incentive to bias it upwards relative to the true θ. This incentive makes it impossible to sustain fully informative cheap talk between the manager and her employees. However, as the following result establishes, there may exist equilibria with partially informative cheap talk.

Proposition 2. *Suppose Assumption 1 holds and there is leakage i.e, $\rho > 0$. Then there exists a positive integer $\bar{N} \geq 1$ such that, for every N with $1 \leq N \leq \bar{N}$, there exists an equilibrium characterized by cutoffs $l = a_0 < ... < a_N = q_N^{Leak} \leq h$, where an informed manager (i) discloses θ to shareholders if and only if $\theta \geq q_N^{Leak}$, (ii) perfectly reveals θ to her employees if and only if $\theta \geq q_N^{Leak}$, and (iii) reports only the partition $\theta \in [a_n, a_{n+1}]$ to her employees when $\theta < q_N^{Leak}$. There always exists an equilibrium for $N = 1$. Moreover, for a fixed $N \leq \bar{N}$, the disclosure cutoff q_N^{Leak} is unique.*

We leave the details of the proof to the appendix, but provide an outline of the steps here. As in the case with no leakage, note that conditional on disclosure, the value of the firm and its price are given by θ, since both shareholders and employees can observe fundamentals perfectly. In this case, the payoff to the manager is $U_D(\theta) = \theta$.

In contrast to the case with no leakage, however, when the manager does not disclose her signal to shareholders, her communication to employees is also coarse. Instead of revealing θ

to her employees, the manager only reports the partition $[a_n, a_{n+1}]$ that it is in. This implies that, conditional on no disclosure, the value of the firm is given by

$$V_I(\theta) = \theta - \beta\left(\tfrac{a_n + a_{n+1}}{2} - \theta\right)^2, \tag{15}$$

which reflects the lack of coordination within the firm due to less informative internal communication.

The price of the firm depends on whether or not a leak happens. Conditional on a leak, the price is given by the conditional expectation of the value of the firm, given that θ is in the reported partition:

$$P_{I,L} = \mathbb{E}\left[V_I(\theta)\,\middle|\theta \in [a_n, a_{n+1}]\right]. \tag{16}$$

Conditional on no leakage, the price reflects a weighted average of two possible cases: (i) the manager is uninformed (i.e., $\xi = 0$) or (ii) the manager is informed, did not disclose and the information was not leaked (i.e., $\xi = 1$, $d = \emptyset$ and $\lambda = 0$). As a result, the price conditional on no leakage is given by

$$P_{NL} = \frac{(1-\pi)\,\mathbb{E}\left[V\middle|\xi = 0\right] + \pi(1-\rho)\Pr\left(\theta < q_N^{Leak}\right)\mathbb{E}\left[V\middle|\xi = 1, d = \emptyset, \lambda = 0\right]}{1 - \pi + \pi(1-\rho)\Pr\left(\theta < q_N^{Leak}\right)}. \tag{17}$$

By not disclosing her information, the expected price that the manager gets is therefore given by:

$$P_{ND} = \rho P_{I,L} + (1-\rho)\,P_{NL}, \tag{18}$$

and so her expected payoff from not disclosing is:

$$U_{ND}(\theta, [a_n, a_{n+1}]) = (1-\delta)\,P_{ND} + \delta\left(\theta - \beta\left(\tfrac{a_n + a_{n+1}}{2} - \theta\right)^2\right). \tag{19}$$

Finally, we can use the manager's indifference between disclosure and no disclosure at $\theta = q_N^{Leak}$ to characterize the disclosure cutoff i.e.,

$$U_D\left(q_N^{Leak}\right) = U_{ND}\left(q_N^{Leak}, \left[a_{N-1}, q_N^{Leak}\right]\right), \tag{20}$$

and indifference between adjacent messages $\{\theta \in [a_{n-1}, a_n]\}$ and $\theta \in [a_n, a_{n+1}]$ at $\theta = a_n$ i.e.,

$$U_{ND}(a_n, [a_{n-1}, a_n]) = U_{ND}(a_n, [a_n, a_{n+1}]), \tag{21}$$

to characterize the sequence of cheap-talk cutoffs $\{a_n\}_{n=0}^{N}$. The sufficient condition for the existence of these cutoffs produces an upper bound on the informativeness of the internal communication.

Proposition 2 highlights the main channel through which leaks affects the manager's communication strategy. Because her internal communication to employees is leaked to shareholders with positive probability, the manager has an incentive to distort her message in an effort to push the price up. This misalignment in incentives makes fully informative cheap-talk impossible, and as a result, internal communication in equilibrium is coarse. As we discuss in the next section, this can lead to a decrease in real efficiency.

4 Comparative statics and real efficiency

In this section, we first characterize how the informativeness of internal and external communication changes with the parameters of the model. This analysis reveals a key tradeoff between internal and external communication in our setting. An increase in the probability of leaks decreases the cost of public disclosure, but also makes informative internal communication more difficult to sustain. As a result, when leaks are more likely, public disclosure is higher, but internal communication is less informative. We then describe how the possibility of leaks affect real efficiency. In our setting, the source of inefficiency is the lack of coordination by employees. We show that real efficiency is maximized either when there are no leaks, or when leaks happen with probability one. However, for intermediate probabilities, real efficiency is lower than at either extreme.

4.1 Comparative statics

As Proposition 2 highlights, there exist multiple equilibria, each with a different level of informativeness of internal communication (i.e., a different N). Moreover, the level of external disclosure (i.e., the cutoff q_N^{Leak}) itself depends on the informativeness of internal communication (i.e., N). To develop our intuition for how these equilibrium measures of informativeness depend on the underlying parameters, we first characterize how q_N^{Leak} changes as a function of parameters, treating N as a (fixed) parameter. We then study how $\bar{N}$, the upper bound on N that characterizes the equilibrium with the most informative internal communication, changes as a function of parameters. We conclude with a set of numerical illustrations of the overall effects on the cutoff q_N^{Leak} that highlight the interaction of these two effects.

The next result characterizes how q_N^{Leak} changes with parameters for a fixed N.

Proposition 3. *Suppose there is leakage (i.e., $\rho > 0$). For a fixed $N < \bar{N}$, the disclosure cutoff decreases in the probability of a leak (ρ), the probability that the manager is informed (π), and the value relevance of coordination (β). Moreover, holding other parameters fixed, the* *disclosure cutoff q_N^{Leak} decreases as the informativeness of internal communication decreases (i.e., N decreases).*

The above result characterizes the impact of leaks on public disclosure. The manager's incentives to communicate are driven by two considerations: she would like to strategically withhold negative information from shareholders in order to increase the price P, but she would prefer to be as informative to employees as possible to encourage coordination. The presence of leaks affects both these channels. Leaks decrease the benefits of withholding information, since there is a positive probability this information is made public anyways. Leaks also decrease the informativeness of internal communication since they create incentives for the manager to distort her message to employees. On the margin, this encourages the manager to disclosure more information publicly in an effort to mitigate the effect of noisier internal communication.

Fixing the informativeness of internal communication, this implies that an increase in the probability of a leak increases public disclosure by the manager (decreases q^{Leak}) since it is less beneficial to withhold information. Moreover, for a fixed probability of leaks, when the informativeness of internal communication is lower (i.e., N decreases), the manager partially compensates the decrease in coordination by increasing disclosure. As a result, the disclosure threshold q_N^{Leak} decreases as N decreases.

The effects of π and β on the disclosure threshold are more direct. An increase in the ex-ante likelihood of being informed, π, decreases the incentives for informed managers to pool with the uninformed managers by withholding information, and as a result, leads to greater disclosure. For a fixed level of informativeness of internal communication, an increase in the relative importance of coordination β encourages the manager to disclose more information publicly.

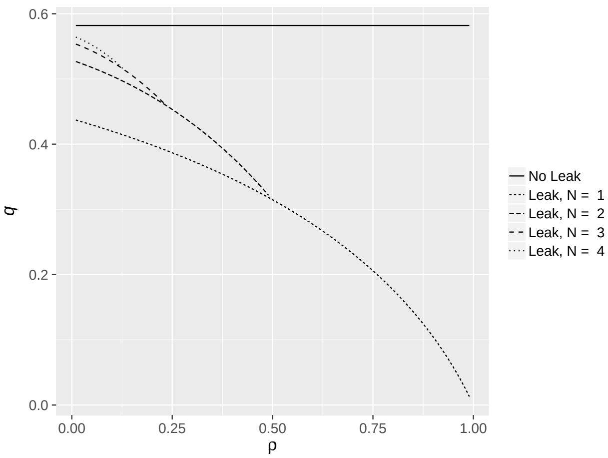
Next, we characterize the impact of leaks on the maximal informativeness of internal communication.

Proposition 4. *The maximum informativeness of internal communication that can be sustained (i.e., $\bar{N}$) weakly decreases with the probability of a leak (ρ) and the probability that the manager is informed (π).*

Note that while an increase in N leads to an increase in the corresponding threshold q_N^{Leak} , the maximal informativeness $\bar{N}$ decreases when the manager is more likely to disclose information. Intuitively, $\bar{N}$ measures how finely the range of undisclosed θ can be partitioned. When there is more disclosure, the range of the undisclosed θ is smaller, and consequently, $\bar{N}$ is lower.

Figure 1: Disclosure versus leakage

The figure plots the disclosure threshold q, when (i) there are no leaks and (ii) when there are leaks for different values of N. Benchmark parameters are $h = 2$, $l = 0$, $\delta = 0.75$, $\beta = 1$, and $\pi = 0.5$.



