Choosing to Disagree in Financial Markets

Snehal Banerjee, Jesse Davis and Naveen Gondhi*

January 2020

Abstract

A large literature in psychology documents that people derive utility from their beliefs through the anticipation of future outcomes. We show that this leads to predictable deviations from rational expectations in financial markets: investors systematically choose to distort their interpretation of both private and price information. When aggregate risk tolerance is low, there exists a unique, symmetric equilibrium where investors optimally choose to exhibit overconfidence in their private information but dismiss the information in prices. However, when risk tolerance is sufficiently high, such symmetric equilibria do not exist. Instead, investors endogenously choose different interpretations: one type ignores the information in prices, while the other chooses to overweight the price signal. Our model predicts how diversity in investment strategies, return predictability, volatility and price informativeness can vary with economic conditions.

JEL Classification: D8, G1

Keywords: difference of opinions, dismissiveness, overconfidence, wishful thinking, return predictability.

---

*Banerjee (snehalb@ucsd.edu) is at the University of California, San Diego; Davis (Jesse_Davis@kenan-flagler.unc.edu) is at the University of North Carolina, Chapel Hill; and, Gondhi (naveen.gondhi@insead.edu) is at INSEAD. We would like to thank Bradyn Breon-Drish, Jason Donaldson, David Hirshleifer, Joey Engelberg, Sergei Glebkin, Chris Parsons, Joel Peress, Geoff Tate and Liyan Yang, Francesca Zucchi, conference participants at the 2019 Behavioral Finance Working Group Conference, 2019 FTG Summer Meeting, EFA 2019 Meeting, 2019 Colorado Finance Summit, 10th Tel Aviv University Finance Conference and seminar participants at Queen Mary University of London, the London School of Economics, Purdue University, University of Miami, Tilburg University, Indiana University, University of Toronto, Stockholm School of Economics, and Stanford GSB for valuable feedback. All errors are our own.

1 Introduction

Prices are an essential source of information because they aggregate disperse data about payoffs and preferences (see Hayek (1945) for an early discussion). As such, the standard approach in economics assumes that market participants have rational expectations and, therefore, learn efficiently from prices. Yet there is ample evidence that people do not behave this way: returns exhibit excess predictability and volatility, investors are overconfident and trade too often, and individuals appear to under-react to prices in some settings but over-react in others.[1] To explain this evidence, the existing literature has explored how informational frictions, endowed behavioral biases, and cognitive limits affect how investors process private and public signals. However, they provide limited guidance in understanding when such distortions arise because market participants are constrained to exhibit such behavior by assumption. In this paper, we ask a more fundamental question: given a *choice*, how do investors interpret the information available to them, including that reflected in prices?

To address this question, we require a model of subjective belief choice. In standard expected utility theory, an investor is *endowed* with objective beliefs that affect utility only through their role in the weighing of future outcomes. However, a large literature in psychology and behavioral economics demonstrates that individuals also derive direct, immediate utility from their beliefs through the anticipation of future events (see Bénabou and Tirole (2016) for a recent survey). For instance, while the likelihood of actually winning may be extremely low, an individual may also value a lottery ticket for the anticipatory thrill of winning. On the other hand, the anticipation of an undesirable experience can generate disutility immediately even if the actual event occurs at a future date (e.g., feelings of anxiety about an upcoming dental appointment).[2]

In such cases, an individual may optimally choose her actions *and* her subjective beliefs to maximize her "anticipated utility," while accounting for the costs of deviating from the rational expectations benchmark. Intuitively, this leads individuals to exhibit "wishful thinking": they tend to choose beliefs that make them happy about the future, provided

[1] For instance, Shiller (1981) documents that stock returns exhibit excess volatility relative to fundamentals, and Jegadeesh and Titman (1993) document that stocks exhibit momentum. Odean (1999) documents that individual investors exhibit over-confidence as evidenced by excessive trading. Recent work by Coibion and Gorodnichenko (2012) suggests professional forecasters are slow to update their beliefs about macroeconomic variables, while Greenwood and Shleifer (2014) document that investor expectations of future returns exhibit extrapolation.

[2] Kocher, Krawczyk, and van Winden (2014) show that a substantial minority of lottery ticket buyers prefer delayed resolution of the lottery so that they can "savor" the thrill of anticipation for longer. On the other hand, Loewenstein (1987) argues how disutility from negative experiences can lead to a preference for early resolution — one might choose the earliest possible date for a dental appointment to "get it over with."

that they are not too far from the objective truth. We analyze whether such individuals choose to adopt rational expectations in a market setting and, if not, study which biases arise endogenously and under what conditions.

We show that investors who experience anticipated utility always choose to deviate from holding objective beliefs; moreover, these deviations vary predictably with their environment. When aggregate risk tolerance is low, there exists a unique, symmetric equilibrium in which investors choose to overweight their private information but dismiss the information in prices.[3] However, when aggregate risk tolerance is high, such symmetric equilibria cannot exist. Instead, we find that investors endogenously choose different interpretations: while most investors continue to dismiss price information, a small fraction overweight this information. We show that the subjective belief choice equilibria exhibit higher expected returns, trading volume and return predictability than the corresponding rational expectations equilibrium, and our model predicts how diversity in investment strategies and return predictability varies with economic conditions.

