Motivated Beliefs in Coordination Games

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Abstract

We characterize how wishful thinking affects the interpretation of information in economies with strategic and external effects. Optimal subjective beliefs generally exhibit over- or under-reaction to public information, depending on the relative impact of non-fundamental volatility on payoffs. Furthermore, we establish conditions under which players endogenously "agree to disagree" about public information, even though players are ex-ante symmetric. In contrast to models with rational expectations, better public information can increase belief dispersion, and the predictability of forecast errors using forecast revisions varies across economic environments. We explore implications for strategic investment, Cournot and Bertrand competition, and beauty contest models.

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1 Introduction

There is a large literature that documents dispersion in beliefs across investors and professional forecasters and, in particular, disagreement about the interpretation of public information.[1] Understanding the nature of this disagreement is important for understanding the functioning of markets and the macroeconomy. Yet, most models simply assume that agents have heterogeneous priors or "agree to disagree" about public news.[2] As such, they provide little guidance about when such disagreement can arise, and why it varies over time and across economic conditions.

We show how such disagreement about the interpretation of public information can arise *endogenously* within a generalized coordination game. In our setting, players face uncertainty about the precision of the public signal and optimally choose a subjective interpretation from a set of reasonable beliefs. We build on the extensive evidence from psychology and behavioral economics that individuals experience direct utility flows, called anticipatory utility, from their beliefs about future events (e.g., excitement about an upcoming celebration). In such cases, people tend to engage in "wishful thinking": they distort their interpretation of information to increase their current well-being, even when such distortions are costly.[3]

Even though players are ex-ante symmetrically informed and start with a common prior, we show that wishful thinking can lead them to endogenously agree to disagree about the precision of public information. We characterize the explicit conditions that give rise to such disagreement and show when subjective beliefs exhibit under- or over-reaction to public news. Such deviations from rational expectations depend not on the direction of strategic externalities (i.e., whether actions are complements or substitutes), but on their intensity. These optimal beliefs give rise to novel predictions about the aggregate response to public news, dispersion in beliefs, and forecast error predictability.

Intuition. We build on the framework of Angeletos and Pavan (2007) in which players'

[1]Kandel and Zilberfarb (1999) use inflation forecasts to find evidence of heterogeneous processing of public information. Patton and Timmermann (2010) use the term-structure of GDP and inflation forecasts to conclude that observed disagreement stems from heterogeneity in forecaster models. Manzan (2011) estimates there is significant heterogeneity in the processing of news in the Survey of Professional Forecasters. In financial markets, Kandel and Pearson (1995) show that analyst forecast dispersion increase around earnings announcements. Banerjee and Kremer (2010) and Bollerslev, Li, and Xue (2018) document evidence on the volume-volatility relation that is suggestive of investors agreeing to disagree about public announcements. Finally, Fedyk (2021) uses a direct measure of information consumption by professional investors to find direct evidence consistent with disagreement about public information.

[2]For instance, see Harrison and Kreps (1978), Harris and Raviv (1993), Morris (1994), Kandel and Pearson (1995), Hong and Stein (1999), Dumas, Kurshev, and Uppal (2009), Banerjee and Kremer (2010).

[3]For example, Oster, Shoulson, and Dorsey (2013) provides evidence of individuals choosing not to learn about their risk of a deadly disease, even when the test was effectively costless. More generally, our discussion in the next section and the recent survey by Bénabou and Tirole (2016) highlight the ubiquity of motivated beliefs.

optimal actions depend not only upon the state of the world (or "fundamentals") but also on strategic considerations. The precision of public information affects payoffs via two channels. Via the "fundamental uncertainty" channel, more precise information necessarily increases payoffs as the reduction in uncertainty leads to more efficient actions. The effect of the "non-fundamental volatility" channel depends upon whether correlated errors in the actions of others, driven by the common noise in the public signal, increase or decrease each player's payoffs. On the one hand, reducing the noise in the aggregate action affects individual payoffs through direct payoff externalities. On the other, since all players condition on the public information, more variability in the public signal can make it easier to predict others' actions. Moreover, this latter effect is particularly important when strategic considerations are stronger and when others put more weight on the public signal.[4]

The key quantity of interest for our analysis is the relative impact of non-fundamental volatility, denoted by χ, which reflects how important the second channel is with respect to the first. When χ is sufficiently large (small), the overall impact of more precise public information is to reduce (increase) payoffs. As a result, wishful thinking leads all players to under-estimate (over-estimate) the precision of the public information. For intermediate χ, however, we show there may not exist a symmetric equilibrium.[5] Instead, the unique equilibrium features mixed strategies so that players endogenously choose to "agree to disagree" about the public information: while some under-estimate the public precision others potentially over-react to it.

Implications. Since players' subjective beliefs about the public signal endogenously depend on the economic environment, our model's predictions (Section 5) differ not only from settings with rational expectations (including those with costly information acquisition or rational inattention) but also from models in which players are exogenously assumed to exhibit behavioral biases (e.g., dismissiveness or overconfidence).

Aggregate response to public information. A large empirical literature documents evidence of "sluggishness" in response to certain types of public macroeconomic news, but over-reaction to others. In contrast to models of rational inattention or costly information acquisition, we show that aggregate beliefs can *both* under- or over-react to public information, irrespective of whether actions are substitutes or complements.[6] Instead, the aggregate

[4]Predicting others' actions is important both when actions are substitutes and when they are complements - the intensity of the externality matters, not its sign. For example, in Section 6, we show that in settings with either Bertrand or Cournot competition, $\chi > 0$.

[5]Intuitively, if all other players under-react to the public signal, lowering the importance of the non-fundamental channel, a given player has an incentive to deviate and over-estimate its precision since it reduces perceived fundamental uncertainty. The opposite incentive arises when all other players over-weight the public signal as this increases the relative strength of the non-fundamental channel.

[6]This is distinct from the result that players' *actions* can put too much or too little weight on public infor-

reaction is monotonic in χ: players under-react to public information when χ is sufficiently large, but over-react to it otherwise.

Dispersion in beliefs. Under rational expectations, dispersion in beliefs arises because players have access to dispersed, private information. This generally implies that when public information precision increases, dispersion in beliefs falls as players put more weight on the public signal. In our setting, belief dispersion can also arise because players endogenously choose to disagree about the quality of the public signal. As a result, dispersion in beliefs can increase with the precision of public information because such changes endogenously affect the extent to which they disagree. Moreover, in contrast to models in which players are exogenously assumed to "agree to disagree" about the public signal, we find that belief dispersion is hump-shaped in the relative importance of non-fundamental volatility, χ.

Forecast error predictability. Coibion and Gorodnichenko (2012, 2015) and Bordalo, Gennaioli, Ma, and Shleifer (2020) use regressions of forecast errors on forecast revisions to document (i) under-reaction of consensus forecasts and (ii) over-reaction of individual forecasts for macroeconomic variables. Our model can reconcile both sets of results when χ is neither too large (which ensures over-reaction at the individual level) nor too small (which ensures under-reaction at the consensus level). We show that both aggregate and individual regression coefficients are increasing in χ, which distinguishes our model from settings where forecasters exogenously disagree about public information. Moreover, because players endogenously disagree, our model predicts that there should be *dispersion* in individual CG regression coefficients and that this dispersion is hump-shaped in χ, which further distinguishes our model.[7]

Applications and Extensions. In Section 6, we map our findings into a series of applications, nested within the generalized model. These include models of strategic investment, industrial organization models featuring Cournot and Bertrand competition, and the canonical beauty contest models. Specifically, we show how parameters in each application determine the impact of non-fundamental volatility, χ. We show that the necessary conditions for endogenous disagreement arise naturally in such settings, providing a novel explanation for why observed behavior (e.g., investment across firms, prices within an industry, macroeconomic forecasts) not only exhibits dispersion but also why disagreement varies with the economic and information environment.

mation depending on whether actions are strategic complements or substitutes (e.g., Morris and Shin (2002)) as players' *beliefs* in these models are generally rational and so incorporate public information efficiently.

[7]Specifically, when players have rational expectations, the individual CG coefficients are zero. When players are assumed to exhibit behavioral biases that are ex-ante symmetric, individual CG coefficients may not be zero, but are identical across players. Finally, if players exogenously agree to disagree, the dispersion in CG coefficients may be non-zero, but does not vary with χ.

While our benchmark model focuses on public information, in Section 7 we discuss how our analysis extends to subjective beliefs about private signals. We show that when players entertain subjective beliefs about the precision of their own private information, they tend to exhibit over-confidence: they endogenously choose to over-estimate the precision of the private information. In contrast, when players entertain subjective beliefs about the precision and correlation among others' private information, subjective beliefs depend on the relative impact of aggregate volatility versus dispersion in others' actions on payoffs.

2 Related Literature

The concept of anticipatory utility, or current subjective expected utility, dates to at least Jevons (1905) who considers agents who derive contemporaneous utility not simply from current actions but also the anticipation of future utility flows.[8] For example, an individual who anticipates a negative, future experience (e.g., a risky medical procedure) experiences a negative, contemporaneous utility flow (e.g., anxiety about potential bad outcomes). In contrast, beliefs about future, positive events can increase an agent's current utility (e.g., excitement about a long-awaited vacation). There is now an extensive literature that incorporates anticipatory utility into models of belief choice (e.g., Akerlof and Dickens (1982), Loewenstein (1987), Caplin and Leahy (2001), Eliaz and Spiegler (2006)). Importantly, individuals do not exhibit "multiple selves" but consciously hold a single set of beliefs about the world so that an individual's subjective beliefs affect his actions, too. As Bénabou and Tirole (2016) emphasize, this generates tension between holding "accurate" beliefs, which lead to ex-post optimal actions, and "desirable" beliefs, which increase contemporaneous utility.

The most closely related papers are Brunnermeier and Parker (2005) and Caplin and Leahy (2019). Brunnermeier and Parker (2005) show how subjective belief choice ("optimal expectations") can help explain a preference for skewness, portfolio under-diversification, and consumption/savings patterns. Caplin and Leahy (2019) shows how anticipatory utility and belief choice can generate a large number of common behavioral biases, such as confirmation bias, and procrastination. Our analysis builds on this earlier work, but our focus is different: we are interested in understanding how payoff externalities, strategic interaction, and the information environment, affect the interpretation of public and private information.

Banerjee, Davis, and Gondhi (2019) explores how wishful thinking affects investors' inter-

[8] We utilize the term anticipatory utility as distinct from the concept of *anticipated* utility, whereby agents hold fixed the parameters they learn about when choosing actions (e.g., Kreps (1998), Cogley and Sargent (2008)).

pretation of endogenous price information in a stylized financial market setting. In contrast, this paper focuses on the interpretation of *exogenous* public information in a more general setting, featuring both strategic substitutability and complementarity. Moreover, the generalized setting we analyze allows us to highlight how subjective beliefs can give rise to consistent patterns of disagreement and overconfidence across a wide range of applications.

A complementary approach to modeling subjective belief choice is ambiguity aversion and robust control (e.g., Hansen and Sargent (2001, 2008)). One approach to modeling the belief choice of an ambiguity-averse (or robust control) player is to assume that he chooses action a and subjective beliefs μ to solve

$$\min_{\mu \in \left[\underline{\mu}, \bar{\mu}\right]} \max_{a} \mathbb{E}_{\mu}\left[u\left(a\right)\right],$$

where $\mathbb{E}_{\mu}\left[u\left(a\right)\right]$ reflects the subjective expected utility from action a under "worst case" beliefs, where μ is chosen from a set of reasonable beliefs $\left[\underline{\mu}, \bar{\mu}\right]$.[9] In our setting, players solve an analogous problem: they choose action a and beliefs μ to solve:

$$\max_{\mu \in \left[\underline{\mu}, \bar{\mu}\right]} \max_{a} \mathbb{E}_{\mu}\left[u\left(a\right)\right].$$

In this case, however, the optimal μ reflects the "wishful thinking" that the agent engages in to increase anticipatory utility $\mathbb{E}_{\mu}\left[u\left(a\right)\right]$ – in a sense, the agent chooses actions that perform well given "best case" outcomes.[10] While both wishful thinking and ambiguity aversion are likely to be relevant to how individuals interpret information in strategic situations, we leave a model that incorporates both approaches for future work.

Our paper contributes to the rich literature on coordination games by highlighting the importance of subjective belief choice for understanding both individual and aggregate actions. We adopt the framework of Angeletos and Pavan (2007), who generalize the canonical "beauty contest" model of Morris and Shin (2002) to a large class of quadratic-Gaussian economies. Our paper is related to the literature following Hellwig and Veldkamp (2009), Myatt and Wallace (2012), and Colombo, Femminis, and Pavan (2014) that analyzes how strategic considerations affect information acquisition. In more recent work, Hébert and La'O (2020) explore the impact of costly rational inattention on players' actions and be-

[9] For example, Dupraz (2015) considers the impact of such preferences in the canonical beauty contest model of Morris and Shin (2002))

[10] It is worth noting that an alternate interpretation of robust control is that the agent's beliefs coincide with the objective distribution, but she faces uncertainty about these beliefs and chooses actions that perform well given adverse outcomes. Analogously, one could interpret the choices of a "wishful thinker" as actions that perform well given favorable outcomes.

liefs in a generalized coordination game. Our analysis is complementary to this literature and approach: we show that strategic considerations and, in particular, the impact of non-fundamental volatility on payoffs, can affect how players choose to interpret public information. Relative to this literature, our model generates novel, endogenous disagreement about public information even though players are ex-ante symmetric and share a common prior.

3 Model

3.1 Setup

Our analysis builds on the generalized setting formalized in Angeletos and Pavan (2007). There is a unit measure continuum of players indexed by $i \in [0,1]$. Each player chooses an action, $k_i \in \mathbb{R}$, to maximize his expected payoff. This payoff, U_i, also depends upon the true state of the world, θ, as well as the actions of all other players, denoted by the vector k_{-i}. We assume that U_i is (i) quadratic in its arguments and (ii) symmetric across the actions of other players (i.e., $U_i\left(k_i, k_{-i}, \theta\right) = U_i\left(k_i, k'_{-i}, \theta\right)$ for any permutation k'_{-i} of k_{-i}). Let $K \equiv \int_0^1 k_j dj$ denote the average action of all other players and $\sigma_k \equiv \left(\int_0^1 \left(k_j - K\right)^2 dj\right)^{\frac{1}{2}}$ denote the dispersion of others' actions.

As Angeletos and Pavan (2007) show, the above implies that payoffs can be expressed as a function $U_i \equiv u\left(k_i, K, \sigma_k, \theta\right)$, where $u\left(\cdot\right)$ is quadratic and its partials satisfy $u_{k\sigma} = u_{K\sigma} = u_{\theta\sigma} = 0$ and $u_\sigma(k, K, 0, \theta) = 0$ for all (k, K, θ). This generalized functional form ensures tractability while still preserving flexibility for our analysis.[11] For instance, this payoff structure includes settings in which aggregate activity can create positive or negative externalities ($u_K \neq 0$ or $u_\sigma \neq 0$) and allows for strategic substitutability or complementarity ($u_{kK} < 0$ or $u_{kK} > 0$, respectively). We make the following assumptions about player payoffs to ensure that the equilibrium action is unique and bounded.

Assumption 1. *$u\left(k_i, K, \sigma_K, \theta\right)$ satisfies both (i) $u_{kk} < 0$, and (ii) $-u_{kK}/u_{kk} < 1$.*

Equilibrium actions depend upon fundamentals, i.e., $u_{k\theta} \neq 0$, but players have incomplete information when choosing their actions. Specifically, $\theta \sim N\left(0, 1/\tau\right)$ and each player observes both a private signal s_i and a public signal s, where

$$s_i = \theta + \varepsilon_i, \quad \text{and} \quad s = \theta + \eta, \tag{1}$$

[11] As noted in Angeletos and Pavan (2007), one can also interpret this setting as a second-order approximation of a much broader class of economies.

where the common error $\eta \sim N(0, 1/\tau_\eta)$, and individual errors $\varepsilon_i \sim N(0, 1/\tau_e)$ are independent of θ and across each other.

We extend this generalized setting by assuming players face uncertainty about the quality of public information and allowing them to entertain subjective beliefs about the precision of such information.[12] Specifically, we allow player i to perceive the error in the public signal to be:

$$\eta \sim_i N\left(0, \frac{1}{\delta_{\eta,i}\tau_\eta}\right). \tag{2}$$

If $\delta_{\eta,i} = 1$, player i's beliefs coincide with the objective distribution: he exhibits **rational expectations**. When $\delta_{\eta,i}$ is greater than one, player i overweights the public signal when forming expectations: he believes the signal contains less noise than it actually does. The opposite is true when $\delta_{\eta,i}$ is less than one. These subjective beliefs, however, are bounded. Specifically, we assume the following.

Assumption 2. *Player i's subjective beliefs are bounded above and below i.e., there exists $0 < \underline{\delta} < 1$ and $1 < \bar{\delta} < \infty$, such that $\underline{\delta} \leq \delta_{\eta,i} \leq \bar{\delta}$ for all i.*

These bounds assume that players cannot completely ignore the public signal nor believe that it is perfectly informative. As we discuss in the next subsection, this is motivated by the evidence that individuals must hold "reasonable beliefs", i.e., the distance between their beliefs and the objective distribution cannot be too large.

In all other ways, the players are rational: in particular, they (i) take as given other players' actions and beliefs and (ii) update using Bayes' rule. For ease of notation, we denote the expectation and variance of random variable X, given player i's subjective beliefs $\delta_{\eta,i}$, by $\mathbb{E}_i[X]$ and $\text{var}_i[X]$, respectively.

Each player's optimal action, $k_i^*(\delta_{\eta,i})$, maximizes his expected payoff, given his subjective beliefs, i.e.,

$$k_i^*(\delta_{\eta,i}) \equiv \arg\max_{k_i} \mathbb{E}_i\left[u(k_i, K, \sigma_k, \theta)\,\middle|\, s_i, s\right]. \tag{3}$$

Given this optimal action, we denote player i's **anticipatory utility**, i.e., the contemporaneous utility flow he receives from his beliefs about his expected payoff, by:

$$AU_i(\delta_{\eta,i}) \equiv \mathbb{E}_i\left[u\left(k_i^*(\delta_{\eta,i}), K, \sigma_k, \theta\right)\right], \tag{4}$$

[12] In Section 7.1, we allow players to entertain subjective beliefs about their own private signal. In Section 7.2, we consider the implications when players hold subjective beliefs about other players' private information. The joint distribution of beliefs analyzed in the benchmark model can be found in equation (31).

whereas his expected **experienced utility**, given his chosen action, is

$$EU_i\left(\delta_{\eta,i}\right) = \mathbb{E}\left[u\left(k_i^*\left(\delta_{\eta,i}\right), K, \sigma_k, \theta\right)\right]. \tag{5}$$

Each player chooses his subjective beliefs to maximize a weighted average of his anticipatory utility and his experienced utility. Formally, player i's objective function is

$$\max_{\delta_{\eta,i}\in[\underline{\delta},\bar{\delta}]} \frac{1}{1+\psi}\left\{AU_i(\delta_{\eta,i}) + \psi EU_i(\delta_{\eta,i})\right\} \equiv TU_i(\delta_{\eta,i}), \tag{6}$$

where $\psi \geq 0$ scales the relative importance of experienced utility versus anticipatory utility.

The specification of the objective in (6) highlights the tradeoff that players face between "desirable" models (that increase anticipatory utility) against "accurate" models (that increase experienced utility) in a transparent and tractable manner.[13] When $\psi \to 0$, the player ignores the impact of her subjective belief distortion on her experienced utility - this leads to a pure focus on "desirable" models and maximal belief distortion within the set of reasonable beliefs. On the other hand, when $\psi \to \infty$, the player effectively only cares about the experienced utility and the optimal choice of beliefs converges to the rational expectations benchmark i.e., $\delta_{\eta,i} = 1$.

An equilibrium consists of actions $\{k_i^*\}_i$ and subjective beliefs $\{\delta_{\eta,i}\}_i$ such that (i) player i's action k_i^* maximizes (3) given his subjective beliefs, (ii) player i's subjective belief choice $\delta_{\eta,i}$ maximizes (6), and (iii) player i's subjective beliefs are consistent with the equilibrium choices of other players and satisfy Bayes rule, given subjective beliefs. In what follows, we distinguish between two types of equilibria: (i) a pure strategy, or symmetric, equilibrium in which all players have the same interpretation of the public signal (i.e., $\delta_{\eta,i}$ is the same), and (ii) a mixed strategy, or asymmetric, equilibrium in which players disagree about the interpretation of the public signal (i.e., the equilibrium $\delta_{\eta,i}$ differs across players).

3.2 Discussion of Assumptions

A natural interpretation of the set of "reasonable beliefs" is that they arise from the confidence intervals associated with their estimates of the joint distribution of fundamentals and signals. In their survey, Epley and Gilovich (2016) provide supportive evidence: individuals form a single, subjective model of the world through motivated reasoning, but it must be "reasonable", i.e., this model is naturally bound by the limits of the observable evidence. Moreover, incorporating experienced utility naturally imposes a cost on players who choose

[13] This is related to the specification of "psychological expected utility" in Caplin and Leahy (2001) and Caplin and Leahy (2004).

to deviate from rational expectations – this is analogous to the optimal expectations approach of Brunnermeier and Parker (2005) and is the basis of our general analysis in Section 4.3. An alternative approach would be to assume the cost of choosing subjective beliefs is dependent upon a measure of statistical distance (e.g., the K-L distance), as in Caplin and Leahy (2019). In Section 4.4, we show how this approach can be embedded in our model by endogenizing the set of "reasonable beliefs" in terms of the statistical (KL) distance from the objective distribution. By considering these cases sequentially, we make clear which implications arise from the effects of anticipatory utility (Section 4.2) and which are due to different assumptions about the cost of belief distortion.[14]

We restrict attention to subjective belief choice about public information for tractability and expositional clarity. In Section 7.1, we discuss the implications when we allow players to choose their subjective beliefs about the precision of their private signals. We find that players choose to exhibit over-confidence about their private information, and characterize how this over-confidence endogenously depends on strategic considerations and how it interacts with their subjective beliefs about the public signal. In Section 7.2, we summarize our results on players' optimal subjective beliefs about *other* players' private information. While such beliefs do not affect equilibrium behavior, we show that each player chooses to distort his beliefs about the precision of, and correlation across, others' private signals depending on the relative impact of volatility and dispersion on his perceived payoff. In Appendix C.4, we characterize how players choose subjective beliefs about the means of each signal. While players never distort their belief about their private signal, we show that across many applied settings, players may endogenously disagree about the mean of the public signal.