This observations helps explain the intuition for how π and ρ affect $\bar{N}$. When the prior probability that the manager is informed (π) increases, the benefit from withholding information decreases. All else equal, the manager is more likely to disclosure θ, but this implies the non-disclosure region is smaller, and as a result, $\bar{N}$ is lower. Similarly, increasing the probability of a leak (ρ) decreases the manager's cost of public disclosure, but this makes it more difficult to sustain a higher level of N. Intuitively, a higher ρ increases the manager's incentives to distort internal communication, since it is more likely to be leaked to shareholders and, thereby, affect the price.

Figure 1 provides a numerical illustration of the overall effect of leakage on the disclosure threshold. First, note that disclosure is always lower without leakage, as evidenced by the solid line in the figure. Second, consistent with Proposition 3, when there is leakage, the cutoff q_N^{Leak} increases with N but decreases with ρ for a fixed N. Moreover, the maximal level of informativeness, $\bar{N}$, decreases with ρ as implied by Proposition 4, since informative communication is more difficult to sustain with higher leakage. As such, the effects on q_N^{Leak} and $\bar{N}$ reinforce each other: an increase in ρ decreases q_N^{Leak} for a fixed N, and decreases $\bar{N}$, which leads the disclosure threshold in the most internally informative equilibrium to fall.

4.2 The effect of leaks on efficiency

The source of inefficiency in our setting is the lack of coordination by the employees. Specifically, real efficiency is given by

$$RE(\rho, N) = -\mathbb{E}\left[\beta \int_{i \in I} (x_i^* - \theta)^2\right] \tag{22}$$

$$= \begin{cases} 0 & \text{with no leakage} \\ -\beta F\left(q_N^{Leak}\right) \mathbb{E}\left[\int_{i \in I} (x_i^* - \theta)^2 \,\middle|\, \theta < q_N^{Leak}, \{a_n\}_{n=1}^N\right] & \text{with leakage} \end{cases}. \tag{23}$$

This implies the following result.

Proposition 5. *Real efficiency is maximized when either (i) there is no leakage (i.e., $\rho = 0$), or (ii) leaks occur with probability 1 (i.e., $\rho = 1$).*

On the one hand, when there is no leakage, Proposition 1 implies that the manager perfectly reveals her information to employees, and as a result, coordination is perfect. On the other hand, as we show in the proof of the above result, when leaks occur with probability 1 (i.e., when $\rho = 1$), there is no benefit from withholding information from shareholders, and so the manager discloses all her information (i.e., disclosure threshold is $q_N^{Leak} = l$).[5] As a result, employees observe fundamentals perfectly, and again, coordination is perfect. When the probability of leaks is positive, but less than one, real efficiency is lower than in the extremes since neither public disclosures nor internal communication are perfectly informative. As a result, employees only have noisy information about fundamentals and the firm incurs an efficiency loss due to decreased coordination.

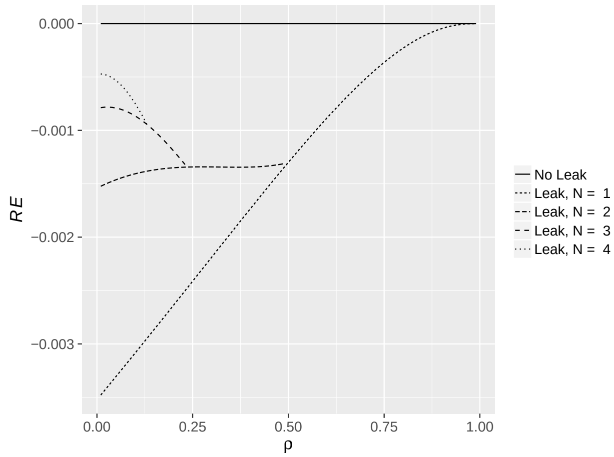
Figure 2 illustrates how real efficiency changes with the probability of leakage. The overall effect of ρ on real efficiency is non-monotonic, and depends on the level of internal communication (i.e., N). This relation is driven by two effects. First, consistent with Proposition 4, informative internal communication is more difficult to sustain as ρ increases i.e., $\bar{N}$ falls with ρ. Second, for a fixed N, efficiency tends to decrease with ρ when informativeness is high (e.g., $N = 3, 4$), but increases with ρ when informativeness is low (e.g., $N = 1$).

To better understand the relation between efficiency and ρ for a fixed N, note that an increase in ρ has two offsetting effects. On the one hand, an increase in the probability of leaks (higher ρ) increases public disclosure (decreases q), which improves efficiency. On the other hand, more leakage increases the manager's incentives to misreport her information internally and, consequently, makes internal communication less informative. The first effect

[5] Arguably, leakage with probability one is an unrealistic benchmark in general, since there are other (unmodeled) constraints on what information can be revealed publicly.

Figure 2: Real efficiency versus leakage

The figure plots real efficiency RE as a function of parameters. Benchmark parameters are $h = 2$, $l = 0$, $\delta = 0.75$, $\beta = 1$, and $\pi = 0.5$.



dominates when internal communication is not very informative (i.e., N is low), but the second effect dominates when internal communication is informative (e.g., N is high). As a result, real efficiency initially decreases but eventually increases with ρ and reaches its maximum at $\rho = 1$, when public disclosure is perfectly informative. In general, the analysis suggests that leaks are more likely to improve efficiency when internal communication is less informative. This observation has potentially important implications for regulatory policy, which we return to in our concluding remarks.

The results of this section imply that leaks have a negative effect: the possibility of leakage tends to decrease real efficiency, even though it can lead to more public disclosure. The next section considers a setting in which the possibility of leaks can play a more positive role.

5 The effect of leaks with managerial bias

Leaks are often ascribed to have a disciplinary role: the possibility of leaks discourages managers from misbehaving because they are concerned about the consequences of their behavior becoming public. Our benchmark model does not capture this aspect, since the incentives of the manager are well aligned with those of other stakeholders of the firm. As a result, leaks are counterproductive because they limit the manager's ability to communicate with her employees.

However, we can analyze the disciplinary role of leaks by introducing a conflict of interest between the manager and the firm's other stakeholders. In particular, suppose the value of the firm is given by (1), the employee's optimal action is given by (3), and the shareholders' pricing rule is given by (4). However, now assume the manager's payoff is given by

$$U = (1-\delta) P + \delta \tilde{V}, \text{ where} \tag{24}$$

$$\tilde{V} = V(\theta + b) = \theta + b - \beta \int_{i \in I} (x_i - (\theta + b))^2 \, di, \tag{25}$$

for $b \geq 0$.[6] The parameter b parsimoniously captures the degree to which the manager's incentives differ from those of the employees. This bias may arise because the manager has different priors over the firm's fundamentals (e.g., she is more optimistic about the firm so that $b > 0$), or if she prefers coordinating employees' actions towards a different objective. The additive specification is a standard approach to introducing such conflicts of interest, and is chosen primarily for tractability (e.g., Crawford and Sobel, 1982).

In this case, the equilibrium is characterized by the following result.

Proposition 6. *Suppose Assumption (1) holds and $b > 0$. Then there exists a positive integer $\bar{N} \geq 1$ such that for every N with $1 \leq N \leq \bar{N}$, there exists an equilibrium characterized by cutoffs $l = a_0 < \ldots < a_N = q_N^{Bias} \leq h$, where an informed manager (i) discloses θ to shareholders if and only if $\theta \geq q_N^{Bias}$, (ii) perfectly reveals θ to her employees if and only if $\theta \geq q_N^{Bias}$, and (iii) reports only the partition $\theta \in [a_n, a_{n+1}]$ to her employees when $\theta < q_N^{Bias}$. There always exists an equilibrium for $N = 1$. Moreover, for a fixed $N \leq \bar{N}$, the disclosure cutoff q_N^{Bias} is unique.*

The proof of this result extends that of Proposition 2 by accounting for the bias in the manager's payoff. The key observation is that even when there is no leakage (i.e., $\rho = 0$), the manager has an incentive to distort her report to employees, and consequently, internal communication is not perfectly informative.

When the manager publicly discloses her signal, both employees and shareholders observe θ. In this case, $P = \theta$, and so the manager's payoff is

$$U_D(\theta) = (1-\delta)\theta + \delta\left(\theta + b - \beta b^2\right). \tag{27}$$

Conditional on no disclosure, the manager reports that $\theta \in [a_n, a_{n+1}]$ to her employees and

[6] The proof of the results in this section allow for a slightly more general specification for $\tilde{V}$, given by:

$$\tilde{V} = \theta + b_0 - \beta \int_{i \in I} (x_i - (\theta + b))^2 \, di \tag{26}$$

where b and b_0 can potentially be different. The key parameter is the bias in the quadratic term b since this affects the manager's cheap talk strategy.

her resulting payoff is

$$U_{ND}\left(\theta,[a_n,a_{n+1}]\right)=(1-\delta)\,P_{ND}+\delta\left(\theta+b-\beta\left(\tfrac{a_n+a_{n+1}}{2}-\theta-b\right)^2\right). \tag{28}$$

Note that these expressions mirror the corresponding expressions for the benchmark model in Section 3.3, but account for the bias terms b_0 and b. As before, the disclosure cutoff is pinned down by the indifference between disclosure and not at $\theta=q_N^{Bias}$ i.e.,

$$U_D\left(q_N^{Bias}\right)=U_{ND}\left(q_N^{Bias},\left[a_{N-1},q_N^{Bias}\right]\right), \tag{29}$$

while the indifference condition between adjacent messages $\{\theta\in[a_{n-1},a_n]\}$ and $\{\theta\in[a_n,a_{n+1}]\}$ at $\theta=a_n$ i.e.,

$$U_{ND}\left(a_n,[a_{n-1},a_n]\right)=U_{ND}\left(a_n,[a_n,a_{n+1}]\right), \tag{30}$$

pins down the sequence of cheap-talk cutoffs $\{a_n\}_{n=0}^N$. An immediate consequence of the above indifference conditions is that the equilibrium disclosure and communication policies does not depend on b_0.

