The Man(ager) Who Knew Too Much

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Abstract

The "curse of knowledge" is a pervasive cognitive bias that causes better-informed individuals to overweight their private knowledge when forecasting the beliefs of others. We study how this bias affects communication choices and investment decisions within a firm. A principal chooses the optimal level of investment in a risky project, conditional on the information she receives from a better informed, but "cursed," manager. The curse of knowledge leads the manager to overestimate the informativeness of his communication. This misperception amplifies the information loss from strategic communication when the manager and principal's incentives are misaligned. However, this same distortion in the manager's perception leads him to over-invest in acquiring private information. We characterize the overall impact on firm value and on the choice of whether to delegate the investment decision to the manager.

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"The single biggest problem in communication is the illusion that it has taken place." — George Bernard Shaw

1 Introduction

Experts are often poor communicators. Successful entrepreneurs fail to convince early-stage investors that their latest product is the "next big thing." Wonkish political candidates rarely keep the attention of voters when discussing important policy issues. Brilliant researchers are often unable to explain their cutting-edge work at seminars and conferences. And many professors fail to effectively teach students in their introductory classes, despite being thought leaders in their fields.[1]

We argue that these difficulties are a natural outcome when better-informed individuals attempt to communicate with others. There is overwhelming evidence that better-informed individuals exhibit the "curse of knowledge," i.e., they fail to ignore their private information when predicting the beliefs of others and, as a result, overestimate the extent to which others' beliefs are aligned with their own.[2] Though this bias in perception can arise in any strategic setting with asymmetric information, we focus on how it affects decisions within a firm. While managers and employees "on the ground" are likely to be better informed about product demand, operational constraints, and new technologies, "top level" managers are usually responsible for deciding in which projects to invest and how much capital should be invested. As such, the production and communication of dispersed information within the organization is a critical determinant of a firm's performance.

We build on a standard model of intra-firm communication and investment (e.g., Dessein, 2002). The principal chooses how much to invest in a risky project after communicating with her manager, who is privately informed about the project's productivity but is also biased towards higher investment. Importantly, the manager is subject to the curse of knowledge: his perception of the principal's beliefs about the project is tilted towards his own expectation. When the manager is exogenously endowed with private information, we show that the curse of knowledge decreases the effectiveness of communication and, therefore, leads to less informed investment decisions and lower firm value.

[1] In his first semester teaching at the University of Bern, Albert Einstein was able to enroll only three students in his thermodynamics course; in his second semester, his class was cancelled because only one student signed up (see Grant (2018)). Such anecdotes about common misperceptions (and difficulties) in communication are supported by a number of papers in psychology, some of which we describe below.

[2] The curse of knowledge, coined by Camerer, Loewenstein, and Weber (1989), is closely related to the notion of "hindsight bias," introduced by Fischhoff (1975), which reflects the inability to correctly remember one's own prior expectations after observing new information. Our focus is on communication, and therefore, on how private information biases one's beliefs about others expectations i.e., the curse of knowledge.

However, with endogenous information acquisition, we show that the "cursed" manager may choose to acquire more information than an unbiased one.When equilibrium communication is sufficiently informative, we show that the latter channel can cause firm value to increase with the manager's curse of knowledge. Finally, we characterize conditions under which the principal prefers to delegate to a cursed manager while retaining control with an unbiased one. Thus, understanding the source of the manager's inefficient communication (misaligned incentives versus a bias in beliefs) is essential to ensure the efficient allocation of control rights within the firm.

Section 3 introduces the model. The principal faces uncertainty about the productivity of a risky project and must choose the optimal level of (costly) investment. The manager observes a noisy, private signal about the return on the project and can send a message to the principal before she makes the investment decision. Importantly, the manager exhibits the curse of knowledge: he believes that the principal's conditional expectation about firm productivity, given his message, is closer to his conditional expectation than it actually is. Utility for both agents increases with firm value. However, the manager also derives a non-pecuniary, private benefit which increases with the size of the investment. This private benefit introduces a wedge between the principal's and the manager's preferred level of investment.

In Section 4, we consider the case in which the manager's message is not verifiable, i.e., he can only engage in "cheap talk." As in Crawford and Sobel (1982), when the manager's private benefit from investment is sufficiently small, there are informative, partition equilibria in which the same message is sent for all signals in a given interval. When the wedge between their incentives is large, however, only a babbling (uninformative) equilibrium exists. We show that the curse of knowledge reduces the manager's ability to communicate along two dimensions. First, an increase in the degree of the curse of knowledge decreases both the maximal number of partitions and therefore lowers the likelihood that any informative equilibrium exists.[3] Second, holding fixed the number of partitions, less information is conveyed, in expectation. As a result, holding fixed the precision of the manager's information, the curse of knowledge reduces the expected value of the firm.

Section 5 considers a strategic disclosure setting (e.g., Dye, 1985) in which the manager does not observe any information with positive probability. While the manager can verifiably disclose his signal (if observed), he cannot verifiably disclose that he is uninformed. When the manager's bias towards over-investment is sufficiently large, he chooses to withhold information when it is sufficiently negative, i.e., the equilibrium features one-sided,

[3]Specifically, holding fixed the private benefit from investment, a sufficiently large increase in the curse of knowledge implies only the babbling equilibrium exists.

"disclosure on top." When the private benefit is sufficiently small, however, there exists a more informative equilibrium that features "disclosure at extremes," in which the manager discloses information that is sufficiently good or sufficiently bad, but withholds information at intermediate levels. In this case, the relatively small non-pecuniary benefit enjoyed by the manager is outweighed by the loss in firm value due to over-investment for sufficiently low signals which leads to the manager to disclose these realizations.[4] As with cheap talk, when the manager's information precision is fixed, we show that the curse of knowledge decreases firm value because it makes communication less informative. Specifically, an increase in the degree of the curse of knowledge (i) increases the non-disclosure region in the more informative equilibrium and (ii) for a fixed bias, decreases the likelihood that the more informative equilibrium can be sustained.

The intuition for both cases is similar. The manager's communication strategy trades off his desire to inflate investment (due to the private benefit he receives) against the incentive to credibly convey information (to maximize firm value). We show that the curse of knowledge causes the manager to overestimate the informativeness of his communication and, as a result, he under-estimates the cost of distorting his message. This tilts the perceived tradeoff faced by the manager and pushes him to further distort his message in an effort to increase investment. In fact, we show that an increase in the degree of the manager's curse of knowledge has the same effect on communication as increasing his private benefit, and consequently, the curse of knowledge increases his "effective bias." All else equal, this leads to less informative communication.

When the manager optimally chooses the precision of his private signal, however, the expected value of the firm can increase with the curse of knowledge. Because the manager perceives his communication to be more informative than it actually is, his perceived marginal utility of acquired information precision increases with the curse of knowledge. The net effect on firm value depends, on both the precision chosen by the manager, and how much of this private information is lost through communication with the principal. Firm value can increase when the first effect dominates the second. This is more likely to be true when incentives are better aligned. i.e., the manager's private benefit from investment is not too large, and so equilibrium communication is sufficiently informative.

Section 6 considers a setting in which the manager can commit to sending the principal a (noisy) signal, conditional on her private information. In addition to choosing the precision of his acquired information, the manager can also increase the precision of his message by

[4] If the support of the distribution of project productivity were unbounded, the private benefit would always be "small enough" and so only the two-sided equilibrium would arise (see, e.g., Che and Kartik (2009)).

exerting costly effort. Though the manager commits, ex-ante, to the signal he sends the principal, we show that the effects of the curse of knowledge on communication and information acquisition are qualitatively similar in this setting. In particular, the marginal utility of message precision decreases with the curse of knowledge, while the marginal utility of information acquisition increases with it. However, unlike the settings without commitment, the manager's incentive to communicate is unaffected by his private benefit from investment, highlighting an important distinction between the two frictions.

Moreover, the optimal choice of precisions clarifies a channel that underlies all three settings: the complementarity between communication and information acquisition. Specifically, the marginal value of acquiring more precise information increases in the precision of the message, and vice versa. Generally, the relative effect of this complementarity is small, and we find that an increase in the curse of knowledge increases information acquisition but decreases message precision. However, when the complementarity channel dominates, an increase in the curse of knowledge can increase both precisions and improve firm value. In these cases, the cursed manager is both an expert and an effective communicator.

Given the difficulties in communication generated by the curse of knowledge, we analyze whether the principal would prefer to delegate the investment decision to the manager in Section 7. If the principal delegates, she allows the manager to utilize his more precise information to make the investment decision. The tradeoff, however, is that the principal knows that the manager will invest more than she would consider optimal. With both cheap talk and costly communication, the principal follows a threshold strategy: she delegates if the bias is sufficiently small, and otherwise retains control. Interestingly, in these cases, the threshold does not depend on the curse of knowledge. The principal only wants to retain control when the manager's communication is sufficiently uninformative.[5]

With verifiable disclosure, the principal's decision to delegate depends upon whether the equilibrium features one-sided or two-sided disclosure. In the former case, the manager's disclosure threshold does not depend on the curse of knowledge, and so neither does the decision to delegate: if the private benefit from investment is sufficiently small, the principal delegates. With the two-sided disclosure, however, the decision to delegate depends upon the effective bias which, in turn, depends upon the curse of knowledge. In this setting, the manager's message is distorted by his misperception of the principal's beliefs while his preferred investment level is not. As a result, when the curse of knowledge is the primary driver of the manager's effective bias, the principal prefers to delegate.

The curse of knowledge is a robust and pervasive phenomenon. As discussed in the

[5]This mirrors the result from Dessein (2002), who considers the decision of delegation versus communication in the absence of the curse of knowledge.

surveys by Hawkins and Hastie (1990), Blank, Musch, and Pohl (2007) and Ghrear, Birch, and Bernstein (2016), and the papers detailed within, the bias has been documented in many information structures, across different cultures, and in a variety of settings. Particular to our theoretical setting, there is ample evidence that a range of communication methods can give rise to the curse of knowledge. While the original research which identified this bias focused on written communication (Fischhoff, 1975), there is substantial evidence that individuals exhibit the curse of knowledge with respect to oral communication (Keysar, 1994), graphical messages (Xiong, van Weelden, and Franconeri, 2019) and visual illustrations (Bernstein, Atance, Loftus, and Meltzoff, 2004). There is evidence that the curse of knowledge affects individuals at any age (e.g., Birch and Bloom (2007)) and, importantly, even experts, warned of the effect of the curse of knowledge, are not immune. Finally, there is substantial evidence that traditional methods of debiasing have limited, if any impact: a series of papers (e.g., Pohl and Hell (1996), Kennedy (1995)) show that even individuals with prior experience, who receive feedback on their performance and are accountable for their actions, and who are provided with direct warnings about the bias still exhibit the curse of knowledge.[6]

This large body of evidence suggests the curse of knowledge has important consequences for decisions within firms. Our stylized model provides a first step in better understanding them. For instance, our analysis suggests that the negative effects of the curse of knowledge on communication, and consequently firm value, are most severe when the manager is simply endowed with information. These negative effects are more likely to arise in situations where the manager simply aggregates and reports existing information instead of exerting effort to produce new information (e.g., in accounting, risk management, or auditing departments). In these situations, the value of the firm may be improved by better aligning incentives, and establishing formal, internal communication systems that enhance the manager's ability to commit to an informative communication strategy.

However, when the manager exerts costly effort in generating the relevant information (e.g., market research, product development, or R&D), our results imply that an intermediate level of the curse of knowledge can actually be value-enhancing, especially in organizations with formal communication systems in place. Fostering a system that encourages experts to believe that they are better communicators can even increase firm value directly when this perception feeds back into their incentives to acquire better information and more expertise.

[6]For instance, Arkes, Wortmann, Saville, and Harkness (1981) show that physicians, given identical symptoms and several possible diagnoses, overestimate the ex-ante likelihood of the "correct" ailment even when instructed to ignore this information. In Anderson, Jennings, Lowe, and Reckers (1997), judges who are asked to evaluate the quality of an auditors' ex-ante decision are influenced by their ex-post knowledge of the outcome. These are two of many papers on expert decision making and debiasing surveyed in Harley (2007).

Finally, while our model focuses on the implications for decisions within a firm, the analysis applies more generally to other settings where communication by experts plays an important role. For instance, one possible application of the model is to teaching and research. Our analysis suggests that expert researchers tend to overestimate their ability to teach and, therefore, are less likely to communicate their knowledge effectively.[7] Moreover, consistent with informal intuition, our results suggest that teaching and research are complementary activities, and encouraging better teaching practices (i.e., encouraging communication with commitment) can enhance incentives to do research. Other settings in which we expect our results to apply include government officials consulting policy advisers, portfolio managers soliciting information from research analysts, or a consultant providing feedback to firm management.

The next section discusses the related literature, and Section 3 presents the model. Section 4 considers cheap talk communication, Section 5 considers verifiable disclosure and Section 6 studies costly communication with commitment. Section 7 characterizes the delegation decision of the principal and Section 8 concludes. Proofs and extensions can be found in Appendix A and B, respectively.

2 Related Literature

Camerer et al. (1989) is the first paper to explore the implications of the curse of knowledge in economic decision-making. Using an experimental design, the authors document that the bias is a robust feature of individual forecasts and is not eliminated by incentives or feedback. Based on their analysis, the authors conclude that the curse of knowledge can help "alleviate the inefficiencies that result from information asymmetries."[8] Our analysis leads to somewhat different conclusions. In our setting, the same distortion in beliefs exacerbates the effects of asymmetric information: because the manager perceives a smaller information asymmetry, his strategic communication becomes less informative in the presence of the curse of knowledge and investment decisions can be less informationally efficient. More recently, Biais and Weber (2009), Cheng and Hsiaw (2019) and Kocak (2018) explore how sequential updating and hindsight bias can lead individuals to form distorted beliefs. In both papers, however, the bias affects individuals recollections of their own priors, whereas in our setting,

[7] This does not imply that their communication is less effective in absolute terms but that, relatively speaking, more information is lost in translation.

[8] For example, because better informed agents do not exploit their informational advantage fully, the seller of a lemon (peach) sets the price lower (higher, respectively) than they otherwise would. As a result, the likelihood of market failure highlighted by Akerlof (1970) is alleviated by the curse of knowledge. Moreover, they conclude that better informed agents may suffer larger losses, and so "more information can actually hurt."

the bias leads individuals to misestimate the beliefs of others.