3.3 Equilbrium actions

We begin by first characterizing the equilibrium actions given an arbitrary choice of subjective beliefs. If all players observed θ perfectly, the optimal action would be $\kappa(\theta)$, where:

$$\kappa(\theta) = \underbrace{-\frac{u_k(0,0,0,0)}{u_{kk}+u_{kK}}}_{\equiv \kappa_0} \underbrace{-\frac{u_{k\theta}}{u_{kk}+u_{kK}}}_{\equiv \kappa_1} \theta \tag{7}$$

[14] As Caplin and Leahy (2019) point out, these different approaches do not require players to know, or believe, the objective distribution but simply that they find it difficult to hold beliefs that deviate too far from it. This may be because such deviations lead to suboptimal decisions (in terms of experienced utility, as we model it), or because it is more difficult to empirically collect supportive evidence for such beliefs, or because they have a preference for social conformity.

However, players have incomplete information. Given player $i's$ subjective beliefs and the realization of s_i and s, his optimal action, k_i is

$$k_i^*(\delta_{\eta,i}) = r\mathbb{E}_i[K|s_i, s] + (1-r)\mathbb{E}_i[\kappa(\theta)|s_i, s], \tag{8}$$

where $r \equiv -\frac{u_{kK}}{u_{kk}}$ is the **equilibrium degree of coordination** across players. This term captures the extent to which each player chooses to align his action with his expectation of others' actions, K, relative to his expectation of the full-information target, $\kappa(\theta)$. Given our assumptions on the joint distribution of fundamentals and signals, Bayesian updating implies that player i's conditional beliefs about θ are given by

$$\mathbb{E}_i[\theta|s_i, s] = A_i s_i + B_i s, \text{ and } \text{var}_i[\theta|s_i, s] = \frac{1 - A_i - B_i}{\tau}, \tag{9}$$

where player i's weights on the private and public signals are given by:

$$A_i \equiv \frac{\tau_e}{\tau + \tau_e + \delta_{\eta,i}\tau_\eta}, \text{ and } B_i \equiv \frac{\delta_{\eta,i}\tau_\eta}{\tau + \tau_e + \delta_{\eta,i}\tau_\eta}. \tag{10}$$

We define the aggregate weight on private information, $A \equiv \int_i A_i di$, and the public information, $B \equiv \int_i B_i di$, analogously. The following lemma characterizes equilibrium actions.

Lemma 1. *Given a choice of subjective beliefs $\{\delta_{\eta,i}\}_i$ for each player i, there always exists a unique, linear equilibrium in which player i's optimal action is given by: $k_i(\delta_{\eta,i}) = \kappa_0 + \alpha_i s_i + \beta_i s$ and the aggregate action is given by: $K = \kappa_0 + \alpha\theta + \beta s$, where $\alpha_i = \frac{1-r}{1-rA}A_i\kappa_1$, $\beta_i = \frac{(1-r)B_i + Br}{1-rA}\kappa_1$, $\alpha = \frac{A(1-r)}{1-rA}\kappa_1$, and $\beta = \frac{B}{1-rA}\kappa_1$.*

While the form of this solution is standard, the distinguishing feature of our setting is that the relative weights placed on the private and public signal (α_i and β_i, respectively) are distorted by each player i's subjective beliefs (through A_i and B_i). Moreover, when there is a benefit to coordination, i.e., if $r \neq 0$, these weights also depend upon the subjective beliefs of *other* players through the aggregate weights on private and public information (A and B, respectively) which player i takes as given. We show below that both dimensions play a critical role in the equilibrium choice of beliefs.

4 Optimal subjective beliefs

This section presents our main analysis. We first characterize the key trade-off that a player faces when choosing his subjective beliefs (Section 4.1). To build intuition, we then solve the model when players maximize anticipatory utility only i.e., when $\psi = 0$ (Section 4.2). In

Section 4.3, we characterize the equilibrium in the general setting, i.e., when $\psi \neq 0$. Finally, Section 4.4 discusses how the type of equilibrium depends on the set of reasonable beliefs $\left[\underline{\delta}, \bar{\delta}\right]$, and how one can endogenize this set.

4.1 Relative impact of non-fundamental volatility

We begin with a characterization of player i's anticipatory utility, which depends on his beliefs about both fundamentals and the equilibrium actions of others.

Lemma 2. *Given player i's subjective beliefs, anticipatory utility can expressed as:*

$$AU_i\left(\delta_{\eta,i}\right) \propto \left[\underbrace{u_{kk}\left(1-r\right)^2 var_i\left[\theta|s_i, s\right]}_{\text{fundamental uncertainty channel}} + \underbrace{\left(u_{KK} - r^2 u_{kk}\right) B^2 var_i\left[s|\theta\right]}_{\text{non-fundamental volatility channel}}\right]. \tag{11}$$

A player's subjective beliefs affect his anticipatory utility through two channels. First, when the player believes that his fundamental forecast is more precise, i.e., when $\text{var}_i\left[\theta|s_i, s\right]$ decreases, his anticipatory utility increases (since $u_{kk} < 0$ in equation (11)) - we refer to this as the **fundamental uncertainty channel**. Second, the player's anticipatory utility is affected by his beliefs about the error in the average action. We refer to this as the **non-fundamental volatility channel**, since a noisier public signal, i.e., an increase in $\text{var}_i\left[s|\theta\right]$, increases his subjective volatility of the aggregate action, which is increasing in the weight others place on the signal, B.[15]

The net impact of non-fundamental volatility depends on the direct effect of aggregate volatility on player i's utility captured by u_{KK} relative to the indirect effect driven by strategic considerations, reflected by $-r^2 u_{kk}$. Since $u_{kk} < 0$, the latter term is always positive, irrespective of whether actions are strategic complements or substitutes. Intuitively, since all players condition on public information, more volatility in the public signal makes it easier to forecast what others will do in equilibrium. The ability to forecast others' behavior is more useful when actions are strong complements *or* strong substitutes (i.e., if $|r|$ is large).

The non-fundamental volatility channel highlights a novel mechanism through which players may prefer to believe the public signal is *noisier* than it actually is. To capture the relative importance of this channel, we introduce the coefficient

$$\chi \equiv \frac{u_{KK} - r^2 u_{kk}}{-u_{kk}\left(1-r\right)^2} = \frac{r^2 - \frac{u_{KK}}{u_{kk}}}{\left(1-r\right)^2}, \tag{12}$$

[15] Notably, since players have objective beliefs about the private information of others, subjective beliefs do not affect anticipated utility via their impact on the dispersion of others' actions (i.e., $u_{\sigma\sigma}$). In Section 7.2, we relax this assumption.

which we refer to as the **relative impact of non-fundamental volatility**. It is worth noting that while χ depends on the equilibrium degree of coordination, r, the two are distinct. Specifically, note that the sign of χ does not depend upon whether players' actions are strategic complements (i.e., $r > 0$) or substitutes ($r < 0$), and χ can be positive or negative even when there are no strategic considerations (i.e., when $r = 0$). Indeed, the sign of χ depends on the net impact of strategic considerations (captured by r^2) as well as the magnitude of the direct effect of non-fundamental volatility (reflected by $-\frac{u_{KK}}{u_{kk}}$).[16]

Importantly, the sign of χ determines the effect of the player's subjective beliefs since

$$\frac{\partial AU_i}{\partial \delta_{\eta,i}} \propto \left(-\frac{\partial \text{var}_i \left[\theta | s_i, s\right]}{\partial \delta_{\eta,i}} + \chi B^2 \frac{\partial \text{var}_i \left[s|\theta\right]}{\partial \delta_{\eta,i}} \right) = \frac{\tau_\eta}{\left(\tau + \tau_e + \delta_{\eta,i}\tau_\eta\right)^2} - \frac{\chi B^2}{\delta_{\eta,i}^2 \tau_\eta}. \tag{13}$$

When $\chi < 0$, anticipatory utility decreases in non-fundamental volatility, so an increase in $\delta_{\eta,i}$ unambiguously increases anticipatory utility. When $\chi > 0$, the fundamental uncertainty and non-fundamental volatility channels operate in opposite directions. If $\delta_{\eta,i}$ is sufficiently low or χB^2 is sufficiently high, the non-fundamental channel dominates and so anticipatory utility decreases with $\delta_{\eta,i}$. When $\delta_{\eta,i}$ is sufficiently high or χB^2 is sufficiently low, the informational channel dominates and so anticipatory utility increases with $\delta_{\eta,i}$.

Given its novelty and the key role it plays in determining equilibrium behavior, we characterize the determinants of χ in a series of applications in Section 6. We find that χ is positive across many different types of models. For example, in a competitive setting with incomplete markets, households benefit from volatility in aggregate production ($u_{KK} > 0$) since, in equilibrium, they are able to purchase more of the good at a lower price. In contrast, in a setting with investment complementarities, $\chi > 0$ because u_{KK} is zero: aggregate volatility imposes no cost on players. We also show that $\chi > 0$ in both a Cournot game in which firms compete in quantities and actions are strategic substitutes ($r < 0$) and a Bertrand game in which firms compete in prices and actions are strategic complements ($r > 0$). One exception to this pattern arises in the beauty contest models of Morris and Shin (2002). In this setting, the impact of aggregate volatility (u_{KK}) depends on whether actions are substitutes or complements, and so χ inherits the same sign as well.

[16] Recall that since $u_{kk} < 0$, the sign of the second term in the numerator is determined by u_{KK}.

4.2 Benchmark: Maximizing Anticipatory Utility

In this section, we characterize the equilibrium when $\psi = 0$ i.e., when players ignore the impact of their chosen beliefs on experienced utility. Let

$$\underline{B} = \frac{\underline{\delta}\tau_\eta}{\tau + \tau_e + \underline{\delta}\tau_\eta}, \quad \bar{B} = \frac{\bar{\delta}\tau_\eta}{\tau + \tau_e + \bar{\delta}\tau_\eta}, \quad \text{and} \quad B_{RE} = \frac{\tau_\eta}{\tau + \tau_e + \tau_\eta}. \tag{14}$$

A player puts weight $\bar{B}$, $\underline{B}$ and B_{RE} on the public signal if he chooses $\delta_{\eta,i} = \bar{\delta}$ (he maximally over-reacts), $\delta_{\eta,i} = \underline{\delta}$ (he maximally under-reacts) or $\delta_{\eta,i} = 1$ (he exhibit rational expectations), respectively.

We begin by characterizing the optimal belief choice of player i when all other players exhibit rational expectations. The following is an immediate consequence of Lemma 2.

Corollary 1. *Suppose $\psi = 0$ and all other players exhibit rational expectations i.e., for all $j \neq i$, $\delta_{\eta,j} = 1$. Then, player i chooses $\delta_{\eta_i} = \underline{\delta}$ if $\chi > \frac{\bar{B}\underline{B}}{B_{RE}^2}$, and $\delta_{\eta,i} = \bar{\delta}$ otherwise.*

Corollary 1 says that even if all other players exhibit rational expectations, an individual player finds it optimal to deviate and exhibit wishful thinking. When χ is sufficiently high, the non-fundamental volatility channel dominates and the player chooses to believe that the public signal is noisier leading to under-reaction; otherwise, he over-reacts to the public signal. This intuition carries over into the equilibrium in which *all* players choose their beliefs to maximize anticipatory utility.

Proposition 1. *Suppose $\psi = 0$ and all players optimally choose their subjective beliefs.*

(i) If $\chi > \bar{B}/\underline{B}$, then a unique equilibrium exists in which all players choose $\delta_{\eta,i} = \underline{\delta}$.

(ii) If $\chi < \underline{B}/\bar{B}$, then a unique equilibrium exists in which all players choose $\delta_{\eta,i} = \bar{\delta}$.

(iii) If $\chi \in \left[\underline{B}/\bar{B}, \bar{B}/\underline{B}\right]$, then a unique equilibrium exists in which a fraction λ of players choose $\delta_{\eta,i} = \bar{\delta}$ while all others choose $\delta_{\eta,i} = \underline{\delta}$, where

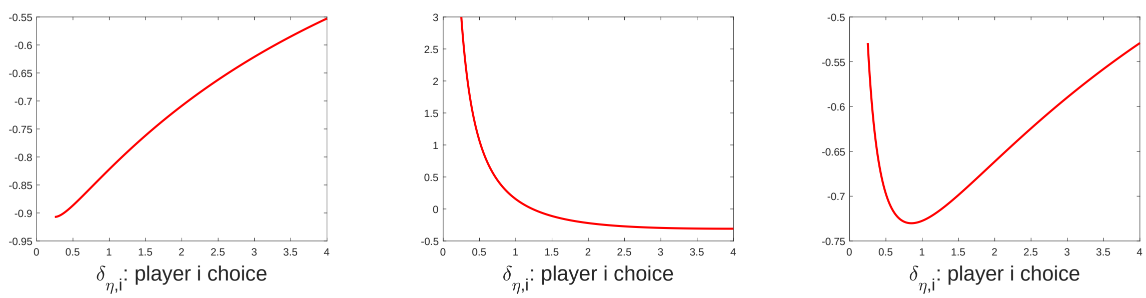
$$\lambda = \frac{\sqrt{\frac{\bar{B}\underline{B}}{\chi}} - \underline{B}}{\bar{B} - \underline{B}}, \tag{15}$$

Moreover, $\bar{B}/\underline{B}$ is increasing in τ and τ_e, but decreasing in τ_η.

As in Corollary 1, it is always optimal to over-react (under-react) to public information when χ is sufficiently low (high). However, when χ is in an intermediate region, there cannot exist a pure-strategy, symmetric equilibrium in which all players interpret the public information in the same way. This is due to the impact that the equilibrium beliefs of the other players have on player i's incentives. For instance, if all other players choose to under-react to the public signal so that $B = \underline{B} < B_{RE}$, then player i's incentive to choose $\underline{\delta}$ falls: the non-fundamental volatility channel is less important, as seen in (13). If $\chi > \underline{B}/\bar{B}$, player i has an incentive to deviate and chooses to over-react (i.e., choose $\delta_{\eta,i} = \bar{\delta}$).

As a result, when $\chi \in \left[\underline{B}/\bar{B},\, \bar{B}/\underline{B}\right]$, the unique equilibrium features *endogenous disagreement* about the interpretation of the public signal: a fraction λ of players choose to over-react to the public signal, while the remaining players under-react to it. This optimal λ ensures that the relative impact of the non-fundamental volatility and fundamental uncertainty channels exactly offset so that each player is indifferent between the two choices: as χ increases, λ and, as a result, B decreases so that χB^2 stays constant in this interval.[17] Intuitively, as χ increases, the measure of players who choose $\underline{\delta}$ increases, reflecting the increased importance of the non-fundamental volatility channel.

Figure 1: Anticipatory utility versus $\delta_{\eta,i}$ when $\chi \in \left[\underline{B}/\bar{B},\, \bar{B}/\underline{B}\right]$
The figure plots the anticipatory utility for player i as a function of $\delta_{\eta,i}$. Other parameters: $\tau = \tau_e = \tau_\eta = 1$, $\chi = 1$, $\psi = 0$ and $\bar{\delta} = 1/\underline{\delta} = 4$.



(a) $\delta_\eta = \underline{\delta}$ (b) $\delta_\eta = \bar{\delta}$ (c) Mixed eqm.

Figure 1 provides a numerical illustration of this intuition. The panels show player i's anticipatory utility as a function of $\delta_{\eta,i}$, given the beliefs of others. In panel (a), all other agents choose $\delta_\eta = \underline{\delta} = 1/4$. Since $B = \underline{B}$ is sufficiently low, the information channel dominates and player i deviates by over-weighting the public signal (chooses $\delta_{\eta,i} = 4$). In panel (b), all other agents choose $\delta_\eta = \bar{\delta} = 4$. Now, the non-fundamental channel dominates, i.e., $B = \bar{B}$ is sufficiently large, and player i strictly prefers to believe the public signal is (relatively) uninformative (chooses $\delta_{\eta,i} = 1/4$). In both cases, a symmetric equilibrium is ruled out. Panel (c) illustrates the mixed strategy equilibrium in which a fraction $\lambda = 0.25$

[17] Moreover, the equilibrium B is a continuous function of the underlying parameters: if $\chi = \underline{B}/\bar{B}$, $\lambda = 1$, consistent with the symmetric equilibrium that obtains when $\chi < \underline{B}/\bar{B}$; if $\chi = \bar{B}/\underline{B}$, $\lambda = 0$ consistent with the symmetric equilibrium that obtains when $\chi > \bar{B}/\underline{B}$.

of players overweight the public signal while the remaining fraction $1-\lambda = 0.75$ underweight it. As is clear, given the choice of all other agents, player i is indifferent between these two (sets of) beliefs.

Finally, the proposition's comparative statics with respect to $\bar{B}/\underline{B}$ imply that the range of χ for which a mixed equilibrium obtains increases with τ and τ_e, but decreases with τ_η. This suggests that, all else equal, players are more likely to endogenously disagree about public information when prior variance about fundamentals is low, when private information is precise, and when public information is noisy. Crucially, these predictions distinguish our analysis from models in which agents are (exogenously) assumed to disagree about public information.

4.3 Subjective beliefs about public information

The analysis in 4.2 highlights the economic channels through which subjective belief choice can endogenously give rise to disagreement about public information. However, when players maximize their anticipatory utility only ($\psi = 0$), their belief choices are extreme within the set of "reasonable beliefs". When $\psi > 0$, each player faces an additional trade-off: over- or under-weighting the public signal reduces the informational efficiency of his action, reducing his experienced utility. The following proposition characterizes equilibria in this setting.

Proposition 2. *Let* $B_{opt} = \max\left\{\min\left\{\frac{B_{RE}(2\psi+1)-\psi\underline{B}}{2\psi}, \bar{B}\right\}, \underline{B}\right\}$ *and* $\Gamma = 1+\psi\left(2-\frac{\underline{B}+B_{opt}}{B_{RE}}\right) > 0$.

(i) If $\chi > \frac{B_{opt}}{\underline{B}}\Gamma$*, then a unique equilibrium exists in which all players choose* $\delta_{\eta,i} = \underline{\delta}$.

(ii) If $\chi < \frac{\underline{B}}{B_{opt}}\Gamma$*, then a unique equilibrium exists in which all players choose*

$$\delta_{\eta,i} = \min\left\{\bar{\delta}, \frac{2\psi+1-\chi}{2\psi - \frac{(1-\chi)\tau_\eta}{\tau_e+\tau}}\right\} \equiv \delta_{sym}. \tag{16}$$

(iii) If $\chi \in \left[\frac{\underline{B}}{B_{opt}}\Gamma, \frac{B_{opt}}{\underline{B}}\Gamma\right]$*, then a unique equilibrium exists in which a fraction* λ *of players choose* $\delta_{\eta,i} = \delta_{opt}$*, while all others choose* $\delta_{\eta,i} = \underline{\delta}$*, where*

$$\lambda = \frac{\sqrt{\frac{\Gamma \underline{B} B_{opt}}{\chi}} - \underline{B}}{B_{opt} - \underline{B}}, \quad \text{and} \quad \delta_{opt} = \frac{B_{opt}}{1-B_{opt}} \frac{(\tau_e + \tau)}{\tau_\eta}. \tag{17}$$

When $\chi = 0$*, a unique equilibrium exists in which all players choose* $\delta_{\eta,i} = \delta_{sym} > 1$.

In the absence of strategic externalities ($r = 0$) and when there is no direct impact of aggregate volatility on payoffs ($u_{KK} = 0$), then $\chi = 0$. In this case, players choose to over-estimate the precision of the public signal as this reduces perceived fundamental uncertainty. As strategic considerations become more important (i.e., $|r|$ and hence χ increases), players have an incentive to believe that public information is noisier. On the other hand, if the cost of aggregate volatility increases (i.e., u_{KK} and hence χ becomes more negative), players have an incentive to believe the signal is more precise.

More generally, the above result mirrors Proposition 1, but adjusted for the fact that players now account for their experienced utility loss. When χ is sufficiently high, all players still choose to under-react to the public information by choosing $\delta_{\eta,i} = \underline{\delta}$. When χ is sufficiently low, there also exists a unique symmetric equilibrium; however, when ψ is sufficiently high, this equilibrium choice is interior, i.e., the optimal $\delta_{sym} < \bar{\delta}$ due to the increased cost of deviating from rational expectations. Moreover, this interior δ_{sym} reflects the impact of non-fundamental volatility on optimal beliefs; holding fixed ψ, δ_{sym} decreases in χ in this region, i.e., agents choose to believe the public signal is less informative as χ increases.

For intermediate χ, players endogenously disagree: players mix between δ_{opt} and $\underline{\delta}$, and the measure λ who choose δ_{opt} decreases in χ as before. Now, δ_{opt} (and the corresponding expressions for B_{opt} and λ) reflects the trade-off between desirable and accurate beliefs. When ψ is relatively low, the fraction λ of players over-react maximally (since $B_{opt} = \bar{B}$, we have $\delta_{\eta,i} = \delta_{opt} = \bar{\delta}$).[18] However, when experienced utility is more important (i.e., for larger ψ), players optimally choose an interior weight: $\delta_{opt} < \bar{\delta}$, reflecting the cost of holding extreme beliefs.

Figure 2 illustrates how the region of endogenous disagreement changes as a function of underlying precisions. Specifically, each panel illustrates the range of χ (the shaded region) in which the unique equilibrium features mixing. Consistent with the results in Section 4.2 and holding fixed ψ, endogenous disagreement is more likely to arise when aggregate prior uncertainty is low, private information is precise, or public information is noisy.