Intuitively, when the manager's incentives are less aligned with those of the employees (i.e., when b is larger), internal communication is less informative, even in the absence of leakage. As a result, $\bar{N}$ decreases with b. Moreover, this decrease in informativeness decreases the value of the firm due to worse coordination, which in turn, increases the relative benefit from publicly disclosing information. As a result, for a fixed N, the disclosure cutoff q_N^{Bias} falls with the bias b i.e., a larger bias leads to more disclosure. These results are summarized in the next result.

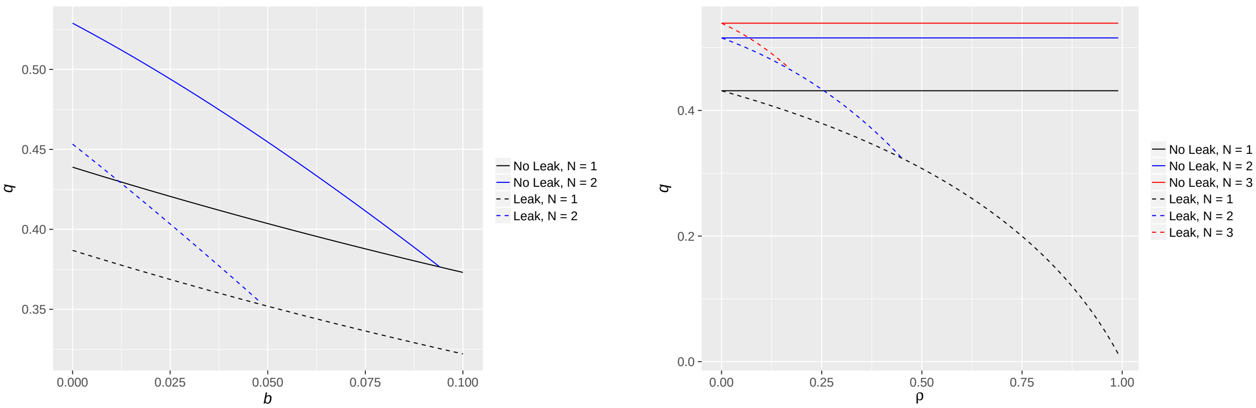
Proposition 7. *Suppose $N\leq\bar{N}$ and the manager is biased. For a fixed N, the disclosure cutoff q_N^{Bias} decreases in the bias b. Moreover, the upper bound on the informativeness of internal communication (i.e., $\bar{N}$) decreases in the bias b.*

Panel (a) of Figure 3 provides an illustration of the above results. The figure plots the disclosure cutoffs as a function of the bias b when there are no leaks (solid lines) and when there are leaks ($\rho=0.25$, dotted lines). The plots suggest that in either case: (i) the cutoffs are decreasing in b, and (ii) the maximal informativeness $\bar{N}$ is decreasing in b. Moreover, as in the benchmark base (with a bias $b=0$), the disclosure cutoff is lower in the presence of leaks (i.e, dotted lines are lower than solid lines).

Panel (b) of Figure 3 also confirms that the effect of ρ on the disclosure cutoff and the informativeness of internal communication are robust to the introduction of the managerial bias b. Specifically, an increase in the probability of leakage (i.e., ρ) leads to a decrease in the maximal informativeness of internal communication (i.e., $\bar{N}$) and, for a fixed N, a decrease

Figure 3: Disclosure cutoffs with a biased manager

The figure plots the disclosure cutoff q_N^{Bias} as a function of parameters. Benchmark parameters are $h = 2$, $l = 0$, $\delta = 0.75$, $\beta = 1$, $\pi = 0.5$, $\rho = 0.25$ and $b = 0.01$.



(a) q_N^{Bias} vs. b, with $\rho = 0.25$ (b) q_N^{Bias} vs. ρ with $b = 0.01$

in the disclosure threshold q_N^{Bias}. As before, leaks make internal communication coarser, but increase public disclosure.

Next, we turn to real efficiency when the manager is biased. When the manager's bias is b, the probability of leakage is ρ and the number of partitions of the internal communication is N, denote real efficiency by $RE(\rho, N, b)$, where

$$RE(\rho, N, b) = -\mathbb{E}\left[\beta \int_{i\in I} (x_i^* - \theta)^2\right] = -\beta F\left(q_N^{Bias}\right)\mathbb{E}\left[\int_{i\in I} (x_i^* - \theta)^2 \left|\theta < q_N^{Bias}, \{a_n\}_{n=1}^N\right.\right] \tag{31}$$

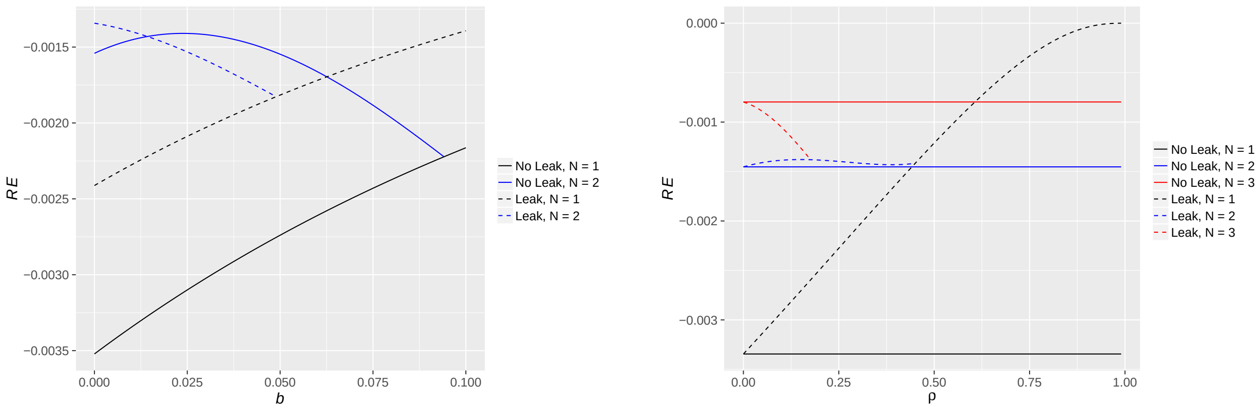
Since internal communication is noisy when the manager is biased, real efficiency is no longer maximized when there is no leakage (i.e., $\rho = 0$) as in the benchmark case. Moreover, as the following result highlights, real efficiency may be higher with some leakage than with no leakage.

Proposition 8. *Real efficiency is maximized when leaks occur with probability 1 (i.e., $\rho = 1$). Moreover, for a fixed $b > 0$, there exists a threshold $\bar{\rho} > 0$, such that for $\rho \geq \bar{\rho}$, $RE(\rho, N = 1, b) > RE\left(0, N = \bar{N}, b\right)$.*

In contrast to our benchmark model, the above result implies that leaks can have a disclipinary role when there are conflicts of interest between the manager and the employees. The bias in the manager's payoff induces coarse internal communication even in the absence of the leaks. While increasing the probability of a leak decreases the informativeness of internal communication further, it increases public disclosure. If internal communication is sufficiently informative initially (i.e., N is high), then the first effect dominates, and an

Figure 4: Real efficiency with a biased manager

The figure plots real efficiency RE as a function of parameters. Benchmark parameters are $h = 2$, $l = 0$, $\delta = 0.75$, $\beta = 1$, $\pi = 0.5$,$\rho = 0.25$ and $b = 0.01$.



(a) RE vs. b, with $\rho = 0.25$

(b) RE vs. ρ with $b = 0.01$

increase in ρ decreases efficiency. However, when the manager's bias (or the probability of leaks) is sufficiently high, informative internal communication can no longer be sustained (i.e., $N = 1$). In this case, the second effect dominates and, as a result, an increase in ρ increases efficiency.

Figure 4 illustrates these effects for a specific parameterization. Panel (a) compares real efficiency for varying levels of managerial bias when there is zero or positive ($\rho = 0.25$) probability of leaks. When more informative internal communication is possible (e.g., $N = 2$), real efficiency tends to be lower with leakage (blue, dashed line) than without (blue, solid line), unless the manager's bias is extremely small. In contrast, when internal communication is not informative (i.e., $N = 1$), the equilibrium with leaks is more efficient for all levels of b, since public disclosure is higher. Similarly, for a fixed bias $b > 0$, panel (b) illustrates how real efficiency eventually increases in the likelihood of leaks. Intuitively, a non-zero bias in the manager's incentives limits the maximal informativeness of internal communication ($\bar{N} = 3$ in our numerical example). When the likelihood of leaks is sufficiently high, informative internal communication cannot be sustained (i.e., $N = 1$), but in this case, efficiency improves with ρ.