These results highlight a key takeaway of our approach: an individual's choice of subjective beliefs, and the resulting "behavioral bias," depends on her environment and the equilibrium behavior of others. Investors choose to be over-confident about private information irrespective of whether others exhibit rational expectations or over-confidence, but their decision to dismiss price information depends crucially on whether others are doing so. The difference in the interpretation of private versus price information is particularly striking given that prices are incrementally informative, and investors trade in a perfectly competitive market and face neither informational nor cognitive frictions. In fact, we show that investors may choose to dismiss price information even when it is arbitrarily precise.[4] Moreover, the endogenous heterogeneity in the asymmetric equilibrium is not a matter of degree but of type: ex-ante identical investors choose to adopt opposing interpretations of the information in prices. These results may help us better understand the empirically observed heterogeneity in investment styles between "fundamental" or "value" investors and "technical" or "momentum" traders.

[3] An overconfident investor, who believes her signal is more informative than it is, necessarily puts relatively less weight on the information in the price when updating her beliefs. We show that, in addition to such an effect, the investor optimally chooses to believe that the price is less informative than it objectively is, which further amplifies this distortion in her updating process.

[4] In our benchmark model, the price is a noisy signal about fundamentals because the aggregate supply of the asset is noisy. Investors can condition on the price costlessly, are fully attentive to it, and perfect competition implies the beliefs of any individual do not affect the price. However, as we discuss in Section 7, subjective beliefs about the price signal can be re-interpreted as subjective beliefs about the information that other traders possess. We show that even in a model where the aggregate supply of the asset is fixed and the price perfectly reveals the fundamental value, individual investors can increase their anticipated utility by perceiving the price to be noisy.

Formally, our analysis extends a standard setting in which a continuum of symmetric, risk-averse investors trade a single risky asset in a perfectly competitive market (à la Hellwig (1980)). Investors are privately informed about the risky payoff and can condition on prices when submitting their demand schedules. Each investor chooses her perceived precision of both her private signal and the information contained in the price, subject to a cost of distorting her beliefs away from the objective distribution. For instance, an investor's *anticipated* utility is higher if she believes that her signal is more informative than it actually is. Such beliefs increase the utility she derives today from her anticipation of future trading gains. But, the deviation carries an objective cost: the investor's trading position will be suboptimal from the perspective of the objective distribution and hence the average *ex-post,* experienced utility will be lower in the future.[5]

The key to understanding our results is that, in a speculative setting, each investor prefers not only to be better informed, but also to have information that is not shared by others, i.e., she *chooses to disagree* with other investors. In general, when she believes her information to be more precise than it actually is, an investor perceives less uncertainty and can trade more aggressively, which increases her anticipated utility. We refer to this as the **information channel**. In equilibrium, this channel leads investors to exhibit overconfidence in their private signal.

The information channel also applies when the information available in prices improves, e.g., when other investors' aggregated information is more informative. However, subjective beliefs about such information affect anticipated utility through an additional channel. Believing that the price is more informative necessarily implies a price that tracks fundamentals more closely; for instance, because other investors are trading on better, or less correlated, information. This reduces the potential gains from speculating against others which, in turn, reduces anticipated utility — we term this the **speculative channel**.[6] The offsetting effect of this speculative channel can actually dominate the informational channel, leading investors to dismiss the information contained in prices.

[5]We refer to this as the *experienced utility* penalty. More concretely, an investor's anticipated utility is her expected utility from her (distorted) trading strategy, under the subjective distribution, while her average ex-post utility is her expected utility from the same (distorted) trading strategy, under the objective distribution. The investor's optimal choice of subjective beliefs trades off these two measures of expected utility. As we discuss in Section 3, this is analogous to the "optimal expectations" approach of Brunnermeier and Parker (2005). For robustness, we also consider an alternative approach which imposes a cost that depends only on the statistical distance between the subjective and objective beliefs. As we discuss in the next section, this is in line with the approach taken by Caplin and Leahy (2019), and mirrors the approach taken in the large literature on robust control.

[6]In the limit, if the price is perfectly informative, there are no speculative gains from trading. There may still be a gain from the risk premia earned by holding the risky asset, which reflects the compensation for bearing aggregate risk.

The relative effect of these two channels on an individual's optimal subjective beliefs depends on the beliefs and behavior of other market participants in equilibrium. We show that in any symmetric equilibrium, with any well-behaved cost function, investors overweight their private information but underweight the information in prices.[7] Intuitively, an investor chooses not to condition on the information in prices if others are doing so. Moreover, we show that such a symmetric equilibrium can only exist when aggregate risk tolerance is sufficiently low: in this case, the (objective) cost of dismissing the information in prices is not very high because the price signal is sufficiently noisy, and the speculative channel dominates the information channel.

However, when aggregate risk tolerance is sufficiently high, we find that there cannot exist a symmetric equilibrium. As risk tolerance rises, investors trade more aggressively: this increases the informativeness of the price so that the information channel is amplified. Moreover, if most other investors are choosing to ignore the price, it serves almost as a private signal for those who condition on it: the speculative channel is weakened. As a result, an asymmetric equilibrium can arise in which the majority of investors ignore the price, but a minority overweight the information in the price. This equilibrium highlights an important consequence of allowing investors to choose their subjective beliefs: endogenous investor polarization can arise.[8] Such heterogeneity does not commonly arise in standard models where investors are simply assumed (constrained) to exhibit rational expectations or specific behavioral biases.