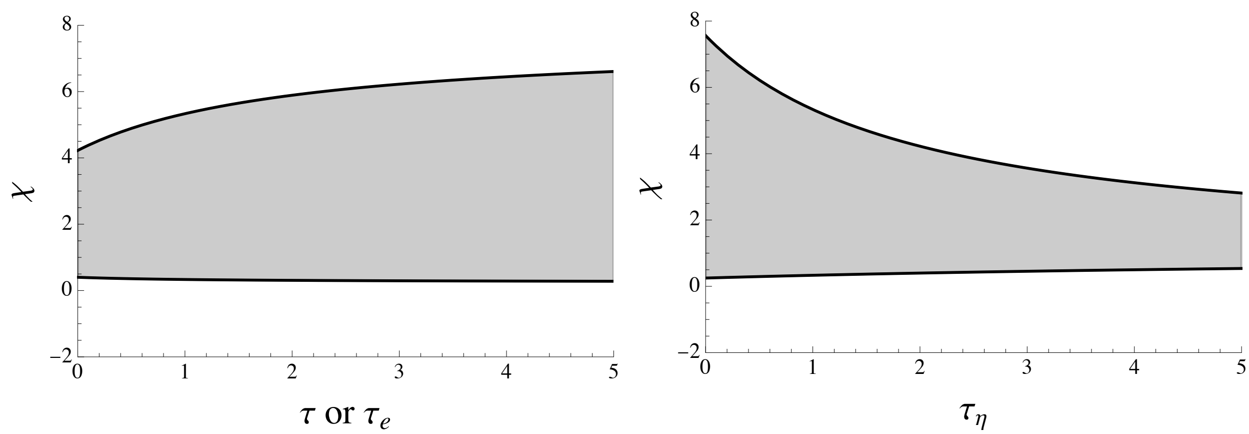
Moreover, the region of endogenous disagreement now also depends on the relative importance of experienced utility, ψ. We show that the impact of increasing ψ on the likelihood of endogenous disagreement depends on how large the set of reasonable beliefs is, and in particular, how small $\underline{\delta}$ can be.

Corollary 2. *If $\underline{\delta} \leq \frac{2(\tau_e+\tau)}{3(\tau_e+\tau)+\tau_\eta}$, then for any ψ, there exists a range of χ for which the unique equilibrium features endogenous disagreement. If $\underline{\delta} > \frac{2(\tau_e+\tau)}{3(\tau_e+\tau)+\tau_\eta}$, then for $\psi > \frac{B_{RE}}{3\underline{B}-2B_{RE}}$, the equilibrium does not feature disagreement about the public signal.*

[18] Note that as $\psi \to 0$, $B_{opt} \to \bar{B}$ and $\Gamma \to 1$ so that the threshold χ and equilibrium fraction λ coincide with the analogous expressions in Proposition 1.

Figure 2: Region of endogenous disagreement versus τ, τ_η, τ_e

The shaded area of each plot corresponds to the region of the parameter space in which the unique equilibrium exhibits endogenous disagreement about the public signal. Unless otherwise mentioned, the parameters are fixed at $\tau = \tau_e = \tau_\eta = \psi = 1$ and $\bar{\delta} = 1/\underline{\delta} = 4$.



Note that for $\psi = 0$, $B_{opt} = \bar{B} > \underline{B}$, and that B_{opt} (weakly) decreases as ψ increases. If $\underline{\delta}$ (and consequently, $\underline{B}$) is sufficiently low, then $B_{opt} > \underline{B}$ for all ψ. Intuitively, in this case, there always exists some sufficiently high χ such that choosing a sufficiently low $\underline{\delta}$ is optimal and so there is endogenous disagreement for all ψ.[19] Otherwise, there exists some ψ such that $B_{opt} = \underline{B}$ and, as a result, endogenous disagreement disappears above this threshold ψ.

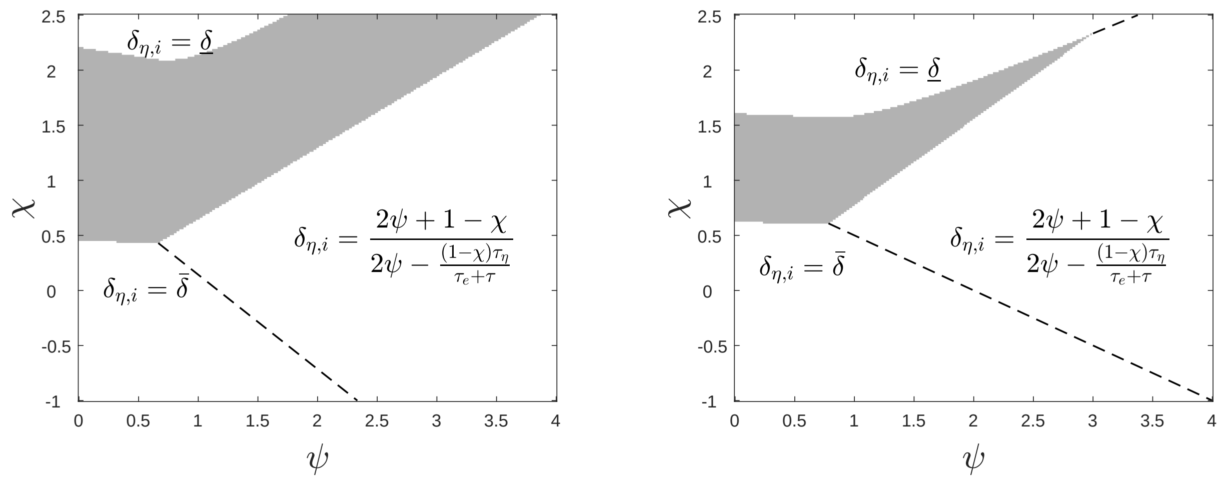
Figure 3 provides an illustration of this result. Specifically, the left panel of the plot considers a parameterization in which $\underline{\delta}$ is sufficiently low (relative to other parameters), and so for any ψ, there is a region of χ for which players exhibit endogenous disagreement (the shaded area). This might seem at odds with the intuition that as $\psi \to \infty$, all players converge to exhibiting rational expectations. But note that as ψ increases, the region of endogenous disagreement eventually moves above any given χ (e.g., if we hold fixed the strategic payoffs), and the optimal $\delta_{\eta,i} \to 1$. In contrast, in the right panel, $\underline{\delta}$ is insufficiently small so that endogenous disagreement does not exist for ψ sufficiently high. The figures also show the subjective belief choices (i.e., $\delta_{\eta,i}$) in the symmetric equilibria (unshaded areas).

4.4 The set of reasonable beliefs

Having characterized how equilibria depend on other parameters for a fixed set of reasonable beliefs, we now turn to the impact of changes in $[\underline{\delta}, \bar{\delta}]$.

[19] This is straightforward to see since by taking $\psi \to \infty$, $B_{opt} \to \max\{B_{RE} - \underline{B}/2, \underline{B}\}$ and $\Gamma \neq 0$.

Figure 3: Region of Endogenous disagreement versus ψ
The shaded area of each plot corresponds to the region of the parameter space in which the unique equilibrium exhibits endogenous disagreement about the public signal. The figures also show the choice of $\delta_{\eta,i}$ in the symmetric equilibria (unshaded area). Unless otherwise mentioned, the parameters are fixed at $\tau = \tau_e = \tau_\eta = 1$. The left panel corresponds to $\underline{\delta} = 1/\bar{\delta} = 0.55$, while the right panel corresponds to $\underline{\delta} = 1/\bar{\delta} = 0.70$.



(a) $\underline{\delta} = 0.55$ (b) $\underline{\delta} = 0.70$

Corollary 3. *The lower bound for the mixed equilibrium increases in $\underline{\delta}$ and decreases in $\bar{\delta}$. The upper bound for the mixed equilibrium decreases in $\underline{\delta}$ and increases in $\bar{\delta}$.*

This corollary implies that as the range of reasonable beliefs $\left[\underline{\delta}, \bar{\delta}\right]$ increases, the region of endogenous disagreement about public information also increases. While we treat these bounds as exogenous for tractability, in practice, we expect them to be endogenously determined as part of a richer specification of experimentation, learning, and experience. For instance, suppose player i only entertains subjective beliefs which are not "too far" from the objective distribution as measured by the K-L distance. This can be written as[20]

$$KL(s|\theta; \delta_{\eta,i} = \delta) \equiv \frac{1}{2}\left(\log\left(\frac{\text{var}[s|\theta]}{\text{var}_i[s|\theta]}\right) + \frac{\text{var}_i[s|\theta]}{\text{var}[s|\theta]} - 1\right) = \frac{1 - \delta + \delta \log(\delta)}{2\delta}.$$

If player i's notion of reasonable beliefs are such that $KL(s|\theta; \delta_{\eta,i} = \delta) \leq \kappa$, then there exists $0 < \underline{\delta} < 1 < \bar{\delta}$ such that for any κ, the player must choose from $\delta_{\eta,i} \in [\underline{\delta}, \bar{\delta}]$. Here, κ serves as a measure of the model uncertainty, or ambiguity, that the player faces: as it increases, the player entertains a larger set of models i.e., the interval $\left[\underline{\delta}(\kappa), \bar{\delta}(\kappa)\right]$ increases. In turn, this implies that the potential for endogenous disagreement about public information increases given the above corollary. This is consistent, for instance, with Dovern (2015) which finds

[20] This specific K-L distance specification is similar to the flow model discussed in Caplin and Leahy (2019).

that disagreement amongst professional forecasters is positively correlated with (aggregate) economic uncertainty. The uncertainty that players face can vary across domains: we expect κ to be higher, and the set of reasonable beliefs to be larger, when players are more uncertain about fundamentals (e.g., firms investing in new technology) or are more unsure about the quality information available to them.

In Appendix C.1, we consider an extension where players can choose to increase κ at a cost. As in the benchmark model, when χ is sufficiently low (high), all players optimally choose to over-react (under-react) to public information. We find that during periods of high prior uncertainty, agents are willing to consider a larger set of subjective beliefs: the chosen κ and the interval, $\left[\underline{\delta}, \bar{\delta}\right]$, is larger.

5 Implications

In this section, we characterize some implications of our model which distinguish it from both the rational expectations benchmark as well as other exogenous belief specifications. In Section 5.1, we characterize the equilibrium response to public information; in Section 5.2 we analyze the cross-sectional dispersion of beliefs across players; in Section 5.3, we characterize conditions under which our model can reconcile the existing empirical evidence on forecast error regressions (e.g., Coibion and Gorodnichenko (2012, 2015), Bordalo et al. (2020)).

5.1 Aggregate response to public information

There is ample empirical evidence that beliefs, quantities, and prices respond sluggishly to public information and macroeconomic news (e.g., Lorenzoni (2011) for a survey, and Angeletos, Huo, and Sastry (2020) for survey evidence). However, a number of recent studies have argued that forecasts over-react to new realizations of the variable of interest (e.g., Afrouzi, Kwon, Landier, Ma, and Thesmar (2020) and Azeredo da Silveira and Woodford (2019)).

Our model can generate both under- *and* over-reaction. Specifically, the aggregate response to public information is given by $B = \int_i B_i di$, which measures the average weight players put on the public signal. Given the equilibrium characterization in Proposition 2,

$$B = \begin{cases} \max\left(\min\left(\bar{B}, \frac{B_{RE}(2\psi+1-\chi)}{2\psi}\right), \underline{B}\right) & \text{in symmetric equilibrium} \\ \sqrt{\frac{\Gamma \underline{B} B_{opt}}{\chi}} & \text{in mixed equilibrium} \end{cases} \tag{18}$$

The average response to public information is "sluggish" when $B < B_{RE}$, since beliefs respond less to public information than they would under rational expectations. The following result characterizes when this arises as a result of equilibrium belief choice.

Proposition 3. *The average weight on public information (B) decreases with χ. In a symmetric equilibrium, players under-react to public information (i.e., $B < B_{RE}$) if and only if $\chi > 1$. In a mixed equilibrium, players under-react to public information (i.e., $B < B_{RE}$) if and only if $\chi > \frac{\Gamma \underline{B} B_{opt}}{B_{RE}^2} > 0$.*

Figure 4 provides an illustration of this result. Panel (a) plots the contour plot of B as a function of ψ and χ, and panel (b) plots the corresponding region of the mixed equilibrium. Note that $B_{RE} = 0.33$ for the parameters chosen. Panel (a) illustrates that in a symmetric equilibrium (unshaded regions of panel (b)), players under-react to public information if and only if $\chi > 1$. For the mixed equilibrium (shaded region in (b)), players under-react even when χ is lower than 1, so long as it satisfies the bound in proposition 3. Finally, holding fixed ψ, the average response decreases, i.e., B falls, as χ increases

It is worth noting that the "sluggishness" we characterize is in terms of players' beliefs and not their actions, which distinguishes our results from earlier work. As highlighted by Morris and Shin (2002) and the ensuing literature, in settings with strategic coordination, players' *actions* may put too much or too little weight on public information depending on whether actions are strategic complements or substitutes. But in these models, players exhibit rational expectations, and so their beliefs about fundamentals are correct $B = B_{RE}$.[21]

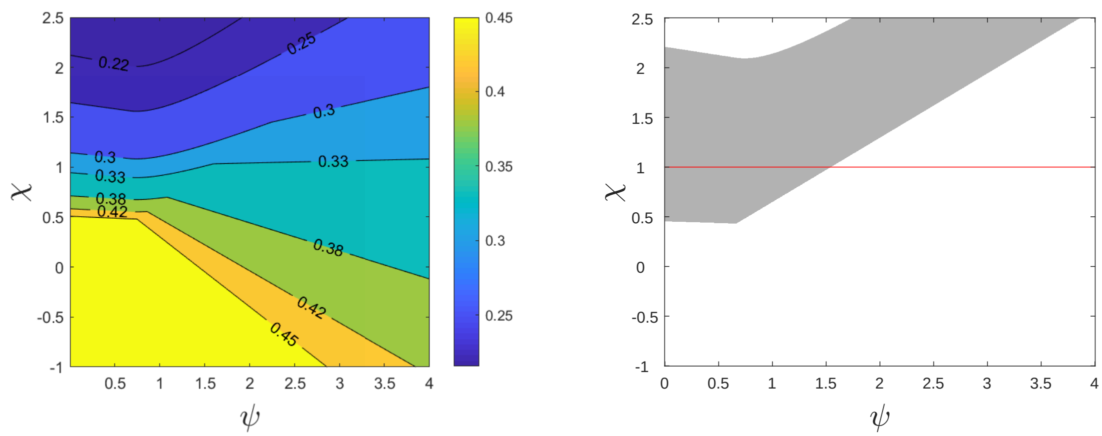
In contrast, our model predicts that players' aggregate *beliefs* can under-react to public information, irrespective of whether actions are strategic substitutes or complements, and that the degree of over-reaction decreases with the relative importance of non-fundamental volatility χ. Lemma 1 shows how these beliefs feed into players' equilibrium actions, highlighting how strategic considerations can amplify or dampen the actual response, given beliefs.

5.2 Belief dispersion and public information

In our setting, players have dispersed beliefs about fundamentals because they have different (private) information and because they interpret the same (public) information differently. However, in contrast to existing models where players are exogenously assumed to behave like this, our model provides novel predictions on how belief dispersion *endogenously* changes with the underlying parameters of the model. Our contribution is identifying a clear structure

[21]Sluggishness can also arise, for example, in models with rational inattention, but since players are assumed to be rational in these models, they do not generate over-reaction to public information.

Figure 4: Aggregate response to public information
Panel (a) of the figure plots the average response B to public information as a function of ψ and χ. Panel (b) shows the region of mixed equilibrium. Other parameters: $\tau = \tau_e = \tau_\eta = 1$, and $\underline{\delta} = 1/\bar{\delta} = 0.55$. For these parameters, $B_{RE} = 0.33$.



(a) B contour plot   (b) Region of mixed equilibrium

for "what happens when". In this subsection, we characterize how disagreement varies across equilibria, and with the relative importance of non-fundamental volatility, χ.

Let player i's conditional expectation of θ be denoted by $y_i \equiv \mathbb{E}_i[\theta|s_i, s]$. Then, one measure of the cross-sectional dispersion of beliefs across players is given by the expected variance (across players), conditional on fundamentals and common noise i.e.,

$$\sigma_y^2 = \mathbb{E}\left[\text{var}[y_i|\{\theta, \eta\}]\right]. \tag{19}$$

Let $\sigma_{y,RE}^2$ denote the corresponding measure if players exhibit rational expectations.

The following result characterizes how belief dispersion varies with the precision of public information given players' equilibrium beliefs.

Proposition 4. *(i) If players' exhibit rational expectations, then dispersion in beliefs decreases with public information quality i.e., $\frac{\partial \sigma_{y,RE}^2}{\partial \tau_\eta} < 0$, and is independent of χ.*

(ii) In a symmetric equilibrium with subjective beliefs, dispersion in beliefs is higher than under rational expectations (i.e., $\sigma_y^2 > \sigma_{y,RE}^2$) if and only if $\chi > 1$. Moreover, dispersion decreases with public information quality i.e., $\frac{\partial \sigma_y^2}{\partial \tau_\eta} < 0$ and increases with χ.

(iii) In a mixed equilibrium with subjective beliefs, dispersion in beliefs can increase with public information quality i.e., $\frac{\partial \sigma_y^2}{\partial \tau_\eta} > 0$. Moreover, dispersion is hump-shaped in χ.

Part (i) provides the rational expectations benchmark: when public information is more precise, players rationally put less weight on their private signals, and so disagreement falls.

Moreover, since players' beliefs only depend on the informational content of their signals, disagreement is independent of χ. Part (ii) shows the impact of endogenous subjective beliefs in the symmetric equilibrium. When the relative impact of non-fundamental volatility increases, players endogenously choose to put less weight on the public signal (i.e., δ_{sym} is decreasing in χ), which in turn, increases dispersion in beliefs. As a result, dispersion is higher in the symmetric equilibrium than under rational expectations when χ is sufficiently large (i.e., $\chi > 1$). On the other hand, because players agree on their interpretation of the public signal, we show that dispersion decreases in τ_η, irrespective of how this information affects equilibrium beliefs.

In contrast, part (iii) describes how dispersion behaves when players endogenously *disagree* about the precision of public signal. Recall from Proposition 2 that λ, the fraction of players who maximally over-react to the public signal, is monotonic in χ. As a result, disagreement about the public signal, and dispersion in beliefs, is hump-shaped in χ as the measure of each type shifts endogenously.[22] Moreover, an increase in τ_η can increase belief dispersion *across* the two groups: the distance in conditional expectations between those who choose $\underline{\delta}$ and those who choose δ_{opt} can increase when the precision of the public signal improves.

There is a large literature in macroeconomics and finance that documents how disagreement among investors and professional forecasters varies predictably with the business cycle (e.g., Mankiw, Reis, and Wolfers (2003), Andrade, Crump, Eusepi, and Moench (2016)), and can increase after public announcements (e.g., Kandel and Pearson (1995)). To understand this empirical evidence, a number of papers assume that agents interpret incoming public information differently.[23] Our results provide novel predictions on how such dispersion in beliefs varies with strategic considerations (as captured by χ). Furthermore, the effect of public information in the mixed equilibrium distinguishes our model from one of information acquisition (e.g., Colombo et al. (2014)). In those settings, providing a more precise public signal usually crowds out the acquisition of private information, and consequently, reduces dispersion in beliefs as in the rational expectations benchmark. In contrast, recent studies have found that increased communication by central banks can increase forecast dispersion (e.g., Lustenberger and Rossi (2018)), consistent with our model. While our analysis is not normative, Angeletos and Pavan (2007) note that such dispersion can have welfare effects depending upon the sign of the payoff externalities.

[22]Disagreement is low when λ is close to zero or one, but higher in the middle.

[23]These papers include Harrison and Kreps (1978), Kandel and Pearson (1995), Hong and Stein (1999), Dumas et al. (2009), and Banerjee and Kremer (2010).

5.3 Forecast error predictability

Coibion and Gorodnichenko (2012, 2015) develop a new approach for testing the departures of forecasts from rational expectations by regressing forecast errors on forecast revisions. Under the null of rational expectations and full information, the forecast errors are unpredictable and so the regression coefficient is zero. However, the authors document that the regression coefficient for consensus forecasts is significantly positive in the data, suggesting that aggregate forecasts under-react to available information which they attribute to sticky or noisy information. In contrast, Bordalo et al. (2020) document that individual forecasts tend to exhibit overreaction: the analogous regression coefficients are negative. We characterize conditions under which these puzzling results can arise in our model, and provide novel testable predictions in these cases.

In our model, the forecast revision (FR) and forecast error (FE) of player i are given by

$$FR_i \equiv \mathbb{E}_i\left[\theta|s_i, s\right] - \mathbb{E}_i\left[\theta\right] \quad \text{and} \quad FE_i \equiv \theta - \mathbb{E}_i\left[\theta|s_i, s\right], \tag{20}$$

respectively. Taking averages across i gives us the analogous expressions for the consensus forecast revision $\bar{FR}$ and consensus forecast error $\bar{FE}$. One can then characterize the "Coibon-Gorodnichenko regression coefficient" for individual forecasts as CG_i and for consensus forecasts as CG_a, where

$$CG_i = \frac{\text{cov}(FE_i, FR_i)}{\text{var}(FR_i)}, \quad \text{and} \quad CG_a = \frac{\text{cov}(\bar{FE}, \bar{FR})}{\text{var}(\bar{FR})}. \tag{21}$$

We begin by characterizing conditions under which our model generates the empirically observed patterns for CG_a and CG_i in a symmetric equilibrium.

Proposition 5. *Consider a symmetric equilibrium in which all players choose $\delta_{\eta,i} = \delta^*$. Then,*

$$CG_i = \frac{\delta^*\left(1-\delta^*\right)\tau_\eta}{\left(\tau_e + \delta^*\tau_\eta\right)^2\frac{1}{\tau} + \tau_e + (\delta^*)^2\tau_\eta}, \quad \textit{and} \quad CG_a = \frac{\tau_e + \delta^*\left(1-\delta^*\right)\tau_\eta}{\left(\tau_e + \delta^*\tau_\eta\right)^2\frac{1}{\tau} + (\delta^*)^2\tau_\eta} \tag{22}$$

(i) There is over-reaction at the individual level (i.e., $CG_i < 0$) iff $\chi < 1$.

(ii) There is under-reaction at the consensus level (i.e., $CG_a > 0$) iff

$$\bar{\delta} \leq \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\tau_e}{\tau_\eta}} \quad \textit{or} \quad \frac{1-\chi}{2\psi} < \frac{\sqrt{\frac{1}{4} + \frac{\tau_e}{\tau_\eta}} - \frac{1}{2}}{\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\tau_e}{\tau_\eta}}\right)\frac{\tau_\eta}{\tau+\tau_e} + 1}. \tag{23}$$

Moreover, when $\chi < 1$, both CG_i and CG_a are increasing in χ.

Under rational expectations ($\delta^* = 1$), note that $CG_i = 0$ because individuals condition correctly on all their information. Since players have dispersed private signals with uncorrelated errors, the aggregate information available to them is more precise than any individual's information, and so the consensus update under-reacts to the aggregate information (i.e., $CG_a > 0$) even with rational expectations.[24]

Proposition 5 implies that our model can reconcile the empirical evidence of consensus underreaction and individual overreaction in macroeconomic forecasts when players overreact to public information (i.e., $\delta^* > 1$), but not too much, consistent with the expressions in equation (22).[25] Whether this arises depends on the impact of non-fundamental volatility since $\delta^* = \delta_{sym} > 1$ only if χ is sufficiently small (see Proposition 2 (ii)). In particular, over-reaction at the individual level requires χ is not too large ($\chi < 1$), while under-reaction at the consensus level requires that χ be sufficiently large (which obtains if either of the conditions in (23) holds).[26] Figure 5 provides a numerical illustration of these results, and establishes that similar results extend to the mixed equilibrium: both regression coefficients increase as χ increases.