The discussion in Section 4.2 on the relationship between efficiency and ρ carries over to the case of biased manager here: efficiency initially decreases with ρ, but eventually increases until it reaches a maximum for $\rho = 1$. However, unlike the benchmark case with no bias (i.e., $b = 0$), the equilibria without leaks (i.e., $\rho = 0$) do not dominate the ones with $\rho < 1$, since internal communication is imperfect even in the absence of leakage. In fact, when internal communication is not informative (i.e., $N = 1$), an increase in the probability of leakage improves efficiency for all values of ρ. As in the case with no conflicts of interest (i.e., with $b = 0$), the possibility of leaks is more likely to improve efficiency when internal communication is less informative. We turn to the implications of this observation in our final section.

6 Implications and Concluding Remarks

As our analysis highlights, leaks introduce an important tradeoff: more leakage makes public disclosure more informative, but internal communication less informative. As a result, the effect of leaks on real efficiency (welfare) is non-monotonic, and depends crucially on the informativeness of internal communication within the firm. While identifying appropriate empirical measures of the underlying parameters is extremely challenging, our model makes a number of testable predictions.

First, the average price reaction to information leaks is negative. This is intuitive: an informed manager publicly discloses good news, but withholds bad news (as in standard disclosure models). Hence, any leaks reveal negative news to the market. This is consistent with a number of papers in the literature, including Bowen et al. (2010) and Dyck et al. (2010).

Second, firms respond to an increase (decrease) in the probability of leaks by increasing (decreasing, respectively) public voluntary disclosures. For instance, our model predicts that voluntary disclosures by firms should increase following the implementation of the Whistleblower Program of the Dodd-Frank Act (see http://www.sec.gov/spotlight/dodd-frank/whistleblower.shtml). However, a key challenge in testing such a prediction empirically is to separately identify the impact of such regulation on the likelihood of leakage, while controlling for other aspects that might affect firms' disclosure policies directly.

Third, all else equal, a higher likelihood of leakage is associated with less informative internal communication. Empirical tests of this prediction are confounded by the fact that following (the incidence of) a leak, firms usually make changes to the internal governance and communication policies. As such a test of this prediction requires identifying firms that are ex-ante more likely to have leaks, and comparing their internal communication to a control group. Bowen et al. (2010) provide some preliminary evidence consistent with this prediction: in their sample, targets of employee whistle-blowing allegations are more likely to have unclear internal communications channels.

Fourth, an increase in the likelihood of leakage affects real efficiency across firms differently. Specifically, real efficiency decreases in firms with fewer conflicts of interest and informative internal communication, but can increase in firms with poor internal communication and bad governance. To the extent that firms with better governance and internal communication perform better ex ante, our model predicts that the cross-sectional variation in firm performance should decrease in response to regulatory changes that encourage leakage.

The above result is an important consideration when evaluating regulatory policies that improve whistleblower protection. An increased likelihood of leakage can have a disciplinary effect on firms, but not always. Such policies are more likely to be beneficial for firms that have severe agency problems, uninformative internal communication and more short-term compensation for managers (e.g., through stock / option grants). To the extent possible, targeting such firms may be important in ensuring that regulations have the intended effect.

Our current model is stylized for tractability, but provides a first step in understanding how external and internal communication is affected by leakage. Studying how leaks affect the aggregation of information in a setting where employees have private information would be a natural next step. It would also be interesting to develop a richer model of employee / insider incentives to study the effect of strategic leakage and to endogenize what information is leaked. We hope to explore these directions in future work.

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Appendices

A Proofs of main results

Proof of Proposition 1. By Lemma 2, we know that the optimal disclosure strategy must be a cutoff strategy where the manager discloses θ only if $\theta \geq q$ for some cutoff q. Since there is no leakage, an informed manager optimally communicates θ perfectly to her employees. This implies that

$$V(\xi = 1) = \theta, \quad V(\xi = 0) = \mathbb{E}[\theta] - \beta \text{var}[\theta]. \tag{32}$$

Conditional on disclosing her information to shareholders, this implies $U_D(\theta) = \theta$, which is strictly increasing in θ. Conditional on no disclosure, the price is then given by:

$$P_{ND}(q) = \frac{(1-\pi)V(\xi = 0) + \pi F(q)\mathbb{E}[V(\xi = 1, \theta)|\theta < q]}{(1-\pi) + \pi F(q)} \tag{33}$$

$$= \frac{(1-\pi)\left(\frac{h+l}{2} - \beta\frac{(h-l)^2}{12}\right) + \pi\frac{q-l}{h-l}\frac{l+q}{2}}{(1-\pi) + \pi\frac{q-l}{h-l}} \tag{34}$$

At $\theta = q$, the manager should be indifferent between disclosing or not, and so:

$$q = \delta q + (1-\delta)P_{ND}(q) \tag{35}$$

$$\leftrightarrow q = \frac{(1-\pi)\left(\frac{h+l}{2} - \beta\frac{(h-l)^2}{12}\right) + \pi\frac{q-l}{h-l}\frac{l+q}{2}}{(1-\pi) + \pi\frac{q-l}{h-l}} \tag{36}$$

$$\leftrightarrow q = \frac{6\pi h - (h-l)(6 \pm \sqrt{6(1-\pi)(6-\pi\beta(h-l))}}{6\pi}. \tag{37}$$

To ensure that the cutoff $q \in [l, h]$ we must select the larger root i.e.,

$$q = h - (h-l)\frac{6 - \sqrt{6(1-\pi)(6-\pi\beta(h-l))}}{6\pi} \equiv q^{NoLeak}. \tag{38}$$

Finally, note that when Assumption 1 holds, we have:

$$\frac{h+l}{2} - \frac{\beta}{12}(h-l)^2 > l \Rightarrow 6 - \beta(h-l) > 0, \tag{39}$$

which ensures q^{NoLeak} is well-defined. Moreover, the above establishes that q^{NoLeak} is uniquely determined. □

Proof of Proposition 2. Conditional on a message $m_n = \{\theta \in [a_n, a_{n+1}]\}$, note that

$$\mathbb{E}[\theta|m_n] = \frac{a_n + a_{n+1}}{2}, \quad \text{var}[\theta|m_n] = \frac{(a_{n+1} - a_n)^2}{12}. \tag{40}$$

Next, note that disclosure must follow a cutoff strategy (Lemma 2) and when θ is disclosed publicly, the manager's payoff from disclosure is $U_D(\theta) = \theta$. When there is no disclosure, there are two possibilities:

- The manger is uninformed, in which case, the value of the firm is:

$$V_{NI} = \tfrac{h+l}{2} - \tfrac{\beta}{12}(h-l)^2 = P_{NI}. \tag{41}$$

- The manager is informed, and only sends message m_n to her employees. In this case, the value of the firm is

$$V_I(\theta, m_n) = \theta - \beta\left(\tfrac{a_n + a_{n+1}}{2} - \theta\right)^2. \tag{42}$$

  If the message is leaked to shareholders, the price is given by

$$P_{I,L}(m_n) = \mathbb{E}\left[V_I \middle| m_n\right] = \tfrac{a_{n+1}+a_n}{2} - \tfrac{\beta}{12}(a_{n+1} - a_n)^2. \tag{43}$$

  On the other hand, if the message is not leaked, the price is given by

$$P_{I,NL}(m_n) = \mathbb{E}\left[V_I \middle| \xi = 1, d = \emptyset\right] = \mathbb{E}\left[V_I \middle| \theta < q\right] = \tfrac{q_N + l}{2} - \tfrac{\beta}{12}\sum_i \tfrac{(a_{n+1}-a_n)^3}{q_N - l}. \tag{44}$$

As a result, conditional on no disclosure, the expected price is then given by:

$$P_{ND}(m_n) = \rho P_{I,L} + \rho\left(\gamma P_{NI} + (1-\gamma) P_{I,NL}\right), \tag{45}$$

where ρ is the probability of a leak, and γ reflects the probability that the manager is uninformed, given that there is no disclosure (and no leak) i.e.,

$$\gamma \equiv \frac{1-\pi}{1 - \pi + \pi(1-\rho)\Pr(\theta < q_N)}. \tag{46}$$

As a result, an informed manager's payoff from not disclosing her information is

$$U_{ND}(\theta, m_n) = \delta V_I(\theta, m_n) + (1-\delta) P_{ND}(m_n). \tag{47}$$

The disclosure cutoff is pinned down by indifference for $\theta = q$ i.e.,

$$U_D(q) = U_{ND}(q, \{\theta \in [a_{N-1}, q]\}), \tag{48}$$

while the cheap-talk partitions are characterized by indifference at the cutoffs $\theta = a_n$ i.e.,

$$U_{ND}(a_n, \{\theta \in [a_{n-1}, a_n]\}) = U_{ND}(a_n, \{\theta \in [a_n, a_{n+1}]\}). \tag{49}$$

The latter indifference condition implies that a_n satisfies the difference equation:

$$\tfrac{a_{n-1}-a_{n+1}}{2}\left(\rho(1-\delta) - \beta(3\delta + (1-\delta)\rho)\tfrac{(a_{n-1}+a_{n+1}-2a_n)}{6}\right) = 0 \tag{50}$$

$$\Rightarrow a_{n+1} = \left\{a_{n-1},\ 2a_n - a_{n-1} + \tfrac{6\rho(1-\delta)}{\beta(3\delta+\rho(1-\delta))}\right\} \tag{51}$$