The asymmetric equilibrium may also help us better understand the empirically observed heterogeneity in investment styles, and how they vary with economic conditions. Fundamental, or "value", investors that try to identify "mispriced" securities and tend to rely more on their private valuation of stocks are analogous to the over-confident, dismissive investors in our model. On the other hand, "technical" analysis and momentum strategies are based on extrapolating price changes, which is reminiscent of the overweighting of price information by the minority in our asymmetric equilibrium. Our model suggests that the latter strategy is more likely to be profitable when aggregate risk tolerance is high and prices are very

[7] In our benchmark, with the experienced utility penalty, agents exhibit behavior akin to the "fully cursed" agents of Eyster, Rabin, and Vayanos (2018) or that found in a pure "differences of opinion" framework (e.g., found in Banerjee, Kaniel, and Kremer (2009), Banerjee (2011)). Agents do not distort their beliefs in their private signal but fail to condition on the information found in prices. Under alternative frameworks, we show that overconfidence arises along with dismissiveness ("cursedness").

[8] In Brunnermeier and Parker (2005), general equilibria are analyzed in which some investors choose to be optimistic about an asset's payoff. If such behavior distorts the price sufficiently upward, some agents choose to be pessimistic about the asset's payoff which is equivalent to be being *optimistic* about the potential trading gains. Effectively, beliefs are distorted for the same reason (though in different directions). In our asymmetric equilibrium, ex-ante well-being is the same across both types, but the channels through which this arises are distinct across the two types of investor.

informative.

Relatedly, our model generates a number of predictions that distinguish it from the rational expectations benchmark. For instance, the subjective beliefs equilibrium exhibits higher expected returns and higher volume than the corresponding rational expectations equilibrium. Such equilibria can even exhibit positive return predictability (more positive serial correlation in returns) in a setting where the rational expectations equilibrium always leads to reversals. To the extent that bull markets and macroeconomic expansions reflect higher risk tolerance, our model also predicts these episodes should generate higher diversity in investment strategies, higher return predictability, a positive relation between return predictability and volatility, and higher price informativeness. On the other hand, periods of high market stress and high volatility are more likely to be associated with higher correlation in investment styles, low (or even negative) serial correlation in returns, a negative relation between predictability and volatility, and lower price informativeness. As we discuss below, a number of our predictions are broadly consistent with the existing empirical evidence on momentum returns and crashes (e.g., Moskowitz, Ooi, and Pedersen (2012), Daniel and Moskowitz (2016)) and price informativeness (e.g., Bai, Philippon, and Savov (2016), and Dávila and Parlatore (2019)), while others offer novel implications for future empirical work.

Note that subjective belief choice is not just of theoretical interest — there is substantial, direct empirical evidence that suggests individuals experience anticipated utility, and as a result, distort their subjective beliefs in systematic ways. Individuals engage in information avoidance, for instance, by choosing not to learn about the risk of deadly disease even if the test is approximately costless (Oster, Shoulson, and Dorsey (2013)). At the same time, individuals may actively seek (and pay) to learn about potential good news, such as the outcome of a lottery-like event (Ganguly and Tasoff (2017)) or the performance of their portfolios on days when the market has done well (Karlsson, Loewenstein, and Seppi (2009)). Individuals also update asymmetrically when information is revealed: more weight is placed on good news (e.g., a positive signal about one's IQ in Mobius, Niederle, Niehaus, and Rosenblat (2014)) than bad news (e.g. a negative signal about one's attractiveness in Eil and Rao (2011)). Finally, many individuals interpret information in ways which are favorable to their current well-being, updating in ways consistent with their political beliefs (Kahan (2013)) or interpreting uninformative signals of ability as positive indicators (Exley and Kessler (2019)). This literature suggests that such wishful thinking is not generated by individuals inability to understand their environment; in fact, both Kahan (2013) and Kahan, Peters, Dawson, and Slovic (2017) show that cognitive ability can even exacerbate the effect because more sophisticated individuals can better "rationalize" their beliefs and interpretations. Given this evidence, we expect our analysis to apply, to varying degrees, to both retail and institutional investors, and believe it is important to understand the impact of such behavior on market outcomes.[9]

The rest of the paper is as follows. Section 2 briefly discusses the phenomenon of anticipatory utility and reviews the related literature. Section 3 introduces the model setup, discusses our assumptions, and characterizes the financial market equilibrium, given investor beliefs. Section 4 studies the tradeoffs associated with deviating from rational expectations. Section 5 presents the characterization of subjective beliefs in equilibrium: 5.1 studies a setting in which the investor chooses beliefs only about her private signal while Section 5.2 considers the more general setting. Section 6 summarizes some of the empirical implications of the model. Section 7 presents a summary of the supplementary analysis and extensions we consider, including allowing for subjective beliefs about other investors and (exogenous) public signals as well as an analysis of welfare. Section 8 concludes, and proofs and extensions can be found in Appendices A and B, respectively.

2 Background and related literature

2.1 Anticipated utility and subjective belief choice

Bénabou and Tirole (2016) survey the now extensive literature on belief choice (e.g., Akerlof and Dickens (1982), Loewenstein (1987), Caplin and Leahy (2001), Eliaz and Spiegler (2006)). The concept of anticipatory utility, or current subjective expected utility, dates to at least Jevons (1905) who considers agents who derive contemporaneous utility not simply from current actions but also the anticipation of future utility flows. For instance, an agent who anticipates a positive *future* experience (e.g., the purchase of a new home) will experience a positive, *contemporaneous* utility flow (e.g., excitement about your new neighborhood). On the other hand, the prospects of a negative future outcome (e.g., the inevitability of declining health) can decrease an agent's current utility (e.g., through anxiety or fear).