The above result also helps shed light on how consensus over-/under-reaction varies with economic conditions. From Section 4.4, recall that the set of reasonable beliefs is larger, and $\bar{\delta}$ is higher, when players face more uncertainty. All else equal, Proposition 5 (ii) implies that consensus under-reaction is less likely in such settings.[27] This is consistent with Figure 3 of Coibion and Gorodnichenko (2015) who show that consensus CG coefficients are negatively related to the standard deviation of GDP growth.[28]

[24] This is consistent, for instance, with the noisy information model of Woodford (2003), which Coibion and Gorodnichenko (2015) propose as an explanation for their evidence on aggregate under-reaction.

[25] Our analysis complements other recent approaches to resolving this puzzling evidence. For instance, Bordalo et al. (2020) show that a diagnostic expectations variant of a model with dispersed information can help reconcile this evidence, Angeletos et al. (2020) do so utilizing a model with dispersed noisy information and over-extrapolation, while Da Silveira, Sung, and Woodford (2020) argue that they can reconcile this empirical puzzle when players face a memory constraint.

[26] The results in Proposition 5 also hold qualitatively in a mixed equilibrium. For example, one can prove that the average CG_i coefficient will be negative when χ is less than a threshold. We omit this for brevity.

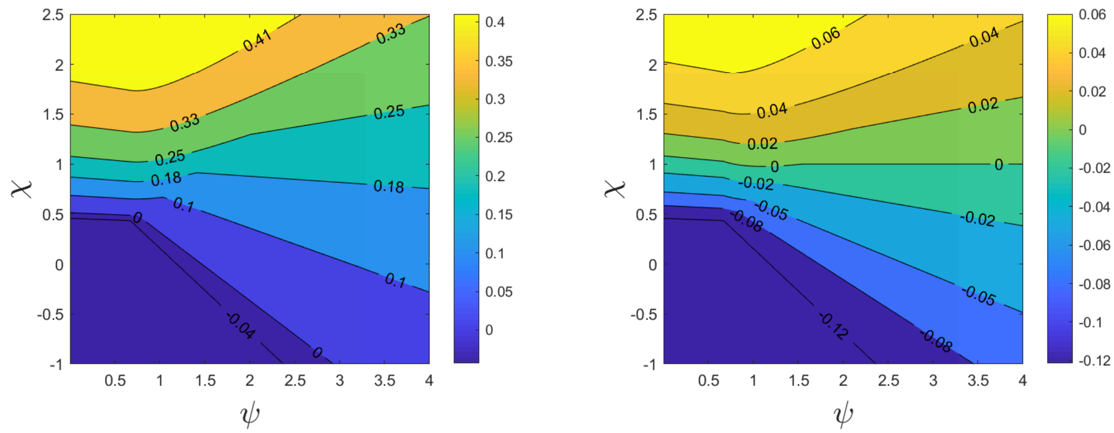
[27] Coibion and Gorodnichenko (2012, 2015) and Bordalo et al. (2020) document under-reaction for consensus macro-economic forecasts, while Bordalo, Gennaioli, Porta, and Shleifer (2019) document consensus over-reaction for stock market forecasts. To the extent that forecasters face more uncertainty when predicting stock returns than when predicting macro-economic variables, our results suggest that consensus over-reaction is more likely in the former case.

[28] This follows, since, in our model,

$$\frac{\partial CG_a}{\partial \bar{\delta}} = \begin{cases} 0 & \text{if} \quad \delta_{sym} < \bar{\delta} \\ -\frac{\tau \tau_\eta \left(\bar{\delta}^2 \tau_\eta (2\tau_e + \tau_\eta + \tau) + 2\bar{\delta}^2 \tau_e (\tau_e + \tau_\eta + \tau) + \tau_e^2\right)}{\left(\tau \bar{\delta}^2 \tau_\eta + \left(\bar{\delta}^2 \tau_\eta + \tau_e\right)^2\right)^2} < 0 & \text{otherwise} \end{cases}.$$

Figure 5: Average CG_a and CG_i

The figure plots the CG_a and CG_i as a function of ψ and χ. Other parameters: $\tau = \tau_e = \tau_\eta = 1$, and $\underline{\delta} = 1/\bar{\delta} = 0.55$.



(a) CG_a contour plot (b) CG_i contour plot

The dependence of the regression coefficients on the relative importance of non-fundamental volatility (i.e., χ) distinguishes our model from settings in which players are exogenously assumed to under-react or over-react to information. Importantly, another distinctive prediction of our model is the endogenous dispersion in individual CG_i coefficients.

Proposition 6. *In a symmetric equilibrium, the dispersion of CG_i coefficient is zero. In a mixed equilibrium, the dispersion of CG_i coefficients is positive and hump-shaped in χ.*

The above result further distinguishes our model from rational models with endogenous information acquisition or rational inattention. Recall that dispersion in beliefs (and actions) can arise either because players have private signals, or because they disagree about the interpretation of the public signal, and so dispersion in beliefs cannot be used to distinguish these settings. However, even when private signal precisions differ across players (e.g., due to different information acquisition decisions, or staggered attention), there cannot be a dispersion in CG_i coefficients when players exhibit rational expectations, since they use all their information efficiently (and so $CG_i = 0$). In contrast, such dispersion arises in our model when players endogenously choose to disagree about public information. Moreover, in contrast to models that exogenously assume that players agree to disagree about public signals, our model predicts that this dispersion is hump-shaped in the relative impact of non-fundamental volatility χ.

6 Applications

In this section, we consider the implications of our analysis for a number of common applications that nest within the generalized model. In each case, we highlight how parameters of the model affect the relative importance of non-fundamental volatility, χ which, together with the predictions from Section 5, generates specific predictions for each application.[29]

6.1 Strategic Investment

We begin by deriving χ in two stylized settings: in the first, there is complementarity in investment across firms, while in the second, production exhibits substitutability.

First, consider a setting in which the terminal value of the firm is given by $V(R, k_i) = Rk_i - \frac{1}{2}k_i^2$ where R measures the return on investment of the project available and k_i represents the scale of investment in the project. Suppose $R = (1-a)\theta + aK$ where θ represents the firm's exogenous productivity, K denotes the investment at the aggregate level, and $1 > a > 0$ measures the degree of complementarity in investment decisions.[30] In this case, $\chi = \frac{a^2}{(1-a)^2} > 0$.

Second, consider an incomplete-market, competitive economy in which there are two goods and a continuum of households (who act as consumers and producers). Consumer i chooses q_{1i} and q_{2i}, the respective quantities of each good, to maximize

$$u_i = \theta q_{1i} - \frac{bq_{1i}^2}{2} + q_{2i}$$

subject to the budget constraint $pq_{1i} + q_{2i} = e + \pi_i$, where θ is a shock to the relative demand for the two goods, p is the relative price of good one (good two is the numeraire), e is the endowment of good two, and $\pi_i = pk_i - \frac{k_i^2}{2}$ are the profits of producer i who produces quantity k_i of good one. Market clearing implies that the equilibrium price is given by $p = \theta - bK$, where $K = \int k_i di$ is the aggregate quantity of good one produced. The parameter $b > 0$ captures the impact of aggregate production on the price, and so reflects the degree of strategic substitutability in this economy. In this setting, $\chi = \frac{b}{1+b} > 0$.

In both settings, χ is always positive irrespective of whether actions are complements or substitutes, and increasing in the intensity of strategic considerations (i.e., in the magnitude of a and b, respectively). Propositions 2 and 3 imply that when strategic considerations, i.e., a and b, are sufficiently low (sufficiently high), there exists a unique symmetric equilibrium

[29] Detailed descriptions of these applications, including derivations of χ, are in Online Appendix B.

[30] Angeletos and Pavan (2007) assume that $a < 1/2$ to ensure that the first-best allocation is unique and bounded which, we can show, implies that $\chi < 1$.

in which firms over-react (under-react) to the public information about θ relative to the rational expectations benchmark. In contrast, for intermediate a (or b), firms choose to respond differently to public information. Our analysis also implies that an increase in strategic considerations leads to a decrease in the average response to public information (Proposition 3), but can have a non-monotonic impact on the dispersion of firm beliefs when strategic concerns are not too high or too low (Proposition 4).

6.2 Cournot versus Bertrand

Similarly, we can derive χ in the canonical linear industrial organization models with a large number of firms, which feature either substitutability or complementarity in actions. Specifically, consider a setting in which a continuum of firms compete in a single product market. Each firm maximizes profits $u = pq - C(q)$, where p is the price of the good, q is the quantity produced and $C(q) = c_1 q + c_2 q^2$ (with $c_1, c_2 > 0$) is the cost associated with producing quantity q.

In a Cournot setting, firms compete on quantity. Demand for a single firm's good is (implicitly) given by $p = a_0 + a_1\theta - a_2 q - a_3 Q$ (with $a_0, a_1, a_2, a_3 > 0$), where θ is a fundamental shock to demand and Q is the aggregate quantity produced across all firms. We show that $r = -\frac{a_3}{2(a_2+c_2)}$, $u_{KK} = 0$, and so

$$\chi = \left(\frac{\frac{a_3}{2(a_2+c_2)}}{1 + \frac{a_3}{2(a_2+c_2)}}\right)^2 = \left(\frac{a_3}{2\left(a_2 + c_2\right) + a_3}\right)^2 > 0,$$

increases in the degree of strategic substitutability.

In a Bertrand setting, firms compete on price. Demand for a single firm's good is given by $q = b_0 + b_1\theta - b_2 p + b_3 P$ (with $b_0, b_1, b_2, b_3 > 0$), where P is the average price in the market and θ is again a shock to demand. Let $b \equiv \frac{b_3}{b_2}$ and $c \equiv c_2 b_2$. Then it can be shown that $r = \frac{b(1+2c)}{2(1+c)}$, $u_{KK} = -2cb^2$, $u_{kk} = -2(1+c)$ and so

$$\chi = \frac{\left(\frac{b(1+2c)}{2(1+c)}\right)^2 - \frac{cb^2}{1+c}}{\left(1 - \frac{b(1+2c)}{2(1+c)}\right)^2} = \left(\frac{b}{2\left(1+c\right) - b\left(1+2c\right)}\right)^2 > 0.$$

The relative importance of non-fundamental volatility (χ) is positive even though actions are strategic substitutes in the first case, but strategic complements in the second. This leads to empirical implications similar to those described in Section 6.1. Unique to this set of models, however, is that χ is now decreasing in the costs, both implicit and explicit, of players' production/pricing decisions (i.e., a_2 and c_2 in Cournot, b_2 and c_2 in Bertrand). When these costs are relatively high (i.e., χ is low), our model predicts that all firms agree on the interpretation of public signals. Moreover, dispersion in beliefs decreases with the magnitude of these costs and with public information quality. However, when these costs are muted (i.e., closer to zero), our model predicts that firms will disagree on the interpretation of public signals.[31]

6.3 Beauty Contest

In this section, we consider the canonical beauty contest model of Morris and Shin (2002). The payoff of player i is given by

$$u_i \equiv -\rho \left(k_i - K\right)^2 - (1-\rho)\left(k_i - \theta\right)^2 + \rho\sigma_k^2, \tag{24}$$

where $r = \rho \in (-1, 1)$ is a measure of strategic complementarity. In contrast to the earlier applications, in this case, we can show that $\chi = -\frac{\rho}{1-\rho}$ and is positive (negative) when players' actions are substitutes (complements). We emphasize, however, that this is because, ρ captures also captures the direct impact of non-fundamental volatility in this setting, i.e., $u_{KK} = -\rho$ and this is what determines the impact of ρ on χ.[32]

The canonical beauty contest model provides a natural and parsimonious way to capture the important strategic incentives that professional forecasters have to distort their reports. As suggested by Croushore (1997) forecasters may shade their bid towards consensus to avoid being classified as "too wrong," while others may make extreme forecasts to stand out from the crowd. For instance, Ottaviani and Sørensen (2006) and Marinovic, Ottaviani, and Sorensen (2013) provide a model of forecasting contests in which the most accurate forecaster gains a disproportionate amount of rewards (e.g., attention). Such a game can be modeled as a beauty contest game with strategic substitutability in actions (i.e., $\rho < 0$), and consequently, $\chi > 0$.[33] These papers document empirical evidence of exaggeration consistent with these incentives using individual, real GNP forecasts from Business Week Investment Outlook and from the Survey of Professional Forecasters.

[31] In the Cournot setting, $\chi \in (0, 1)$. In the Bertrand setting, imposing that $b_3 < b_2$, so that an equal increase in p and P reduces q, implies that $b < 1$ and so $\chi \in (0, 1)$ also. Under this restriction, no symmetric equilibrium exists in which players under-react to the public signal.

[32] The benefit of non-fundamental volatility which arises from increased covariance with the actions of others (i.e., $u_{kk}r^2$) is ρ^2. Since $|\rho| < 1$, the effect of u_{KK} always dominates and determines the sign of χ.

[33] In a related paper, Marinovic, Ottaviani, and Sørensen (2010) model a prediction market as a beauty contest game with strategic complementarities. Even if one believes that professional forecasters don't have strategic incentives (i.e., they just minimize the mean squared error with $\rho = 0$), our model implies that they still deviate from rational expectations and all forecasters overreact to public information.

While the original paper of Morris and Shin (2002) emphasized how strategic considerations distort players' actions, our analysis shows that such considerations can endogenously distort beliefs, too. For instance, when ρ is sufficiently large (i.e., χ is low), there exists a unique symmetric equilibrium in which all forecasters over-react to public information, and dispersion in beliefs decreases with ρ. In contrast, when ρ is sufficiently close to minus one (so that $\chi \approx 0.5$), the unique equilibrium can feature disagreement about the interpretation of public information, and dispersion in forecasters is U-shaped in ρ. Moreover, across all equilibria, we expect the average response to public information (B) increases with ρ.

7 Subjective beliefs about private information

In this section, we consider the implications of allowing players to choose subjective beliefs about the precision of their own private signals (Section 7.1) and the quality of others' private signals (Section 7.2).

7.1 Subjective beliefs about one's own private information

In this section, we summarize the results of Appendix C.2, in which we allow players to choose subjective beliefs about the quality of their own private information. Specifically, player i's subjective belief about the error in his private signal is given by

$$\varepsilon_i \sim_i N\left(0, \frac{1}{\delta_{e,i}\tau_e}\right), \tag{25}$$

where $\delta_{e,i} \in \left[\underline{\delta}, \bar{\delta}\right]$. In Proposition 7, we show that when players choose $\delta_{e,i}$ (only) to maximize their total utility, as in (6), there exists a unique, symmetric equilibrium. All players choose to exhibit over-confidence in their private information i.e., set $\delta_{e,i} > 1$ since beliefs about the quality of one's private signal affect only the fundamental uncertainty channel. While managerial over-confidence has been extensively documented, our model predicts that over-confidence endogenously increases in prior uncertainty (i.e., decreases in τ), consistent with the empirical evidence on CFO beliefs found in Ben-David, Graham, and Harvey (2013).

In Proposition 8, we consider a setting in which players choose their subjective beliefs about both private and public information. In particular, we are able to fully characterize the equilibrium for all χ when the relative importance of experienced utility (ψ) is sufficiently low. The equilibria which arise are analogous to our benchmark. In particular, for intermediate χ, there exists a unique mixed strategy equilibrium that features endogenous disagreement about public information.

7.2 Subjective beliefs about the private information of others

In this section, we summarize the results of Appendix C.3, in which we allow players to choose their beliefs about others' private signals. Specifically, we assume that player i believes that the error in player j's private signal is given by

$$\varepsilon_j =_i \sqrt{1-p_i^2}\eta + p_i\varepsilon_j, \text{ where } \quad \eta, \varepsilon_j \sim N\left(0, \tfrac{1}{\delta_{j,i}\tau_e}\right), \tag{26}$$

η and ε_j are independent of each other and all other variables and $p_i \in [0,1]$ and $\delta_{j,i} \in \left[\underline{\delta}, \overline{\delta}\right]$ are chosen by player i to maximize his anticipatory utility. Notably, player i has subjective beliefs about the precision of others' signals ($\delta_{j,i}$) and the correlation (or commonality) across them (p_i). To highlight the impact of beliefs about others, we shut down the public signal, s, and players hold objective beliefs about the precision of their own signal.

Proposition 9 characterizes the equilibrium subjective beliefs. We show that subjective belief choices depend on the relative importance of dispersion versus aggregate volatility. When dispersion is preferable ($u_{\sigma\sigma} > u_{KK}$), player i benefits from believing that others' signals are more independent (i.e., distorting p_i upwards). Moreover, if $u_{\sigma\sigma}$ is positive, then he wants to believe that $\delta_{j,i}$ is lower, since this further increases the dispersion in players' actions; otherwise he chooses to increase $\delta_{j,i}$. On the other hand, when aggregate volatility is preferable ($u_{\sigma\sigma} < u_{KK}$), player i prefers to distort p_i downwards, and if $u_{KK} > 0$, player i lowers $\delta_{j,i}$. These distortions serve to increase the perceived volatility of aggregate actions.

We note that a player's action (i.e., k_i) does not depend on his subjective beliefs about others' private signals. As a result, such choices do not affect aggregate observables, such as realized dispersion and non-fundamental volatility. In a more general setting, where players also hold subjective beliefs about their own signal (or the public signal), beliefs about others may have an indirect effect.[34] While such interactions are potentially interesting, the main economic channels remain the same as in our main analysis, and so we leave characterizing equilibrium beliefs along these additional dimensions for future work.

8 Conclusions

In a standard model of externalities where players are uncertain about the precision of public information, we show that "wishful thinking" can lead to endogenous disagreement about public news. We show that the nature of the equilibrium depends not on whether actions are complements or substitutes, but on the relative impact of non-fundamental volatility

[34] For example, if player i believes that $p_i = 0$, he anticipates more non-fundamental volatility which will change the marginal benefit of believing the public signal is noisier.

on payoffs. We derive a number of testable predictions about the aggregate response to public information, dispersion in beliefs, and implications for individual-level forecast error regressions that distinguish our model from the existing literature. We map these into specific predictions for applications of our model including strategic investment decisions, Bertrand and Cournot competition, and the canonical beauty contest model.

There are natural opportunities for future work. First, it would be interesting to study how information acquisition or attention interacts with players' endogenous perception in the presence of externalities. Second, it would be useful to extend the analysis to dynamic settings and characterize the implications of wishful thinking for the term structure of forecasts and disagreement. Finally, we view a preference for robust control as complementary to our own - individuals are likely to exhibit wishful thinking in some settings, but robust control under others - and hope to add to a budding literature (e.g., Bhandari, Borovicka, and Ho (2019)) which allows both types to arise.

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A Proofs

A.1 Proof of Lemma 1

Suppose player i conjectures that the average action has the form $K = \kappa_0 + \alpha\theta + \beta s$, where α and β are determined in equilibrium. Then we solve for the equilibrium by (i) plugging in the conjecture in the optimal action for player i, (ii) aggregating across players, and (iii) matching terms. This yields:

$$\alpha = \frac{A\,(1-r)\,\kappa_1}{1-rA}, \quad \beta = \frac{B}{1-rA}\kappa_1. \tag{27}$$

where $A \equiv \int_i A_i di$ and $B \equiv \int_i B_i di$ reflect the average weights players put on their private and public signals, respectively.[35]

A.2 Proof of Lemma 2

Let $\mathcal{K} \equiv rK + (1-r)\,\kappa$ denote the target action for each player. Given the assumptions about payoffs, note that we can express the utility as:

$$u(k,K,\theta,\sigma) = \begin{array}{c} u_0 + u_k k + u_K K + u_\theta \theta + \frac{1}{2}\left(u_{kk}k^2 + u_{KK}K^2 + u_{\theta\theta}\theta^2 + u_{\sigma\sigma}\sigma^2\right) \\ + u_{k\theta}k\theta + u_{K\theta}K\theta + u_{kK}kK \end{array}$$

A Taylor expansion around the target $(\kappa,\kappa,\theta,0)$ yields:

$$u\left(k,K,\theta,\sigma\right) = \begin{array}{c} u(\kappa,\kappa,\theta,0) + \frac{\partial}{\partial k}u\left(\kappa,\kappa,\theta,0\right)\cdot\left(k-\kappa\right) + \frac{\partial}{\partial K}u\left(\kappa,\kappa,\theta,0\right)\cdot\left(K-\kappa\right) \\ +\frac{1}{2}\left(u_{\sigma\sigma}\sigma^2 + u_{kk}\left(k-\kappa\right)^2 + u_{KK}\left(K-\kappa\right)^2 + 2u_{kK}\left(k-\kappa\right)\left(K-\kappa\right)\right) \end{array} \tag{28}$$

Note that $\frac{\partial}{\partial k}u\left(\kappa,\kappa,\theta,0\right) = u_k + u_{kk}\kappa + u_{k\theta}\theta + u_{kK}\kappa = 0$ and

$$\left(k-\kappa\right)^2 = \left(k-\mathcal{K}\right)^2 + r^2\left(K-\kappa\right)^2 + 2r\left(k-\mathcal{K}\right)\left(K-\kappa\right)$$

Let $\mathbb{E}_j\left[\cdot\right]$ denote expectations w.r.t. arbitrary beliefs - we will later plug in subjective and objective beliefs. Then,

$$\mathbb{E}_j\left[u\left(k,K,\theta,\sigma\right)\right] = \mathbb{E}_j\left[\begin{array}{c} u\left(\kappa,\kappa,\theta,0\right) + \left(u_K + ru_{kk}\left(k-\mathcal{K}\right) + u_{kK}\left(k-\kappa\right)\right)\cdot\left(K-\kappa\right) \\ +\frac{1}{2}\left(u_{\sigma\sigma}\sigma^2 + u_{kk}\left(k-\mathcal{K}\right)^2 + \left(u_{KK} + r^2u_{kk}\right)\left(K-\kappa\right)^2\right) \end{array}\right] \tag{29}$$

[35] Implicitly, we are assuming that the law of large numbers implies: $\int_i A_i s_i di = \int_i A_i di \times \int_i s_i di = A \times \theta$. Specifically, this assumes there is no cross-sectional correlation between A_i and s_i, but this is valid because $(\delta_{e,i}, \delta_{\eta,i})$ are chosen before s_i (and s) are observed.