Our paper is most closely related to Madarász (2011). He considers a setting in which a biased *receiver* evaluates experts using ex post information. When the receiver exhibits the curse of knowledge (or "information projection"), she overestimates how much experts could have known ex ante, and underestimates their ability on average.[9] In an application to costly communication, the paper shows that a biased speaker speaks too rarely, is difficult to understand, and underestimates the ability of her audience when they do not understand her. In a related paper, Madarász (2015) shows that in a persuasion game with costly verification and a biased receiver, the equilibrium may feature credulity or disbelief.[10]

Our analysis complements these results. We focus on settings in which the sender exhibits the curse of knowledge, not the receiver. With exogenous information, we establish that biased experts are poor communicators not only with costly communication but also with cheap talk and verifiable disclosure. Moreover, because of the complementarity between communication and information acquisition, we show that the bias can lead to higher information acquisition and greater efficiency.

Our paper is also related to the broader literature on communication within a firm, and the resulting efficiency of investment choices. To our knowledge, we are the first paper to study the impact of the curse of knowledge on standard variants of communication studied in the literature: cheap talk (e.g., Crawford and Sobel (1982)) and and voluntary disclosure (e.g., Dye (1985)). We also complement the analysis in Dessein (2002), by characterizing how the curse of knowledge affects the tradeoff between delegation and communication. While much of this literature considers rational behavior on the part of both the principal and manager, there is a growing list of papers that introduces behavioral biases (e.g., see Malmendier (2018) for a recent survey).[11] Like us, Campbell, Gallmeyer, Johnson, Rutherford, and Stanley (2011) argue that some level of overconfidence can lead to value-maximizing policies. In the context of cheap talk models, Kawamura (2015) argues that overconfidence can lead to more information transmission and welfare improvement. Relatedly, Ashworth and Sasso (2019) show that the optimal mechanism delegates the decision to an overconfident, un-biased agent agent for moderate signal realizations but retains control otherwise.

Austen-Smith (1994) was the first to analyze costly information acquisition in the tra-

[9] In response, strategic experts overproduce information that is a substitute for the evaluators' ex post information and underproduce information that is a complement.

[10] In this case, the receiver believes that the sender knows her cost of verification and either overestimates the truthfulness of the message when it is cheaper for her to verify it (credulity) or underestimates it (disbelief).

[11] The managerial biases considered include overconfidence (e.g., Gervais, Heaton, and Odean (2011)) , reference-dependence (see Baker, Pan, and Wurgler (2012)), experience effects (see Malmendier, Tate, and Yan (2011)) and confirmation bias (see Martel and Schneemeier (2019)).

ditional "cheap talk" setting. Since then, several papers have analyzed how strategic communication influences information acquisition.[12] A closely related paper is Che and Kartik (2009), who study how differences of opinion between a decision maker and adviser affect communication and information acquisition. Similar to our analysis, they show that a difference of opinion reduces the informativeness of (verifiable) strategic communication, but increase the incentives for information acquisition. In their setting, increased investment in information acquisition is a result of (i) a motivation to persuade the decision maker, and (ii) an incentive to avoid rational prejudice.

Argenziano, Severinov, and Squintani (2016) also consider endogenous information acquisition in a cheap talk model and show that a biased expert may acquire more precise information than the decision marker, even when they have access to the same information technology. This (relative) over-investment in information acquisition is driven, in part, by the decision maker's threat to ignore messages when the expert deviates.[13] While the conclusions are similar, in our setting, the curse of knowledge leads the manager to increase his information acquisition because he overestimates the informativeness of his communication, and hence the value of acquiring more information. Moreover, we show that similar tradeoffs arise not only in settings with strategic, costless communication, but also when the manager can commit to costly communication before observing her information.

3 Model Setup

We begin with a description of the general model.

Payoffs and Technology. There are two dates $t \in \{0, 1\}$ and a single firm. The terminal value of the firm is given by $V \equiv V(R, k)$ where R measures the return on investment, or productivity, of the project available and k represents the scale of investment in the project.

[12] In Dur and Swank (2005), a principal chooses an adviser with opposing priors because this maximizes information acquisition: the adviser exerts more effort in an attempt to convince the principal to flip his beliefs. Di Pei (2015) shows that a biased expert may fully share his information when he can optimally design the signal he receives. Frug (2018) analyzes a dynamic setting in which the ability to reveal information over time can positively affect information acquisition and transmission.

[13] This is the key channel with *overt* information acquisition i.e., when the decision maker can observe the precision choice of the expert. In a related paper, Deimen and Szalay (2019) find similar results when the bias is endogenous and information acquisition is costless. Our benchmark analysis also focuses on overt acquisition in that the we assume the principal can observe the precision choice of the manager. In Appendix B, we consider how covert information acquisition (i.e., when the principal cannot observe precision choice, but infers it in equilibrium) affects cheap talk communication in our setting.

For analytical tractability, we assume

$$V(R,k) = Rk - \tfrac{1}{2}k^2, \tag{1}$$

$$R = \mu + \theta, \tag{2}$$

with μ is the expected productivity, while $\theta \sim U\left[-\frac{\sigma}{2}, \frac{\sigma}{2}\right]$ is the learnable shock to the return on investment.

Beliefs. The firm consists of a principal p (she) and a manager m (he). The manager observes a noisy, private signal, x, about productivity and can send a message d to the principal. Specifically, the manager observes a "truth or noise" signal: she observes θ with probability p and an independent shock η with probability $1-p$, i.e.,

$$x = \begin{cases} \theta & \text{with probability } p \\ \eta & \text{with probability } 1-p \end{cases}, \tag{3}$$

where $\eta \sim U\left[-\frac{\sigma}{2}, \frac{\sigma}{2}\right]$ and is independent of θ. The manager chooses the probability he successfully observes the true shock by exerting effort at a cost $c(p)$, where $c'(0) = c(0) = 0$, $c''(\cdot) > 0$ and $c'(p) \to \infty$ as $p \to 1$. We will refer to the probability p as the **precision** of the private signal x.

Let $\mathcal{I}_m$ and $\mathcal{I}_p$ denote the information sets of the manager and principal, respectively and note that $\mathcal{I}_m$ is finer than $\mathcal{I}_p$. We assume that both agents have common priors about the joint distribution of fundamentals and signals, and that these priors are consistent with the objective joint distribution. However, we assume that the manager exhibits the **curse of knowledge**. In particular, when (1) the manager forecasts the principal's conditional expectation of a random variable X and (2) the principal's information set is coarser, the manager's conditional expectation is given by

$$\mathbb{E}_m\left[\mathbb{E}\left[X|\mathcal{I}_p\right]\big|\mathcal{I}_m\right] = (1-\omega)\,\mathbb{E}\left[X|\mathcal{I}_p\right] + \omega\mathbb{E}\left[X|\mathcal{I}_m\right] \text{ for all } \mathcal{I}_p \subseteq \mathcal{I}_m. \tag{4}$$

We distinguish between the expectations operator $\mathbb{E}_m[\cdot]$, which reflects the "cursed" (biased) expectation of the manager, and the expectations operator $\mathbb{E}[\cdot]$ without the subscript, which corresponds to the expectation under objective beliefs. The parameter $\omega \in [0,1]$ measures the degree to which agent i exhibits the curse of knowledge. When $\omega = 0$, the manager correctly applies the law of iterated expectations; however, as ω increases, the manager's forecast is biased toward his conditional expectation (given his private information). The specification in (4) matches the one utilized by Camerer, Loewenstein, and Weber (1989).

Preferences. The principal prefers an investment level which maximizes the expected value of the firm, given her information set, $\mathcal{I}_p$. Specifically, her desired level of investment, given her information set $\mathcal{I}_p$ is

$$k^p \equiv \arg\max_k \mathbb{E}\left[Rk - \tfrac{1}{2}k^2 \middle| \mathcal{I}_p\right] = \arg\max_k \mathbb{E}\left[\tfrac{1}{2}R^2 - \tfrac{1}{2}(R-k)^2 \middle| \mathcal{I}_p\right] \tag{5}$$

$$= \mathbb{E}\left[R|\mathcal{I}_p\right] = \mu + \mathbb{E}\left[\theta|\mathcal{I}_p\right] \tag{6}$$

The principal would like the level of investment, k, to be as close as possible to the firm's productivity, R, (i.e., she wants to decrease $(R-k)^2$) since this maximizes the value of the firm.

The manager, however, also derives a non-pecuniary, private benefit from investment. As a result, his desired level of investment given his beliefs ($\mathcal{I}_m$) is

$$k^m \equiv \arg\max_k \mathbb{E}\left[Rk - \tfrac{1}{2}k^2 + bk \middle| \mathcal{I}_m\right] = \arg\max_k \mathbb{E}\left[\tfrac{1}{2}R^2 - \tfrac{1}{2}(R-k)^2 + bk \middle| \mathcal{I}_m\right], \tag{7}$$

where $b \geq 0$ reflects the manager's private benefit from investment. Intuitively, all else equal, he prefers that the principal invest (weakly) more than optimal since he receives private benefits from managing a larger project. The manager's desired level of investment reflects a tradeoff between his preference for higher investment (the bk term) and more efficient investment (the $(R-k)^2$) term.

In the following sections, we consider the impact of the bias in the manager's preferences, b, and in his beliefs, ω, on communication. Specifically, we assume that the principal chooses her preferred level of investment, k^p; however, since she does not directly observe the signal about fundamentals, x, her decision relies on the manager's message, d, i.e., $k^p = k^p(d)$. As a result, and given his preferences, the manager's optimal message, given his information, is

$$d(x) \equiv \arg\max_d \mathbb{E}_m\left[(R+b)\,k^p(d) - \tfrac{1}{2}\left(k^p(d)\right)^2 \middle| x, d\right]. \tag{8}$$

To measure the impact of the curse of knowledge on the quality of the manager's communication, we utilize a standard measure of informativeness: the expected reduction in the receiver's uncertainty after observing the message.

Definition 1. The message function $d(x)$ is **more informative** than $\hat{d}(x)$ if $\mathbb{E}\left[\operatorname{var}(\theta|d)\right] < \mathbb{E}\left[\operatorname{var}\left(\theta|\hat{d}\right)\right]$.

In the following sections we show how the curse of knowledge affects the principal's investment (and therefore, firm value) through the distortions it creates in both communication and information acquisition. We begin with settings in which the message is *costless* to send, and consider both non-verifiable communication (Section 4) and verifiable disclosure (Section 5). We then consider the impact on *costly* communication and allow the manager to commit to send a noisy message, with precision of his choice, to the principal (6). Before concluding, we consider whether the investment distortions that arise due to biased communication (both costly and costless) can be alleviated via delegation (in Section 7). Specifically, we analyze under what conditions the principal would delegate the investment decision to the manager, i.e., allow the manager to choose his desired level of investment, k^m, given his beliefs.

4 Non-verifiable communication (Cheap talk)

We begin by studying how curse of knowledge affects strategic communication in a setting where the manager can engage in "cheap talk": costless and non-verifiable communication. Suppose that, after observing his signal x, the manager can send a costless but non-verifiable message, $d = d(x)$, to the principal. As is standard in the class of "cheap talk" models introduced by Crawford and Sobel (1982), we begin by conjecturing that if an informative communication equilibrium exists, it follows a partition structure. Specifically, conjecture that there exists a partition characterized by cutoffs $-\frac{\sigma}{2} = s(0) < s(1) < s(2) \ldots s(N) = \frac{\sigma}{2}$, such that for all $x \in [s(n-1), s(n)]$, the manager sends the same message $d(n)$. In such an equilibrium, a message $d(n)$ induces the principal to optimally set

$$k^p(d(n)) = \mathbb{E}[\theta | x \in [s(n-1), s(n)]] + \mu \tag{9}$$

$$= p\frac{s(n-1)+s(n)}{2} + \mu. \tag{10}$$

This expression is similar to that found in standard cheap talk models with one modification: the principal knows that the manager's signal is only informative with probability p, and so discounts the information provided accordingly.

Moreover, the manager exhibits the curse of knowledge, and so believes that the principal's action will hew more closely to his beliefs about θ i.e., his conditional expectation of her action is given by:

$$\mathbb{E}_m[k^p(d(n)) | \mathcal{I}_m] = [(1-\omega)\mathbb{E}[\theta | d(n)] + \omega \mathbb{E}[\theta | x]] + \mu \tag{11}$$

$$= p\left[(1-\omega)\frac{s(n-1)+s(n)}{2} + \omega x\right] + \mu. \tag{12}$$

As such, the manager mistakenly believes that the principal's action will be better aligned with his conditional expectation of the true productivity, $\mu + \theta$. As the following proposition shows, this distortion in beliefs limits the manager's ability to convey information in equilibrium.

Proposition 1. *There exists a positive integer* $N_{max} \equiv ceil\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1 + 2\sigma\frac{p(1-\omega)}{b}}\right)$*, such that for every* N*, with* $1 \leq N \leq N_{max}$*, there exists at least one cheap talk equilibrium with* N *partitions and cutoffs*

$$s(n) = \sigma\left(\frac{n}{N} - \frac{1}{2}\right) + 2n(n-N)\frac{b}{p(1-\omega)}. \tag{13}$$

When $b > \frac{\sigma p(1-\omega)}{4}$*, then the only equilibrium is uninformative (i.e.,* $N_{max} = 1$*).*

All proofs are in the appendix. The above result corresponds directly that in Crawford and Sobel (1982), except that the manager's *effective* bias is now $\frac{b}{p(1-\omega)}$. The effective bias reflects the manager's inability to communicate effectively due to (i) the bias b in preferences, (ii) the (imperfect) precision p of his private signal, and (iii) the degree ω of curse of knowledge.

First, the effective bias increases with the bias b in preferences - the stronger this bias, the less the manager is able to convey in equilibrium. Next, note that the effective bias decreases with the precision p of the manager's signal. This is intuitive: when the manager's signal is more noisy, the principal correctly puts less weight on his message, which limits the extent of communication between the two. Finally, the effective bias is increasing in the degree of the manager's curse of knowledge, ω. This reflects the manager's mistaken beliefs about the principal's conditional expectation: as (4) makes clear, the manager perceives that the principal's beliefs place more weight on his private information and less weight on his message as the curse of knowledge grows. This creates a stronger incentive for the manager to distort his message in an effort to increase investment which, in turn, makes his message less informative (less credible) to the principal.

Note that the curse of knowledge reduces the ability of the manager to communicate effectively along two dimensions. First, as the curse of knowledge increases (i.e., ω increases), the maximum bias for which informative communication is feasible (i.e., $\frac{\sigma p(1-\omega)}{4}$), shrinks. In other words, informative cheap talk is less likely to arise. Second, even when informative communication is feasible, an increase in the degree of distortion, ω, increases the size of the partitions (for all but the last interval), which reduces the expected amount of information that is conveyed via cheap talk. In turn, this implies that the expected value of the firm decreases with the degree to which the manager exhibits the curse of knowledge, as summarized by the following corollary.