Given the monotonicity of a_i, the second solution solution must hold. The recursive equation of the form:

$$a_{i+2} = 2a_{i+1} - a_i + 2c, \quad a_0 = l,$$

has the solution:

$$a_n = l + A_0 n + cn(n-1)$$

for a constant A_0 and

$$c \equiv \frac{3(1-\delta)\rho}{\beta(3\delta + (1-\delta)\rho)}. \tag{52}$$

Since $a_N = q$, we must have:

$$q - l = A_0 N + cN(N-1) \geq cN(N-1). \tag{53}$$

The constant A_0 can be pinned down using $a_N = q$, which implies:

$$A_0 = \tfrac{q-l}{N} - c(N-1), \tag{54}$$

$$\Rightarrow a_n = \tfrac{(n-N)(nNc-l)+nq}{N}, \quad a_{N-1} = \tfrac{l-(N-1)(cN-q)}{N}. \tag{55}$$

Finally, note that the above implies:

$$\Sigma(q) \equiv \tfrac{1}{12(q-l)} \sum_{i=1}^{N} (a_i - a_{i-1})^3 = \tfrac{c^2\left(N^2-1\right)}{12} + \tfrac{(q-l)^2}{12N^2}. \tag{56}$$

Given these expressions, one can solve for q (using (48)) as the fixed point to $q = g(q)$, where

$$g(q) \equiv U_{ND}(q, \{\theta \in [a_{N-1}, q]\}) \tag{57}$$

$$= \delta\left(q - \beta\left(\tfrac{q+a_{N-1}}{2} - q\right)^2\right) + (1-\delta)\begin{pmatrix} \rho\left(\frac{q+a_{N-1}}{2} - \frac{\beta}{12}(q-a_{N-1})^2\right) \\ +(1-\rho)\gamma\left(\frac{h+l}{2} - \frac{\beta}{12}(h-l)^2\right) \\ +(1-\rho)(1-\gamma)\left(\frac{q_N+l}{2} - \beta\Sigma(q)\right) \end{pmatrix} \tag{58}$$

$$= \tfrac{q+l}{2} - \beta\Sigma(q) + \delta\left(\tfrac{q-l}{2} - 2\beta\Sigma(q)\right) + (1-\delta)(1-\rho)\gamma\left(\tfrac{h-q}{2} - \beta\left(\tfrac{(h-l)^2}{12} - \Sigma(q)\right)\right) \tag{59}$$

Fix an $N \geq 1$ and recall that $q - l \geq cN(N-1)$. To show the existence of a solution to $q = g(q)$, where $h > q > cN(N-1) + l$, it is sufficient to establish conditions for $g(h) < h$ and $g(l + cN(N-1)) > l + cN(N-1)$. When $q = h$, note that

$$g(h) = \delta V_I + (1-\delta)(\rho P_{I,L} + (1-\rho)(\gamma P_{NI} + (1-\gamma)P_{I,NL})) \tag{60}$$

for V_I, $P_{I,L}$, P_{NI} and $P_{I,L}<h$, and so $g(h) < h$. Next, note that $\beta c = \frac{3(1-\delta)\rho}{(3\delta+(1-\delta)\rho)} \leq 3$, and so

$$g(cN(N-1)+l) = \begin{matrix} l + (1-\delta)(1-\rho)\underline{\gamma}\left(\frac{h-l}{2} - \beta\frac{(h-l)^2}{12}\right) \\ +\frac{cN(N-1)}{2}\left(1+\delta-(1-\delta)(1-\rho)\underline{\gamma}\right) \\ -\frac{\beta c^2\left((N-1)^2+N^2-1\right)}{12}\left(1+2\delta-(1-\delta)(1-\rho)\underline{\gamma}\right) \end{matrix} \tag{61}$$

$$
\begin{array}{ll}
\geq & \begin{array}{c} l+(1-\delta)(1-\rho)\underline{\gamma}\left(\frac{h-l}{2}-\beta\frac{(h-l)^2}{12}\right) \\ +\frac{cN(N-1)}{2}\left(1+\delta-(1-\delta)(1-\rho)\underline{\gamma}\right) \\ -\frac{3c(2N(N-1))}{12}\left(1+2\delta-(1-\delta)(1-\rho)\underline{\gamma}\right) \end{array} \quad (62) \\
& = l+(1-\delta)(1-\rho)\underline{\gamma}\left(\frac{h-l}{2}-\beta\frac{(h-l)^2}{12}\right)-\frac{cN(N-1)}{2}\delta, \quad (63)
\end{array}
$$

where

$$
\underline{\gamma}=\frac{1-\pi}{1-\pi+\pi(1-\rho)\frac{cN(N-1)}{h-l}}. \tag{64}
$$

To ensure that $g(cN(N-1)+l)>l+cN(N-1)$, we must have:

$$
l+\frac{(1-\delta)(1-\rho)(1-\pi)}{1-\pi+\pi(1-\rho)\frac{cN(N-1)}{h-l}}\left(\frac{h-l}{2}-\beta\frac{(h-l)^2}{12}\right)-\frac{cN(N-1)}{2}\delta>l+cN(N-1) \tag{65}
$$

$$
\frac{(1-\delta)(1-\rho)(1-\pi)\left(\frac{h-l}{2}-\beta\frac{(h-l)^2}{12}\right)}{1-\pi+\pi(1-\rho)\frac{cN(N-1)}{h-l}}>cN(N-1)\left(1+\frac{\delta}{2}\right) \tag{66}
$$

or equivalently,

$$
cN(N-1)\left(1+\frac{\delta}{2}\right)\left(1-\pi+\frac{\pi(1-\rho)}{h-l}cN(N-1)\right)<(1-\delta)(1-\rho)(1-\pi)\left(\frac{h-l}{2}-\beta\frac{(h-l)^2}{12}\right). \tag{67}
$$

Since the LHS of the above is always increasing in N, the above condition is equivalent to ensuring that $N\leq\bar{N}$ for some bound. Moreover, note that the above condition holds for $N=1$, and so $\bar{N}\geq 1$.

For a fixed $N\leq\bar{N}$, a sufficient condition for the uniqueness of q_N^{Leak} is that $\frac{\partial g}{\partial q}<1$, since this implies $g(q)$ intersect the 45 degree line (at most once) from above. Note that

$$
\frac{\partial g}{\partial q}=\begin{array}{c}\frac{1}{2}\left(1+\delta-\gamma(1-\delta)(1-\rho)\right)-\beta\Sigma_q\left(1+2\delta-(1-\delta)(1-\rho)\gamma\right) \\ +\gamma_q(1-\delta)(1-\rho)\left(\frac{h-q}{2}-\beta\left(\frac{(h-l)^2}{12}-\Sigma\right)\right)\end{array} \tag{68}
$$

where

$$
\Sigma_q=\frac{2(q-l)}{12N^2}>0,\quad \gamma_q=-\frac{\pi(1-\pi)(1-\rho)}{(h-l)\left(1-\pi+\pi(1-\rho)\frac{q-l}{h-l}\right)^2}<0. \tag{69}
$$

At any point of intersection, we have $q=g(q)$, and so

$$
\begin{aligned}
(1-\delta)(1-\rho)\left(\frac{h-q}{2}-\beta\left(\frac{(h-l)^2}{12}-\Sigma\right)\right) &= \frac{1}{\gamma}\left(q-\left(\frac{q+l}{2}-\beta\Sigma(q)+\delta\left(\frac{q-l}{2}-2\beta\Sigma(q)\right)\right)\right) && (70)\\
&= \frac{1}{\gamma}\left(\frac{q-l}{2}(1-\delta)+(1+2\delta)\beta\Sigma(q)\right)\geq 0 && (71)
\end{aligned}
$$

Since $1+2\delta-(1-\delta)(1-\rho)\gamma>0$,

$$
\frac{\partial g}{\partial q}<\frac{1}{2}(1+\delta)<1. \tag{72}
$$

This establishes that any point of intersection $q=g(q)$, $g(q)$ intersects the 45-degree line from below, which in turn implies there can be only one such intersection for $q \in [l + cN(N-1), h]$. $\square$

Proof of Proposition 3. We begin by proving a useful Lemma.

Lemma 3. *Fix an $N < \bar{N}$. The unique solution q_N^{Leak} to $q = g(q; N)$ satisfies:*

$$q_N^{Leak} - l \geq cN(N+1) > cN(N-1). \tag{73}$$

Proof. Uniqueness of q_N^{Leak} is established in the proof of Proposition 2. Specifically, recall that for a fixed N, $g(q)$ intersects the 45-degree line from above. Next, note that

$$G(q) \equiv g(q, N) - g(q, N-1) = -(\beta(1 + 2\delta - (1-\delta)(1-\rho)\gamma))\left[\Sigma(N) - \Sigma(N-1)\right] \tag{74}$$

$$\text{where } \Sigma(N) - \Sigma(N-1) = \tfrac{1}{12}\left(c^2(N^2 - (N-1)^2) + (q-l)^2\tfrac{(N-1)^2 - N^2}{N^2(N-1)^2}\right) \tag{75}$$

$$= \tfrac{2N-1}{12}\left(c^2 - \tfrac{(q-l)^2}{N^2(N-1)^2}\right) \tag{76}$$

This implies that for $q = cN(N-1)$, we have $g(q, N) = g(q, N-1)$. Since $q_N^{Leak} - l \geq cN(N-1)$, we have:

$$G(q_N^{Leak}) = g(q_N^{Leak}, N) - g(q_N^{Leak}, N-1) = q_N^{Leak} - g(q_N^{Leak}, N-1) > 0 \tag{77}$$