Since $ru_{kk} = -u_{kK}$,we have that

$$ru_{kk}\left(k-\mathcal{K}\right)+u_{kK}\left(k-\kappa\right)=u_{kK}\left(\mathcal{K}-\kappa\right)=-r^{2}u_{kk}\left(K-\kappa\right)$$

which implies

$$\mathbb{E}_{j}\left[u\left(k,K,\theta,\sigma\right)\right]=\mathbb{E}_{j}\left[\begin{array}{c} u\left(\kappa,\kappa,\theta,0\right)+u_{K}\left(K-\kappa\right) \\ +\frac{1}{2}\left(u_{\sigma\sigma}\sigma^{2}+u_{kk}\left(k-\mathcal{K}\right)^{2}+\left(u_{KK}-r^{2}u_{kk}\right)\left(K-\kappa\right)^{2}\right)\end{array}\right] \tag{30}$$

Player i's subjective beliefs about the joint distribution of $(\theta, s_i, \{s_j\}_{j\neq i}, s)$ are given by

$$\begin{pmatrix}\theta\\ s_i\\ s_j\\ s\end{pmatrix}\sim_i N\left(\begin{pmatrix}0\\0\\0\\0\end{pmatrix},\begin{pmatrix}\frac{1}{\tau} & \frac{1}{\tau} & \frac{1}{\tau} & \frac{1}{\tau}\\ \frac{1}{\tau} & \frac{1}{\tau}+\frac{1}{\tau_e} & \frac{1}{\tau} & \frac{1}{\tau}\\ \frac{1}{\tau} & \frac{1}{\tau} & \frac{1}{\tau}+\frac{1}{\tau_e} & \frac{1}{\tau}\\ \frac{1}{\tau} & \frac{1}{\tau} & \frac{1}{\tau} & \frac{1}{\tau}+\frac{1}{\delta_{\eta,i}\tau_\eta}\end{pmatrix}\right). \tag{31}$$

Let $u_0 \equiv u(\kappa, \kappa, \theta, 0)$ and note that the unconditional expectations $\mathbb{E}_i[u_0] = \mathbb{E}[u_0]$ and $\mathbb{E}_i[u_K(K-\kappa)] = \mathbb{E}[u_K(K-\kappa)]$ do not depend on player i's subjective beliefs. Then, player i's anticipatory utility is given by:

$$AU_i=\mathbb{E}_i\left[u_i\right]=\begin{array}{c}\mathbb{E}\left[u_0\right]+\mathbb{E}[u_K(K-\kappa)]+\frac{u_{\sigma\sigma}}{2}\sigma_i^2\\ +\frac{1}{2}\left(u_{kk}\mathbb{E}_i\left[\left(k-\mathcal{K}\right)^2\right]+\left(u_{KK}-r^2u_{kk}\right)\mathbb{E}_i\left[\left(K-\kappa\right)^2\right]\right)\end{array} \tag{32}$$

Moreover,

$$\mathbb{E}_i\left[\left(k-\mathcal{K}\right)^2\right]=\left(\frac{\kappa_1\left(1-r\right)}{1-rA}\right)^2\operatorname{var}_i\left[\theta|s_i,s\right]$$

$$\mathbb{E}_i\left[\left(K-\kappa\right)^2\right]=\left(\frac{\kappa_1\left(1-A-B\right)}{1-rA}\right)^2\operatorname{var}\left[\theta\right]+\left(\frac{\kappa_1 B}{1-rA}\right)^2\operatorname{var}_i\left[s|\theta\right]$$

Substituting all the above expressions into equation (32) and simplifying gives us

$$AU_i\left(\delta_{\eta,i}\right)=L_1+\left(\frac{\kappa_1}{1-rA}\right)^2\left[u_{kk}\left(1-r\right)^2\operatorname{var}_i\left[\theta|s_i,s\right]-\left(u_{KK}-r^2u_{kk}\right)B^2\operatorname{var}_i\left[s|\theta\right]\right], \tag{33}$$

where L_1 does not depend upon player i's choice of beliefs.

A.3 Proof of Corollary 1

Since all other players exhibit rational expectations ($\delta_{\eta,j} = 1 \forall j \neq i$), $B = \int_i B_i di = B_{RE}$, the objective of player i (from (6) and equation (33)) is

$$\max_{\delta_{\eta,i}\in\{\underline{\delta},\bar{\delta}\}}\frac{B_{RE}^2\chi}{\delta_{\eta,\mathrm{i}}\tau_\eta}-\frac{1}{\delta_{\eta,\mathrm{i}}\tau_\eta+\tau_e+\tau}$$

where player i takes $-u_{kk}\left(\frac{\kappa_1(1-r)}{1-rA}\right)^2 > 0$ as given. It is easy to see that the above objective is either downward sloping or U shaped in $\delta_{\eta,i}$ which implies that player i will choose either

$\delta_{\eta,i} = \bar{\delta}$ or $\delta_{\eta,i} = \underline{\delta}$. Player i will choose $\delta_{\eta,i} = \bar{\delta}$ iff

$$\frac{B_{RE}^2 \chi}{\bar{\delta}\tau_\eta} - \frac{1}{\bar{\delta}\tau_\eta + \tau_e + \tau} > \frac{B_{RE}^2 \chi}{\underline{\delta}\tau_\eta} - \frac{1}{\underline{\delta}\tau_\eta + \tau_e + \tau}$$

which simplifies to the condition in Corollary 1.

A.4 Proof of Proposition 1

Since $\psi = 0$, the objective of player i (from (6) and equation (33)) can be rewritten as

$$\max_{\delta_{\eta,i} \in \{\underline{\delta}, \bar{\delta}\}} \frac{B^2 \chi}{\delta_{\eta,\mathrm{i}}\tau_\eta} - \frac{1}{\delta_{\eta,\mathrm{i}}\tau_\eta + \tau_e + \tau},$$

since player i takes $-u_{kk}\left(\frac{\kappa_1(1-r)}{1-rA}\right)^2 > 0$ as given. It is easy to see that the above objective is either downward/upward sloping or U shaped in $\delta_{\eta,i}$ which implies that player i will choose either $\delta_{\eta,i} = \bar{\delta}$ or $\delta_{\eta,i} = \underline{\delta}$. Conjecture that all other players choose $\delta_{\eta,j} = \bar{\delta} \forall j \neq i$. In this case, $B = \bar{B}$ and player i will also choose $\delta_{\eta,i} = \bar{\delta}$ iff

$$\frac{\bar{B}^2 \chi}{\bar{\delta}\tau_\eta} - \frac{1}{\bar{\delta}\tau_\eta + \tau_e + \tau} > \frac{\bar{B}^2 \chi}{\underline{\delta}\tau_\eta} - \frac{1}{\underline{\delta}\tau_\eta + \tau_e + \tau}$$

which is true iff $\chi < \underline{B}/\bar{B}$. Similarly, conjecture that all players choose $\delta_{\eta,i} = \underline{\delta}$. Player i will also choose $\delta_{\eta,i} = \underline{\delta}$ iff

$$\frac{\underline{B}^2 \chi}{\bar{\delta}\tau_\eta} - \frac{1}{\bar{\delta}\tau_\eta + \tau_e + \tau} < \frac{\underline{B}^2 \chi}{\underline{\delta}\tau_\eta} - \frac{1}{\underline{\delta}\tau_\eta + \tau_e + \tau}$$

which is true iff $\chi > \bar{B}/\underline{B}$. This also implies that, if $\chi \in \left[\underline{B}/\bar{B}, \bar{B}/\underline{B}\right]$, then a symmetric equilibrium cannot exist. Conjecture an equilibrium in which a fraction λ of players choose $\delta_{\eta,j} = \bar{\delta}$ and the remaining choose $\delta_{\eta,j} = \underline{\delta}$. This will be the equilibrium iff player i also is indifferent between these choices i.e.,

$$\frac{B^2 \chi}{\bar{\delta}\tau_\eta} - \frac{1}{\bar{\delta}\tau_\eta + \tau_e + \tau} = \frac{B^2 \chi}{\underline{\delta}\tau_\eta} - \frac{1}{\underline{\delta}\tau_\eta + \tau_e + \tau}$$

which will be true iff $B = \sqrt{\frac{\bar{B}\underline{B}}{\chi}}$. By definition, $B = \lambda \bar{B} + (1-\lambda)\underline{B}$, which implies $\lambda = \frac{\sqrt{\frac{\bar{B}\underline{B}}{\chi}} - \underline{B}}{\bar{B} - \underline{B}}$. Finally, note that $\lambda \in (0,1) \iff \chi \in \left[\frac{\underline{B}}{\bar{B}}, \frac{\bar{B}}{\underline{B}}\right]$. This implies that the equilibrium is continuous. Define

$$\bar{\chi} \equiv \frac{\bar{B}}{\underline{B}} = \frac{\bar{\delta}\left(\underline{\delta}\tau_\eta + \tau_e + \tau\right)}{\underline{\delta}\left(\bar{\delta}\tau_\eta + \tau_e + \tau\right)} = \frac{1}{\underline{\chi}}$$

which implies that

$$\frac{\partial\bar{\chi}}{\partial\tau}=\frac{\bar{\delta}\left(\bar{\delta}-\underline{\delta}\right)\tau_{\eta}}{\underline{\delta}\left(\bar{\delta}\tau_{\eta}+\tau_{e}+\tau\right)^{2}}>0,\quad \frac{\partial\bar{\chi}}{\partial\tau_{\eta}}=-\frac{\bar{\delta}\left(\bar{\delta}-\underline{\delta}\right)(\tau_{e}+\tau)}{\underline{\delta}\left(\bar{\delta}\tau_{\eta}+\tau_{e}+\tau\right)^{2}}<0,\quad \frac{\partial\bar{\chi}}{\partial\tau_{e}}=\frac{\bar{\delta}\left(\bar{\delta}-\underline{\delta}\right)\tau_{\eta}}{\underline{\delta}\left(\bar{\delta}\tau_{\eta}+\tau_{e}+\tau\right)^{2}}>0. \tag{34}$$

A.5 Lemma 3 and its proof

Lemma 3. *The experienced utility is the expected utility that the player incurs under the objective distribution and is given by*

$$EU_{i}\left(\delta_{\eta,i}\right)=L_{2}+u_{kk}\left(\frac{\kappa_{1}\left(1-r\right)}{1-rA}\right)^{2}\left(\frac{\tau+\tau_{e}+\delta_{\eta,i}^{2}\tau_{\eta}}{\left(\tau+\tau_{e}+\delta_{\eta,i}\tau_{\eta}\right)^{2}}\right). \tag{35}$$

where L_2 does not depend upon player i's beliefs.

Proof. Applying equation (30) to objective beliefs, we get $EU_{i}\left(\delta_{\eta,i}\right)=\mathbb{E}\left[u\left(k_{i}\left(\delta_{\eta,i}\right),K,\theta,\sigma\right)\right].$ Note that the only term which depends upon player i's beliefs is

$$\mathbb{E}\left[\left(k-\mathcal{K}\right)^{2}\right]=\left(\frac{\kappa_{1}\left(1-r\right)}{1-rA}\right)^{2}\left(\frac{\tau+\tau_{e}+\delta_{\eta,i}^{2}\tau_{\eta}}{\left(\tau+\tau_{e}+\delta_{\eta,i}\tau_{\eta}\right)^{2}}\right)$$

since it is dependent upon his action $k_i\left(\delta_{\eta,i}\right)$. This immediately implies (35). ☐

A.6 Proof of Proposition 2 and Corollary 2

Each player maximizes (6), which by (33) and (35) is equivalent to maximizing

$$OU\left(B_{i},\chi,B\right)\equiv-\frac{1}{\tau+\tau_{e}+\delta_{\eta,i}\tau_{\eta}}+\frac{\chi B^{2}}{\delta_{\eta,i}\tau_{\eta}}-\psi\frac{\tau+\tau_{e}+\delta_{\eta,i}^{2}\tau_{\eta}}{\left(\tau+\tau_{e}+\delta_{\eta,i}\tau_{\eta}\right)^{2}} \tag{36}$$

$$=-\frac{1-B_{i}}{\tau+\tau_{e}}\left(1-\frac{\chi B^{2}}{B_{i}}\right)-\frac{\psi}{\tau+\tau_{e}}\left[\left(1-B_{i}\right)^{2}+B_{i}^{2}\left(\frac{1-B_{RE}}{B_{RE}}\right)\right] \tag{37}$$

where $B_{i}\equiv\frac{\delta_{\eta,i}\tau_{\eta}}{\tau+\tau_{e}+\delta_{\eta,i}\tau_{\eta}}$ denotes the weight player i chooses to place on s. This follows since player i takes $-u_{kk}\left(\frac{\kappa_{1}(1-r)}{1-rA}\right)^{2}>0$ as given. Then,

$$\frac{\partial OU_{i}}{\partial B_{i}}=\frac{1}{B_{i}^{2}\left(\tau+\tau_{e}\right)}\left[\underbrace{\left(1+2\psi\left[1-\frac{B_{i}}{B_{RE}}\right]\right)B_{i}^{2}-\chi B^{2}}_{\equiv f\left(B_{i},B,B_{RE},\chi,\psi\right)}\right]. \tag{38}$$

Whether OU_i is increasing or decreasing in player i's subjective belief, B_i depends upon the sign of $f\left(B_{i},B,B_{RE},\chi,\psi\right)$ defined in equation (38). Note that $\frac{\partial f}{\partial B_{i}}=-\frac{6\psi}{B_{RE}}B_{i}^{2}+2\left(2\psi+1\right)B_{i}$ and equals zero at $B_i=0$ and $B_{i}=\frac{B_{RE}(2\psi+1)}{3\psi}$. Moreover, $\frac{\partial^{2}f}{\partial B_{i}^{2}}$ is positive at $B_i=0$ (i.e., it is a minimum) and negative at $B_{i}=\frac{B_{RE}(2\psi+1)}{3\psi}$ (i.e., it is a maximum). This implies that, if $\frac{B_{RE}(2\psi+1)}{3\psi} < 1$, $f(B_i, B, B_{RE}, \chi, \psi)$ is increasing in the range $B_i \in (0, \frac{B_{RE}(2\psi+1)}{3\psi})$ and decreasing for higher B_i. If $\frac{B_{RE}(2\psi+1)}{3\psi} > 1$, $f(B_i, B, B_{RE}, \chi, \psi)$ is increasing in the range $B_i \in (0, 1)$.

When $\chi \leq 0$, the objective function is increasing at $B_i = 0$ since $f(0, B, B_{RE}, \chi, \psi) = -\chi B^2 > 0$.Given our observations above, this gives rise to two possibilities. Either the objective function is (i) always increasing for any feasible B_i or (ii) there is an interior, global maximum (when $f(B_i, B, B_{RE}, \chi, \psi) = 0$ for some $B_i < 1$.)

When $\chi > 0$, the objective function is decreasing at $B_i = 0$. This implies that there are three possible cases. Either the objective function is (i) always decreasing, (ii) U-shaped, or (iii) has an interior (local) maximum. The latter two cases arise when $\exists$ some B_0 such that $f(B_0, B, B_{RE}, \chi, \psi) > 0$ and $0 < B_0 \leq 1$. Case (ii) occurs if $f(B_i, B, B_{RE}, \chi, \psi) > 0$ for all feasible $B_i \geq B_0$; otherwise, case (iii) occurs.

Given this, we can establish conditions for the different equilibria over the interval $B_i \in [\underline{B}, \bar{B}]$. First, note that there is, at most, a single interior maximum. Second, both $\underline{B}$ and $\bar{B}$ are potential global maxima. Third, there are only two feasible mixed strategy equilibria: a player can mix between $\underline{B}$ and either (i) $\bar{B}$ or (ii) an interior maximum, derived below.

Step 1: Characterize the mixed-strategy equilibria

There are two cases to consider. Suppose that $f(\bar{B}, B, B_{RE}, \chi, \psi) \geq 0$. In this setting, the objective function is (weakly) increasing at the upper bound. As a result, the objective function is either upward-sloping or U-shaped, i.e., there are no interior maxima by the argument laid out above that the f function is hump-shaped. To determine the conditions on χ for each type of equilibrium, we will directly compare the expected utility of the two possible choices, $\underline{B}$ and $\bar{B}$, given the behavior of all other players, B. In a mixed-strategy equilibrium, each player must be indifferent between choosing $\underline{B}$ and $\bar{B}$, i.e.,

$$OU(\underline{B}, \chi, B) = OU(\bar{B}, \chi, B) \tag{39}$$

Simplifying this utilizing (37) yields

$$\chi B^2 = \left(\psi\left[2 - \left(\frac{\bar{B} + \underline{B}}{B_{RE}}\right)\right] + 1\right)\underline{B}\bar{B}. \tag{40}$$

This implies that in any mixed-strategy equilibrium utilizing $\underline{B}$ and $\bar{B}$

$$B = \sqrt{\frac{\left(\psi\left[2 - \left(\frac{\bar{B}+\underline{B}}{B_{RE}}\right)\right] + 1\right)\underline{B}\bar{B}}{\chi}} \text{ and } \lambda = \frac{B - \underline{B}}{\bar{B} - \underline{B}}, \tag{41}$$

where λ denotes the measure of players who choose $\bar{B}$. Note that B is decreasing in χ. Since it must be the case that $\underline{B} \leq B \leq \bar{B}$, we can substitute these bounds on B into the indifference condition to yield the bounds on χ such that this mixed equilibrium can be sustained:

$$\left(\psi\left[2 - \left(\frac{\bar{B} + \underline{B}}{B_{RE}}\right)\right] + 1\right)\frac{\bar{B}}{\underline{B}} > \chi > \left(\psi\left[2 - \left(\frac{\bar{B} + \underline{B}}{B_{RE}}\right)\right] + 1\right)\frac{\underline{B}}{\bar{B}}. \tag{42}$$

It remains, however, for us to confirm our conjecture that $f\left(\bar{B}, B, B_{RE}, \chi, \psi\right) \geq 0$. Given the equilibrium B specified above, this is true as long as

$$\left(1+2\psi\left[1-\frac{\bar{B}}{B_{RE}}\right]\right)\bar{B}^2 \geq \chi B^2 \iff \psi \leq \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}. \tag{43}$$

On the other hand, suppose that $f\left(\bar{B}, B, B_{RE}, \chi, \psi\right) < 0$. Under this conjecture, the objective function is decreasing at the upper bound and so the only feasible mixed-strategy equilibrium is where players are indifferent between $\underline{B}$ and an interior maximum, which we will denote as B_η. Utilizing the same steps as above, indifference between $\underline{B}$ and B_η requires

$$\chi B^2 = \left(\psi\left[2-\left(\frac{B_\eta+\underline{B}}{B_{RE}}\right)\right]+1\right)\underline{B}B_\eta. \tag{44}$$

This yields expressions analogous to those found in (41) for the equilibrium B and λ and analogous cutoffs for χ as those found in (42) with B_η replacing $\bar{B}$. It remains, however, to identify the equilibrium B_η which arises in this equilibrium. Note that, for an interior maximum, the first-order condition must hold given the equilibrium B. This implies that $f\left(B_\eta, B, B_{RE}, \chi, \psi\right) = 0$ is true if and only if

$$\left(1+2\psi\left[1-\frac{B_\eta}{B_{RE}}\right]\right)B_\eta^2 = \left(\psi\left[2-\frac{B_\eta+\underline{B}}{B_{RE}}\right]+1\right)\underline{B}B_\eta \iff B_\eta = \frac{B_{RE}\left(1+2\psi\right)-\psi\underline{B}}{2\psi}. \tag{45}$$

It is easy to verify that the SOC holds at $B_i = B_\eta$. Confirming our conjecture that $f\left(\bar{B}, B, B_{RE}, \chi, \psi\right) < 0$ is equivalent to showing that $B_\eta < \bar{B}$ given our observations on the properties of $f\left(\cdot\right)$. Using (45), this is the case only if

$$\psi > \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}, \tag{46}$$

which exactly corresponds to our threshold from Case 1 (given in equation (43)). Moreover, when $\psi = \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$, $B_\eta = \bar{B}$ so that both the upper and lower bounds of the two types of mixed-strategy equilibria are continuous.

Finally, feasibility also requires that B_η be greater than or equal to $\underline{B}$. If $3\underline{B} \leq 2B_{RE}$, then this is true for any $\psi > 0$. Otherwise, it must be the case that

$$\psi \leq \frac{B_{RE}}{3\underline{B}-2B_{RE}}. \tag{47}$$

If $3\underline{B} > 2B_{RE}$, for ψ above this cutoff, mixed equilibrium doesn't exist.

Step 2: Characterize the possible symmetric equilibria

There are only three possible symmetric equilibria: everyone chooses $\underline{B}$, $\bar{B}$, or an interior maximum which we denote B_s. For B_s to be an equilibrium, the first-order condition must hold when everyone else chooses B_s, i.e., $f(B_s, B_s, B_{RE}, \chi, \psi) = 0$. This is true as long as

$$\left(1 + 2\psi\left[1 - \frac{B_s}{B_{RE}}\right]\right) B_s^2 = \chi B_s^2 \implies B_s = B_{RE}\left[\frac{1 + 2\psi - \chi}{2\psi}\right]. \tag{48}$$

Note that feasibility requires that

$$B_s < \bar{B} \iff 2\psi(\bar{B} - B_{RE}) > B_{RE}(1 - \chi) \tag{49}$$

$$B_s > \underline{B} \iff 2\psi(\underline{B} - B_{RE}) < B_{RE}(1 - \chi) \tag{50}$$

Placing weight B_s on the public signal is equivalent to choosing

$$\delta_{\eta,i} = \frac{2\psi + 1 - \chi}{2\psi - \frac{\tau_\eta}{\tau + \tau_e}(1 - \chi)} \equiv \delta_s \tag{51}$$

To ensure that this is a local maximum (we will derive conditions under which it is the global maximum below), the second-order condition must also hold, i.e., $\frac{\partial^2}{\partial B_i^2} OU(B_s, \chi, B_s) < 0$. This condition can be written as

$$-\frac{\tau_\eta(-3\chi + 2\psi + 1)(2\psi(\tau_e + \tau) + (\chi - 1)\tau_\eta)^4}{8\psi^3(\tau_e + \tau)^3(-\chi + 2\psi + 1)(\tau_e + \tau_\eta + \tau)^3} < 0 \tag{52}$$

which is true if either (i) $\chi > 2\psi + 1$ or (ii) $\chi < \frac{2\psi+1}{3}$. Note that if the former cutoff holds, then $B_s < 0$ and so the latter will be the relevant constraint.