Corollary 1. *Fixing the precision, p, of the manager's signal, in any cheap talk equilibrium, the informativeness of communication and the expected firm value decrease (weakly) in the degree, ω, of the manager's curse of knowledge.*

As we show in the proof of Corollary 1, and as is standard in "cheap talk" equilibria, the value of the firm is increasing in the informativeness of the manager's communication. Specifically, the expected value of the firm is

$$\mathbb{E}\left[V\left(R,k\right)\right]=\tfrac{1}{2}\left(\mu^{2}+\text{var}\left(\theta\right)-\mathbb{E}\left[\text{var}\left(\theta|d\right)\right]\right) \tag{14}$$

Thus, firm value increases when the principal faces less uncertainty, in expectation, about the firm's productivity, i.e., as $\mathbb{E}\left[\text{var}\left(\theta|d\right)\right]$ falls. Because the curse of knowledge effectively amplifies the manager's bias, it (i) reduces the informativeness of a given partition equilibrium and (ii) can eliminate the existence of the most informative equilibria. Taken together, the expected value of the firm decreases.

For the manager, however, the effects are more nuanced. Let $u^{m}\left(d\left(x\right);x\right)$ denote the manager's utility, conditional on observing x and sending a message $d\left(x\right)$. Then,

$$\mathbb{E}_{m}\left[u^{m}\left(d\left(x\right);x\right)\right]=\tfrac{1}{2}\mathbb{E}_{m}\left[\mathbb{E}_{m}\left[\left(R+b\right)\big|x\right]^{2}-\left(\mathbb{E}_{m}\left[\left(R+b\right)\big|x\right]-k^{p}\left(d\left(x\right)\right)\right)^{2}\right] \tag{15}$$

Holding fixed the bias in his preferences, the manager would like the principal to make a more informed investment decision, i.e., he wants to minimize the distance between $\mathbb{E}_{m}\left[\left(R+b\right)\big|x\right]$ and $k^{p}\left(d\left(x\right)\right)$. As a result, he prefers both a more informative signal (an increase in p) and the most informative equilibrium (where $N=N_{max}$). Somewhat surprisingly, however, this also implies that the manager's expected utility is increasing in the degree to which he exhibits the curse of knowledge, holding fixed the number of partitions. The manager's belief about the principal's investment decision, $k^{p}\left(d\left(x\right)\right)$ is distorted: as ω increases, he expects the principal's beliefs (and therefore, the level of investment she chooses) will be closer to his conditional expectation, i.e., $\mathbb{E}_{m}\left[\left(R+b\right)\big|x\right]$. We establish these results in the proof of the following lemma.

Lemma 1. *The manager's expected utility is increasing in the number of partitions (N). Holding N fixed,* $\frac{\partial\mathbb{E}[u^{m}(x)]}{\partial\omega}$, $\frac{\partial\mathbb{E}[u^{m}(x)]}{\partial p}$, *and* $\frac{\partial^{2}\mathbb{E}[u^{m}(x)]}{\partial\omega\partial p}>0$.

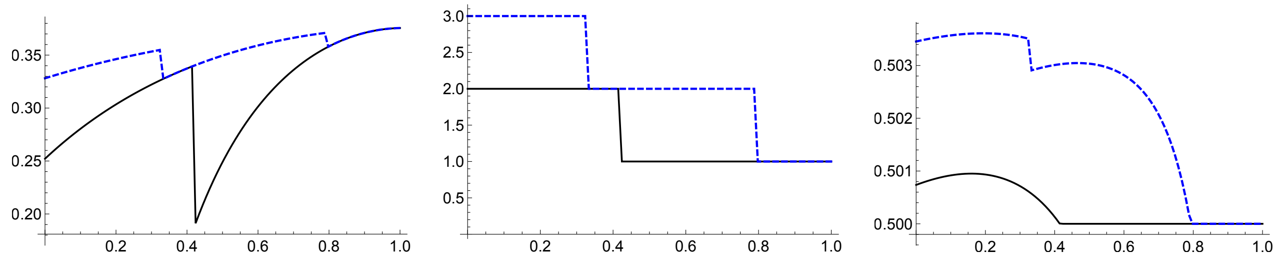
As emphasized above, the nature of the informative communication equilibrium depends upon the effective bias, $\frac{b}{p(1-\omega)}$, suggesting that the manager's bias for over-investment (b) and the curse of knowledge (ω) act as substitutes. However, Lemma 1 highlights one differential impact of these two distortions. Because an increase in the curse of knowledge also distorts how the manager *perceives* his communication, we show that the manager's perception of his expected utility, $\mathbb{E}_m [u^m (d(x); x)]$, rises even as his ability to communicate effectively falls. In short, he fails to fully internalize the information lost when he communicates with the principal. In contrast, the manager fully internalizes the impact of his bias towards over-investment and so, holding fixed the expected non-pecuniary benefits from investment ($b\mu$), an increase in b lowers his expected utility.

In many settings, the manager is not endowed with his private information, but must acquire it through costly effort / investment. Lemma 1 suggests that the curse of knowledge can increase information acquisition. In particular, as long as the number of partitions remains fixed, the manager finds more value in increasing the precision of his private signal as the curse of knowledge grows. This relies on the same channel as above – the manager believes that he will be able to communicate more effectively than he does in practice, which increases the value of the information he acquires.

Thus, with endogenous learning, there is a countervailing, indirect effect to the negative, direct, effect on communication generated by the curse of knowledge: an increase in p can increase N_{max} and lower the effective bias. As a result, depending upon the information technology (i.e., the cost of effort or information acquisition), an increase in the curse of knowledge could increase N_{max} *and* lower the effective bias in communication: firm value could potentially increase. There is, however, a limit to this channel: note that, even if the manager is perfectly informed, i.e., $p = 1$, there always exists a level of ω such that informative communication is not feasible (and so eventually, firm value must fall as ω increases).

Figure 1: Cheap talk communication versus degree of cursedness

The figure plots the expected value of the firm with cheap talk communication as a function of the degree of cursedness ω, where the manager optimally chooses p subject to a cost $c(p) = c_0 \frac{p^2}{1-p}$. The dashed line corresponds to $b = 0.02$ and the solid line corresponds to $b = 0.05$. The other parameters of the model are set to: $\mu = 1$, $\sigma = 1$, $c_0 = 0.02$.



(a) Optimal precision p vs ω (b) N_{max} vs ω (c) Expected Value vs ω

Figure 1 provides an illustration of the impact of endogenous learning on the expected value of the firm. We assume that the manager optimally chooses p subject to a cost of the form: $c(p) = c_0 \frac{p^2}{1-p}$. The figure plots the optimally chosen precision and the expected value of the firm, under the maximally informative (i.e., $N = N_{max}$), feasible cheap talk equilibrium, for different values of b, as a function of the cost parameter ω. The dashed line corresponds to a bias of $b = 0.02$, while the solid line corresponds to a bias of $b = 0.05$. The left panel confirms Lemma 1: holding fixed the informativeness of communication (i.e., the level of N), the optimal precision, p, increases with the degree of the curse of knowledge. However, when cursedness increases sufficiently, this leads to a decrease in the informativeness of communication, as illustrated by the discrete drops in N_{max}. As suggested by the analysis above, the overall effect on expected value is non-monotonic: as the right panel shows, the expected value of firm may be higher when the manager exhibits moderate levels of the curse of knowledge than for a rational ($\omega = 0$) manager.

5 Verifiable Disclosure

We now analyze how curse of knowledge affects strategic communication in a setting where the manager can disclose a costless but verifiable message. Suppose that the manager observes x with probability q (i.e., $s = x$) and nothing with probability $1 - q$ (i.e., $s = \emptyset$). An informed manager (one who observed $s = x$) can choose to either disclose nothing (i.e., $d = \emptyset$) or to disclose his information truthfully (i.e., $d = x$). An uninformed manager (one who observed $s = \emptyset$) cannot, however, verifiably disclose that he did not observe an informative signal.

Let $\mu_\emptyset \equiv \mathbb{E}[\theta | d = \emptyset]$ denote the principal's equilibrium belief about θ when the manager discloses no information. The optimal action for the principal is

$$k^p(d) = \mathbb{E}[R|d] = \begin{cases} px + \mu & \text{if } d = x \\ \mu_\emptyset + \mu & \text{if } d = \emptyset \end{cases} \tag{16}$$

However, because he suffers from the curse of knowledge, an informed manager believes

$$\mathbb{E}_m [k^p(d) | x, d = \emptyset] = (1 - \omega)\mu_\emptyset + \omega p x + \mu. \tag{17}$$

The following result characterizes the verifiable disclosure equilibria in this setting.

Proposition 2. *There exist cutoffs $x_l, x_h \in \left[-\frac{\sigma}{2}, \frac{\sigma}{2}\right]$ and $x_l \leq x_h$ such that an informed manager does not disclose her signal x (i.e., sends a message $d(x) = \emptyset$) iff $x \in [x_l, x_h]$.*

(i) If $\frac{b}{p(1-\omega)} \geq \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$, *then the cutoffs are* $x_h = \sigma\frac{2\sqrt{1-q}-(2-q)}{2q}$ *and* $x_l = -\frac{\sigma}{2}$.

(ii) If $\frac{b}{p(1-\omega)} < \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$, *then the cutoffs are* $x_h = -\frac{2q}{(1-q)\sigma}\left(\frac{b}{p(1-\omega)}\right)^2$ *and* $x_l = x_h - \frac{2b}{p(1-\omega)}$.

To gain some intuition for the nature of these equilibria, it is useful to consider the difference in the manager's expected utility from disclosing versus not, given a signal x. Specifically, given the expression for expected utility in (15), one can show that the expected utility benefit from not disclosing can be expressed as:

$$\Delta(\emptyset, x; x) \equiv \mathbb{E}_m\left[u^m(d=\emptyset; x)\right] - \mathbb{E}_m\left[u^m(d=x; x)\right] \tag{18}$$

$$= \mathbb{E}_m\left[\left(k^p(d=\emptyset) - k^p(d=x)\right)\left(R + b - \frac{k^p(d=\emptyset)+k^p(d=x)}{2}\right)\right] \tag{19}$$

$$= (1-\omega)^2(\mu_\emptyset - px)\left(\tfrac{b}{(1-\omega)} + px - \tfrac{\mu_\emptyset + px}{2}\right) \tag{20}$$

The above expression is a concave, quadratic function of the manager's signal x. Intuitively, it reflects the two components of the manager's utility characterized by equation (7): (i) the private benefits he receives from higher investment (i.e., a higher bk term), and (ii) higher firm value generated by more efficient investment (i.e., a lower $(R-k)^2$ term). Not disclosing his signal is only optimal if the benefit from higher investment from being pooled with uninformed managers offsets the loss due to less efficient investment. This implies that managers with extreme signals are more likely to prefer disclosure while managers with intermediate signals are more likely to prefer pooling, as suggested by the shape of Δ in (20).

The above result highlights that the verifiable disclosure equilibrium is one of two types. When the effective bias $\frac{b}{p(1-\omega)}$ is sufficiently large (case (i)), there is disclosure by managers who have sufficiently high signals. This is similar to the equilibria characterized by Dye (1985) and others. On the other hand, when the effective bias is sufficiently small (case (ii)), there is disclosure by managers with extreme signals but no disclosure for those with intermediate signals.[14]

The region of non-disclosure (and therefore, pooling) is driven by the magnitude of the effective bias $\frac{b}{p(1-\omega)}$. First, note that managers with very high signals always prefer to disclose — this ensures higher and more efficient investment than pooling with lower types. As such, the upper boundary of nondisclosure x_h is always strictly below $\sigma/2$. Second, when the effective bias is sufficiently large (i.e., $\frac{b}{p(1-\omega)} \geq \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$), managers with the lowest possible signal ($x = -\sigma/2$) prefer pooling to disclosure because the benefit from pooling

[14] Case (ii) is analogous to the verifiable disclosure equilibrium in Che and Kartik (2009). In their case, the distribution of fundamentals and signals is unbounded, and so the bias is always "effectively" small enough.

with higher types more than offsets the cost from inefficient investment. This leads to the single cutoff equilibrium characterized by case (i) of Proposition 2. On the other hand, when the bias is sufficiently small, low types prefer disclosing their signal — the loss from lower investment is dominated by the gain from more efficient investment. In the limit, as $b \to 0$, note that $x_l = x_h = 0$. Intuitively when the bias is arbitrarily small, almost all managers prefer disclosure to pooling.

Analogous to the equilibria with cheap talk, the curse of knowledge ω affects the informativeness of communication through two channels. First, a higher curse of knowledge increases the effective bias, which increases the likelihood of the less informative equilibrium arising (i.e., more likely to have case (i)). Second, even when the bias is low enough to sustain the more informative equilibrium, an increase in ω reduces the informativeness of the manager's disclosure policy, as summarized by the following corollary.

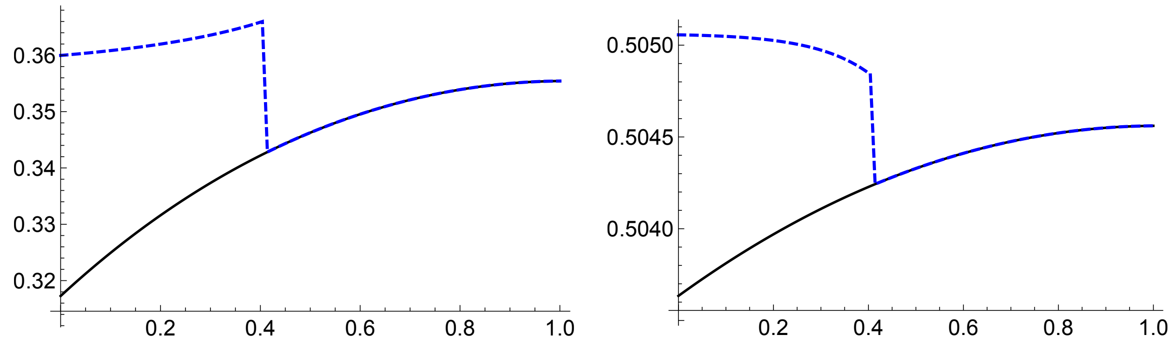
Corollary 2. *If* $\frac{b}{p(1-\omega)} < \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$ *and the manager's communication is verifiable, the informativeness of communication and the expected firm value decrease in the degree,* ω*, of the manager's curse of knowledge.*

This expansion of the non-disclosure interval is one reason why the curse of knowledge reduces the expected quality of the manager's message to the principal: in expectation, it is more likely that the manager chooses not to share his private information. The change in the effective bias also changes *when* the manager chooses not to disclose. In particular, as $\frac{b}{p(1-\omega)}$ grows, the manager chooses not to reveal increasingly negative information about the firm's productivity; however, since such signals are more informative for the principal (since these realizations are further from his prior belief, $\mathbb{E}[\theta]$), this reduces the informativeness of the manager's message. As above, when the informativeness of the manager's message falls, $\mathbb{E}[\text{var}(\theta|d)]$ increases, which decreases the expected value of the firm: the principal faces more uncertainty and invests less efficiently in expectation.