As a result, an agent's subjective beliefs about future events will affect not just an agent's actions but also her current utility. This creates a tension between holding beliefs that are "accurate" (and therefore will lead to optimal actions) and beliefs that are "desirable" (and therefore will increase current utility). We emphasize that agents do not suffer from a "multiple selves" problem but instead choose to hold a *single* set of beliefs which accounts for this implicit tradeoff.[10]

[9]For instance, it is likely that sophisticated traders are particularly adept at justifying their favored bets, attributing their successes to skill and ability, while blaming losses to bad luck.

[10]In particular, our assumption that deviations from the objective distribution are costly is a modeling convenience — we do not interpret the use of the objective distribution in specifying the cost as the agent

Like us, Caplin and Leahy (2019) also consider a setting in which agents engage in wishful thinking - they choose subjective beliefs to maximize anticipatory utility subject to a cost. In their case, the cost of choosing distorted beliefs depends upon the distance between the subjective and objective distributions. They show that subjective belief choice can help explain a number of behavioral biases, including as optimism, procrastination, confirmation bias, polarization and the endowment effect. We allow for general cost functions and show how endogenous belief choice can give rise to both overconfidence and under-reaction to price information (e.g., cursedness or dismissiveness) in financial markets.

In another closely related paper, Brunnermeier and Parker (2005) propose a model of optimal expectations in which agents maximize their expected wellbeing. In their model, agents choose prior beliefs to maximize anticipatory utility, and choose optimal actions subject to these priors. Unlike the "statistical distance" approach of Caplin and Leahy (2019), the cost of deviating from the objective distribution in Brunnermeier and Parker (2005) is the loss in experienced utility as a result of actions that are sub-optimal under the objective distributions. This paper (along with Brunnermeier, Gollier, and Parker (2007)) apply this framework to understand risk-taking, preference for skewness, portfolio under-diversification and consumption/savings patterns. Our analysis considers similar behavior in a setting with asymmetric information where investors form beliefs not only about exogenous variables (fundamentals, signals) but also endogenous objects (equilibrium prices).[11]

2.2 Differences of opinion and overconfidence

Our paper contributes to two strands of the literature studying the financial market impact of deviations from rational expectations. The first strand focuses on differences of opinion, whereby investors "agree to disagree" about the joint distribution of payoffs and signals and therefore, incorrectly condition on the information in prices (e.g., Harrison and Kreps (1978), Kandel and Pearson (1995), Banerjee et al. (2009) and Banerjee (2011)). Other approaches that lead investors to discount the information in prices include models that feature rational inattention (e.g., Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016)), cursedness (e.g.,

"knowing" the true distribution. Instead, we wish to capture the notion that the agent behaves as if deviating too far from accurate beliefs is costly, either due to experience or because it reflects a departure from consensus. See Caplin and Leahy (2019) for a further discussion.

[11] As we discuss in Section 3, we specify the cost of deviating from objective beliefs as the disutility the agent experiences from choices made under subjective beliefs. As such, while the overall objective function resembles that of Brunnermeier and Parker (2005), we interpret it somewhat differently. As Caplin and Leahy (2019) suggest, the Brunnermeier and Parker (2005) approach may be interpreted as one with divided selves: the agent chooses subjective beliefs by evaluating outcomes under the objective distribution (at date zero), and chooses actions in the following periods by evaluating outcomes under the chosen subjective distribution. In contrast, agents in our model (as in Caplin and Leahy (2019)) evaluate outcomes only under the subjective beliefs, but their choice of subjective beliefs is anchored to be "close" to the objective distribution.

Eyster et al. (2018)) and costly learning from prices (e.g., Vives and Yang (2017)).[12] Notably, in the models of Kacperczyk et al. (2016) and Vives and Yang (2017), updating one's beliefs using prices is costly (in an attention or utility sense), and, as such, investors choose to discount this information. We view our analysis as complementary to this earlier work. We show that even when learning (efficiently) from prices is costless, investors may choose to dismiss price information when they experience anticipated utility.[13]

The second, related strand focuses on the impact of overconfidence: specifically, settings in which agents believe their private information is more precise than it objectively is (e.g., Odean (1998a); Daniel, Hirshleifer, and Subrahmanyam (1998, 2001); Gervais and Odean (2001); Scheinkman and Xiong (2003)). These two strands highlight how such deviations can explain a number of stylized facts about financial markets that are difficult to reconcile in the rational expectations framework, including excess trading volume and return predictability. In contrast to the existing literature, however, we do not assume that investors exhibit differences of opinions or overconfidence. Instead, investors are allowed to choose their beliefs, and importantly, exhibiting rational expectations is within their choice set. As such, our analysis sheds light on the economic forces that may lead to behavior (e.g., overconfidence and dismissiveness) that deviates from the rational expectations benchmark.

3 Model setup and financial market equilibrium

This section discusses the key assumptions of our model and characterizes the financial market equilibrium, taking subjective belief choices as given. We explore how these subjective beliefs are determined in the following sections.

3.1 Model setup

Asset payoffs. There are two securities. The gross return on the risk-free security is normalized to one. The terminal payoff (fundamental value) of the risky security is F, which

[12]While Eyster et al. (2018) show that cursedness can generate distinct predictions from a model of differences of opinions (which they term dismissiveness) when there is imperfect competition and no noise trading, our setting features perfectly competitive markets and noise in prices, and so cursedness and differences of opinions are effectively isomorphic.