Step 3: Establish uniqueness for the mixed strategy equilibria

To establish uniqueness, we will show that under the conditions which give rise to the mixed-strategy equilibrium (derived in step 1), no symmetric equilibrium can exist. We start with the case where $\psi \le \frac{B_{RE}}{2\bar{B} + \underline{B} - 2B_{RE}}$ where the mixed-strategy equilibrium is mixing between $\underline{B}$ and $\bar{B}$.

Conjecture that everyone chooses $\underline{B}$. To show that this cannot be an equilibrium in this region, note that player i finds it profitable to deviate to $\bar{B}$ when $OU\left(\bar{B}, \chi, \underline{B}\right) > OU\left(\underline{B}, \chi, \underline{B}\right)$, which is true if $\chi < \left(\psi\left[2 - \left(\frac{\bar{B} + \underline{B}}{B_{RE}}\right)\right] + 1\right)\frac{\bar{B}}{\underline{B}}$. This corresponds to the upper bound for the mixed strategy equilibrium found in (42), i.e., everyone choosing $\underline{B}$ cannot be an equilibrium in this region. Similarly, everyone choosing $\bar{B}$ cannot be an equilibrium in this region since $OU\left(\underline{B}, \chi, \bar{B}\right) > OU\left(\bar{B}, \chi, \bar{B}\right)$ if $\chi > \left(\psi\left[2 - \left(\frac{\bar{B} + \underline{B}}{B_{RE}}\right)\right] + 1\right)\frac{\underline{B}}{\bar{B}}$, the lower bound in (42).

Lastly, note that player i finds it beneficial to deviate from B_s to $\underline{B}$ if $OU\left(\underline{B}, \chi, B_s\right) > OU\left(B_s, \chi, B_s\right)$, which is the case when $\chi > \psi\frac{\underline{B}}{B_{RE}}$. The symmetric equilibrium cannot arise then as long as this threshold is below the lower bound found in (42), i.e.,

$$\psi\frac{\underline{B}}{B_{RE}} < \left(\psi\left[2 - \left(\frac{\bar{B} + \underline{B}}{B_{RE}}\right)\right] + 1\right)\frac{\underline{B}}{\bar{B}} \iff \psi < \frac{B_{RE}}{2\bar{B} + \underline{B} - 2B_{RE}} \tag{53}$$

which defines the cutoff for this mixed-strategy equilibrium. Together, this implies that the conjectured mixed-strategy equilibrium when $\psi \le \frac{B_{RE}}{2\bar{B} + \underline{B} - 2B_{RE}}$ is unique.

We next consider the case where $\psi > \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$, i.e., where the mixed-strategy equilibrium features mixing between $\underline{B}$ and B_η. The steps are isomorphic to those above. First, player i deviates and places weight B_η on s when others choose $\underline{B}$ as long as $\chi < \left(\psi\left[2-\left(\frac{B_\eta+\underline{B}}{B_{RE}}\right)\right]+1\right)\frac{B_\eta}{\underline{B}}$, which is the upper bound for this equilibrium. Second, player i deviates and places weight $\underline{B}$ on s when others choose $\bar{B}$ if $\chi > \left(\psi\left[2-\left(\frac{\bar{B}+\underline{B}}{B_{RE}}\right)\right]+1\right)\frac{\underline{B}}{\bar{B}}$, which is less than the lower bound for this equilibrium.

Lastly, note that we can rewrite the lower bound for the mixed-strategy equilibrium when $\psi > \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$ by substituting in the equilibrium B_η. This yields the following lower bound: $\chi \geq \psi\frac{\underline{B}}{B_{RE}}$ for the conjectured mixed-strategy equilibrium to arise. But this rules out the symmetric equilibrium B_s because, as we noted above, player i would deviate to $\underline{B}$ if everyone else chooses B_s in this region. Together, this implies that the conjectured mixed-strategy equilibrium when $\psi > \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$ is unique.

Step 4: Establish existence and uniqueness for the symmetric equilibria

We start by defining the relative utility player i receives from two choices of B_i (B_1 and B_2), given χ and B (the aggregate weight placed by others):

$$\Delta\left(B_1, B_2, \chi, B\right) \equiv \int_{B_1}^{B_2} \frac{\partial OU}{\partial B_i} dB_i. \tag{54}$$

If $\Delta\left(B_{1,2}, \chi, B\right) = 0$, then player i's payoff from B_1 and B_2 is the same, i.e., he is indifferent between choosing B_1 and B_2 given χ and B. If $\Delta\left(B_1, B_2, \chi, B\right) > 0$, he prefers B_2; if $\Delta\left(B_1, B_2, \chi, B\right) < 0$, he prefers B_1. We will construct the existence and uniqueness of the symmetric equilibrium in a piecemeal fashion.

1. Suppose that $\psi \leq \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$ and $\chi > \left(\psi\left[2-\left(\frac{\bar{B}+\underline{B}}{B_{RE}}\right)\right]+1\right)\frac{\bar{B}}{\underline{B}}$. First, we show that a symmetric equilibrium in which all players choose $\underline{B}$ exists under these conditions. Note that when $\chi = \left(\psi\left[2-\left(\frac{\bar{B}+\underline{B}}{B_{RE}}\right)\right]+1\right)\frac{\bar{B}}{\underline{B}}$, the analysis in Step 1 above implies that players are indifferent between $\underline{B}$ and $\bar{B}$ and that $\lambda = 0$ which implies $B = \underline{B}$ and so $\Delta\left(\underline{B}, \bar{B}, \chi, \underline{B}\right) = 0$. Moreover, this implies that $\Delta\left(\underline{B}, \hat{B}, \chi, \underline{B}\right) < 0$ if $\underline{B} < \hat{B} < \bar{B}$. Second, note that (38) implies that an increase in χ uniformly decreases $\frac{\partial OU}{\partial B_i}$. Thus, for any $\chi > \left(\psi\left[2-\left(\frac{\bar{B}+\underline{B}}{B_{RE}}\right)\right]+1\right)\frac{\bar{B}}{\underline{B}}$, it must be the case that $\Delta\left(\underline{B}, \hat{B}, \chi, \underline{B}\right) < 0$ for any feasible $\hat{B}$ not equal to $\underline{B}$. That is, a symmetric equilibrium in which everyone chooses $\underline{B}$ exists in this region. Moreover, the arguments found above in Step 3 rule out potential symmetric equilibria in which all players choose $\bar{B}$ or B_s in this region, establishing uniqueness.

2. Suppose that $\psi \leq \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$ and $\chi < \left(\psi\left[2-\left(\frac{\bar{B}+\underline{B}}{B_{RE}}\right)\right]+1\right)\frac{\underline{B}}{\bar{B}}$. We show that a symmetric equilibrium in which all players choose $\bar{B}$ exists under these conditions. If $\chi = \left(\psi\left[2-\left(\frac{\bar{B}+\underline{B}}{B_{RE}}\right)\right]+1\right)\frac{\underline{B}}{\bar{B}}$, the analysis in Step 1 above implies that players are indifferent between $\underline{B}$ and $\bar{B}$ and that $\lambda = 1$ which implies $B = \bar{B}$ and so $\Delta\left(\underline{B}, \bar{B}, \chi, \bar{B}\right) = 0$. Moreover, this implies that $\Delta\left(\hat{B}, \bar{B}, \chi, \bar{B}\right) > 0$ if $\underline{B} < \hat{B} < \bar{B}$. Again, (38) implies that a

decrease in χ uniformly increases $\frac{\partial OU}{\partial B_i}$. Thus, for any $\chi < \left(\psi\left[2-\left(\frac{\bar{B}+\underline{B}}{B_{RE}}\right)\right]+1\right)\frac{\underline{B}}{\bar{B}}$, it must be the case that $\Delta\left(\hat{B},\bar{B},\chi,\bar{B}\right) > 0$ for any feasible $\hat{B}$ not equal to $\bar{B}$. That is, a symmetric equilibrium in which everyone chooses $\bar{B}$ exists in this region. The arguments found above in Step 3 rule out potential symmetric equilibria in which all players choose $\underline{B}$. Moreover, under these conditions on ψ and χ, $B_s > \bar{B}$ by (49) and so it is not a feasible equilibrium. This establishes uniqueness.

3. Suppose that either (i) $3\underline{B} \leq 2B_{RE}$ and $\psi > \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$ or (ii) $3\underline{B} > 2B_{RE}$ and $\frac{B_{RE}}{3\underline{B}-2B_{RE}} \geq \psi > \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$. Then when $\chi = \left(\psi\left[2-\left(\frac{B_\eta+\underline{B}}{B_{RE}}\right)\right]+1\right)\frac{B_\eta}{\underline{B}}$, by step 1 above, players are indifferent between $\underline{B}$ and B_η. For any χ above this threshold, the unique symmetric equilibrium is $\underline{B}$. Establishing this follows the same logic as above. For any $\chi > \left(\psi\left[2-\left(\frac{B_\eta+\underline{B}}{B_{RE}}\right)\right]+1\right)\frac{B_\eta}{\underline{B}}$, it must be the case that $\Delta\left(\underline{B},\hat{B},\chi,\underline{B}\right) < 0$ for any feasible $\hat{B} \neq \underline{B}$: thus, a symmetric equilibrium in which everyone chooses $\underline{B}$ exists in this region. Moreover, the arguments found above in Step 3 rule out potential symmetric equilibria in which all players choose $\bar{B}$ or B_s in this region, establishing uniqueness.

4. Suppose that either (i) $3\underline{B} \leq 2B_{RE}$ and $\psi > \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$ or (ii) $3\underline{B} > 2B_{RE}$ and $\frac{B_{RE}}{3\underline{B}-2B_{RE}} \geq \psi > \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$. For $\chi < \left(\psi\left[2-\left(\frac{B_\eta+\underline{B}}{B_{RE}}\right)\right]+1\right)\frac{\underline{B}}{B_\eta}$ (which simplifies to $\chi < \psi\frac{\underline{B}}{B_{RE}}$ by substituting B_η from equation 45), symmetric equilibrium in which everyone chooses $\underline{B}$ cannot exist under these conditions by the arguments in Step 3. Thus, for any χ below this threshold, the unique symmetric equilibrium is either B_s or $\bar{B}$.

First, we establish the existence of a symmetric equilibrium in which everyone chooses B_s. Under the conjectured conditions on ψ and χ above, $\chi < \frac{2\psi+1}{3}$, so that the second-order condition holds, i.e, B_s is a local maximum. If everyone chooses B_s, the only possible profitable deviation is to $\underline{B}$ but that is ruled out since $\chi < \psi\frac{\underline{B}}{B_{RE}}$ as discussed in Step 3. Thus, the optimal choice for player i is to choose B_s when others choose B_s, i.e., it is a global maximum. It is feasible only when $\underline{B} \leq B_s \leq \bar{B}$. Equation (50) always holds under these conditions and so $B_s > \underline{B}$; however, (49) implies that $B_s \leq \bar{B}$ if and only if

$$\chi \geq 2\psi + 1 - \frac{2\psi\bar{B}}{B_{RE}}. \tag{55}$$

That $B_s = \bar{B}$ is an equilibrium at this cutoff implies that $\Delta\left(\hat{B},\bar{B},\chi = 2\psi+1-\frac{2\psi\bar{B}}{B_{RE}},\bar{B}\right) > 0$ for all $\hat{B} < \bar{B}$. A decrease in χ uniformly increases $\frac{\partial OU}{\partial B_i}$ and so for all χ failing to satisfy (55) and $\hat{B} < \bar{B}$, $\Delta\left(\hat{B},\bar{B},\chi,B_{RE},\bar{B}\right) > 0$: $\bar{B}$ is an equilibrium.

In this region, however, it cannot be that $\bar{B}$ is an equilibrium above the cutoff in (55). For $\bar{B}$ to be an equilibrium, it must be that $f\left(\bar{B},\bar{B},B_{RE},\chi,\psi\right) \geq 0$. This is only the case as long as

$$\chi \leq 2\psi + 1 - \frac{2\psi\bar{B}}{B_{RE}}. \tag{56}$$

Together, this establishes uniqueness since the two thresholds are the same: if (56) holds, the only equilibrium is when everyone chooses $\bar{B}$; if $2\psi + 1 - \frac{2\psi\bar{B}}{B_{RE}} < \chi < \psi\frac{\underline{B}}{B_{RE}}$, the only equilibrium is when everyone chooses B_s.

5. Suppose that $3\underline{B} > 2B_{RE}$ and $\psi > \frac{B_{RE}}{3\underline{B}-2B_{RE}}$. Under these conditions, no mixed-strategy equilibrium exists. First, we establish the existence of a symmetric equilibrium in which everyone chooses B_s. When everyone chooses B_s, no one deviates to $\underline{B}$ as long as

$$\chi < 2\psi + 1 - \frac{2\psi\underline{B}}{B_{RE}}. \tag{57}$$

If this inequality holds and given the conjecture that $\psi > \frac{B_{RE}}{3\underline{B}-2B_{RE}}$, then $\chi < \frac{2\psi+1}{3}$, i.e., the second-order condition holds. Thus, a symmetric equilibrium in which everyone chooses B_s exists.

Next, we establish the existence of a symmetric equilibrium in which everyone chooses $\underline{B}$. Equation (48) implies that $B_s = \underline{B}$ if and only if $\chi = 2\psi + 1 - \frac{2\psi\underline{B}}{B_{RE}}$. Following the same logic as above, it is straightforward to show that $\underline{B}$ must be an equilibrium for χ above this cutoff. To show that it is only an equilibrium above this cutoff, note that $f\left(\underline{B}, \underline{B}, B_{RE}, \chi, \psi\right) < 0$ only if $\chi > 2\psi + 1 - \frac{2\psi\bar{B}}{B_{RE}}$.

Finally, establishing the cutoff between the unique symmetric equilibrium in which everyone chooses B_s and the unique equilibrium in which everyone chooses $\bar{B}$ follows the same steps as above.

A.7 Proof of Corollary 3

Suppose we are in a region in which $\psi < \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$. In this case, the lower threshold $\chi_l \equiv \left(\psi\left(2 - \frac{\underline{B}+\bar{B}}{B_{RE}}\right) + 1\right)\frac{\underline{B}}{\bar{B}}$ and the upper threshold $\chi_h \equiv \left(\psi\left(2 - \frac{\underline{B}+\bar{B}}{\gamma_0}\right) + 1\right)\frac{\bar{B}}{\underline{B}}$. In this region,

$$\frac{\partial \chi_l}{\partial \underline{B}} = \frac{B_{RE}\left(2\psi + 1\right) - \psi\left(\bar{B} + 2\underline{B}\right)}{B_{RE}\bar{B}} > 0 \iff B_{RE} > \psi\left(\bar{B} + 2\underline{B} - 2B_{RE}\right)$$

Note that $\psi\left(\bar{B} + 2\underline{B} - 2B_{RE}\right) < \psi\left(2\bar{B} + \underline{B} - 2B_{RE}\right)$. This implies $\frac{\partial \chi_l}{\partial \underline{B}} > 0$ in the region $\psi < \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$. Similarly,

$$\frac{\partial \chi_l}{\partial \bar{B}} = \frac{\underline{B}\left(\psi\underline{B} - B_{RE}\left(2\psi + 1\right)\right)}{B_{RE}\bar{B}^2} < 0 \iff \psi\underline{B} < B_{RE}\left(2\psi + 1\right)$$

which is always true. Similarly,

$$\frac{\partial \chi_h}{\partial \underline{B}} = \frac{\bar{B}\left(\psi\bar{B} - B_{RE}\left(2\psi + 1\right)\right)}{B_{RE}\underline{B}^2} < 0 \iff \psi\left(\bar{B} - 2B_{RE}\right) < B_{RE}$$

Note that $\psi\left(\bar{B}-2B_{RE}\right) < \psi\left(2\bar{B}+\underline{B}-2B_{RE}\right)$. This implies $\frac{\partial\chi_h}{\partial\underline{B}} < 0$ in the region $\psi < \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$. Similarly,

$$\frac{\partial\chi_h}{\partial\bar{B}} = \frac{B_{RE}\left(2\psi+1\right)-\psi\left(2\bar{B}+\underline{B}\right)}{B_{RE}\underline{B}} > 0 \iff B_{RE} > \psi\left(2\bar{B}+\underline{B}-2B_{RE}\right)$$

which is true in the region $\psi < \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$.

Suppose we are in a region where $\psi > \frac{B_{RE}}{2\bar{B}+\underline{B}-2B_{RE}}$ and $\psi\max\left(3\underline{B}-2B_{RE},0\right) < B_{RE}$. Then, $\chi_l = \frac{\psi\underline{B}}{B_{RE}}$ and $\chi_h = \frac{\left(2\psi B_{RE}+B_{RE}-\psi\underline{B}\right)^2}{4\psi B_{RE}\underline{B}}$. These don't depend on $\bar{B}$ which implies $\frac{\partial\chi_l}{\partial\bar{B}} = \frac{\partial\chi_h}{\partial\bar{B}} = 0$. In this region, $\frac{\partial\chi_l}{\partial\underline{B}} > 0$ and

$$\frac{\partial\chi_h}{\partial\underline{B}} = \frac{\psi}{4B_{RE}} - \frac{B_{RE}\left(2\psi+1\right)^2}{4\psi\underline{B}^2} < 0 \iff \psi\underline{B} < B_{RE}\left(2\psi+1\right)$$

which is always true.

A.8 Proof of Proposition 3

Note that the average response to public information is given by equation (18). In a symmetric equilibrium, agents underreact to public information if $B < B_{RE} \iff \chi > 1$ and overreact otherwise. Finally, $\frac{\partial B}{\partial\chi} \leq 0$.

In a mixed equilibrium, $B < B_{RE} \iff \frac{\Gamma\underline{B}B_{opt}}{B_{RE}^2} < \chi$ and $\frac{\partial B}{\partial\chi} = \frac{\partial}{\partial\chi}\left(\sqrt{\frac{\Gamma\underline{B}B_{opt}}{\chi}}\right) < 0$.

A.9 Proof of Proposition 4

The dispersion of beliefs is given by $\sigma_y^2 = \mathbb{E}\left[\text{var}[y_i|\{\theta,\eta\}]\right]$.

If players have rational expectations, the dispersion of beliefs is $\sigma_{y,RE}^2 = \frac{\tau_e}{(\tau+\tau_e+\tau_\eta)^2}$ and this decreases as public information precision increases i.e., $\frac{\partial\sigma_{y,RE}^2}{\partial\tau_\eta} < 0$

In a symmetric equilibrium in which all players choose δ^*, dispersion of beliefs is $\sigma_y^2 = \frac{\tau_e}{(\tau+\tau_e+\delta^*\tau_\eta)^2}$ and

$$\sigma_y^2 > \sigma_{y,RE}^2 \iff \delta^* < 1 \iff \chi > 1.$$

Moreover, if $\delta^* = \bar{\delta}$ or $\underline{\delta}$, $\frac{\partial\sigma_y^2}{\partial\tau_\eta} < 0$ and $\frac{\partial\sigma_y^2}{\partial\chi} = 0$. In an interior equilibrium,

$$\frac{\partial\sigma_y^2}{\partial\tau_\eta} = -\frac{\tau_e(-\chi+2\psi+1)\left(2\psi-\frac{(1-\chi)\tau_\eta}{\tau_e+\tau}\right)}{2\psi^2\left(\tau_e+\tau_\eta+\tau\right)^3} < 0 \qquad \frac{\partial\sigma_y^2}{\partial\chi} = \frac{\tau_e\tau_\eta\left(2\psi-\frac{(1-\chi)\tau_\eta}{\tau_e+\tau}\right)}{2\psi^2\left(\tau_e+\tau\right)\left(\tau_e+\tau_\eta+\tau\right)^2} > 0.$$

In a mixed equilibrium,

$$\sigma_y^2 = \frac{\lambda A_{opt}^2+(1-\lambda)\underline{A}^2}{\tau_e} + \lambda(1-\lambda)\left(\frac{\left(A_{opt}-\underline{A}+B_{opt}-\underline{B}\right)^2}{\tau} + \frac{\left(B_{opt}-\underline{B}\right)^2}{\tau_\eta}\right) \qquad (58)$$

where $\underline{A} = \frac{\tau_e}{\tau+\tau_e+\underline{\delta}\tau_\eta}$, $\bar{A} = \frac{\tau_e}{\tau+\tau_e+\bar{\delta}\tau_\eta}$, and $A_{opt} = \frac{\tau_e}{\tau+\tau_e+\delta_{opt}\tau_\eta}$. The first term in equation (58) captures the dispersion of beliefs within the group and the second term captures the dispersion across groups. Finally,

$$\frac{\partial\sigma_y^2}{\partial\chi} = \underbrace{\frac{\partial\lambda}{\partial\chi}}_{<0}\left[\underbrace{\frac{A_{opt}^2-\underline{A}^2}{\tau_e}}_{<0} + (1-2\lambda)\underbrace{\left(\frac{(A_{opt}-\underline{A}+B_{opt}-\underline{B})^2}{\tau} + \frac{(B_{opt}-\underline{B})^2}{\tau_\eta}\right)}_{>0}\right]$$

which in turn implies that the effect of χ on σ_y^2 can be hump shaped.