Lemma 2. *The manager's expected utility is higher if* $\frac{b}{p(1-\omega)} < \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$ *(i.e., if the manager utilizes a two-sided disclosure policy). Holding the type of equilibrium fixed,* $\frac{\partial \mathbb{E}[u^m]}{\partial \omega}$*,* $\frac{\partial \mathbb{E}[u^m]}{\partial p}$*, and* $\frac{\partial^2 \mathbb{E}[u^m]}{\partial \omega \partial p} > 0$.

Just as in the setting with cheap talk, Lemma 1 implies that the curse of knowledge can increase information acquisition when the manager's disclosure is verifiable. The curse of knowledge leads the manager to believe that he communicates more effectively: as a result, the marginal value of the information he acquires (and communicates to the principal) increases. As with cheap talk, increasing the curse of knowledge increases the manager's effective bias (and can shift the equilibrium disclosure policy), while endogenous learning generates a countervailing effect and so the impact of an increase in ω can be reduced, or even reversed, by an increase in p.

Figure 2: Verifiable disclosure versus degree of cursedness
The figure plots the expected value of the firm with verifiable disclosure as a function of the degree of cursedness ω, where the manager optimally chooses p subject to a cost $c(p) = c_0 \frac{p^2}{1-p}$. The dashed line corresponds to $b = 0.02$ and the solid line corresponds to $b = 0.05$. The other parameters of the model are set to: $\mu = 1$, $\sigma = 1$, $c_0 = 0.02$ and $q = 0.95$.



(a) Optimal precision p vs ω (b) Expected Value vs ω

Figure 2 illustrates how the curse of knowledge affects the expected value of the firm. As in the example above, we assume that the manager optimally chooses p subject to a cost of the form: $c(p) = c_0 \frac{p^2}{1-p}$. The dashed line corresponds to a bias of $b = 0.02$, the solid line corresponds to a bias of $b = 0.05$, and all other parameter values remain the same. In this case, the figure plots the optimal choice of precision and the expected value of the firm for different values of b, as a function of curse of knowledge, ω.

For a given equilibrium, the marginal value of acquiring information increases with the curse of knowledge, and so the optimal choice of information precision p increases with ω (as illustrated by the left panel), consistent with the result above. However, an increase in ω can also cause the equilibrium to switch from the more informative disclosure equilibrium to the less informative one: this corresponds to the discrete jump in the dashed line. As in the case with cheap talk communication, the overall effect on expected value is non-monotonic. The plots suggest that when the bias is sufficiently high (so that we are in the less informative disclosure equilibrium), increasing the degree to which manager's exhibit the curse of knowledge increases the expected value of the firm by increasing the equilibrium precision of acquired information. However, when the bias is relatively low, communication is more informative, and the expected value of the firm is highest when managers exhibit lower levels of the curse of knowledge.

6 Costly communication with commitment

Our analysis so far has focused on how the curse of knowledge affects strategic, costless communication in the absence of commitment. In this section, we explore a setting in which the manager can commit to sending the principal a (potentially noisy) message by incurring a private cost.

The model setup is the same as that described in Section 3 with the exception of how the manager communicates. Instead of sending a message after observing her signal (as in equation 8), we assume that the manager can incur a cost $\kappa(\rho)$ to commit to send the principal a noisy signal y about her information x with precision ρ. In particular, the manager sends a message, y, where

$$y = \begin{cases} x & \text{with probability } \rho \\ \xi & \text{with probability } 1-\rho \end{cases}, \tag{21}$$

and where $\xi \sim U\left[-\frac{\sigma}{2}, \frac{\sigma}{2}\right]$ is independent of θ and η.[15]

Given the information structure, after receiving the message, the principal chooses to invest

$$k^p(y;\rho,p) = \mu + \mathbb{E}[\theta|y] = \mu + p\rho y. \tag{22}$$

This implies that the manager's optimal choice of **message precision** ρ^* and **information precision** p^* are given by

$$\rho^*, p^* \equiv \arg\max_{\rho,p} \bar{u}_m(\rho,p) - \kappa(\rho) - c(p), \text{ where} \tag{23}$$

$$\bar{u}_m(\rho,p) = \mathbb{E}_m\left[(R+b)\,k^p(y;\rho,p) - \tfrac{1}{2}\left(k^p(y;\rho,p)\right)^2\right], \tag{24}$$

and where $\kappa(\rho)$ is the cost of increasing message precision, ρ, and $c(p)$ is the cost of increasing the precision, p, of acquired information. The following result summarizes how communication, information acquisition and the curse of knowledge affect the manager's expected utility and the expected value of the firm.

Proposition 3. *The manager's expected utility and expected firm value are given by*

$$\bar{u}_m = b\mu + \tfrac{1}{2}\mu^2 + \tfrac{\sigma^2}{24}p^2\left(1-(1-\omega)^2\left(1-\rho^2\right)\right), \text{ and } \mathbb{E}[V(R,k^p)] = \tfrac{1}{2}\mu^2 + \tfrac{\sigma^2}{24}\rho^2 p^2, \tag{25}$$

respectively. This implies:

[15] We focus on this information structure to maintain tractability. While studying the effect of the curse of knowledge on the broader, optimal information design problem (as in Kamenica and Gentzkow (2011) and Gentzkow and Kamenica (2014)) is very interesting, it is beyond the scope of this paper.

(i) The manager's expected utility $\bar{u}_m$ increases with both the message precision and the precision of acquired information. The marginal utility of message precision decreases with the curse of knowledge, but the marginal utility of acquired information precision increases with the curse of knowledge and with message precision i.e.,

$$\frac{\partial}{\partial \rho}\bar{u}_m \geq 0, \quad \frac{\partial}{\partial p}\bar{u}_m \geq 0, \quad \frac{\partial^2}{\partial \rho \partial \omega}\bar{u}_m \leq 0, \quad \frac{\partial^2}{\partial p \partial \omega}\bar{u}_m \geq 0, \text{ and } \frac{\partial^2}{\partial p \partial \rho}\bar{u}_m \geq 0. \tag{26}$$

(ii) The expected value of the firm $\mathbb{E}[V(R,k)]$ increases with both the message precision and the precision of acquired information, but holding fixed both precisions, is not impacted by the curse of knowledge, i.e.,

$$\frac{\partial}{\partial \rho}\mathbb{E}[V(R,k)] \geq 0, \quad \frac{\partial}{\partial p}\mathbb{E}[V(R,k)] \geq 0, \text{ and } \frac{\partial}{\partial \omega}\mathbb{E}[V(R,k)] = 0. \tag{27}$$

Intuitively, the curse of knowledge leads the manager to overestimate the informativeness of her message to the principal. Specifically, an informed manager who suffers from the curse of knowledge believes that,

$$\mathbb{E}_m[k^p(y;\rho,p)|y,x] = (1-\omega)p\rho y + \omega p x + \mu. \tag{28}$$

Since the message y is a strictly noisier version of the manager's private information, this is equivalent to believing that the principal's investment decision conditions on a more precise message than is objectively sent. As (28) suggests and (25) makes clear, this implies that an increase in the curse of knowledge leads to a decrease in the manager's marginal utility of message precision, ρ. As a result, and analogous to the setting with costless communication, the curse of knowledge decreases the informativeness of communication and firm value when the information precision, p, is fixed.

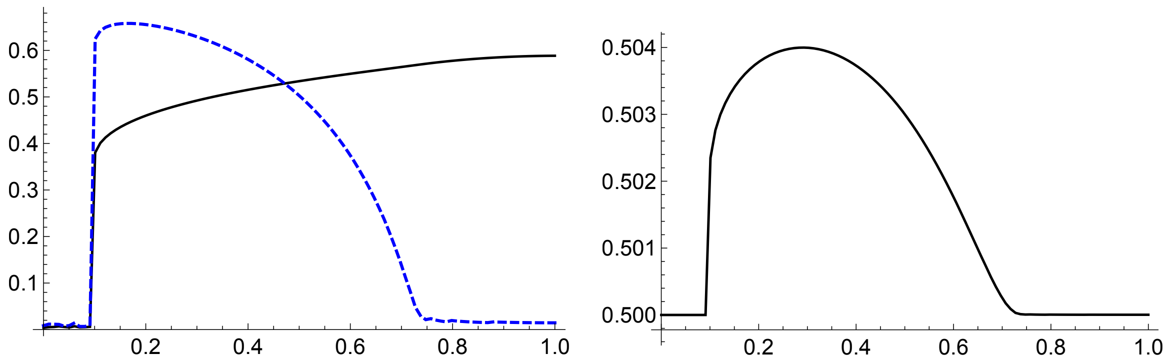
However, the marginal utility of information precision, p, increases in the extent to which the manager exhibits the curse of knowledge. Moreover, there is complementarity in the choice of information and message precision: an increase in one type of precision increases the marginal utility of the other type, since

$$\frac{\partial^2}{\partial p \partial \rho}\bar{u} = \frac{1}{6}p\rho\sigma^2(1-\omega)^2 > 0. \tag{29}$$

This suggests that, as with costless communication, an increase in the curse of knowledge can increase firm value when the precision of the manager's private signal is endogenous.

Figure 3 illustrates how these channels drive optimal precision choice and, consequently, can alter the expected value of the firm. In addition to information acquisition costs of

Figure 3: Optimal communication and information acquisition under costly communication
The figure plots the choice of message precision ρ (dashed), acquired information precision p (solid), and expected value of the firm $\mathbb{E}\left[V\left(R,k\right)\right]$ under costly communication, as a function of the degree of cursedness. The manager optimally chooses message precision ρ subject to a cost $\kappa\left(\rho\right)=\kappa_0\frac{\rho^2}{1-\rho}$ and chooses acquired information precision p subject to a cost $c\left(p\right)=c_0\frac{p^2}{1-p}$. The other parameters of the model are set to: $\mu=1$, $b=0.02$, $\sigma=1$ and $c_0=0.01$ and $\kappa_0=0.001$.



(a) ρ (dashed) and p (solid) vs ω (b) Expected value vs ω

$c\left(p\right)=c_0\frac{p^2}{1-p}$, as before, we assume that the manager's cost of communicating with precision ρ is give by $\kappa\left(\rho\right)=\kappa_0\frac{\rho^2}{1-\rho}$. The figure plots the optimal choice for both precisions, ρ (dashed) and p (solid), and the expected firm value as a function of the manager's curse of knowledge, ω.

In this example, there is very little information acquisition or communication when the manager is rational (i.e., when $\omega=0$). As ω increases, the marginal utility from information acquisition increases and the marginal utility from message precision decreases. When ω is sufficiently large (when $\omega\approx 0.1$ in panel (a)), however, this is offset by the complementarity across precisions and so both p and ρ quickly increase with ω. As panel (b) illustrates, this leads to an increase in the firm's expected value since the manager is now acquiring and communicating more precise information. Notably, this suggests that small changes in the manager's bias can lead to large changes in informational efficiency and firm value.

As the curse of knowledge increases further, the offsetting impact of the complementarity begins to dissipate: eventually (when $\omega\approx 0.2$) this leads to (i) higher information acquisition, and (ii) lower message precision. Note that, even in this region, the expected value of the firm continues to rise, until the rate at which the message precision decreases outweighs the informativeness of the manager's signal (near $\omega\approx 0.3$). Eventually, the manager's curse of knowledge is sufficiently high to drive the optimal choice of message precision to nearly zero. From this point forward, the expected value of the firm remains relatively insensitive to changes in the manager's bias.

While the specifics of the example in Figure 3 depend on the choice of information / communication costs and parameter values, they illustrate the robustness of our results even in settings in which the communication is costly. As before, the curse of knowledge hampers communication but enhances information acquisition: taken together, the manager's distorted beliefs have a nuanced impact ton both informational efficiency and firm value.

7 Delegation versus Communication

In this section, we consider the implications for firm value when the principal delegates the investment decision to the manager. While the principal knows that manager is biased towards over-investment, delegation allows the manager to utilize his private signal instead of forcing him to communicate a noisy version to the principal.

Recall from (7) that the manager optimally chooses the investment to maximize firm value while also accounting for the non-pecuniary benefits he receives. As a result, given his information set $\mathcal{I}_m$, he chooses to invest

$$k^m = \mathbb{E}_m\left[R + b|\mathcal{I}_m\right] = \mu + \mathbb{E}_m\left[\theta|\mathcal{I}_m\right] + b. \tag{30}$$

Thus, the expected value of the firm when the manager invests is

$$\begin{aligned}\mathbb{E}\left[V\left(R, k^m\right)\right] &= \mathbb{E}\left[Rk^m - \tfrac{1}{2}\left(k^m\right)^2\right] &(31)\\ &= \tfrac{1}{2}\left(\mu^2 - b^2 + \mathrm{var}\left(\theta\right) - \mathbb{E}\left[\mathrm{var}\left(\theta|\mathcal{I}_m\right)\right]\right). &(32)\end{aligned}$$

Notably, the expected value of the firm under delegation is unaffected by the degree of the manager's curse of knowledge ω. Comparing this equation to firm value under communication, found in (14), makes stark the principal's tradeoff. On the one hand, the manager over-invests which decreases firm value by $\frac{b^2}{2}$. On the other hand, the manager bases his investment decision off more precise information, which increases firm value by $\frac{\mathbb{E}[\mathrm{var}(\theta|d)] - \mathbb{E}[\mathrm{var}(\theta|\mathcal{I}_m)]}{2}$.