[13]Our analysis also highlights that the benefits and costs of using the information in prices depends on how other investors use this information. This is reminiscent of, but distinct from, the complementarity discussed by Vives and Yang (2017), who show that there can exist complementarity in learning from prices: an investor may choose to learn more from prices when others become more sophisticated about learning from prices because prices become more informative. In contrast, our analysis suggests that when others are learning from prices, one's incentive to do the same decreases.

is normally distributed with mean m and prior precision τ i.e.,

$$F \sim \mathcal{N}\left(m, \frac{1}{\tau}\right). \tag{1}$$

We denote the market-determined price of the risky security by P, and the aggregate supply of the risky asset by $Z + z$, where

$$z \sim \mathcal{N}\left(0, \frac{1}{\tau_z}\right), \tag{2}$$

and we normalize the mean aggregate supply to $Z = 0$.

Information. There is a continuum of investors, indexed by $i \in [0, 1]$. Before trading, each investor is endowed with a private signal s_i, where a

$$s_i = F + \varepsilon_i \qquad \varepsilon_i \sim N\left(0, \frac{1}{\tau_e}\right) \tag{3}$$

and ε_i is independent and identically distributed across investors so that $\int \varepsilon_i \, di = 0$. Moreover, investors can update their beliefs about F by conditioning on the information in the price P.

Beliefs and preferences. Each investor i is endowed with initial wealth W_0 and zero shares of the risky security, and exhibits CARA utility with coefficient of absolute risk aversion γ over terminal wealth W_i:

$$W_i = W_0 + x_i (F - P), \tag{4}$$

where x_i denotes her demand for the risky security. We allow each investor to interpret the quality of the information in both her private signals and the price subjectively. Specifically, we assume that investor i believes that the noise in her private signal is given by[14]

$$\varepsilon_i \sim_i \mathcal{N}\left(0, \frac{1}{\delta_{e,i}\tau_e}\right), \tag{5}$$

and believes the distribution of the asset's aggregate supply, which as we show below is proportional to the noise in the signal investors can extract from the price, is given by

$$z \sim_i \mathcal{N}\left(0, \frac{1}{\delta_{z,i}\tau_z}\right). \tag{6}$$

[14]Here $\sim_i$ denotes "distributed as, according to investor i's beliefs."

In what follows, we will denote the expectation and variance of random variable X under investor i's beliefs about the information environment, i.e., $\delta_{e,i}$ and $\delta_{z,i}$ by $\mathbb{E}_i[X]$ and $\text{var}_i[X]$, respectively. As is standard, we will denote the expectation and variance of X under the true (objective) distribution by $\mathbb{E}[X]$ and $\text{var}[X]$, respectively.

The parameters $\delta_{e,i}, \delta_{z,i} \in [0, \infty]$ reflect the degree to which investor i over- or under-estimates the precision of the private signal and aggregate noise, respectively. When $\delta_{e,i} = \delta_{z,i} = 1$, investor i's beliefs satisfy **rational expectations**: her beliefs coincide with the objective distribution of the underlying shocks. On the other hand, when $\delta_{e,i} > 1$, investor i is overconfident about her private signal: she believes her private signal is more informative than it objectively is and she overweights it in forming her beliefs. The opposite is true when $\delta_{e,i}$ is less than one. Similarly, when $\delta_{z,i} > 1$ ($\delta_{z,i} < 1$), investor i believes the price to be more informative (less informative, respectively) about fundamentals. We assume that such deviations from the objective distribution impose a utility cost, denoted by $C(\delta_{e,i}, \delta_{z,i})$.

Given her choice of subjective beliefs, each investor optimally chooses her position in the risky security. Thus, optimally chosen subjective beliefs maximize her anticipated utility, net of cost $C(\cdot)$. Formally, denote investor i's optimal demand, given her beliefs, by:

$$x_i^*(\delta_{e,i}, \delta_{z,i}) = \arg\max_{x_i} \mathbb{E}_i\left[-\gamma \exp\left\{-\gamma x_i (F - P) - \gamma W_0\right\} \middle| s_i, P\right]. \tag{7}$$

and denote investor i's **anticipated utility** by

$$AU_i(\delta_{e,i}, \delta_{z,i}) \equiv \mathbb{E}_i\left[\mathbb{E}_i\left[-\gamma \exp\left\{-\gamma x_i^* (F - P) - \gamma W_0\right\} \middle| s_i, P\right]\right]. \tag{8}$$

Then, investor i optimally chooses subjective beliefs $\delta_{e,i}$ and $\delta_{z,i}$ to maximize:

$$\max_{\delta_{e,i}, \delta_{z,i}} AU_i(\delta_{e,i}, \delta_{z,i}) - C(\delta_{e,i}, \delta_{z,i}). \tag{9}$$

We assume that the cost function $C(\cdot)$ penalizes deviations from the objective distribution (i.e., from $\delta_{e,i} = \delta_{z,i} = 1$) and is well-behaved as defined below.

Definition 1. A cost function $C(\delta_{e,i}, \delta_{z,i})$ is **well-behaved** if $C(1,1) = \frac{\partial C}{\delta_{e,i}}(1,1) = \frac{\partial C}{\delta_{z,i}}(1,1) = 0$, and C is strictly convex (i.e., its global minimum is at $(1,1)$).

While many of our results apply to general cost functions, our main analysis focuses on a special case in which the cost each investor incurs by distorting her subjective beliefs is the reduction in expected utility (under the *objective* distribution) when her position in the risky asset, $x_i^*(\delta_{e,i}, \delta_{z,i})$, is determined by the chosen *subjective* distribution. As is well-established, any deviation from the rational expectations benchmark ($\delta_{e,i} = \delta_{z,i} = 1$) is objectively inefficient: the investor is over- or under-weighting the information she receives. We refer to this specification as the "experienced utility" penalty benchmark.