A.10 Proof of Proposition 5

In the symmetric equilibrium in which all players choose $\delta_{\eta,i} = \delta^*$, the CG coefficient at the individual level is

$$CG_i = \frac{\text{cov}\left(\theta - \mathbb{E}_i\left[\theta|s_i,s\right], \mathbb{E}_i\left[\theta|s_i,s\right]\right)}{\text{var}\left(\mathbb{E}_i\left[\theta|s_i,s\right]\right)} = \frac{\delta^*\left(1-\delta^*\right)\tau_\eta}{\left(\tau_e+\delta^*\tau_\eta\right)^2\frac{1}{\tau}+\tau_e+(\delta^*)^2\tau_\eta}. \quad (59)$$

Moreover, $CG_i < 0 \iff \delta^* = \delta_{sym} > 1 \iff \chi < 1$ (since $\delta^* = \underline{\delta} < 1$). If $\delta^* = \bar{\delta}$ or $\underline{\delta}$, then $\frac{\partial CG_i}{\partial\chi} = 0$. Otherwise, substituting the optimal interior $\delta^* = \frac{2\psi+1-\chi}{2\psi-\frac{(1-\chi)\tau_\eta}{\tau_e+\tau}}$ in the above expression and differentiating wrt χ, we get

$$\frac{\partial CG_i}{\partial\chi} = \frac{2\tau\psi\tau_\eta\left(4\psi\tau_e\left(\tau_\eta+\tau+\tau_e\right)\left(\psi+1-\chi\right)+\tau\tau_\eta(\chi-2\psi-1)^2\right)}{\left(4\psi^2\tau_e\left(\tau_\eta+\tau\right)+4\psi^2\tau_e^2+\tau\tau_\eta(\chi-2\psi-1)^2\right)^2} > 0 \quad \text{in the region } \chi < 1.$$

The CG coefficient at the aggregate level is

$$CG_a = \frac{\text{cov}\left(\theta - \frac{\tau_e\theta+\delta^*\tau_\eta s}{\tau+\tau_e+\delta^*\tau_\eta}, \frac{\tau_e\theta+\delta^*\tau_\eta s}{\tau+\tau_e+\delta^*\tau_\eta}\right)}{\text{var}\left(\frac{\tau_e\theta+\delta^*\tau_\eta s}{\tau+\tau_e+\delta^*\tau_\eta}\right)} = \frac{\tau_e+\delta^*\left(1-\delta^*\right)\tau_\eta}{\left(\tau_e+\delta^*\tau_\eta\right)^2\frac{1}{\tau}+(\delta^*)^2\tau_\eta} \quad (60)$$

Moreover, $CG_a > 0 \iff \tau_e + \delta^*\left(1-\delta^*\right)\tau_\eta > 0 \iff \delta^* < \frac{1}{2} + \sqrt{\frac{1}{4}+\frac{\tau_e}{\tau_\eta}}$. This is true if either $\bar{\delta} \leq \frac{1}{2} + \sqrt{\frac{1}{4}+\frac{\tau_e}{\tau_\eta}}$, or if $\delta^* = \frac{2\psi+1-\chi}{2\psi-(1-\chi)\frac{\tau_\eta}{\tau+\tau_e}}$ satisfies the bound, which requires

$$\frac{1-\chi}{2\psi} < \frac{\sqrt{\frac{1}{4}+\frac{\tau_e}{\tau_\eta}}-\frac{1}{2}}{\left(\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{\tau_e}{\tau_\eta}}\right)\frac{\tau_\eta}{\tau+\tau_e}+1}.$$

If $\delta^* = \bar{\delta}$ or $\underline{\delta}$, then $\frac{\partial CG_a}{\partial\chi} = 0$. Otherwise, in an interior symmetric equilibrium, $\frac{\partial CG_a}{\partial\chi} = \frac{\partial CG_a}{\partial\delta^*} \times \frac{\partial\delta^*}{\partial\chi}$ where

$$\frac{\partial CG_a}{\partial\delta^*} = -\frac{\tau\tau_\eta\left(\tau_\eta(\delta^*)^2\left(2\tau_e+\tau_\eta+\tau\right)+2\delta^*\tau_e\left(\tau_e+\tau_\eta+\tau\right)+\tau_e^2\right)}{\left(\tau(\delta^*)^2\tau_\eta+\left(\delta^*\tau_\eta+\tau_e\right)^2\right)^2} < 0$$

$$\frac{\partial \delta^*}{\partial \chi} = -\frac{2\psi\left(\tau_e+\tau\right)\left(\tau_e+\tau_\eta+\tau\right)}{\left(2\psi\left(\tau_e+\tau\right)+(\chi-1)\tau_\eta\right)^2} < 0$$

which implies $\frac{\partial CG_a}{\partial \chi} \geq 0$.

A.11 Proof of Proposition 6

In a symmetric equilibrium, all players have the same interpretation of the public signal (i.e., $\delta_{\eta,i} = \delta^*$ is the same). Combined with equation (22), this implies that the dispersion of CG coefficients is zero in symmetric equilibrium.

In a mixed equilibrium, a fraction λ of players choose $\delta_{\eta,i} = \delta_{opt}$, while the remaining fraction of players choose $\delta_{\eta,i} = \underline{\delta}$. This implies that the individual CG coefficients can take one of two values:

$$CG_{i,1} = \frac{\underline{\delta}(1-\underline{\delta})\tau_\eta\tau}{(\tau_e+\underline{\delta}\tau_\eta)^2+\tau\tau_e+\underline{\delta}^2\tau\tau_\eta} \qquad CG_{i,2} = \frac{\delta_{opt}(1-\delta_{opt})\tau_\eta\tau}{(\tau_e+\delta_{opt}\tau_\eta)^2+\tau\tau_e+\delta_{opt}^2\tau\tau_\eta}$$

This implies that the dispersion of CG coefficients is $\lambda(1-\lambda)(CG_{i,1}-CG_{i,2})^2 > 0$. Note that the individual CG coefficients don't depend on χ. As χ increases, λ decreases (from equation (17)), and hence the effect on the dispersion of CG coefficients is hump-shaped.

Online Appendix

B Applications

In this section, we consider the implications of our analysis for a few applied models.

B.1 Efficient Competitive Economies

Consider an incomplete-market competitive economy in which agents' choices are strategic substitutes. There are two goods and a continuum of households (who act as consumers and producers). Let q_{1i} and q_{2i} denote the respective quantities purchased by consumer i (the consumer living in household i). The preferences of this consumer are given by

$$u_i = \theta q_{1i} - \frac{bq_{1i}^2}{2} + q_{2i}$$

where the random variable θ represents a shock in the relative demand for the two goods and the budget is

$$pq_{1i} + q_{2i} = e + \pi_i$$

where p is the price of good one, good two serves as numeraire , e is an exogenous endowment of good 2, and π_i are the profits of producer i. Profits in turn are given by $\pi_i = pk_i - \frac{k_i^2}{2}$ where k_i denotes the quantity of good 1 produced by household i.

Consumer i chooses the optimal bundle (q_{1i}, q_{2i}) so as to maximize utility subject to budget constraint, which gives $p = \theta - bq_{1i}$. Households are ex-ante identical which, together with market clearing, implies that $q_{1i} = K$ for all i and therefore $p = \theta - bK$ where $K = \int k_i di$. This example is thus nested in our model with the utility

$$U(k, K, \sigma_k, \theta) = (\theta - bK)\, k - k^2/2 + bK^2/2 + e$$

which implies that

$$u_{kk} = -1, u_{kK} = -b, u_{k\theta} = 1, u_{KK} = b$$

and $\kappa_0 = 0$;$\kappa_1 = \frac{1}{1+b}$ and

$$\chi \equiv \frac{u_{KK} - u_{kk}r^2}{-u_{kk}\,(1-r)^2} = \frac{b}{1+b} > 0.$$

Since $\chi \in (0, 1)$ increases in b, Proposition 2 shows that whether the equilibrium belief choice is symmetric or mixed depends upon the level of b. If b is too low, then the equilibrium is symmetric. For high enough b, the equilibrium is mixed. Proposition 3 implies that, in symmetric equilibrium (low b), the players overreact to public information. In a mixed equilibrium, players under-react to information if $\chi = \frac{b}{1+b} > \frac{\Gamma BB_{opt}}{B_{RE}^2}$ i.e., if b is high. Moreover, the average weight on public information decreases in b.

Proposition 4 implies that, in a symmetric equilibrium (i.e., low b), dispersion of beliefs increases with b. In a mixed equilibrium (when b is high), the dispersion of beliefs is hump-shaped in b. Proposition 5 implies that, in a symmetric equilibrium, there is over-reaction at the individual level and there will be under-reaction at the consensus level if either the set of reasonable beliefs is small or if b is high. Moreover, both CG_i and CG_a are increasing in b. Proposition 7, implies that, when b is high, the dispersion of individual CG coefficients is hump-shaped in χ.

B.2 Investment complementarities

Consider a setting in which the terminal value of the firm is given by $V \equiv V(R, k)$ where R measures the return on investment, or productivity, of the project available and k represents the scale of investment in the project. For analytical tractability, let

$$V(R, k) = Rk - \tfrac{1}{2}k^2, \tag{61}$$

$$R = (1-a)\theta + aK, \tag{62}$$

where $\theta \in R$ represents the firm's exogenous productivity. This implies that firm i's utility is

$$U_i(k_i, K, \theta) = (1-a)\theta k_i + aKk_i - \tfrac{1}{2}k_i^2$$

which falls into the class of general objective functions we analyzed. This implies that

$$u_{kk} = -1, u_{kK} = a, u_{k\theta} = 1 - a.$$

which implies that $r = a$ and $\kappa_1 = 1$ and so

$$\chi = -\frac{(u_{KK} - r^2 u_{kk})}{u_{kk}(1-r)^2} = \frac{a^2}{(1-a)^2} > 0$$

In this economy, χ increases with a. Moreover, $\chi < 1 \iff a < \frac{1}{2}$. Proposition 2 shows that whether the equilibrium belief choice is symmetric or mixed depends upon the level of a. If a is too low or too high, then the equilibrium is symmetric. For medium a, the equilibrium is mixed. Proposition 3 implies that, in symmetric equilibrium, the players under-react to public information if $a > \frac{1}{2}$. In a mixed equilibrium, players under-react to information if $\chi = \frac{a}{1-a} > \frac{\Gamma \underline{B} B_{opt}}{B_{RE}^2}$ i.e., if a is high enough. Moreover, the average weight on public information decreases in a.

Proposition 4 implies that, in a symmetric equilibrium, the dispersion of beliefs increases with a. In a mixed equilibrium, the dispersion of beliefs is hump-shaped in a. Proposition 5 implies that, in a symmetric equilibrium, there is over-reaction at the individual level iff $a < \frac{1}{2}$ and there will be under-reaction at the consensus level if either the set of reasonable beliefs is small or if a is high. Moreover, both CG_i and CG_a are increasing in a. Proposition 7, implies that, in a mixed equilibrium, the dispersion of individual CG coefficients is hump-shaped in a.

B.3 Cournot versus Bertrand

We next study two IO applications with a large number of firms: a Cournot-like game, where firms compete in quantities and actions are strategic substitutes; and a Bertrand-like game, where firms compete in prices and actions are strategic complements.

First, consider a Cournot setting in which firms compete on quantity. For any firm, consumer demand is $p = a_0 + a_1\theta - a_2 q - a_3 Q$ (with $a_0, a_1, a_2, a_3 > 0$), where p denotes the price at which the firm sells each unit of its product, q is the quantity it produces, Q is the average quantity produced across all firms, and θ is a fundamental demand shock/shifter. Each firm's profits are given by $u = pq - C(q)$, where $C(q) = c_1 q + c_2 q^2$ is the cost function (with $c_1, c_2 > 0$). This model is nested in our general framework with $k \equiv q, K \equiv Q$, and

$$U(k, K, \sigma_k, \theta) = (a_0 - c_1 + a_1\theta - a_3 K) k - (a_2 + c_2) k^2.$$

This implies that $U_{kk} = -2(a_2 + c_2), U_{kK} = -a_3, U_{k\theta} = a_1, U_k = a_0 - c_1$ which in turn implies that $r = \frac{-a_3}{2(a_2+c_2)}$,$\kappa_1 = -\frac{a_1}{-2(a_2+c_2)-a_3}$ and

$$\chi = -\frac{(u_{KK} - r^2 u_{kk})}{u_{kk}(1-r)^2} = \frac{\frac{a_3^2}{4(a_2+c_2)^2}}{\left(1 + \frac{a_3}{2(a_2+c_2)}\right)^2} > 0$$

which implies that $\chi \in (0, 1)$ and is increasing in $b \equiv \frac{a_3}{2(a_2+c_2)}$. This, in turn implies that χ increases in a_3 but decreases in a_2 and c_2.

Next, consider a Bertrand setting in which firms compete on price. Consumer demand for each firm is given by $q = b_0 + b_1\theta' - b_2 p + b_3 P$, where q denotes the quantity sold by the firm, p is the price the firm sets, P is the average price in the market, and θ' is an exogenous demand shifter $(b_0, b_1, b_2, b_3 > 0)$; we naturally impose $b_3 < b_2$, so that an equal increase in p and P reduces q. Individual profits are $u = pq - C(q)$, where $C(q) = c_1 q + c_2 q^2$ (with $c_1, c_2 > 0$). This model is nested in our framework with $k \equiv p - c_1, K \equiv P - c_1$ (actions are now prices), and

$$U(k, K, \sigma_k, \theta) = \theta k (1 + 2c) - k^2 (1 + c) + Kk (b + 2bc) - c\theta^2 - cb^2 K^2 - 2bc\theta K$$

where $\theta = \frac{b_0}{b_2} + \frac{b_1}{b_2}\theta' - c_1(1 - b)$, $b \equiv \frac{b_3}{b_2}$ and $c \equiv c_2 b_2$. This implies that $U_{kk} = -2(1 + c), U_{kK} = b(1 + 2c), U_{k\theta} = 1 + 2c, U_{KK} = -2cb^2, U_{K\theta} = -2bc, U_{\theta\theta} = -2c$ which in turn implies that $r = \frac{b(1+2c)}{2(1+c)}$, $\kappa_1 = \frac{1+2c}{2(1+c)-b(1+2c)}$ and

$$\chi = -\frac{(u_{KK} - r^2 u_{kk})}{u_{kk}(1-r)^2} = \frac{b^2}{(2(1+c) - b(1+2c))^2} > 0.$$

Since $b < 1$, one can show that $\chi \in (0, 1)$. Moreover, χ is increasing in $b = b_3/b_2$ and decreases in $c = c_2 b_2$. This implies that χ increases in b_3, decreases in b_2 and c_2.

B.4 Information Spillovers

The model of Angeletos, Lorenzoni, and Pavan (2018) considers a novel channel through which the information in the real sector affects behavior in the financial sector. The real sector is comprised of entrepreneurs making investment decisions, and the financial sector is comprised of investors who provide liquidity to the "real" economy. All players are risk-neutral and the discount rate is zero. Time is divided in three periods, $t \in \{1, 2, 3\}$. At

$t = 1$, a new investment opportunity becomes available with productivity $\theta \sim N\left(0, \frac{1}{\tau_\theta}\right)$. This investment pays off at $t = 3$. There is a continuum of entrepreneurs who can choose how much to invest in new technology. Let k_i denote the investment of entrepreneur i, and let the cost of this investment be $\frac{k_i^2}{2}$. Entrepreneurs have access to information technology that generates both a private and a "public" signal that they utilize when making their investment decision.[36] The joint distribution of fundamentals and signals follows the specification detailed in Section 3.

At t = 2, the "financial market" operates: some entrepreneurs transfer their capital to the traders. Each entrepreneur is hit by an idiosyncratic shock with probability $l \in [0, 1]$. Entrepreneurs hit by this shock do not value consumption at t = 3 and have no choice but to sell all their capital at t = 2, at a price p, to investors. For simplicity, the entrepreneurs not hit by the shock are precluded from trading. Entrepreneur preferences are given by $u_i = c_{i1} + c_{i2} + s_i c_{i3}$, where c_{it} denotes agent i's consumption in period t, while s_i is a random variable that takes value 0 if the agent is hit by a liquidity shock and value 1 otherwise. Each entrepreneur's expected utility at the time of investment is given by

$$\mathbb{E}_i \left[u_i | s_i, s\right] = \mathbb{E}_i \left[(1-l)\,\theta k_i + lpk_i - \frac{1}{2}k_i^2 | s_i, s\right]. \tag{63}$$

The financial market is competitive and the market-clearing price is denoted by p. Investors do not have access to their own information technology but, given the assumptions above, update their beliefs about the productivity of the technology utilizing the information contained in the supply of capital to be liquidated. It can be shown that, given the distributional assumptions and the risk-neutrality of traders, that $p = \mathbb{E}\left[\theta | K\right] = \alpha_1 K$, where α_1 is pinned down in equilibrium. This implies that expected utility of entrepreneur is given by

$$\mathbb{E}\left[u_i\left(k_i, K, \theta\right) | s_i, s\right] = \mathbb{E}_i \left[(1-l)\,\theta k_i + lk_i\alpha_1 K - \frac{1}{2}k_i^2 | s_i, s\right]. \tag{64}$$

and therefore entrepreneurs optimal action is given by $k_i = \mathbb{E}_i\left[(1-l)\,\theta + l\alpha_1 K\right]$. Note that this objective falls into the class of general objective functions we studied. Aggregating across all entrepreneurs, this implies that aggregate investment can be written as

$$K = \frac{1-l}{1-lA}\kappa_1 A\theta + \frac{B}{1-lA}\kappa_1 s \tag{65}$$

$$= \frac{\kappa_1\left((1-l)\,A + B\right)}{1-lA}\left(\theta + \frac{B}{(1-l)\,A + B}\eta\right). \tag{66}$$

It is straightforward to see that the aggregate level of capital reveals a signal of the form $\xi = \theta + \frac{B}{((1-r)A+B)}\eta$ to investors which, given the linear-normal structure, verifies the

[36] It is public in the sense that all entrepreneurs observe the same signal; however, as we discuss below, it is assumed that investors do not observe the signal and can only learn about θ through the aggregate investment level.

conjectured functional form for the price of capital. This implies that α_1 solves the equation

$$\frac{\alpha_1(1-l)}{1-l\alpha_1} = \frac{((1-l\alpha_1)A+B)\tau_\eta}{((1-l\alpha_1)A+B)^2\tau_\eta + B^2\tau_\theta}(1-l\alpha_1 A)$$

On the other hand, however, note that

$$\chi \equiv \frac{u_{KK} - u_{kk}r^2}{-u_{kk}(1-r)^2} = \frac{l^2\alpha_1^2}{(1-l\alpha_1)^2} > 0.$$

Note that χ increases with l. Moreover, $\chi < 1 \iff l\alpha_1 < \frac{1}{2}$. Proposition 2 shows that whether the equilibrium belief choice is symmetric or mixed depends upon the level of $l\alpha_1$. If $l\alpha_1$ is too low or too high, then the equilibrium is symmetric. For medium $l\alpha_1$, the equilibrium is mixed. Proposition 3 implies that, in symmetric equilibrium, the players under-react to public information if $l\alpha_1 > \frac{1}{2}$. In a mixed equilibrium, players under-react to information if $\chi = \frac{l\alpha_1}{1-l\alpha_1} > \frac{\Gamma \underline{B} B_{opt}}{B_{RE}^2}$ i.e., if l is high enough. Moreover, the average weight on public information decreases in l.

Proposition 4 implies that, in a symmetric equilibrium, the dispersion of beliefs increases with l. In a mixed equilibrium, the dispersion of beliefs is hump-shaped in l. Proposition 5 implies that, in a symmetric equilibrium, there is over-reaction at the individual level iff $l\alpha_1 < \frac{1}{2}$ and there will be under-reaction at consensus level if either the set of reasonable beliefs is small or if l is high. Moreover, both CG_i and CG_a are increasing in l. Proposition 6, implies that, in a mixed equilibrium, the dispersion of individual CG coefficients is hump-shaped in l.

C Extensions

C.1 Endogenizing the set of reasonable beliefs

In Section 4.4, we considered a scenario where the set of reasonable beliefs $\left[\underline{\delta}, \overline{\delta}\right]$ is specified with an exogenous constraint on the KL cost i.e., $KL(s|\theta; \delta_{\eta,i} = \delta) \leq \kappa$. In this section, we endogenize κ i.e., we let players choose κ subject to a cost. Since κ has a cost, the above inequality is always binding in equilibrium. For simplicity, we assume that $\psi = 0$ i.e., players do not account for experienced utility. The objective of player i is then given by

$$\max_{\delta_{\eta,i}} AU_i(\delta_{\eta,i}) - \phi\left(\frac{1-\delta_{\eta,i} + \delta_{\eta,i}\log(\delta_{\eta,i})}{2\delta_{\eta,i}}\right)$$

The FOC of player i is

$$\Omega\left(\frac{\tau_\eta}{(\tau+\tau_e+\delta_{\eta,i}\tau_\eta)^2} - \frac{\chi B^2}{\delta_{\eta,i}^2\tau_\eta}\right) - \phi\frac{\delta_{\eta,i}-1}{\delta_{\eta,i}^2} = 0 \tag{67}$$

where $\Omega = -u_{kk}\left(\frac{\kappa_1(1-r)}{1-rA}\right)^2$. At $\delta_{\eta,i}=0$, the objective function is increasing iff

$$\phi - \Omega\frac{\chi B^2}{\tau_\eta} > 0$$

which is true if ϕ is high enough. Let us assume that the above condition is satisfied. At $\delta_{\eta,i}=\infty$, the objective function is decreasing since the LHS of the FOC is negative. Moreover, the FOC is a cubic equation. This implies that there are either one or three roots for the FOC. The FOC can be rewritten as

$$\Omega\left(\delta_{\eta,i}^2\tau_\eta^2 - \chi B^2\left(\tau+\tau_e+\delta_{\eta,i}\tau_\eta\right)^2\right) - \phi\tau_\eta\left(\tau+\tau_e+\delta_{\eta,i}\tau_\eta\right)^2\left(\delta_{\eta,i}-1\right) = 0.$$

The FOC has only one root when

$$\frac{1}{3} + \chi B^2 < \frac{\phi}{\Omega}\left(\tau+\tau_e+\tau_\eta\right). \tag{68}$$

If this condition is true, the objective function of each investor is hump shaped. This implies that the only possible equilibrium is symmetric in which all players choose $\delta_{\eta,i}=\delta_\eta$ which satisfies the FOC:

$$\Omega\delta_\eta^2\tau_\eta^2\left(1-\chi\right) - \phi\tau_\eta\left(\tau+\tau_e+\delta_\eta\tau_\eta\right)^2\left(\delta_\eta-1\right) = 0.$$

In this symmetric equilibrium,

$$\delta_\eta > 1 \iff \chi < 1.$$

In summary, when ϕ is sufficiently high, the only equilibrium possible is symmetric. If inequality (68) is not satisfied, then the FOC will have more than one solution.

While an analytical characterization of the model when ϕ is low is not tractable, we numerically explore the implications next. Figure 6 panel (a) shows that when χ is sufficiently low or high, there exists a pure symmetric equilibrium. For interim χ, there exists a mixed equilibrium in which a fraction optimally choose $\delta_{\eta,1} < 1$, while the remaining $1-\lambda$ choose $\delta_{\eta,2} > 1$.