7.1 Delegation versus Non-verifiable Disclosure

Suppose that the principal is considering whether or not to delegate when the manager can only engage in cheap talk communication, as in Section 4. In this case, firm value with delegation, V_m, is

$$V_{m,c} \equiv \tfrac{1}{2}\left(\mu^2 - b^2 + \frac{p^2\sigma^2}{12}\right), \tag{33}$$

while firm value with communication, V_c, which we derive in the proof of Corollary 1, can be written as

$$V_c \equiv \tfrac{1}{2}\left(\mu^2 + \frac{p^2\sigma^2}{12}\left(1 - \left(\frac{1}{N^2} + \tfrac{4b^2\left(N^2-1\right)}{p^2(1-\omega)^2\sigma^2}\right)\right)\right). \tag{34}$$

Taken together, this implies that the principal prefers to delegate as long as

$$\underbrace{\frac{p^2\sigma^2}{12}\left(\frac{1}{N^2} + \tfrac{4b^2\left(N^2-1\right)}{p^2(1-\omega)^2\sigma^2}\right)}_{\text{loss through communication}} > \underbrace{b^2}_{\text{loss due to bias}}. \tag{35}$$

Proposition 4. *The principal retains control over the investment decision if and only if the manager's bias is sufficiently large, i.e., iff* $b > \frac{p\sigma}{2\sqrt{3}}$.

If the manager's bias is too large, the principal prefers to invest, even though she is using a coarser information set. In fact, in this setting, if the principal retains control, it must also the be the case that the manager cannot credibly send an uninformative signal, i.e., there is only a "babbling" communication equilibrium.[16] Equivalently, the principal prefers to invest using only her prior beliefs rather than allow a (biased) manager to invest using his private information.

Proposition 4 also implies that the curse of knowledge does not impact the delegation decision when the precision of the manager's private signal is exogenously specified. The manager's effective bias ($\frac{b}{p(1-\omega)}$) determines the nature of the communication equilibrium. Suppose the curse of knowledge increases sufficiently such that the communication equilibrium shifts from informative ($\frac{b}{p(1-\omega)} < \frac{\sigma}{4}$) to uninformative. Despite this, the principal would still prefer to delegate, because the manager's *actual* bias is unchanged. As (35) makes clear, an increase in ω harms the manager's ability to communicate but does not affect the efficiency of the manager's investment decision.

7.2 Delegation versus Verifiable Disclosure

Suppose instead that the principal can either delegate or allow the manager to send a verifiable message. If the manager observes the signal x with probability q, then firm value with delegation is

[16] This is because the principal retains control if $b > \frac{p\sigma}{2\sqrt{3}} > \frac{p\sigma(1-\omega)}{4}$, where the second inequality implies that any communication from the manager is uninformative, by Proposition 1.

$$V_{m,d} \equiv \tfrac{q}{2}\left(\mu^2 - b^2 + \frac{p^2\sigma^2}{12}\right) + \tfrac{1-q}{2}\left(\mu^2 - b^2\right) \tag{36}$$

$$= \tfrac{1}{2}\left(\mu^2 - b^2 + \frac{qp^2\sigma^2}{12}\right). \tag{37}$$

On the other hand, as we show in the proof of Corollary 2, firm value with verifiable communication, V_d, depends upon the nature of the communication equilibrium. If $\frac{b}{p(1-\omega)} \geq \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$,the manager only discloses sufficiently high values of x. In this case, firm value is

$$V_{d,1} \equiv \tfrac{1}{2}\left(\mu^2 + \frac{qp^2\sigma^2}{12}\left(1 - \left(\frac{(1-q)\left(8\left(1-\sqrt{1-q}\right) - q^2 - 4q\right)}{q^3}\right)\right)\right). \tag{38}$$

If $\frac{b}{p(1-\omega)} < \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$,then there is communication for both low and high values of x. Firm value in this case is

$$V_{d,2} \equiv \tfrac{1}{2}\left(\mu^2 + \frac{qp^2\sigma^2}{12}\left(1 - \left(\frac{48b^4q}{p^4\sigma^4(1-q)(1-\omega)^4} + \frac{32b^3}{p^3\sigma^3(1-\omega)^3}\right)\right)\right). \tag{39}$$

Comparing these expectations yields the following result.

Proposition 5. *(i) If* $\frac{b}{p(1-\omega)} \geq \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$, *the principal retains control over the investment decision if and only if the manager's bias is sufficiently large, i.e., iff*

$$b > \tfrac{p\sigma}{2\sqrt{3}} \frac{\sqrt{(1-q)\left(8\left(1-\sqrt{1-q}\right) - q^2 - 4q\right)}}{q}. \tag{40}$$

(ii) If $\frac{b}{p(1-\omega)} < \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$, *the principal retains control over the investment decision if and only if the curse of knowledge is sufficiently small, i.e., if*

$$(1-\omega)^2 > \frac{4q^2}{\sigma^2(1-q)}\left(\frac{b}{p\,(1-\omega)}\right)^2 + \frac{8q}{3\sigma}\left(\frac{b}{p\,(1-\omega)}\right). \tag{41}$$

As discussed in Section 5, when $\frac{b}{p(1-\omega)} \geq \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$, there is only one-sided disclosure. If that is the case, the manager's threshold for disclosure depends only upon the likelihood he is informed, q, and the prior uncertainty both the principal and manager face, σ. As a result, if the *effective* bias is sufficiently high, then the delegation decision depends only upon the *investment bias* of the manager: both the quality of communication and the degree of over-investment are unaffected by the curse of knowledge. This result is similar to what arises with non-verifiable communication.

On the other hand, when $\frac{b}{p(1-\omega)} < \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$, there is two-sided disclosure, and both (i) the thresholds for communication and (ii) the size of the non-disclosure region depend upon the curse of knowledge. In this setting, the principal evaluates whether the manager's effective bias is driven more by his desire to over-invest (b) or by the curse of knowledge (ω). All else equal, if the manager's *effective* bias is largely driven by the curse of knowledge, i.e., if ω is sufficiently large such that does not (41) hold, then the principal prefers to delegate: the distortion in communication will be larger than the distortion in the manager's investment decision.

Figure 4: Delegation versus verifiable disclosure

The figure plots the region of the $b-\omega$ parameter space in which delegation is preferred to communication (shaded in blue), and the region in which the less informative equilibrium is sustained (shaded in peach). The other parameter values are set to: $\mu = 1$, $p = 0.7$, $\sigma = 1$ and $q = 0.75$.

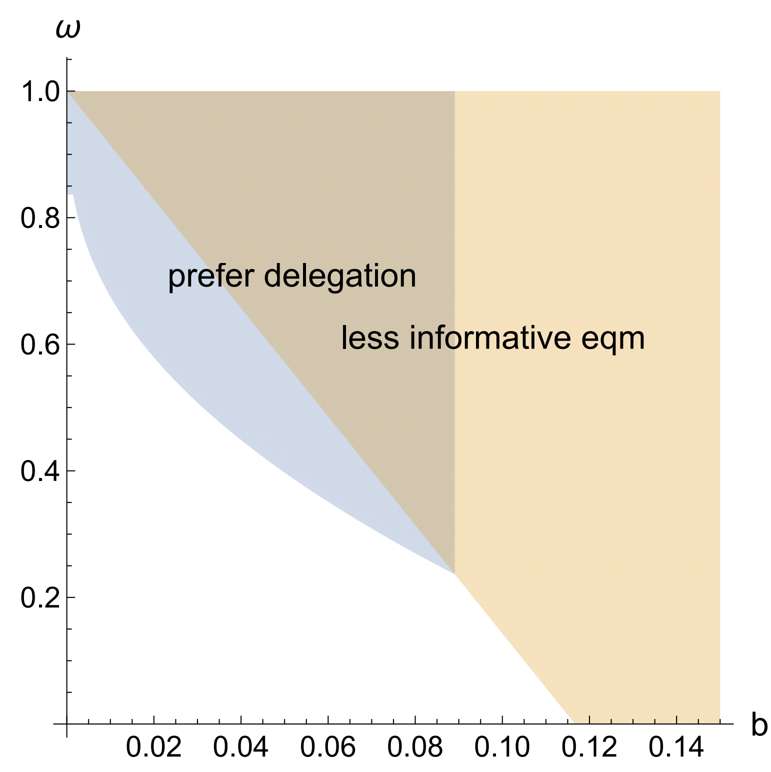


Figure 4 illustrates these results through a numerical example. The figure overlays the region of the $b-\omega$ parameter space in which delegation is preferred to communication, which is shaded in blue, with the region where the less informative equilibrium is sustained (i.e., where $\frac{b}{p(1-\omega)} \geq \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$), which is shaded in peach. In the region of the less informative equilibrium, the decision of whether or not to delegate depends only on whether the bias b is sufficiently large. In this case, when the bias is sufficiently low, the principal prefers to delegate (overlapped region), but when the bias is high, she prefers to take the action herself. In the region of the more informative equilibrium, the delegation decision depends on both the investment bias, b, and the curse of knowledge. For a fixed investment bias, b, the manager prefers to delegate the investment decision when the curse of knowledge is sufficiently severe because the loss from the distortion in communication overwhelms the loss due to the preference bias.

7.3 Delegation versus costly communication

Finally, we consider whether the principal would prefer delegation or costly communication with commitment, as in Section 6. Recall that the expected value of the firm with costly communication is given by

$$V_{cc} = \tfrac{1}{2}\mu^2 + \tfrac{\sigma^2}{24}\rho^2 p^2, \tag{42}$$

while it is straightforward to show that the value of the firm with delegation is given by

$$V_{m,cc} = \tfrac{1}{2}\left(\mu^2 - b^2 + \tfrac{p^2\sigma^2}{12}\right). \tag{43}$$

This immediately implies the following result.

Proposition 6. *The principal retains control over the investment decision if and only if the manager's bias is sufficiently large i.e., iff*

$$b^2 > \tfrac{p^2\sigma^2}{12}\left(1-\rho^2\right). \tag{44}$$

As is the case with strategic communication without commitment, the principal retains control only when the manager's bias is sufficiently large. Moreover, as communication becomes more informative (i.e., message precision ρ increases), the principal is more likely to retain control because the loss from imperfect communication is smaller. On the other hand, an increase in information precision, p, makes delegation more likely since the manager's information advantage is increasing in p.[17]

As in the cheap talk setting analyzed above, the delegation decision with costly communication does not depend on the curse of knowledge when message precision and acquired information precision are exogenously specified. Unlike the cheap talk setting, however,

[17] When ρ is fixed, the manager effectively shares a fixed proportion of his private information, and so the gap between the precision of the manager and the principal's information grows in p.

this independence arises because the curse of knowledge only affects costly communication through the choice of precisions. At first glance, with endogenous learning, an increase in the curse of knowledge would seem to favor delegation: the marginal utility of message precision falls while the marginal utility of information precision rises. Intuitively, a more cursed manager acquires more precise information but communicates less effectively. While the former channel increases value in either case, the latter channel favors delegation.[18] However, the complementarity across p and ρ implies that the optimal $p(\omega)$ and $\rho(\omega)$ can be increasing, decreasing or move in opposite directions as a function of ω. which complicates the analysis in the presence of more general cost functions.

8 Conclusion

We study the effect of the curse of knowledge on communication within a firm and the resulting efficiency of the firm's investment policy. In our setting, a principal, who must choose how much to invest in a new project, communicates with a manager, who is privately informed about the project's productivity and also exhibits the curse of knowledge. We show that the curse of knowledge leads the manager to overestimate the effectiveness of his communication, which decreases the informativeness of equilibrium messages. As a result, when the precision of the manager's information is fixed, the curse of knowledge reduces firm value by reducing investment efficiency.

However, when the manager can exert costly effort to acquire more precise information, the same bias in his beliefs leads him to overestimate the value of his information and, consequently, over-invest in information acquisition. This suggests that, when the manager is responsible for generating information, firms can benefit by choosing managers who have a higher tendency to exhibit the curse of knowledge. We show that when incentives are well-aligned and equilibrium communication is informative, the curse of knowledge can lead to more informed decisions and higher firm value. We also characterize conditions under which the principal may be willing to delegate to a cursed manager while retaining control with an unbiased one.

Our analysis of how the curse of knowledge affects investment efficiency and the decision to delegate suggests a number of directions for future work. It would be interesting to study

[18] The LHS of (44) is unaffected by ω. Moreover, since $p_\omega > 0$ and $\rho_\omega < 0$ when the complementarity effects do not dominate, the RHS is increasing in ω i.e.,

$$\frac{\partial}{\partial \omega} \frac{p^2 \sigma^2}{12} \left(1 - \rho^2\right) = \frac{\sigma^2 p}{6} \left(\left(1 - \rho^2\right) p_\omega - p \rho \rho_\omega\right) > 0 \tag{45}$$

when $p_\omega > 0$ and $\rho_\omega < 0$.

whether one could design an internal reporting system which mitigates the negative effects on informativeness of communication, but amplifies the benefits of more precise information acquisition. Another natural extension would be to explore the implications in a setting with multiple, division managers whose objectives are partially aligned. Finally, in a multi-firm setting with strategic complementarities and public information, one would expect the curse of knowledge to affect not only communication within a given firm, but also investment decisions across firms in the economy. We hope to explore these ideas in future work.