Definition 2. Investor i incurs the **experienced utility penalty** if the cost of choosing $\delta_{e,i}, \delta_{z,i}$ is given by:

$$C_{obj}\left(\delta_{e,i}, \delta_{z,i}\right) \equiv \begin{array}{c} \mathbb{E}\left[-\gamma \exp\left\{-\gamma x_i^*\left(1,1\right) \times \left(F-P\right) - \gamma W_0\right\}\right] \\ -\mathbb{E}\left[-\gamma \exp\left\{-\gamma x_i^*\left(\delta_{e,i}, \delta_{z,i}\right) \times \left(F-P\right) - \gamma W_0\right\}\right] \end{array} \tag{10}$$

As we show in the appendix, when investors incur the experienced utility penalty, their subjective belief choice problem can be represented as:

$$\max_{\delta_{e,i},\delta_{z,i}} AU_i\left(\delta_{e,i}, \delta_{z,i}\right) + \mathbb{E}\left[-\gamma \exp\left\{-\gamma x_i^*\left(\delta_{e,i}, \delta_{z,i}\right) \times \left(F-P\right) - \gamma W_0\right\}\right]. \tag{11}$$

This is closely related to the objective function in the "optimal expectations" approach of Brunnermeier and Parker (2005). Under this approach, an investor optimally chooses actions under subjective beliefs $\mathbb{E}_i[\cdot]$, and the optimal choice of beliefs maximizes the investor's well-being under the objective distribution i.e.,

$$\max_{\delta_{e,i},\delta_{z,i}} \mathbb{E}\left[\begin{array}{c} \mathbb{E}_i\left[-\gamma \exp\left\{-\gamma x_i^*\left(\delta_{e,i}, \delta_{z,i}\right)\left(F-P\right) - \gamma W_0\right\} \middle| s_i, P\right] \\ -\gamma \exp\left\{-\gamma x_i^*\left(\delta_{e,i}, \delta_{z,i}\right) \times \left(F-P\right) - \gamma W_0\right\} \end{array}\right] \tag{12}$$

$$= \max_{\delta_{e,i},\delta_{z,i}} \mathbb{E}\left[AU_i\left(\delta_{e,i}, \delta_{z,i}\right)\right] + \mathbb{E}\left[-\gamma \exp\left\{-\gamma x_i^*\left(\delta_{e,i}, \delta_{z,i}\right) \times \left(F-P\right) - \gamma W_0\right\}\right]. \tag{13}$$

In our setting, $AU_i\left(\delta_{e,i}, \delta_{z,i}\right) = \mathbb{E}\left[AU_i\left(\delta_{e,i}, \delta_{z,i}\right)\right]$ and so the two objectives coincide. We utilize this cost function as our benchmark because of its clear interpretation, intuitive appeal and direct quantitative implications.

3.2 Discussion of assumptions

We emphasize that the penalty function in (10) does not necessarily imply that the investor knows the objective distribution; instead, it should be interpreted as a tractable specification for the utility cost of subjective beliefs, from the perspective of the modeler (or observer). This is in contrast to the interpretation of Brunnermeier and Parker (2005), where agents choose their beliefs with knowledge of the objective distribution and then choose actions using their subjective model of the world. Intuitively, we interpret the above specification (and that of Brunnermeier and Parker (2005)) as one in which investors evaluate their actions and outcomes under a single, subjective model of the world, which is "close to the truth" in the sense that the distortions in behavior do not generate too large of a loss in experienced utility. The subjective model that is chosen may result from a more complicated process of experimentation, learning and experience, which trades off "desirable" models (that increase anticipated utility) against "accurate" models (that increase experienced utility). The specification in (10) provides a tractable characterization of this process from the perspective of economic modeling.

Our paper's focus is on the manner and degree to which investors choose beliefs which imply disagreement with others - as such, we choose to focus on investors' subjective beliefs about private signals and the price. In Appendix B.2, we explore how investors form beliefs about a public signal observed by all agents in the economy. For ease of presentation, we capture each agent's subjective beliefs about others through the parameter $\delta_{z,i}$. As we discuss further in Section 7 and show formally in Appendix B.1, this parameterization of investors' beliefs about the asset's supply effectively captures several alternative interpretations of investors's subjective beliefs about others.

We restrict investors to choose subjective beliefs about second moments (the precision of both private and price information), instead of allowing them to choose their subjective beliefs more flexibly. Moreover, we assume that investors make their subjective belief choices *before* observing the realizations of their signals. These assumptions make the analysis tractable: conditional on choosing their interpretation of s_i and s_p, we can explicitly characterize the financial market equilibrium since the resulting equilibrium price is a linear signal about fundamentals. Moreover, this specification is naturally interpreted as the stage game of a dynamic, repeated setting in which investors experiment with (and update about) different models of the world.[15] In Appendix B.3, however, we explore how our results are affected when investors choose their interpretations *after* observing the signals. While allowing for more flexibility in subjective belief choice may lead to additional implications (e.g., a preference for skewness as in Brunnermeier and Parker (2005)), we expect our main results to be qualitatively similar in these settings. However, a formal analysis is beyond the scope of this paper and left for future work.

We assume that the aggregate supply of the risky asset is zero for analytic tractability. In Appendix B.4, we consider an extension of the model in which the aggregate supply of the risky asset is $Z > 0$. While this extension is not as analytically tractable, we can solve it numerically and find that the resulting equilibria are very similar to the case with $Z = 0$. Moreover, as we discuss in Section 6, this allows us to characterize the implications of subjective beliefs on unconditional expected returns.