Moreover, since $g(x; N)$ intersects the 45-degree line from above, we have:

$$l + cN(N-1) \leq g(l + cN(N-1); N), \tag{78}$$

and since $G(l + cN(N-1)) = 0$, we have:

$$l + cN(N-1) \leq g(l + cN(N-1); N-1) \tag{79}$$

But this implies that the solution q_{N-1}^{Leak} to $q = g(q; N-1)$ must satisfy:

$$l + cN(N-1) \leq q_{N-1}^{Leak} \leq q_N^{Leak} \tag{80}$$

which completes the result. $\blacksquare$

Note that the above result immediately implies that q_N^{Leak} is increasing in N. Since $q = g(q; \rho)$ and $g_q \equiv \frac{\partial g}{\partial q} < 1$, we have

$$\frac{\partial q}{\partial \rho} = \frac{g_\rho}{1 - g_q} < 0 \quad \Leftrightarrow \quad g_\rho \equiv \frac{\partial g}{\partial \rho} < 0. \tag{81}$$

So to establish the comparative statics results for ρ, (and analogously π and β) it is sufficient to show $g_\rho < 0$ (and analogously $g_\pi < 0$ and $g_\beta < 0$). Given the expression for $g(\cdot)$ above, we have:

$$g_\rho = -\Sigma_\rho\left(1 + 2\delta - (1-\delta)(1-\rho)\gamma\right) + (1-\delta)\left((1-\rho)\gamma_\rho - \gamma\right)\left(\tfrac{h-q}{2} - \beta\left(\tfrac{(h-l)^2}{12} - \Sigma\right)\right) < 0,$$

since $\gamma_\rho = \frac{\gamma(1-\gamma)}{1-\rho} \geq 0$ (see (46)), $\frac{h-q}{2} - \beta\left(\frac{(h-l)^2}{12} - \Sigma\right) > 0$ (see the argument for (71) above) and $\Sigma_\rho = \frac{\partial \Sigma}{\partial c}\frac{\partial c}{\partial \rho} > 0$, given (52) and (56). Similarly, note that

$$g_\pi = (1-\delta)(1-\rho)\left(\frac{h-q}{2} - \beta\left(\frac{(h-l)^2}{12} - \Sigma\right)\right)\gamma_\pi < 0 \tag{82}$$

since $\gamma_\pi < 0$ (see (46)) and $\frac{h-q}{2} - \beta\left(\frac{(h-l)^2}{12} - \Sigma\right) > 0$, and that

$$g_\beta = -(1+2\delta - (1-\delta)(1-\rho)\gamma)(\Sigma + \beta\Sigma_\beta) - (1-\delta)(1-\rho)\gamma\frac{(h-l)^2}{12}, \text{and} \tag{83}$$

$$\Sigma + \beta\Sigma_\beta = \frac{1}{12}\left(\frac{\left(q_N^{Leak} - l\right)^2}{N^2} - c^2(N^2-1)\right) \geq \frac{1}{12}\left(c^2(N+1)^2 - c^2(N^2-1)\right) > 0, \tag{84}$$

for $N < \bar{N}$ from Lemma 3, and so $g_\beta < 0$ for $N < \bar{N}$. □

Proof of Proposition 4. As seen in the proof of Proposition 3, for all $N \leq \bar{N}$, we have

$$l + cN(N-1) \leq g(l + cN(N-1)), \tag{85}$$

and for $\bar{N}$ we also have

$$l + c\bar{N}(\bar{N}+1) > g(l + c\bar{N}(\bar{N}+1)). \tag{86}$$

As π becomes larger, the left-hand side does not change for either inequality, while the right-hand side decreases since $g_\pi < 0$. For (86), this implies that the inequality holds for higher values of π, and so $\bar{N}$ cannot increase with an increase in π. However, (85) may be violated if we increase π sufficiently and $\bar{N}$ is reduced by one. This implies that $\bar{N}$ is non-increasing in π. Since $c_\rho > 0$ and $g_\rho < 0$, similar arguments across (85) and (86) imply that $\bar{N}$ is non-increasing in ρ. □

Proof of Proposition 5. Recall from the proof of Proposition 2 that q_N^{Leak} is the solution to $q = g(q)$, where $g(q)$ is defined by

$$g(q) = \frac{q+l}{2} - \beta\Sigma(q) + \delta\left(\frac{q-l}{2} - 2\beta\Sigma(q)\right) + (1-\delta)(1-\rho)\gamma\left(\frac{h-q}{2} - \beta\left(\frac{(h-l)^2}{12} - \Sigma(q)\right)\right), \tag{87}$$

$$\Sigma(q) = \frac{c^2\left(N^2-1\right)}{12} + \frac{(q-l)^2}{12N^2}, \quad c = \frac{3(1-\delta)\rho}{\beta(3\delta + (1-\delta)\rho)}, \quad \gamma = \frac{1-\pi}{1-\pi+\pi(1-\rho)\frac{q-l}{h-l}} \tag{88}$$

and

$$cN(N-1)\left(1+\frac{\delta}{2}\right)\left(1-\pi+\frac{\pi(1-\rho)}{h-l}cN(N-1)\right) \leq (1-\delta)(1-\rho)(1-\pi)\left(\frac{h-l}{2} - \beta\frac{(h-l)^2}{12}\right). \tag{89}$$

When $\rho = 1$, the above condition reduces to:

$$cN(N-1)\left(1+\frac{\delta}{2}\right)(1-\pi) \leq 0 \quad \Rightarrow N = 1, \tag{90}$$

i.e., when $\rho = 1$, $N \leq \bar{N} = 1$. Moreover, this implies that when $\rho = 1$,

$$\Sigma(q) = \tfrac{(q-l)^2}{12}, \qquad c = \frac{3(1-\delta)}{\beta(3\delta + (1-\delta))} \tag{91}$$

$$\Rightarrow g(q) - q = -(q-l) + \tfrac{q-l}{2}(1+\delta) - \beta\tfrac{(q-l)^2}{12}(1+2\delta) \tag{92}$$

$$= (q-l)\left\{-1 + \tfrac{1+\delta}{2} - \tfrac{\beta(1+2\delta)}{12}(q-l)\right\} \tag{93}$$

$$\Rightarrow q = \left\{l,\ l - \frac{6(1-\delta)}{\beta(1+2\delta)}\right\} \tag{94}$$

but since $q \geq l$, the only possible solution is $q = l$. This implies that real efficiency is maximized at $\rho = 1$ since there is full public disclosure. $\square$

Proof of Proposition 6. The proof follows the structure of the proof of Proposition 2. In the case there is disclosure, both insiders and outsiders observe θ. In this case,

$$P_D = \theta, \quad U_D = (1-\delta)\theta + \delta\left(\theta + b_0 - \beta b^2\right) = \theta + \delta\left(b_0 - \beta b^2\right) \tag{95}$$

Next, note that

$$\tilde{V} = \begin{cases} \frac{h+l}{2} - \frac{\beta}{12}(h-l)^2 + b_0 - \beta b^2 & \text{when } \xi = 0 \\ \theta + b_0 - \beta\left(\frac{a_{n+1}+a_n}{2} - \theta - b\right)^2 & \text{when } \xi = 1 \end{cases} \tag{96}$$

and conditional on a message $m_n = \{\theta \in [a_n, a_{n+1}]\}$,

$$P = \begin{cases} \frac{h+l}{2} - \frac{\beta}{12}(h-l)^2 & \text{when } \xi = 0 \\ \frac{a_{n+1}+a_n}{2} - \frac{\beta}{12}(a_{n+1}-a_n)^2 & \text{when } \xi = 1,\ k = 1\ (\textit{leak}) \\ \frac{q+l}{2} - \beta\Sigma(q) & \text{when } \xi = 1,\ k = 0\ (\textit{no leak}) \end{cases} \tag{97}$$

The expected price given no disclosure is:

$$P_{ND}(m_n) = \begin{array}{c} \rho\left(\frac{a_{n+1}+a_n}{2} - \frac{\beta}{12}(a_{n+1}-a_n)^2\right) \\ +(1-\rho)\left(\gamma\left(\frac{h+l}{2} - \frac{\beta}{12}(h-l)^2\right) + (1-\gamma)\left(\frac{q+l}{2} - \beta\Sigma(q)\right)\right) \end{array} \tag{98}$$

where as before, $\gamma = \frac{1-\pi}{1-\pi+\pi(1-\rho)\frac{q-l}{h-l}}$. Conditional on no disclosure, the manager's payoff is given by:

$$U_{ND}(\theta, m_n) = \delta\left(\theta + b_0 - \beta\left(\frac{a_{n+1}+a_n}{2} - \theta - b\right)^2\right) + (1-\delta)P_{ND}(m) \tag{99}$$

The indifference condition to pin down q_N^{Bias} is given by:

$$U_D(q) = U_{ND}(q, [a_{N-1}, q]) \tag{100}$$

$$\Leftrightarrow q = \delta\left(q - \beta\left(\left(\tfrac{q-a_{N-1}}{2}\right)^2 + 2b\left(\tfrac{q-a_{N-1}}{2}\right)\right)\right) + (1-\delta)P_{ND}([a_{N-1}, q]) \tag{101}$$

The indifference conditions to pin down a_i is given by

$$U_{ND}\left(a_n,\left[a_n,a_{n+1}\right]\right)-U_{ND}\left(a_n,\left[a_{n-1},a_n\right]\right)=0 \tag{102}$$

$$\Rightarrow \begin{array}{c} -\delta\beta\left(\frac{a_{n+1}-a_n}{2}-b\right)^2+(1-\delta)\,\rho\left(\frac{a_{n+1}+a_n}{2}-\frac{\beta}{12}\left(a_{n+1}-a_n\right)^2\right) \\ -\left\{-\delta\beta\left(\frac{a_{n-1}-a_n}{2}-b\right)^2+(1-\delta)\,\rho\left(\frac{a_{n-1}+a_n}{2}-\frac{\beta}{12}\left(a_{n-1}-a_n\right)^2\right)\right\} \end{array}=0 \tag{103}$$