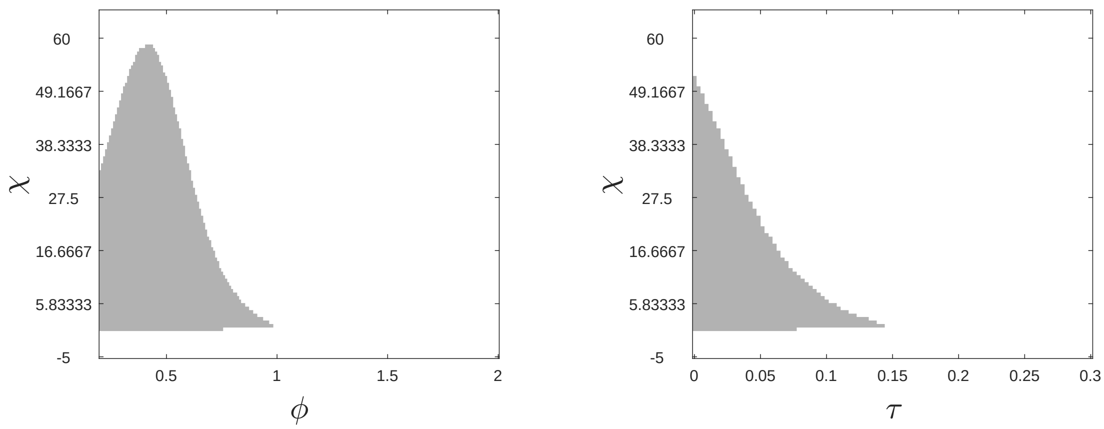
Panel (b) plots how the region of mixed equilibrium changes as a function of ex-ante uncertainty (τ). Endogenous disagreement is more likely to arise when prior uncertainty is high. This is because, during periods of high uncertainty, agents are willing to consider a larger set of subjective beliefs as plausible - periods of high uncertainty are associated with higher κ and wider confidence intervals (i.e., wider $\left[\underline{\delta},\overline{\delta}\right]$). All else equal, this would suggest that subjective beliefs (and the likelihood of disagreement about public information) are more extreme during such episodes.

C.2 Subjective beliefs about private information

We begin with the case when players entertain subjective beliefs about their private information but have rational beliefs about the informativeness of the public signal. From equation (11), a player's anticipatory utility is always increasing in his subjective beliefs about the pri-

Figure 6: Region of endogenous disagreement

The gray area of the plot corresponds to the region of the parameter space in which the unique equilibrium exhibits endogenous disagreement about the public signal. The parameters are fixed at $\tau = \tau_e = \tau_\eta = 0.15$.



vate signal. This implies that there always exists a symmetric equilibrium, as characterized by the following result.

Proposition 7. *Suppose all players have rational beliefs about the public signal i.e., $\delta_{\eta,i} = 1$. There exists a symmetric equilibrium in which all players are over-confident about their private signals and choose $\delta_{e,i}$, where:*

$$\delta_{e,i} = \begin{cases} \frac{2\psi+1}{2\psi - \frac{\tau_e}{\tau+\tau_\eta}} & \text{if} \quad \psi > \frac{1+\bar{\delta}\frac{\tau_e}{\tau+\tau_\eta}}{2\left(\bar{\delta}-1\right)} \\ \bar{\delta} & \text{otherwise.} \end{cases} \tag{69}$$

Moreover, $\delta_{e,i}$ is (weakly) increasing in τ_e, and (weakly) decreasing in τ, τ_η, and ψ.

Consistent with the intuition from Section 4.2, players always want to distort their perception of private signal precision upwards. However, the experienced utility component of the objective now adds a "utility" cost to distorting beliefs. Intuitively, the more player i distorts his beliefs, the larger the expected loss in experienced utility (under the objective measure) from using subjective beliefs when choosing actions. Intuitively, this overestimation is larger when it is less costly to deviate from rational expectations (e.g., when prior information or public information is imprecise, i.e., low τ and low τ_η, or the cost of distorting beliefs is small, i.e., low ψ) and when the available private information is more precise (high τ_e). When the overall costs are sufficiently low (e.g., if ψ is sufficiently low), the player chooses the maximal distortion feasible i.e., $\delta_{e,i} = \bar{\delta}$.

Next, we allow players to entertain subjective beliefs about both their private and public signals, and assume that they maximize a weighted average of anticipatory and experienced utility. While we are not able to offer a complete characterization of equilibria for the entire parameter range, as in the earlier subsections, we establish sufficient conditions for the existence of symmetric and mixed equilibria of each type in the following results.

Proposition 8. *Suppose players choose beliefs about both private and public information and let*

$$B_{opt} = \max\left\{\min\left\{\frac{B_{RE}(2\psi+1)-\psi\underline{B}}{2\psi}, \bar{B}\right\}, \underline{B}\right\} \qquad \Gamma = 1+\psi\left(2-\frac{\underline{B}+B_{opt}}{B_{RE}}\right) \tag{70}$$

$$\underline{B} = \frac{\underline{\delta}\tau_\eta}{\tau+\bar{\delta}\tau_e+\underline{\delta}\tau_\eta}, \quad \bar{B} = \frac{\bar{\delta}\tau_\eta}{\tau+\bar{\delta}\tau_e+\bar{\delta}\tau_\eta}, \quad B_{RE} = \frac{\tau_\eta}{\tau+\tau_e+\tau_\eta}. \tag{71}$$

Assume $\psi < \frac{\tau_e}{2(\tau+\bar{\delta}\tau_e)}$. Then, the following hold.

(i) If $\chi > \frac{B_{opt}}{\underline{B}}\Gamma$, then there exists a unique equilibrium in which all players choose $\delta_{e,i} = \bar{\delta}$ and $\delta_{\eta,i} = \underline{\delta}$.

(ii) If $\chi < \frac{\underline{B}}{B_{opt}}\Gamma$, then there exists a unique equilibrium in which all players choose $\delta_{e,i} = \bar{\delta}$ and

$$\delta_{\eta,i} = \min\left\{\bar{\delta}, \frac{2\psi+1-\chi}{2\psi-\frac{(1-\chi)\tau_\eta}{\bar{\delta}\tau_e+\tau}}\right\}. \tag{72}$$

(iii) For $\chi \in \left[\frac{\underline{B}}{B_{opt}}\Gamma, \frac{B_{opt}}{\underline{B}}\Gamma\right]$, then there exists a unique equilibrium in which a fraction λ of players choose $\delta_{e,i} = \bar{\delta}$ and $\delta_{\eta,i} = \frac{B_{opt}}{1-B_{opt}}\frac{\left(\bar{\delta}\tau_e+\tau\right)}{\tau_\eta}$ while the remaining players choose $\delta_{e,i} = \bar{\delta}$ and $\delta_{\eta,i} = \underline{\delta}$.

The above result shares features of the equilibrium characterization in our main analysis. Specifically, the condition on the relative importance of experienced utility i.e., $\psi < \frac{\tau_e}{2(\tau+\bar{\delta}\tau_e)}$, ensures that in any equilibrium, players choose to maximally distort their beliefs about their private information i.e., set $\delta_{e,i} = \bar{\delta}$.[37] Given this, when χ is sufficiently small (part (i)), we show there exists a symmetric equilibrium in which all players exhibit over-confidence about their private information and over-react to public information. Similarly, when χ is sufficiently large (part (ii)), all players exhibit over-confidence in their private information, but under-react to public information. And for intermediate χ, there exists a unique mixed equilibrium in which some players over-react to the public signals, while others under-react to it.

C.2.1 Proof of Proposition 7

The objective of player i is

$$\max_{\delta_{e,i}\in(\underline{\delta},\bar{\delta})} AU_i + \psi EU_i$$

[37]While a small enough ψ is needed for the proof, numerical simulations show that the qualitative results hold more generally. Thus, we view the requirement of a small ψ in Proposition 8 as technical and not restrictive for the economic mechanism.

The lower constraint is never binding (when players only choose $\delta_{e,i}$) and the objective can be rewritten as

$$\max_{\delta_{e,i}} AU_i + \psi EU_i \quad \text{subject to } \delta_{e,i} < \bar{\delta}$$

Let ω_e denote the Lagrange multiplier for the above inequality. We define the Lagrangian

$$L = -\frac{1}{\tau + \delta_{e,i}\tau_e\tau_\eta} + \chi\frac{B^2}{\tau_\eta} - \psi\frac{\tau + \delta_{e,i}^2\tau_e + \tau_\eta}{\left(\tau + \delta_{e,i}\tau_e + \tau_\eta\right)^2} - \omega_e(\delta_{e,i} - \bar{\delta}).$$

The FOC with respect to $\delta_{e,i}$ for any equilibrium is

$$\frac{\tau_e}{\left(\tau_\eta + \delta_{e,i}\tau_e + \tau\right)^2}\left(1 - 2\psi\delta_{e,i} + \frac{2\psi\left(\tau + \delta_{e,i}^2\tau_e + \tau_\eta\right)}{\tau + \delta_{e,i}\tau_e + \tau_\eta}\right) - \omega_e = 0. \tag{73}$$

Following steps similar to those in the proof of Proposition 2, it can be shown that the objective function has either a single interior maximum or is always increasing. Hence, the solution is

$$\delta_{e,i} \equiv \min\left(\frac{2\psi + 1}{2\psi - \frac{\tau_e}{\tau + \tau_\eta}}, \bar{\delta}\right). \tag{74}$$

C.2.2 Proof of Proposition 8

The objective of player i is

$$\max_{\delta_{\eta,i},\delta_{e,i}\in[\underline{\delta},\bar{\delta}]} -\frac{1}{\tau + \delta_{e,i}\tau_e + \delta_{\eta,i}\tau_\eta} + \chi\frac{B^2}{\delta_{\eta,i}\tau_\eta} - \psi\frac{\tau + \delta_{e,i}^2\tau_e + \delta_{\eta,i}^2\tau_\eta}{\left(\tau + \delta_{e,i}\tau_e + \delta_{\eta,i}\tau_\eta\right)^2}.$$

Let's first focus on the choice of $\delta_{e,i}$. Let $\omega_{e,1}$ and $\omega_{e,2}$ denote the Lagrange multipliers for the inequalities $\underline{\delta} < \delta_{e,i}$ and $\delta_{e,i} < \bar{\delta}$ respectively. We define the Lagrangian

$$L = -\frac{1}{\tau + \delta_{e,i}\tau_e + \delta_{\eta,i}\tau_\eta} + \chi\frac{B^2}{\delta_{\eta,i}\tau_\eta} - \psi\frac{\tau + \delta_{e,i}^2\tau_e + \delta_{\eta,i}^2\tau_\eta}{\left(\tau + \delta_{e,i}\tau_e + \delta_{\eta,i}\tau_\eta\right)^2} + \omega_{e,1}\delta_{e,i} - \omega_{e,2}\delta_{e,i}.$$

The FOC with respect to $\delta_{e,i}$ for any equilibrium is

$$\frac{\tau_e}{\left(\delta_{\eta,i}\tau_\eta + \delta_{e,i}\tau_e + \tau\right)^2}\left(1 - 2\psi\delta_{e,i} + 2\psi\frac{\left(\tau + \delta_{e,i}^2\tau_e + \delta_{\eta,i}^2\tau_\eta\right)}{\tau + \delta_{e,i}\tau_e + \delta_{\eta,i}\tau_\eta}\right) + \omega_{e,1} - \omega_{e,2} = 0.$$

If $\psi < \frac{\tau_e}{2(\tau + \bar{\delta}\tau_e)}$, the constraint $\delta_{e,i} \leq \bar{\delta}$ is binding (irrespective of the choice of $\delta_{\eta,i}$) and all players choose $\delta_{e,i} = \bar{\delta}$. The rest of the proof follows from proposition 2 with τ_e replaced with $\bar{\delta}\tau_e$.

C.3 Subjective beliefs about others' signals

In this section, we modify our benchmark model to allow players to hold subjective beliefs about the private signals observed by other players. As in the benchmark model, each individual observes

$$s_i = \theta + \varepsilon_i \quad \varepsilon_i \sim N\left(0, 1/\tau_e\right), \tag{75}$$

but player i believes that

$$\varepsilon_j =_i \sqrt{1-p_i^2}\eta + p_i\varepsilon_j \quad \varepsilon_j \sim N\left(0, \tfrac{1}{\delta_i \tau_e}\right), \tag{76}$$

where p_i and δ_i are chosen by player i to maximize his anticipatory utility, net of costs. To highlight the role of beliefs about others, we shut down the public signal, s, and we do not allow players to hold subjective beliefs about the precision of their own signal. All other features of the benchmark model are unchanged.

Suppose player i conjectures that $K = \kappa_0 + \alpha\left(\theta + \sqrt{1-p_i^2}\eta\right)$, where α is determined in equilibrium. Then,

$$k_i = \kappa_0 + \left(r\alpha + (1-r)\,\kappa_1\right)\mathbb{E}_i\left[\theta|s_i\right] \tag{77}$$

This implies that, in equilibrium, player i believes that the average action (across all players) is given by

$$K = \int_i k_i di = \kappa_0 + \left(r\alpha + (1-r)\,\kappa_1\right) A\left(\theta + \sqrt{1-p_i^2}\eta\right), \tag{78}$$

Matching terms, we show that α is unchanged from our benchmark model. Players' subjective belief about the private signals of others is given by the following proposition.

Proposition 9. *Suppose players choose subjective beliefs about the private signals of others to maximize a weighted average of anticipatory and experienced utility (as in (6)). Then,*

(i) If $u_{\sigma\sigma} \geq u_{KK}$, all players choose $p_i = 1$. Moreover, players choose $\delta_i = \underline{\delta}$ iff $u_{\sigma\sigma} > 0$, and choose $\delta_i = \bar{\delta}$ otherwise.

(ii) If $u_{\sigma\sigma} < u_{KK}$, all players choose $p_i = 0$. Moreover, players choose $\delta_i = \underline{\delta}$ iff $u_{KK} > 0$, and choose $\delta_i = \bar{\delta}$ otherwise.

Proof. We can rewrite anticipatory utility as

$$AU_i\left(p_i, \delta_i\right) = \begin{array}{c} \mathbb{E}\left[u_0\right] + \mathcal{A}\text{var}\left(\theta\right) + \frac{1}{2}u_{\sigma\sigma}\mathbb{E}_i\left[\sigma_k^2\right] \\ +\frac{1}{2}\left(u_{kk}\mathbb{E}_i\left[\left(k-\mathcal{K}\right)^2\right] + \left(u_{KK} - r^2 u_{kk}\right)\mathbb{E}_i\left[\left(K-\kappa\right)^2\right]\right), \end{array} \tag{79}$$

where the key distinction from our benchmark is that players' subjective beliefs about other's signals impact their expectation of the dispersion in their actions, σ_k^2. Given player i's subjective beliefs,

$$\mathbb{E}_i\left[\sigma^2\right] = \int \left(k_i - K\right)^2\, di = \frac{p_i^2\alpha^2}{\delta_i \tau_e} \tag{80}$$

$$\mathbb{E}_i\left[(k-\mathcal{K})^2\right] = \frac{(r\alpha + (1-r)\,\kappa_1)^2}{\tau} + \frac{r^2\alpha^2\left(1-p_i^2\right)}{\delta_i\tau_e} \tag{81}$$

$$\mathbb{E}_i\left[(K-\kappa)^2\right] = \frac{\left(\frac{\kappa_1(A-1)}{1-rA}\right)^2}{\tau} + \frac{\alpha^2\left(1-p_i^2\right)}{\delta_i\tau_e} \tag{82}$$

This implies that

$$\frac{\partial AU_i}{\partial\delta_i} = -u_{\sigma\sigma}\left(\frac{p_i^2\alpha^2}{2\delta_i^2\tau_e}\right) - u_{KK}\left(\frac{\alpha^2\left(1-p_i^2\right)}{2\delta_i^2\tau_e}\right) \tag{83}$$

$$= -\frac{\alpha^2}{2\delta_i^2\tau_e}\left(p_i^2 u_{\sigma\sigma} + \left(1-p_i^2\right)u_{KK}\right) \tag{84}$$

$$\frac{\partial AU_i}{\partial p_i} = u_{\sigma\sigma}\left(\frac{p_i\alpha^2}{\delta_i\tau_e}\right) - u_{KK}\frac{\alpha^2 p_i}{\delta_i\tau_e} \tag{85}$$

$$= \frac{p_i\alpha^2}{\delta_i\tau_e}\left(u_{\sigma\sigma} - u_{KK}\right) \tag{86}$$

Finally, note that since player i's action does not depend on p_i or δ_i, their experienced utility is also independent of p_i or δ_i. Hence, we have the result. □

C.4 Subjective beliefs about means

In this section, we modify our benchmark model to allow players to hold subjective beliefs about the mean of both their own private signal as well as the public signal. Specifically, player i believes the errors in his signals are

$$\varepsilon_i \sim_i N\left(\mu_{e,i}, \frac{1}{\tau_e}\right), \quad \eta \sim_i N\left(\mu_{\eta,i}, \frac{1}{\tau_\eta}\right).$$

Given player i's subjective beliefs, $\mu_{e,i}$ and $\mu_{\eta,i}$, and the realization of s_i and s, his optimal action, k_i is

$$k_i^*\left(\mu_{e,i}, \mu_{\eta,i}\right) = \mathbb{E}_i\left[rK + (1-r)\,\kappa(\theta)\,|s_i, s\right] = r\mathbb{E}_i\left[K|s_i, s\right] + (1-r)\,\mathbb{E}_i\left[\kappa(\theta)\,|s_i, s\right],$$

Given our assumptions on the joint distribution of fundamentals and signals, Bayesian updating implies that player i's conditional beliefs about θ are given by

$$\mathbb{E}_i\left[\theta|s_i, s\right] = A\left(s_i - \mu_{e,i}\right) + B\left(s - \mu_{\eta,i}\right), \text{ and } \mathrm{var}_i\left[\theta|s_i, s\right] = \frac{1-A-B}{\tau},$$

where player i's weights on the private and public signals are given by:

$$A \equiv \frac{\tau_e}{\tau + \tau_e + \tau_\eta}, \text{ and } B \equiv \frac{\tau_\eta}{\tau + \tau_e + \tau_\eta}.$$

With this, we establish the existence of a unique equilibrium, given players' beliefs.[38]

[38] All the proofs for this section follow similar steps as in the main model and are omitted for brevity.

Lemma 4. *Given a choice of subjective beliefs $\{\mu_{e,i}, \mu_{\eta,i}\}_i$ for all players, there always exists a unique, linear equilibrium in which player i's optimal action is given by:*

$$k_i(\mu_{e,i}, \mu_{\eta,i}) = \kappa_0 + r\gamma_e\mu_e + r\gamma_\eta\mu_\eta + r\beta\mu_{\eta,i} + \alpha(s_i - \mu_{e,i}) + \beta(s - \mu_{\eta,i}),$$

and the aggregate action is given by:

$$K = \kappa_0 + \gamma_e\mu_e + \gamma_\eta\mu_\eta + \alpha\theta + \beta s,$$

where $\alpha = \frac{A(1-r)}{1-rA}\kappa_1, \beta = \frac{B}{1-rA}\kappa_1, \mu_e = \int \mu_{e,i}di,\ \mu_\eta = \int \mu_{\eta,i}di, \gamma_e = -\frac{\kappa_1 A}{1-rA},\ \gamma_\eta = -\frac{\kappa_1 B}{1-rA}$

With the equilibrium established, we then establish how $\mu_{e,i}$ and $\mu_{\eta,i}$ affect both players' anticipatory utility as well as the cost borne (through the experienced utility penalty).

Proposition 10. *Given player i's subjective beliefs, $\mu_{e,i}$ and $\mu_{\eta,i}$, anticipatory utility can be written*

$$AU_i(\mu_{e,i}, \mu_{\eta,i}) = \Gamma + u_K\beta(\mu_{\eta,i} - \mu_\eta) + \frac{1}{2}\left(u_{KK} - r^2u_{kk}\right)(\gamma_e\mu_e + \beta(\mu_{\eta,i} - \mu_\eta))^2 \tag{87}$$

and the experienced utility penalty is

$$EU_i(\mu_{e,i}, \mu_{\eta,i}) = u_{kk}(\alpha\mu_{e,i} + \beta(1-r)\mu_{\eta,i})^2$$

Players' anticipatory utility doesn't depend on $\mu_{e,i}$: intuitively, shifting the mean of the private signal is "accounted for" when updating and so player i's anticipatory utility is unaffected. However, since the true mean is zero, choosing $\mu_{e,i} \neq 0$ is objectively costly. As a result, all players choose $\mu_{e,i} = 0$.

Thus, we rewrite the objective of player i:

$$\max_{\mu_{\eta,i}} AU_i + \psi EU_i.$$

First, we characterize the equilibrium when non-fundamental volatility decreases anticipatory utility, i.e., when $\chi < 0$. In order to characterize the equilibrium, we assume that $\mu_{\eta,i} \in [-\bar{\mu}, \bar{\mu}]$. This is similar to the set of reasonable beliefs imposed in our baseline analysis.

Proposition 11. *If non-fundamental volatility decreases anticipatory utility (i.e., $\chi \leq 0$), then there exists a unique symmetric equilibrium in which*

$$\mu_e = 0 \qquad \mu_{\eta,i} = \mu_\eta = \min\left(\max\left(\frac{u_K}{-2u_{kk}\psi\beta(1-r)^2}, -\bar{\mu}\right), \bar{\mu}\right) \text{ for all } i \in [0,1]. \tag{88}$$

Next, we consider the case when non-fundamental volatility increases anticipatory utility.

Proposition 12. *If non-fundamental volatility increases anticipatory utility (i.e., $\chi > 0$), then there exists a $\bar{\chi} > 0$ such that:*

(i) If $\chi > \bar{\chi}$, the unique equilibrium is mixed. In this equilibrium, a fraction λ of players optimally chooses $\mu_{e,i} = 0$ and $\mu_{\eta,i} = -\bar{\mu}$, while the remaining fraction $1-\lambda$ optimally chooses $\mu_{e,i} = 0$ and $\mu_{\eta,i} = \bar{\mu}$.

(ii) If $\chi < \bar{\chi}$, then the unique equilibrium is symmetric and is given by (88).

While players never choose to distort their beliefs about the mean of their private signals, players always choose to distort their beliefs about the mean of the public signal. Specifically, we find that there exists a symmetric equilibrium when either the effect of non-fundamental volatility on anticipatory utility is not too high. In this symmetric equilibrium, whether players choose to believe that the public signal is biased upward or downward (i.e., positive or negative μ_η, respectively) depends upon whether the aggregate action, u_K, imposes a positive or a negative externality. On the other hand, when non-fundamental volatility increases anticipatory utility sufficiently, there does not exist a symmetric equilibrium - instead, players endogenously choose to disagree about the mean of the public signal.