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A Proofs

Proof of Lemma 1

In the analysis that follows, it will be useful to characterize the difference in the manager's expected utility from sending messages d_1 and d_2. Specifically, let $u^m(d;\theta)$ denote the manager's expected utility from sending a message d, i.e.,

$$u^m(d;x) \equiv \mathbb{E}_m\left[(R+b)\,k^p(d) - \tfrac{1}{2}k^p(d)^2\,\middle|x\right] \tag{46}$$

$$= \mathbb{E}_m\left[\tfrac{1}{2}(R+b)^2 - \tfrac{1}{2}(R+b-k^p(d))^2\,\middle|x\right] \tag{47}$$

and let $\Delta(d_1,d_2;x) \equiv u^m(d_1;x) - u^m(d_2;x)$. A useful characterization is given by

$$\Delta(d_1,d_2;x) = \mathbb{E}_m\left[-\tfrac{1}{2}(R+b-k^p(d_1))^2 + \tfrac{1}{2}(R+b-k^p(d_2))^2\,\middle|x\right] \tag{48}$$

$$= \mathbb{E}_i^m\left[(k^p(d_1)-k^p(d_2))\left(R+b-\tfrac{k^p(d_1)+k^p(d_2)}{2}\right)\middle|\,x\right] \tag{49}$$

Recall that $k^p(d) = \mathbb{E}[\theta|d] + \mu$ and since the manager exhibits the curse of knowledge, we have:

$$\mathbb{E}_m[k^p(d)\,|x] = (1-\omega)\,\mathbb{E}\left[\theta\middle|d_i\right] + \omega\mathbb{E}[\theta|x_i] + \mu. \tag{50}$$

Moreover,

$$\mathbb{E}_m\left[R+b\middle|x\right] = \mu + \mathbb{E}[\theta|x] + b \tag{51}$$

This implies:

$$\Delta_i(d_1,d_2;x_i) = \begin{array}{c}(1-\omega)\left(\mathbb{E}\left[\theta\middle|d_1\right] - \mathbb{E}\left[\theta\middle|d_2\right]\right)\\ \times\left(\mathbb{E}[\theta|x] + b - \frac{\left((1-\omega)\left(\mathbb{E}\left[\theta\middle|d_1\right]+\mathbb{E}\left[\theta\middle|d_2\right]\right)+2\omega\mathbb{E}[\theta|x]\right)}{2}\right)\end{array} \tag{52}$$

$$= \begin{array}{c}(1-\omega)^2\left(\mathbb{E}\left[\theta\middle|d_1\right] - \mathbb{E}\left[\theta\middle|d_2\right]\right)\\ \times\left(\frac{b}{(1-\omega)} + \mathbb{E}[\theta|x] - \frac{\left(\mathbb{E}\left[\theta\middle|d_1\right]+\mathbb{E}\left[\theta\middle|d_2\right]\right)}{2}\right)\end{array} \tag{53}$$

which one can derive by (i) substituting the optimal investment choice, k^p and (ii) recognizing that since the manager exhibits curse of knowledge,

$$\mathbb{E}_m\left[\mathbb{E}[\theta|d]\,\middle|x\right] = (1-\omega)\,\mathbb{E}\left[\theta\middle|d\right] + \omega\mathbb{E}[\theta|x]\,. \tag{54}$$

The cutoffs $s(n)$ are pinned down by the conditions:

$$\Delta\left(d\left(n\right),d\left(n+1\right);s\left(n\right)\right)=0, \tag{55}$$

where $\mathbb{E}\left[\theta|d\left(n\right)\right]=p\frac{s(n-1)+s(n)}{2}$. Imposing that the cutoffs are distinct (i.e., $s\left(n\right)\neq s\left(n+1\right)$) implies that they need to satisfy:

$$0=\tfrac{b}{(1-\omega)}+ps\left(n\right)-\tfrac{1}{2}\left(p\tfrac{s(n-1)+s(n)}{2}+p\tfrac{s(n)+s(n+1)}{2}\right) \tag{56}$$

which implies the sequence satisfies the difference equation:

$$s\left(n+1\right)-s\left(n\right)=s\left(n\right)-s\left(n-1\right)+\tfrac{4b}{p(1-\omega)}, \tag{57}$$

which is analogous to the difference equation in Crawford and Sobel (1982). If $s\left(0\right)=-\frac{\sigma}{2}$, then it is straightforward to show that a solution to this second-order difference equation can be written as

$$s\left(n\right)=ns\left(1\right)-\tfrac{\sigma}{2}+2n\left(n-1\right)\frac{b}{p\left(1-\omega\right)} \tag{58}$$

We also know that $s\left(N\right)=\frac{\sigma}{2}$, which implies that

$$s\left(N\right)=\tfrac{\sigma}{2}=Ns\left(1\right)-\tfrac{\sigma}{2}+2N\left(N-1\right)\frac{b}{p\left(1-\omega\right)}\implies \tag{59}$$

$$s\left(1\right)=\tfrac{\sigma}{N}-2\left(N-1\right)\frac{b}{p\left(1-\omega\right)}\implies \tag{60}$$

$$s\left(n\right)=n\tfrac{\sigma}{N}-\tfrac{\sigma}{2}+2n\left(n-N\right)\frac{b}{p\left(1-\omega\right)} \tag{61}$$

This implies that under the assumption that $s\left(0\right)=-\frac{\sigma}{2}$ such an equilibrium exists for any $N\leq N_{max}$, where we need

$$\tfrac{\sigma}{2}>-\tfrac{\sigma}{2}+2n\left(n-1\right)\frac{b}{p\left(1-\omega\right)}\implies \tag{62}$$

$$N_{max}\equiv\text{ceil}\left(\frac{2\frac{b}{p(1-\omega)}+\sqrt{\left(2\frac{b}{p(1-\omega)}\right)^2+4\sigma\left(2\frac{b}{p(1-\omega)}\right)}}{4\frac{b}{(1-\omega)p}}-1\right) \tag{63}$$

$$=\text{ceil}\left(-\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1+2\sigma\tfrac{p(1-\omega)}{b}}\right) \tag{64}$$

where ceil(x) is the smallest integer greater than or equal to x. In order for there to be an informative equilibrium, N_{max} must be greater than one, which implies that it must be that $b < \sigma\frac{p(1-\omega)}{4}$. □

Proof of Corollary 1

The expected value of the firm is given by:

$$\begin{aligned}
\mathbb{E}\left[V\left(R,k\right)\right] &= \mathbb{E}\left[Rk - \tfrac{1}{2}k^2\right] && (65)\\
&= \tfrac{1}{2}\mathbb{E}\left[R^2\right] - \tfrac{1}{2}\mathbb{E}\left[\left(R-k\right)^2\right] && (66)\\
&= \tfrac{1}{2}\left(\mathbb{E}\left[R^2\right] - \mathbb{E}\left[\left(R - \mathbb{E}\left[R\middle|d\right]\right)^2\right]\right) && (67)\\
&= \tfrac{1}{2}\left(\mathbb{E}\left[R^2\right] - \mathbb{E}\left[\mathrm{var}\left(R\middle|d\right)\right]\right) && (68)\\
&= \tfrac{1}{2}\left(\mu^2 + \mathrm{var}\left(R\right) - \left(\mathrm{var}\left(R\right) - \mathrm{var}\left(\mathbb{E}\left[R\middle|d\right]\right)\right)\right) && (69)\\
&= \tfrac{1}{2}\left(\mu^2 + \mathrm{var}\left(\mathbb{E}\left[R\middle|d\right]\right)\right) \implies && (70)\\
\mathbb{E}\left[V\left(R,k\right)\right] &= \tfrac{1}{2}\left(\mu^2 + \mathrm{var}\left(\mathbb{E}\left[\theta\middle|d\right]\right)\right) && (71)
\end{aligned}$$

Note that, $\mathbb{E}\left[\mathbb{E}\left[\theta|d\right]\right] = 0$ and so

$$\begin{aligned}
\mathrm{var}\left(\mathbb{E}\left[\theta\middle|d\right]\right) &= \sum_{n=1}^{N}\left(\frac{s\left(n\right) - s\left(n-1\right)}{\sigma}\right)\left(p\tfrac{s(n-1)+s(n)}{2} - 0\right)^2 && (72)\\
&= \frac{p^2\sigma^2}{12}\left(N^2-1\right)\left(\frac{1}{N^2} - \tfrac{4b^2}{p^2(1-\omega)^2\sigma^2}\right) && (73)
\end{aligned}$$

It is clear that $\frac{\partial \mathrm{var}\left(\mathbb{E}\left[\theta\middle|d\right]\right)}{\partial\omega} < 0$ and $\frac{\partial \mathrm{var}\left(\mathbb{E}\left[\theta\middle|d\right]\right)}{\partial p} > 0$, which implies that both informativeness and $\mathbb{E}\left[V\left(R,k\right)\right]$ are decreasing (increasing) in the curse of knowledge (in the quality of the manager's signal), holding Nfixed. Finally,

$$\begin{aligned}
\frac{\partial \mathrm{var}\left(\mathbb{E}\left[\theta\middle|d\right]\right)}{\partial N} &\propto \frac{2}{N^3} - \tfrac{8b^2N}{p^2(1-\omega)^2\sigma^2} > 0 \iff && (74)\\
\frac{1}{N^4} &> \tfrac{4b^2}{p^2(1-\omega)^2\sigma^2} && (75)
\end{aligned}$$

Note that $b < \sigma\frac{p(1-\omega)}{4}$ and so

$$1 > \frac{4b}{p\left(1-\omega\right)\sigma} \implies \tag{76}$$

$$1 > \left(\frac{4b}{p\left(1-\omega\right)\sigma}\right)^2 \implies \tag{77}$$

$$\frac{1}{N^4} > \frac{4b^2}{p^2\left(1-\omega\right)^2\sigma^2}. \tag{78}$$

As a result, firm value is highest when $N = N_{max}$. Because the curse of knowledge weakly lowers N_{max}, it also decreases firm value by reducing the maximum number of partitions. Finally, note that firm value can be written

$$\mathbb{E}\left[V\left(R,k\right)\right] = \tfrac{1}{2}\left(\mu^2 + \frac{p^2\sigma^2}{12}\left(1 - \left(\frac{1}{N^2} + \frac{4b^2\left(N^2-1\right)}{p^2\left(1-\omega\right)^2\sigma^2}\right)\right)\right), \tag{79}$$

which completes the proof. □

Proof of Lemma 1

Conditional on observing x, the manager's expected utility is given by:

$$u^m\left(x\right) \equiv \mathbb{E}_m\left[\left(R+b\right)k^p\left(d\left(x\right)\right) - \tfrac{1}{2}k^p\left(d\left(x\right)\right)^2 \middle| x\right] \tag{80}$$

$$= \mathbb{E}_m\left[\left(R+b\right)\middle| x\right]k^p\left(d\left(x\right)\right) - \tfrac{1}{2}k^p\left(d\left(x\right)\right)^2 \tag{81}$$

$$= \tfrac{1}{2}\left(\mathbb{E}_m\left[\left(R+b\right)\middle| x\right]\right)^2 - \tfrac{1}{2}\left(\mathbb{E}_m\left[\left(R+b\right)\middle| x\right] - k^p\left(d\left(x\right)\right)\right)^2 \tag{82}$$

Note that

$$\mathbb{E}_m\left[\left(R+b\right)\middle| x\right] = b + \mu + px \tag{83}$$

and

$$\mathbb{E}_m\left[k^p\left(d\left(n\right)\right)\right] = \left(1-\omega\right)p\left(\tfrac{s(n-1)+s(n)}{2}\right) + \omega px + \mu \tag{84}$$

which implies:

$$u^m\left(x\right) = \tfrac{1}{2}\left(b+\mu+px\right)^2 - \tfrac{1}{2}\left(b + \left(1-\omega\right)p\left(x - \tfrac{s(n-1)+s(n)}{2}\right)\right)^2 \tag{85}$$

Thus,

$$\mathbb{E}\left[u^m\left(x\right)\right] = \mathbb{E}\left[\tfrac{1}{2}\left(b+\mu+px\right)^2\right] - \tfrac{1}{2}\sum_{n=1}^{N}\int_{s(n-1)}^{s(n)}\tfrac{1}{\sigma}\left(b+\left(1-\omega\right)p\left(x-\tfrac{s(n)+s(n+1)}{2}\right)\right)^2 dx \tag{86}$$

$$= \tfrac{1}{2}\left(b+\mu\right)^2 + \tfrac{p^2\sigma^2}{24} - \tfrac{1}{2}b^2 - \tfrac{1}{2}\left(1-\omega\right)^2 p^2 \sum_{i=1}^{N}\tfrac{1}{\sigma}\tfrac{\left(s(n+1)-s(n)\right)^3}{12} \tag{87}$$

$$= \tfrac{1}{2}\left(b+\mu\right)^2 + \tfrac{p^2\sigma^2}{24} - \tfrac{1}{2}b^2 - \tfrac{1}{24}\left(1-\omega\right)^2 p^2\left(\tfrac{\sigma^2}{N^2} + \tfrac{4b^2\left(N^2-1\right)}{p^2(1-\omega)^2}\right) \tag{88}$$

$$= \frac{1}{2}\left(\left(b+\mu\right)^2 - b^2\right) + \frac{p^2}{24}\left(\sigma^2 - \left(1-\omega\right)^2\left(\frac{\sigma^2}{N^2} + \frac{4b^2\left(N^2-1\right)}{p^2\left(1-\omega\right)^2}\right)\right) \tag{89}$$

$$= \frac{1}{2}\left\{2b\mu + \mu^2 + \frac{p^2\sigma^2}{12}\left(1 - \left(1-\omega\right)^2\left(\frac{1}{N^2} + \frac{4b^2\left(N^2-1\right)}{\sigma^2 p^2\left(1-\omega\right)^2}\right)\right)\right\} \tag{90}$$

This implies that, holding fixed the number of partitions, N,

$$\frac{\partial\mathbb{E}\left[u^m\left(x\right)\right]}{\partial\omega} = \frac{p^2\sigma^2\left(1-\omega\right)}{12N^2} > 0 \tag{91}$$

$$\frac{\partial\mathbb{E}\left[u^m\left(x\right)\right]}{\partial p} = \frac{p\sigma^2}{12}\left(1 - \frac{\left(1-\omega\right)^2}{N^2}\right) > 0 \tag{92}$$

$$\frac{\partial^2\mathbb{E}\left[u^m\left(x\right)\right]}{\partial\omega\partial p} = \frac{p\sigma^2\left(1-\omega\right)}{6N^2} > 0 \tag{93}$$

Note that $N_{max} = \text{ceil}\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1 + 2\sigma\frac{p(1-\omega)}{b}}\right)$ also depends on both $\omega$and p; it is decreasing in the former and increasing in the latter. □

Proof of Proposition 2

At each threshold, the manager must be indifferent to disclosing his signal or remaining silent. We can compare the difference in his expected utility under each approach using the expression found in (53):

$$\Delta\left(d_1 = \emptyset, d_2 = x; x\right) = \left(1-\omega\right)^2\left(\mu_\emptyset - px\right)\times\left(\tfrac{b}{(1-\omega)} + px - \tfrac{(\mu_\emptyset + px)}{2}\right) \tag{94}$$

$$= \left(1-\omega\right)^2\left(\mu_\emptyset - px\right)\times\left(\tfrac{b}{(1-\omega)} + \tfrac{px - \mu_\emptyset}{2}\right) \tag{95}$$

Note that

$$\Delta_x \equiv \frac{\partial \Delta\left(d_1=\emptyset, d_2=x; x\right)}{\partial x} = -p\left(1-\omega\right)\left(b+\left(1-\omega\right)\left(px-\mu_{\emptyset}\right)\right) \tag{96}$$

$$\Delta_{xx} \equiv \frac{\partial^2 \Delta\left(d_1=\emptyset, d_2=x; x\right)}{\partial x^2} = -p^2\left(1-\omega\right)^2 < 0 \tag{97}$$