Solving for a_{n+1}, we get

$$a_{n+1}=\left\{a_n,\ 2a_n-a_{n-1}+\frac{6\left((1-\delta)\,\rho+2\beta\delta b\right)}{\beta\left(3\delta+(1-\delta)\,\rho\right)}\right\}. \tag{104}$$

Letting

$$c=\frac{3\left((1-\delta)\,\rho+2\beta\delta b\right)}{\beta\left(3\delta+(1-\delta)\,\rho\right)}, \tag{105}$$

we have a solution of the type:

$$a_n=l+A_0n+cn\,(n-1)\,. \tag{106}$$

Since $a_N=q$, we have: $q-l=A_0N+cN\,(N-1)>cN\,(N-1)$, or equivalently, $N<\frac{1}{2}\left(\sqrt{1+\frac{4(q-l)}{c}}+1\right)$. The expressions for a_{N-1} and Σ correspond to the no bias case with the modified c. Note however, that

$$\lim_{\rho\to 0}c=\frac{3\,(2\beta\delta b)}{\beta\,(3\delta)}=2b\neq 0 \tag{107}$$

and so even when there are no leaks, $N<\frac{1}{2}\left(\sqrt{1+\frac{2(q-l)}{b}}+1\right)$ i.e., internal communication is coarse. Let

$$\tilde{g}\,(q)\equiv\delta\left(q-\beta\left(\left(\frac{q-a_{N-1}}{2}\right)^2+2b\left(\frac{q-a_{N-1}}{2}\right)\right)\right)+(1-\delta)\,P_{ND}\left(\left[a_{N-1},q\right]\right) \tag{108}$$

$$=g\,(q)-\beta b\delta\left(\frac{cN\,(N-1)+q-l}{N}\right) \tag{109}$$

where $g\,(q)$ is given by (57), and note that q_N^{Bias} is the fixed point to $q=\tilde{g}\,(q)$. To show the existence of a solution to $q=\tilde{g}\,(q)$, where $h>q>cN\,(N-1)+l$, it is sufficient to establish conditions for $\tilde{g}\,(h)<h$ and $\tilde{g}\,(l+cN\,(N-1))>l+cN\,(N-1)$. When $q=h$, note that $\tilde{g}\,(h)\leq g\,(h)\leq h$. As before $\beta c\leq 3$, and so analogous calculations to those in the proof of Proposition 2, we have:

$$\tilde{g}\,(l+cN\,(N-1))\geq l+(1-\delta)\,(1-\rho)\,\underline{\gamma}\left(\frac{h-l}{2}-\beta\frac{(h-l)^2}{12}\right)-\frac{cN(N-1)}{2}\delta-\beta b\delta\left(\frac{2cN(N-1)}{N}\right), \tag{110}$$

where $\underline{\gamma} = \frac{1-\pi}{1-\pi+\pi(1-\rho)\frac{cN(N-1)}{h-l}}$. To ensure $\tilde{g}\left(l + cN\left(N-1\right)\right) \geq l + cN\left(N-1\right)$, we must have:

$$cN\left(N-1\right)\left(1+\tfrac{\delta}{2}+\tfrac{2\beta b\delta}{N}\right)\left(1-\pi+\pi\tfrac{(1-\rho)cN(N-1)}{h-l}\right) \leq \left(1-\delta\right)\left(1-\rho\right)\left(1-\pi\right)\left(\tfrac{h-l}{2}-\beta\tfrac{(h-l)^2}{12}\right). \tag{111}$$

Note that this coincides with the corresponding condition in Proposition 2 when $b = 0$. Since the LHS is increasing in N, this corresponds to ensuring that $N \leq \bar{N}$ for some bound. Moreover, the condition always holds for $N = 1$, which implies $\bar{N} \geq 1$. Finally, uniqueness of q_N^{Bias} requires $\frac{\partial \tilde{g}}{\partial q} < 1$, which holds since (72) holds and

$$\frac{\partial \tilde{g}}{\partial q} = \frac{\partial g}{\partial q} - \frac{\beta b\delta}{N} \tag{112}$$

for $b > 0$. □

Proof of Proposition 7. From the proof of Proposition 6, we know that q_N^{Bias} is the fixed point to $q = \tilde{g}\left(q\right)$, where

$$\tilde{g}\left(q\right) = g\left(q\right) - \beta b\delta\left(\frac{cN\left(N-1\right)+q-l}{N}\right) \tag{113}$$

and $g\left(q\right)$, which is given by (57), does not depend on b. This implies that for a fixed N,

$$\tfrac{\partial}{\partial b}q_N^{Bias} = \frac{\frac{\partial \tilde{g}}{\partial b}}{1-\frac{\partial \tilde{g}}{\partial q}} \leq 0 \tag{114}$$

since $\frac{\partial \tilde{g}}{\partial q} < 1$ and $\frac{\partial \tilde{g}}{\partial b} \leq 0$. Next, note that the upper bound $\bar{N}$ is characterized by a strict equality in condition (111). Let

$$L \equiv c\bar{N}\left(\bar{N}-1\right)\left(1+\tfrac{\delta}{2}+\tfrac{2\beta b\delta}{\bar{N}}\right)\left(1-\pi+\pi\frac{(1-\rho)c\bar{N}\left(\bar{N}-1\right)}{h-l}\right) \tag{115}$$

and note that $\frac{\partial L}{\partial b} + \frac{\partial L}{\partial \bar{N}}\frac{\partial \bar{N}}{\partial b} = 0$, or equivalently,

$$\tfrac{\partial \bar{N}}{\partial b} = -\frac{\frac{\partial L}{\partial b}}{\frac{\partial L}{\partial \bar{N}}}. \tag{116}$$

Next, note that

$$\frac{\partial L}{\partial b} = \begin{array}{c} 2\beta\delta(\bar{N}-1)c\left(\frac{\pi(\bar{N}-1)\bar{N}(1-\rho)c}{h-l}-\pi+1\right) \\ +c_b\left(\begin{array}{c}\frac{\pi(\bar{N}-1)^2\bar{N}^2(1-\rho)c\left(\frac{2b\beta\delta}{\bar{N}}+\frac{\delta}{2}+1\right)}{h-l} \\ +(\bar{N}-1)\bar{N}\left(\frac{2b\beta\delta}{\bar{N}}+\frac{\delta}{2}+1\right)\left(\frac{\pi(\bar{N}-1)\bar{N}(1-\rho)c}{h-l}-\pi+1\right)\end{array}\right)\end{array} \tag{117}$$

and $c_b > 0$. Moreover,

$$\frac{\partial L}{\partial \bar{N}} = \begin{array}{c} c\left(2\bar{N}-1\right)\left(1+\frac{\delta}{2}+\frac{2\beta b\delta}{\bar{N}}\right)\left(1-\pi+\pi\frac{(1-\rho)c\bar{N}\left(\bar{N}-1\right)}{h-l}\right) \\ +\frac{\pi(1-\rho)}{h-l}c\bar{N}\left(\bar{N}-1\right)\left(1+\frac{\delta}{2}+\frac{2\beta b\delta}{\bar{N}}\right)c\left(2\bar{N}-1\right) \\ -\frac{2\beta b\delta}{\bar{N}^2}c\bar{N}\left(\bar{N}-1\right)\left(1-\pi+\pi\frac{(1-\rho)c\bar{N}\left(\bar{N}-1\right)}{h-l}\right) \end{array} \tag{118}$$

For $\bar{N}=1$, $\frac{\partial \bar{N}}{\partial b}=\frac{\partial L}{\partial b}=0$. For $\bar{N}>1$, we have $\frac{\partial L}{\partial b}>0$ and

$$\begin{aligned} \frac{\partial L}{\partial \bar{N}} &= \begin{array}{c} c\left(2\bar{N}-1\right)\left(1+\frac{\delta}{2}+\frac{2\beta b\delta}{\bar{N}}\right)\left(1-\pi+\pi\frac{(1-\rho)c\bar{N}\left(\bar{N}-1\right)}{h-l}\right) \\ -\frac{2\beta b\delta}{\bar{N}}c\left(\bar{N}-1\right)\left(1-\pi+\pi\frac{(1-\rho)c\bar{N}\left(\bar{N}-1\right)}{h-l}\right) \\ +\frac{\pi(1-\rho)}{h-l}c\bar{N}\left(\bar{N}-1\right)\left(1+\frac{\delta}{2}+\frac{2\beta b\delta}{\bar{N}}\right)c\left(2\bar{N}-1\right) \end{array} && (119) \\ &\geq \left[\left(2\bar{N}-1\right)\left(1+\frac{\delta}{2}\right)+\frac{2\beta b\delta\left(2\bar{N}-1\right)}{\bar{N}}-\frac{2\beta b\delta\left(\bar{N}-1\right)}{\bar{N}}\right]c\left(1-\pi+\pi\frac{(1-\rho)c\bar{N}\left(\bar{N}-1\right)}{h-l}\right) && (120) \\ &\geq 0 && (121) \end{aligned}$$

which implies $\frac{\partial \bar{N}}{\partial b}<0$. □

Proof of Proposition 8. First, note that when $\rho=1$, the condition that determines $\bar{N}$ (i.e., expression (111)) reduces to:

$$cN\left(N-1\right)\left(1+\frac{\delta}{2}+\frac{2\beta b\delta}{N}\right)\left(1-\pi\right)\leq 0 \Rightarrow \bar{N}=1. \tag{122}$$

$$cN\left(N-1\right)\left(1+\frac{\delta}{2}\right)\left(1-\pi\right)\leq 0 \quad \Rightarrow N=1, \tag{123}$$