[15] In such a rich setting, the cost of distorting beliefs would depend on the history of price and signal realizations and the resulting shocks to experienced utility.

Finally, as emphasized in our discussion of belief formation, motivated beliefs are also constrained beliefs: investors must be able to *plausibly* observe and weigh the available information in a fashion which supports their chosen beliefs. For tractability, we do not directly model this information processing and allow for wide-ranging investor beliefs: for instance, an investor can believe that their signal is perfectly informative. In Appendix B.5, we analyze the impact of bounding investor beliefs on equilibrium outcomes.

As discussed by Caplin and Leahy (2019), the wishful thinking approach we employ has a parallel in the robust control literature (e.g., Hansen and Sargent (2001), Hansen and Sargent (2008)). Agents who exhibit robust control are unsure about their model of the world, and choose actions optimally under the "worst-case" subjective beliefs. As in our setting, the set of plausible beliefs under consideration is restricted to be "close" to the objective distribution (usually, through a statistical penalty function like the Kullback-Leibler divergence). More concretely, a robust control agent chooses action a and subjective beliefs μ to solve

$$\min_{\mu} \max_{a} \mathbb{E}_{\mu} \left[u\left(a\right)\right] + C\left(\mu\right), \tag{14}$$

where $\mathbb{E}_{\mu} \left[u\left(a\right)\right]$ reflects the subjective expected utility from action a under "worst case" beliefs μ and $C\left(\mu\right)$ reflects the penalty of choosing subjective beliefs μ that differ from the reference distribution. On the other hand, a wishful thinking agent chooses action a and subjective beliefs μ to solve:

$$\max_{\mu} \max_{a} \mathbb{E}_{\mu} \left[u\left(a\right)\right] - C\left(\mu\right), \tag{15}$$

where μ reflects the "wishful thinking" that the agent engages in to maximize anticipated utility $\mathbb{E}_{\mu} \left[u\left(a\right)\right]$.

The robust control approach is motivated by the large literature in psychology and economics that documents ambiguity aversion, and has been useful in understanding a number of stylized facts about aggregate financial markets (e.g., the equity premium puzzle). However, as emphasized by Caplin and Leahy (2019) (and suggested by the papers discussed above), there is also substantial evidence for both optimism and motivated beliefs. Each type of behavior is likely to arise in different contexts, and our analysis suggests that accounting for wishful thinking may be an important step in understanding what gives rise to overconfidence and dismissiveness in financial markets. While beyond the scope of the current paper, it would be interesting to explore the implications of investors who exhibit "wishful thinking" in some domains and "robust control" preferences in others.

3.3 Financial market equilibrium

We first solve for the financial market equilibrium, taking investors' chosen subjective beliefs as given. We consider equilibria in which the price P is a linear combination of fundamentals F and noise trading z, and conjecture that observing the price is equivalent to observing a signal of the form:

$$s_p = F + \beta z. \tag{16}$$

The variance of this signal is $\tau_p^{-1} = \beta^2/\tau_z$, and β is a constant determined in equilibrium. Given investor i's subjective beliefs $\delta_{e,i}$ and $\delta_{z,i}$, and conditional on her observed signals, s_i and s_p, investor i's posterior subjective beliefs are given by:

$$F|s_i, s_p \sim_i \mathcal{N}\left(\mu_i, \frac{1}{\omega_i}\right), \text{ where} \tag{17}$$

$$\mu_i \equiv \mathbb{E}_i\left[F|s_i, s_p\right] = m + A_i\left(s_i - m\right) + B_i\left(s_p - m\right), \tag{18}$$

$$\omega_i \equiv \frac{1}{\mathrm{var}_i\left[F|s_i, s_p\right]} = \frac{\tau}{1 - A_i - B_i}, \text{ and} \tag{19}$$

$$A_i \equiv \frac{\delta_{e,i}\tau_e}{\tau + \delta_{e,i}\tau_e + \delta_{z,i}\tau_p}, \text{ and } B_i \equiv \frac{\delta_{z,i}\tau_p}{\tau + \delta_{e,i}\tau_e + \delta_{z,i}\tau_p}. \tag{20}$$

The optimal demand for investor i, given her subjective beliefs, is given by

$$x_i^* = \frac{\mathbb{E}_i\left[F|s_i, P\right] - P}{\gamma \mathrm{var}_i\left[F|s_i, P\right]} = \frac{\omega_i}{\gamma}\left(\mu_i - P\right). \tag{21}$$

Equilibrium prices are determined by market clearing:

$$\int_i x_i^* di = z, \tag{22}$$

which implies:

$$P = \frac{\int_i \omega_i \left\{m + A_i\left(F - m\right) + B_i\left(s_p - m\right)\right\} di}{\int_i \omega_i di} - \frac{\gamma}{\int_i \omega_i di} z \tag{23}$$

This verifies our conjecture for functional form of the price and we can write

$$\beta = \frac{-\gamma}{\int_i \omega_i A_i di} = -\frac{\gamma}{\tau_e \int_i \delta_{e,i} di}. \tag{24}$$

The above results are summarized in the following lemma.