This suggests Δ is hump-shaped in x: for sufficiently low x, $\Delta_x > 0$, while for sufficiently high x, $\Delta_x < 0$. Moreover, note that there are two roots $\{x_l, x_h\}$ of $\Delta\left(\emptyset, x; x\right) = 0$, given by

$$x_h = \frac{1}{p}\mu_{\emptyset}, \quad x_l = \frac{1}{p}\left(\mu_{\emptyset} - \tfrac{2b}{(1-\omega)}\right) \tag{98}$$

and note that

$$\Delta_x\left(x_h\right) = -p\left(1-\rho\right)\left(1-\omega\right)b_i < 0 \tag{99}$$

$$\Delta_x\left(x_l\right) = p\left(1-\rho\right)\left(1-\omega\right)b_i > 0 \tag{100}$$

This implies that there are two potential types of equilibria:

Case 1 ($x_l \leq -\frac{\sigma}{2}$): In this case, there would be disclosure above x_h only. As a result,

$$\mu_{\emptyset} = \frac{(1-q)*0 + \left(q\frac{x_h+\sigma/2}{\sigma}\right)\left(p\frac{x_h-\sigma/2}{2}\right)}{1-q+q\frac{x_h+\sigma/2}{\sigma}} \tag{101}$$

and so

$$x_h = \frac{1}{p}\mu_{\emptyset} \tag{102}$$

$$= \sigma\frac{2\sqrt{1-q}-(2-q)}{2q} \tag{103}$$

Moreover, this implies that

$$x_l = \frac{1}{p}\left(\mu_{\emptyset} - \tfrac{2b}{(1-\omega)}\right) \tag{104}$$

$$= \frac{1}{p}\left(p\frac{\sigma}{2}\frac{2\sqrt{1-q}-(2-q)}{q} - \tfrac{2b}{(1-\omega)}\right) \tag{105}$$

$$= \frac{\sigma}{2}\frac{2\sqrt{1-q}-(2-q)}{q} - \frac{2b}{p\left(1-\omega\right)} \tag{106}$$

We need $x_l \le -\frac{\sigma}{2}$, which implies that this is an equilibrium if and only if

$$\frac{\sigma}{2}\frac{2\sqrt{1-q}-(2-q)}{q} - \frac{2b}{p(1-\omega)} \le -\frac{\sigma}{2} \tag{107}$$

$$\Leftrightarrow \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q} \le \frac{b}{p(1-\omega)} \tag{108}$$

Case 2 ($x_l > -\frac{\sigma}{2}$): Suppose that (108) does not hold. Then if there is going to be an equilibrium of the posited form, it must be that we disclose truthfully above x_h and below x_l. As a result,

$$\mu_\emptyset = \frac{(1-q)\,0 + \left(q\frac{x_h-x_l}{\sigma}\right)\left(p\frac{x_h+x_l}{2}\right)}{1-q+q\frac{x_h-x_l}{\sigma}} \tag{109}$$

Note that $x_l = x_h - \frac{2b}{p(1-\omega)}$. Using this and $\mu_\emptyset$ we can solve for

$$\Rightarrow x_h = -\frac{2q}{(1-q)\,\sigma}\left(\frac{b}{p(1-\omega)}\right)^2 \tag{110}$$

which implies that

$$x_l = -\frac{2b}{p(1-\omega)}\left(\frac{q}{(1-q)\,\sigma}\left(\frac{b}{p(1-\omega)}\right)+1\right) \tag{111}$$

For this to be an equilibrium, we need $x_h < \frac{\sigma}{2}$ and $-\frac{\sigma}{2} < x_l$, i.e.,

$$-\frac{2q}{(1-q)\,\sigma}\left(\frac{b}{p(1-\omega)}\right)^2 < \frac{\sigma}{2} \tag{112}$$

which is always the case and

$$-\frac{\sigma}{2} < -\frac{2b}{p(1-\omega)}\left(\frac{q}{(1-q)\,\sigma}\left(\frac{b}{p(1-\omega)}\right)+1\right) \tag{113}$$

$$\frac{\sigma}{4} > \frac{b}{p(1-\omega)}\left(\frac{q}{(1-q)\,\sigma}\left(\frac{b}{p(1-\omega)}\right)+1\right) \tag{114}$$

$$0 > \frac{q}{(1-q)\,\sigma}\left(\frac{b}{p(1-\omega)}\right)^2 + \frac{b}{p(1-\omega)} - \frac{\sigma}{4} \tag{115}$$

This is true if and only if

$$\frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{q} > \frac{2b}{p(1-\omega)} \tag{116}$$

Taken together, this establishes the result. □

Proof of Corollary 2

The informativeness of the manager's disclosure is $var\left(\mathbb{E}\left[\theta|d\right]\right)$. Moreover, as in the proof of corollary 1, the value of the firm can be written:

$$\begin{aligned}\mathbb{E}\left[V\left(R,k\right)\right] &= \mathbb{E}\left[Rk-\tfrac{1}{2}k^2\right] && (117)\\ &= \tfrac{1}{2}\left(\mu^2+\text{var}\left(\mathbb{E}\left[\theta\middle|d\right]\right)\right) && (118)\end{aligned}$$

Note that, $\mathbb{E}\left[\mathbb{E}\left[\theta|d\right]\right]=0$ and so,

$$\begin{aligned}var\left(\mathbb{E}\left[\theta|d\right]\right) &= \tfrac{q}{\sigma}\int_{-\frac{\sigma}{2}}^{x_l}\left(px\right)^2\,dx+\tfrac{q}{\sigma}\int_{x_h}^{\frac{\sigma}{2}}\left(px\right)^2\,dx+\left(q\frac{x_h-x_l}{\sigma}+\left(1-q\right)\right)\left(\mu_\emptyset\right)^2 && (119)\\ &= \tfrac{qp^2}{3\sigma}\left[x_l^3-x_h^3+\tfrac{\sigma^3}{4}\right]+\left(q\frac{x_h-x_l}{\sigma}+\left(1-q\right)\right)\left(\mu_\emptyset\right)^2. && (120)\end{aligned}$$

To simplify, we can rewrite the expectation given no disclosure:

$$\begin{aligned}\mu_\emptyset &= \frac{\left(q\frac{x_h-x_l}{\sigma}\right)\left(p\frac{x_h+x_l}{2}\right)}{1-q+q\frac{x_h-x_l}{\sigma}}=\frac{\left(\frac{qp}{2\sigma}\left(x_h^2-x_l^2\right)\right)}{1-q+q\frac{x_h-x_l}{\sigma}}\implies && (121)\\ var\left(\mathbb{E}\left[\theta|d\right]\right) &= \tfrac{qp^2}{3\sigma}\left[x_l^3-x_h^3+\tfrac{\sigma^3}{4}\right]+\frac{qp}{2\sigma}\left[\left(x_h^2-x_l^2\right)\mu_\emptyset\right] && (122)\\ &= \tfrac{qp^2}{3\sigma}\left[x_l^3-x_h^3\right]+\frac{\left(\frac{qp}{2\sigma}\left(x_h^2-x_l^2\right)\right)^2}{1-q+q\frac{x_h-x_l}{\sigma}}+\tfrac{qp^2\sigma^2}{12}. && (123)\end{aligned}$$

In **case 1** **(**$x_l\le-\frac{\sigma}{2}$**)**, the value of the firm is independent of ω because x_h doesn't depend on ω. In **case 2** **(**$x_l>-\frac{\sigma}{2}$**)**, this is no longer the case. To simplify, we utilize the fact that $x_h-x_l=\frac{2b}{p(1-\omega)}$ which implies

$$\begin{aligned}var\left(\mathbb{E}\left[\theta|d\right]\right) &= -\tfrac{qp^2}{3\sigma}\tfrac{2b}{p(1-\omega)}\left[x_l^2+x_h^2+x_lx_h\right]+\frac{\frac{q^2p^2}{4\sigma^2}\left(\frac{2b}{p(1-\omega)}\right)^2\left(x_h+x_l\right)^2}{1-q+q\frac{2b}{\sigma p(1-\omega)}}+\tfrac{qp^2\sigma^2}{12} && (124)\\ &= \frac{-q\left(\frac{48b^4q}{(1-q)(1-\omega)^4}+\frac{32b^3p\sigma}{(1-\omega)^3}\right)}{12p^2\sigma^2}+\tfrac{qp^2\sigma^2}{12}. && (125)\end{aligned}$$

Thus, in **case 2,**

$$\frac{\partial var\left(\mathbb{E}\left[\theta|d\right]\right)}{\partial\omega}=\frac{-q}{p^2\sigma^2}\left(\frac{16b^4q}{(1-q)(1-\omega)^5}+\frac{8b^3p\sigma}{(1-\omega)^4}\right)<0 \qquad (126)$$

By continuity, this implies that firm value in **case 2** exceeds firm value in **case 1**. Finally, using the expressions above the cutoffs for disclosure, firm value in **case 1 is**

$$\mathbb{E}\left[V\left(R,k\right)\right]=\tfrac{1}{2}\left(\mu^2+\frac{qp^2\sigma^2}{12}\left(1-\left(\frac{\left(1-q\right)\left(8\left(1-\sqrt{1-q}\right)-q^2-4q\right)}{q^3}\right)\right)\right) \tag{127}$$

while in **case 2,**

$$\mathbb{E}\left[V\left(R,k\right)\right]=\tfrac{1}{2}\left(\mu^2+\tfrac{qp^2\sigma^2}{12}\left(1-\left(\frac{48b^4q}{p^4\sigma^4(1-q)(1-\omega)^4}+\frac{32b^3}{p^3\sigma^3(1-\omega)^3}\right)\right)\right) \tag{128}$$

In both cases, it is clear that $\frac{\partial\mathbb{E}[V(R,k)]}{\partial p}>0$. Finally, note that

$$Z\left(q\right)\equiv\left(8\left(1-\sqrt{1-q}\right)-q^2-4q\right)>0\,\forall q\in\left(0,1\right). \tag{129}$$

This can be shown by observing that

$$\frac{\partial Z}{\partial q}=-2q-4+4\left(1-q\right)^{\frac{-1}{2}} \tag{130}$$

$$\frac{\partial^2 Z}{\partial q^2}=-2+2\left(1-q\right)^{\frac{-3}{2}} \tag{131}$$

$$\frac{\partial^3 Z}{\partial q^3}=3\left(1-q\right)^{\frac{-5}{2}} \tag{132}$$

Note that $\frac{\partial Z}{\partial q}$ and $\frac{\partial^2 Z}{\partial q^2}$ are zero when $q=0$, while $\frac{\partial^3 Z}{\partial q^3}>0$ for all $q\in(0,1)$. This implies that $\frac{\partial Z}{\partial q}>0$ for all $q\in(0,1)$ and since $Z\left(0\right)=0$, $Z\left(q\right)>0$ for all $q\in(0,1)$. □

Proof of Lemma 2

As above, we can write the manager's expected utility as a function of his disclosure and information set as

$$u^m\left(\mathcal{I}_m,d\right)\equiv\mathbb{E}_m\left[\left(R+b\right)k^p\left(d\right)-\tfrac{1}{2}k^p\left(d\right)^2\left|\mathcal{I}_m\right.\right] \tag{133}$$

$$=\tfrac{1}{2}\left(\mathbb{E}_m\left[\left(R+b\right)\left|\mathcal{I}_m\right.\right]\right)^2-\tfrac{1}{2}\left(\mathbb{E}_m\left[\left(R+b\right)\left|\mathcal{I}_m\right.\right]-\mathbb{E}_m\left[k^p\left(d\right)\left|\mathcal{I}_m,d\right.\right]\right)^2. \tag{134}$$

There are three cases to consider:

(1) If the manager observes nothing (i.e., $s = \emptyset$), then his utility is

$$\begin{aligned} u^m(\emptyset, \emptyset) &= \mathbb{E}\left[\tfrac{1}{2}(\mu + b)^2 - \tfrac{1}{2}\left(\mu + b - ((1-\omega)\mu_\emptyset + \omega(0) + \mu)\right)^2\right] \\ &= \tfrac{1}{2}\left[(\mu + b)^2 - (b - (1-\omega)\mu_\emptyset)^2\right]. \end{aligned}$$

(2) If the manager observes x and discloses it, then his utility is

$$\begin{aligned} u^m(x, x) &= \mathbb{E}\left[\tfrac{1}{2}(b + \mu + px)^2 - \tfrac{1}{2}(b + \mu + px - (\mu + px))^2\right] \\ &= \tfrac{1}{2}\left[(b + \mu + px)^2 - b^2\right] \end{aligned}$$

(3) If the manager observes x and does not disclose it, then his utility is

$$\begin{aligned} u^m(x, \emptyset) &= \mathbb{E}\left[\tfrac{1}{2}(b + \mu + px)^2 - \tfrac{1}{2}\left(b + \mu + px - (\mu + (1-\omega)\mu_\emptyset + \omega px)\right)^2\right] \\ &= \tfrac{1}{2}\left[(b + \mu + px)^2 - (b + (1-\omega)(px - \mu_\emptyset))^2\right]. \\ &= u^m(x, x) - \tfrac{1}{2}\left[2b(1-\omega)(px - \mu_\emptyset) + ((1-\omega)(px - \mu_\emptyset))^2\right] \end{aligned}$$

Note that if the manager always disclosed, his expected utility would be

$$\begin{aligned} \mathbb{E}\left[u^m(x, x)\right] &= \tfrac{1}{2}\int_{-\frac{\sigma}{2}}^{\frac{\sigma}{2}} \frac{1}{\sigma}\left[(b + \mu + px)^2 - b^2\right] dx && (135) \\ &= \tfrac{1}{2}\left\{2b\mu + \mu^2 + \frac{p^2\sigma^2}{12}\right\} && (136) \end{aligned}$$

Thus,

$$\mathbb{E}\left[u^m | x\right] = \mathbb{E}\left[u^m(x, x)\right] - \tfrac{1}{2}\left(\tfrac{1}{\sigma}\int_{x_l}^{x_h} 2b(1-\omega)(px - \mu_\emptyset) + ((1-\omega)(px - \mu_\emptyset))^2 \, dx\right), \tag{137}$$

and so,

$$\mathbb{E}\left[u^m\right] = (1-q)\, u^m(\emptyset, \emptyset) + q\mathbb{E}\left[u^m | x\right]. \tag{138}$$

In **case 2**, after substituting in the expressions for $x_l, x_h, \mu_\emptyset$, this reduces to

$$\mathbb{E}\left[u^m\right] = \tfrac{1}{2}\left\{2b\mu + \mu^2 + \frac{qp^2\sigma^2}{12}\left(1 - (1-\omega)^2\left(\frac{48b^4q}{p^4\sigma^4(1-q)(1-\omega)^4} + \frac{32b^3}{p^3\sigma^3(1-\omega)^3}\right)\right)\right\}. \tag{139}$$