Moreover, this implies that when $\rho=1$, $\Sigma\left(q\right)=\frac{(q-l)^2}{12}$,

$$\begin{aligned} \Rightarrow \tilde{g}\left(q\right)-q &= \frac{q+l}{2}-q-\beta\Sigma\left(q\right)+\delta\left(\frac{q-l}{2}-2\beta\Sigma\left(q\right)\right)-\beta b\delta\left(q-l\right) && (124) \\ &= -\left(q-l\right)\left\{\frac{1}{2}-\delta+\beta\delta b+\frac{\beta}{12}\left(1+2\delta\right)\left(q-l\right)\right\} && (125) \\ \Rightarrow q &= \left\{l,\ l-\frac{12b\beta\delta+6\left(1-\delta\right)}{\beta(2\delta+1)}\right\} && (126) \end{aligned}$$

but since $q\geq l$, the only possible solution is $q_N^{Bias}=l$, which implies $RE\left(\rho=1\right)=0$.
Fix a $b>0$, and q_N^0 denote the cutoff when there is no leakage. Since $c=2b$ when $\rho=0$, we have

$$\Sigma\left(\rho=0,N\right)=\frac{\left(2b\right)^2\left(N^2-1\right)}{12}+\frac{\left(q_N^0-l\right)^2}{12N^2}. \tag{127}$$

In contrast, when there is leakage with probability ρ but $N=1$, we have

$$\Sigma\left(\rho,N=1\right)=\frac{\left(q_1^{\rho}-l\right)^2}{12}. \tag{128}$$

Efficiency is higher with leakage when

$$\frac{q_N^0 - l}{h-l}\Sigma\left(\rho = 0, N\right) > \frac{q_1^{\rho} - l}{h-l}\Sigma\left(\rho, N = 1\right), \tag{129}$$

or equivalently,

$$\left(q_N^0 - l\right)\left(\frac{(2b)^2\left(N^2-1\right)}{12} + \frac{\left(q_N^0-l\right)^2}{12N^2}\right) > \frac{\left(q_1^{\rho}-l\right)^2}{12}. \tag{130}$$

Note that for a fixed $b > 0$, the LHS is strictly positive and bounded away from 0, since $q_N^0 > l$ and $N \leq \bar{N}$. On the other hand, one can show that q_1 is always decreasing in ρ and $\lim_{\rho\to 1} q = l$. This implies there exists a $\bar{\rho} > 0$ such that for all $\rho > \bar{\rho}$, the above condition holds. □

B Type dependent probability of leakage

In this appendix, we consider an extension of the model in which the probability of leakage ρ depends on the type of the firm θ. Specifically, suppose that the probability ρ is given by a decreasing function of θ, i.e., $\rho = \rho(\theta)$. We assume that there is some leakage i.e., $\rho(\theta) > 0$ for some $\theta \in [l, h]$ and that $\rho(\theta)$ is decreasing in θ. The rest of the setup is as described in Section 2. As before, we consider equilibria characterized by a sequence of cutoffs $l = a_0 < ... < a_N = q_N^{Leak} \leq h$, where an informed manager (i) discloses θ to shareholders if and only if $\theta \geq q_N^{Leak}$, (ii) perfectly reveals θ to her employees if and only if $\theta \geq q_N^{Leak}$, and (iii) reports only the partition $\theta \in [a_n, a_{n+1}]$ to her employees when $\theta < q_N^{Leak}$.

Conditional on disclosure, the manager's utility is given by $U_D(\theta) = \theta$. Conditional on no disclosure,

- If the manager is not informed, the value of the firm is $V_{NI} = P_{NI} = \frac{h+l}{2} - \frac{\beta}{12}(h-l)^2$.
- If the manager is informed, and only sends a message $m_n = \{\theta \in [a_n, a_{n+1}]\}$ to her employees, the value of the firm is

$$V_I(\theta, m_n) = \theta - \beta\left(\tfrac{a_{n+1}+a_n}{2} - \theta\right)^2. \tag{131}$$

Moreover, if the message is leaked to shareholders, the price is given by

$$P_{I,L}(m_n) = \tfrac{a_{n+1}+a_n}{2} - \tfrac{\beta}{12}\left(a_{n+1} - a_n\right)^2, \tag{132}$$

but if it is not leaked to shareholders, the price is given by

$$P_{I,NL}(m_n) = \tfrac{q_N+l}{2} - \tfrac{\beta}{12}\sum_i \tfrac{(a_{n+1}-a_n)^3}{q_N-l}. \tag{133}$$

As a result, conditional on no disclosure, the expected price is given by:

$$P_{ND}(\theta, m_n) = \rho(\theta) P_{I,L}(m_n) + (1-\rho(\theta))\left(\gamma P_{NI} + (1-\gamma) P_{I,NL}\right), \text{ where} \tag{134}$$

$$\gamma \equiv \frac{1-\pi}{1-\pi+\pi\left(1-\rho_{ND}\right)\Pr\left(\theta < q_N\right)}, \text{ and} \tag{135}$$

$$\rho_{ND} = \frac{1}{q_N - l}\int_l^{q_N} \rho(x)\,dx, \tag{136}$$

and the expected utility to the manager from not disclosing is given by

$$U_{ND}(\theta, m_n) = \delta V_I(\theta, m_n) + (1-\delta) P_{ND}(\theta, m_n). \tag{137}$$

The disclosure cutoff is pinned down by indifference for $\theta = q$ i.e.,

$$U_D(q) = U_{ND}(q, \{\theta \in [a_{N-1}, q]\}), \tag{138}$$

while the cheap-talk partitions are characterized by indifference at the cutoffs $\theta = a_n$ i.e.,

$$U_{ND}(a_n, \{\theta \in [a_{n-1}, a_n]\}) = U_{ND}(a_n, \{\theta \in [a_n, a_{n+1}]\}). \tag{139}$$

The latter condition implies that

$$a_{n+1} = \left\{ a_n,\ 2a_n - a_{n-1} + \frac{6(1-\delta)\rho(a_n)}{\beta(3\delta + (1-\delta)\rho(a_n))} \right\}. \tag{140}$$

In general, the solution to the above difference equation, and the resulting equilibrium, depends on the specification for $\rho(\theta)$. However, for $N = 1$, the conditions simplify. Specifically, note that in this case, $a_{N-1} = a_0 = l$, so that an equilibrium exists if there is a solution q_N^{Leak} to $q = g(q)$, where

$$g(q) \equiv \begin{array}{c} \delta\left(q - \frac{\beta}{4}(q-l)^2\right) \\ +(1-\delta)\left[\begin{array}{c} \rho(q)\left\{\frac{q+l}{2} - \frac{\beta}{12}(q-l)^2\right\} \\ +(1-\rho(q))\left(\gamma\left(\frac{h+l}{2} - \frac{\beta}{12}(h-l)^2\right) + (1-\gamma)\left(\frac{q+l}{2} - \frac{\beta}{12}(q-l)^2\right)\right) \end{array}\right] \end{array}. \tag{141}$$

Note that

$$g(l) = \begin{array}{c} \delta l \\ +(1-\delta)\left[\begin{array}{c} \rho(l)\, l \\ +(1-\rho(l))\left(\gamma\left(\frac{h+l}{2} - \frac{\beta}{12}(h-l)^2\right) + (1-\gamma) l\right) \end{array}\right] \end{array} \tag{142}$$

but since $\frac{h+l}{2} - \frac{\beta}{12}(h-l)^2 > l$, we have $g(l) \geq l$. Next, since

$$h - \tfrac{\beta}{4}(h-l)^2 < h, \text{ and } \tfrac{h+l}{2} - \tfrac{\beta}{12}(h-l)^2 \leq h, \tag{143}$$

we have $g(h) < h$. This implies there always exists an equilibrium when $N = 1$.

We numerically explore equilibria for $N > 1$ for a particular specification for $\rho(\theta)$. Specifically, suppose

$$\rho(\theta) = \bar{\rho}\left(\frac{h-\theta}{h-l}\right) \in [0, \bar{\rho}], \tag{144}$$

which implies $\rho_{ND} = \bar{\rho}\frac{h - \frac{q+l}{2}}{h-l}$. Figure 5 plots the equilibrium disclosure threshold for this specification as a function of the maximum probability of a leak $\bar{\rho}$. A comparison of this plot to the one in Figure 1 suggests that the equilibrium threshold responds similarly to the probability of leakage as before: the likelihood of disclosure is higher as the probability of leakage increases, and more informative internal communication is more difficult to sustain in this case. While the numerical results are sensitive to the specification of $\rho(\theta)$, we expect the qualitative implications of our analysis to be similar in this more general case.

Figure 5: Cutoff q as a function of $\bar{\rho}$ when $\rho(\theta) = \bar{\rho}\left(\frac{h-\theta}{h-l}\right)$

The figure plots the disclosure threshold q, when (i) there are no leaks, and (ii) when there are leaks for different values of N. Benchmark parameters are $h = 2$, $l = 0$, $\delta = 0.75$, $\beta = 1$ and $\pi = 0.5$.