Therefore, in **case 2**,

$$\frac{\partial \mathbb{E}\left[u^m\right]}{\partial \omega} = \frac{qp^2\sigma^2}{12}\left(\frac{48b^4q}{p^4\sigma^4(1-q)(1-\omega)^3} + \frac{16b^3}{p^3\sigma^3\left(1-\omega\right)^2}\right) \tag{140}$$

$$\frac{\partial \mathbb{E}\left[u^m\right]}{\partial p} = \frac{q\sigma^2}{12}\left(2p + \frac{48b^4q}{p^3\sigma^4(1-q)(1-\omega)^2} + \frac{16b^3}{p^2\sigma^3\left(1-\omega\right)}\right) > 0 \tag{141}$$

$$\frac{\partial^2 \mathbb{E}\left[u^m\right]}{\partial p \partial \omega} = \frac{q\sigma^2}{12}\left(\frac{96b^4q}{p^3\sigma^4(1-q)(1-\omega)^3} + \frac{16b^3}{p^2\sigma^3\left(1-\omega\right)^2}\right) > 0 \tag{142}$$

In **case 1**, after substituting in the expressions for $x_l, x_h, \mu_\emptyset$, this reduces to

$$\mathbb{E}\left[u^m\right] = \frac{1}{2}\left(2b\mu + \mu^2 + \frac{qp^2\sigma^2}{12}\left(1 - (1-\omega)^2\left(\frac{(1-q)\left(8\left(1-\sqrt{1-q}\right) - q^2 - 4q\right)}{q^3}\right)\right)\right). \tag{143}$$

Therefore, in **case 1**,

$$\frac{\partial \mathbb{E}\left[u^m\right]}{\partial \omega} = \frac{qp^2\sigma^2}{12}\left((1-\omega)\left(\frac{(1-q)\left(8\left(1-\sqrt{1-q}\right) - q^2 - 4q\right)}{q^3}\right)\right) > 0 \tag{144}$$

$$\frac{\partial \mathbb{E}\left[u^m\right]}{\partial p} = \frac{qp\sigma^2}{12}\left(1 - (1-\omega)^2\left(\frac{(1-q)\left(8\left(1-\sqrt{1-q}\right) - q^2 - 4q\right)}{q^3}\right)\right) > 0 \tag{145}$$

$$\frac{\partial^2 \mathbb{E}\left[u^m\right]}{\partial p \partial \omega} = \frac{qp\sigma^2}{6}\left((1-\omega)\left(\frac{(1-q)\left(8\left(1-\sqrt{1-q}\right) - q^2 - 4q\right)}{q^3}\right)\right) > 0 \tag{146}$$

Finally, as in the proof of Corollary 2, by continuity, these expressions also imply that the manager's expected utility in **case 2** exceeds that in **case 1**. □

Proof of Proposition 3

Note that

$$\bar{u}(\rho, p) = \tfrac{1}{2}\mathbb{E}_m\left[\mathbb{E}_m\left[(R+b)\,|x\right]^2 - \left(\mathbb{E}_m\left[(R+b)\,|x\right] - k^p(y)\right)^2\right] \tag{147}$$

$$= \tfrac{1}{2}\mathbb{E}_m\left[(b+\mu+px)^2 - \left(b+\mu+px-\left(p\left[(1-\omega)\rho y+\omega x\right]+\mu\right)^2\right)\right] \tag{148}$$

$$= \tfrac{1}{2}\mathbb{E}_m\left[(b+\mu+px)^2 - (b+p(1-\omega)(x-\rho y))^2\right] \tag{149}$$

$$= \tfrac{1}{2}(b+\mu)^2 + \tfrac{p^2\sigma^2}{24} - \tfrac{1}{2}\left(b^2 + 2b(1-\omega)\mathbb{E}_m\left[(x-\rho y)\right] + p^2(1-\omega)^2\mathbb{E}\left[(x-\rho y)^2\right]\right) \tag{150}$$

$$= \tfrac{1}{2}\left(2b\mu+\mu^2\right) + \tfrac{p^2\sigma^2}{24} - \tfrac{1}{2}\left(p^2(1-\omega)^2\mathbb{E}\left[(x-\rho y)^2\right]\right) \tag{151}$$

$$= \tfrac{1}{2}\left(2b\mu+\mu^2\right) + \tfrac{p^2\sigma^2}{24} - \tfrac{1}{2}\left(p^2(1-\omega)^2\mathbb{E}\left[\mathrm{var}(x|y)\right]\right) \tag{152}$$

Since,

$$\mathbb{E}\left[\mathrm{var}(x|y)\right] = \mathrm{var}(x) - \mathrm{var}(\mathbb{E}[x|y]) = \frac{\sigma^2}{12} - \mathrm{var}(\rho y) = \left(1-\rho^2\right)\frac{\sigma^2}{12}, \tag{153}$$

we have

$$\bar{u}(\rho, p) = \tfrac{1}{2}\left(2b\mu+\mu^2\right) + \frac{p^2\sigma^2}{24}\left(1-(1-\omega)^2\left(1-\rho^2\right)\right), \tag{154}$$

which implies:

$$\frac{\partial}{\partial\rho}\bar{u} = \rho\tfrac{p^2\sigma^2}{12}(1-\omega)^2 > 0, \quad \frac{\partial^2}{\partial\rho\partial\omega}\bar{u} = -\rho\frac{p^2\sigma^2}{6}(1-\omega) < 0 \tag{155}$$

$$\frac{\partial}{\partial p}\bar{u} = \frac{1}{12}p\sigma^2\left(1-\left(1-\rho^2\right)(1-\omega)^2\right) > 0, \quad \frac{\partial^2}{\partial p\partial\omega}\bar{u} = \frac{1}{6}p\left(1-\rho^2\right)\sigma^2(1-\omega) > 0 \tag{156}$$

$$\frac{\partial^2}{\partial p\partial\rho}\bar{u} = \frac{1}{6}p\rho\sigma^2(1-\omega)^2 > 0, \quad \frac{\partial}{\partial\omega}\bar{u} = \frac{1}{12}p^2\left(1-\rho^2\right)\sigma^2(1-\omega) > 0 \tag{157}$$

Similarly,

$$\mathbb{E}\left[V(R,k)\right] = \tfrac{1}{2}\left(\mu^2 + \mathrm{var}(\theta) - \mathbb{E}\left[\mathrm{var}(\theta|y)\right]\right) = \tfrac{1}{2}\left(\mu^2 + \rho^2 p^2\frac{\sigma^2}{12}\right) \tag{158}$$

since $\mathbb{E}\left[\mathrm{var}(\theta|y)\right] = \mathrm{var}(\theta) - \mathrm{var}\left[\mathbb{E}[\theta|y]\right] = \left(1-\rho^2p^2\right)\frac{\sigma^2}{12}$. Inspecting the relevant partial derivatives completes the result. □

Proof of Proposition 4

We can rewrite (35) so that the principal should retain control if and only if

$$\frac{\sigma^2}{12}\left(\frac{p\left(1-\omega\right)}{b}\right)^2 < N^2\left(\left(1-\omega\right)^2-\frac{\left(N^2-1\right)}{3}\right). \tag{159}$$

There are two cases to consider. If $b < \frac{\sigma p(1-\omega)}{4}$, communication is informative i.e., $N \geq 2$. In this case, equation 159 never holds because $\frac{\sigma^2}{12}\left(\frac{p(1-\omega)}{b}\right)^2 > 0 \geq N^2\left(\left(1-\omega\right)^2-\frac{\left(N^2-1\right)}{3}\right)$. As a result, the principal always delegates.

If $b > \frac{\sigma p(1-\omega)}{4}$, then communication is uninformative (as shown in the proof of Proposition 1), i.e., $N = 1$. In this case, equation 159 holds (and the principal retains control) as long as

$$\frac{p\sigma}{2\sqrt{3}} < b. \tag{160}$$

The curse of knowledge could only reduce delegation if $b < \frac{\sigma p}{4}$ so that absent the curse of knowledge, communication would be informative (and therefore the principal delegates). But if this is true, then even if $b > \frac{\sigma p(1-\omega)}{4}$, so that any communication would be uninformative, the principal will still choose to delegate since (160) holds. □

Proof of Proposition 5

There are two cases to consider. If $\frac{b}{p(1-\omega)} \geq \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$, then the principal should retain control if and only if

$$b^2 > \left(\frac{qp^2\sigma^2}{12}\right)\frac{\left(1-q\right)\left(8\left(1-\sqrt{1-q}\right)-q^2-4q\right)}{q^3} \iff \tag{161}$$

$$b = \frac{p\sigma}{2\sqrt{3}}\chi(q) \tag{162}$$

$$\chi(q) \equiv \frac{\sqrt{\left(1-q\right)\left(8\left(1-\sqrt{1-q}\right)-q^2-4q\right)}}{q} \tag{163}$$

If $\frac{b}{p(1-\omega)} < \frac{\sigma\left(\sqrt{1-q}-(1-q)\right)}{2q}$, then the principal should retain control if and only if

$$b^2 > \frac{qp^2\sigma^2}{12}\left(\frac{48b^4q}{p^4\sigma^4(1-q)(1-\omega)^4}+\frac{32b^3}{p^3\sigma^3(1-\omega)^3}\right) \tag{164}$$

$$(1-\omega)^2 > \frac{4b^2q^2}{p^2\sigma^2(1-q)(1-\omega)^2}+\frac{8bq}{3p\sigma(1-\omega)} \tag{165}$$

$$(1-\omega)^2 > \frac{4q^2}{\sigma^2(1-q)}\left(\frac{b}{p\,(1-\omega)}\right)^2+\frac{8q}{3\sigma}\left(\frac{b}{p\,(1-\omega)}\right) \tag{166}$$

which establishes the result. □

B Covert information acquisition in cheap talk

In our benchmark analysis, we study information acquisition assuming that principal observes the quantity of information acquired by the manager, but not its content i.e., information acquisition is *overt*.In this appendix, we show that our results are similar in a setting where the principal observes neither the quantity nor the content of the manager's information i.e., when information acquisition is *covert*.

In the covert setting, unlike in the overt setting, the principal does not observe the amount of information acquired by the manager. Formally, this implies that a Perfect Bayesian Equilibrium of the covert game must additionally specify the principal's beliefs about the manager's information choice. The other elements are the same in both setting. Assume that the principal believes that manager's will choose information precision p^e and managers acquire information with precision p. Given this,

$$\mu_m \equiv \mathbb{E}_m\left[(R+b)\,\middle|x\right] = b+\mu+px \tag{167}$$

and

$$\mathbb{E}_m\left[k^p\left(d\left(n\right)\right)\right] = (1-\omega)\,p_e\left(\tfrac{s(n-1)+s(n)}{2}\right)+\omega px+\mu. \tag{168}$$

Conditional on observing x, the manager's expected utility is given by:

$$u^m\left(x\right) \equiv \mathbb{E}_m\left[(R+b)\,k^p\left(d\left(x\right)\right)-\tfrac{1}{2}k^p\left(d\left(x\right)\right)^2\middle|x\right] \tag{169}$$

$$= \mathbb{E}_m\left[(R+b)\,\middle|x\right]k^p\left(d\left(x\right)\right)-\tfrac{1}{2}k^p\left(d\left(x\right)\right)^2 \tag{170}$$

$$= \tfrac{1}{2}\left(\mu_m+b\right)^2-\tfrac{1}{2}\mathbb{E}_m\left[\left(\mu_m+b-\mu_p\right)^2\right] \tag{171}$$

Substituting μ_m and μ_p into the above expression:

$$u^m(x) = \frac{1}{2}(b+\mu+px)^2 - \frac{1}{2}\left(b+(1-\omega)\left(px - p_e\frac{s(n-1)+s(n)}{2}\right)\right)^2. \tag{172}$$

Thus the utility of the managers choosing information precision p when principal believes it to be p_e is given by ,

$$\begin{aligned} u^m(p,p_e,\omega) &= \mathbb{E}\left[u^m(x)\right] \\ &= \mathbb{E}\left[\frac{1}{2}(b+\mu+px)^2\right] - \frac{1}{2}\sum_{n=1}^{N}\int_{s(n-1)}^{s(n)}\frac{1}{\sigma}\left(b+(1-\omega)\left(px-p_e\frac{s(n-1)+s(n)}{2}\right)\right)^2 dx \\ &= \frac{(b+\mu)^2}{2}+\frac{p^2\sigma^2}{24}-\frac{b^2}{2}-\frac{1}{2}\frac{(1-\omega)^2}{24p\sigma}\sum_{i=1}^{N}\left[\begin{array}{c}(s(n)(2p-p_e)-p_e s(n-1))^3 \\ -(s(n-1)(2p-p_e)-p_e s(n))^3\end{array}\right] \end{aligned}$$

This implies that, holding fixed the number of partitions, N,

$$\frac{\partial u^m(p,p_e,\omega)}{\partial\omega}|_{p=p_e} = \frac{p^2\sigma^2(1-\omega)}{12N^2} > 0 \tag{173}$$

$$\frac{\partial u^m(p,p_e,\omega)}{\partial p}|_{p=p_e} = \frac{p\sigma^2}{12} - \frac{1}{2}\sum_{n=1}^{N}\int_{s(n-1)}^{s(n)}\frac{1}{\sigma}2\left(b+(1-\omega)\left(px-p_e\frac{s(n-1)+s(n)}{2}\right)\right)(1-\omega)\,xdx \tag{174}$$

$$= \frac{p\sigma^2}{12} - \frac{(1-\omega)^2}{\sigma}p\sum_{n=1}^{N}\int_{s(n-1)}^{s(n)}\left(x^2 - x\frac{s(n-1)+s(n)}{2}\right)dx \tag{175}$$

$$= \frac{p\sigma^2}{12} - \frac{(1-\omega)^2}{\sigma}p\sum_{n=1}^{N}\int_{s(n-1)}^{s(n)}\left(\left(x-\frac{s(n-1)+s(n)}{2}\right)^2 - \left(\frac{s(n-1)+s(n)}{2}\right)^2\right)dx \tag{176}$$

$$\frac{\partial^2\mathbb{E}\left[u^m(x)\right]}{\partial\omega\partial p}|_{p=p_e} = \frac{2(1-\omega)}{\sigma}p\sum_{n=1}^{N}\int_{s(n-1)}^{s(n)}\left(x^2 - x\frac{s(n-1)+s(n)}{2}\right)dx > 0 \tag{177}$$

As in the benchmark analysis, the curse of knowledge increases the marginal value of infromation acquisition, and socan increase information acquisition. In particular, as long as the number of partitions remains fixed, the manager finds more value in increasing the precision of his private signal as the curse of knowledge grows.