Lemma 1. *Given investor i's subjective beliefs $\delta_{e,i}$ and $\delta_{z,i}$, there always exists a unique,* *linear, financial market equilibrium in which*

$$P = m + \Lambda (s_p - m), \ \ \textit{where } \Lambda = \frac{\int_i \delta_{e,i}\tau_e + \delta_{z,i}\tau_p di}{\int_i \tau + \delta_{e,i}\tau_e + \delta_{z,i}\tau_p di}, \tag{25}$$

$s_p = F + \beta z$, $\tau_p = \tau_z/\beta^2$, *and* $\beta = -\frac{\gamma}{\tau_e \int_i \delta_{e,i} di}$. *When subjective belief choices are symmetric (i.e.,* $\delta_{e,i} = \delta_e$ *and* $\delta_{z,i} = \delta_z$ *for all* i*), then the price is given by:*

$$P = m + \Lambda (s_p - m), \ \ \textit{where } s_p = \left(F - \tfrac{\gamma}{\tau_e \delta_e} z\right), \Lambda = \frac{\delta_e \tau_e + \delta_z \tau_p}{\tau + \delta_e \tau_e + \delta_z \tau_p}, \ \ \textit{and } \tau_p = \frac{\tau_z \tau_e^2 \delta_e^2}{\gamma^2}. \tag{26}$$

As the above lemma makes apparent, the choice of investor beliefs affect equilibrium prices through two channels. First, an increase in the perceived precision of private signals (higher $\delta_{e,i}$) increases the signal to noise ratio of the signal s_p (since $|\beta|$ is decreasing in $\delta_{e,i}$). Investors trade more aggressively on their private information which is then reflected in the objective quality of the information in the price. Second, an increase in the perceived precision of either private signals (i.e., higher $\delta_{e,i}$) or price information (i.e., higher $\delta_{z,i}$) increases the sensitivity of the price to fundamentals (F) through Λ. As we shall see in Section 6, these channels interact to affect a number of empirically observable features of the financial market equilibrium.

4 Anticipated utility and subjective beliefs

With the financial market equilibrium established, we can now characterize the optimal subjective beliefs of an investor. Importantly, we assume that each investor takes as given the subjective belief distortion chosen by other investors: she does not assume that other investors hold rational expectations.

Given the optimal demand for the risky asset (21), anticipated utility is given by

$$AU_i(\delta_{e,i}, \delta_{z,i}) = \mathbb{E}_i \left[-\exp\left\{ -\frac{(\mathbb{E}_i[F|s_i, P] - P)^2}{2\mathrm{var}_i[F|s_i, P]} \right\} \right]. \tag{27}$$

Moreover, given the characterization of the equilibrium price in Lemma 1, investor i's beliefs about the conditional return are given by:

$$\mathbb{E}_i[\mathbb{E}_i[F|s_i, P] - P] = m - m = 0, \text{ and} \tag{28}$$

$$\mathrm{var}_i[\mathbb{E}_i[F|s_i, P] - P] = \mathrm{var}_i[F - P] - \mathrm{var}_i[F|s_i, P], \tag{29}$$

where the first equality follows from the law of iterated expectations and the second equality follows from the law of total variance.[16] With this in mind, the above expectation reduces to

$$AU_i\left(\delta_{e,i},\delta_{z,i}\right) = -\sqrt{\frac{\mathrm{var}_i\left[F|s_i,P\right]}{\mathrm{var}_i\left[F-P\right]}}. \tag{31}$$

From this, we derive the following result.

Lemma 2. *Anticipated utility is always (weakly) increasing in $\delta_{e,i}$; it is strictly increasing as long as $\delta_{z,i} > 0$. Anticipated utility is non-monotonic in $\delta_{z,i}$: there exists some $\bar{\delta}$ such that for all $\delta_{z,i} < \bar{\delta}$ anticipated utility is decreasing in $\delta_{z,i}$ while for all $\delta_{z,i} > \bar{\delta}$ it is increasing.*

To gain some intuition, we note that anticipated utility is simply a monotonic transformation of

$$\frac{\mathrm{var}_i\left[F-P\right]}{\mathrm{var}_i\left[F|s_i,P\right]} = \mathrm{var}_i\left(\frac{\mathbb{E}_i\left[F-P|s_i,P\right]}{\sqrt{\mathrm{var}_i\left[F-P|s_i,P\right]}}\right) \equiv \mathrm{var}_i\left(SR_i\right) \tag{32}$$

where

$$SR_i \equiv \frac{\mathbb{E}_i\left[F-P|s_i,P\right]}{\sqrt{\mathrm{var}_i\left[F-P|s_i,P\right]}} \tag{33}$$

is investor i's conditional Sharpe ratio, given her beliefs. When the variance of the conditional Sharpe ratio is higher, the investor expects to observe both (i) more profitable trading opportunities and (ii) the opportunity to trade more aggressively. Of course, she also faces an increased likelihood of facing the opposite scenario, but the benefit on the upside always outweighs the reduction in expected profits on the downside.[17]As a result, anticipated utility is higher when the variance in the conditional Sharpe ratio is higher.

Intuitively, reducing the perceived uncertainty (i.e., $\mathrm{var}_i\left[F-P|s_i,P\right]$) about the trading opportunity is valuable - if the investor has better information about the value of the asset this increases her utility. Increasing the perceived precision of the private signal (i.e., increasing $\delta_{e,i}$) has this effect and so anticipated utility increases when the investor inflates her perception of the quality of the private signal.

On the other hand, increasing the perceived precision of the price signal (i.e., increasing $\delta_{z,i}$) has two off-setting effects. First, the **information effect** of learning from prices reduces the conditional variance $\mathrm{var}_i\left[F-P|s_i,P\right]$: the investor has better information about the

[16]The law of total variance implies

$$\mathrm{var}_i\left[F-P\right] = \mathbb{E}_i\left[\mathrm{var}_i\left[F-P|s_i,P\right]\right] + \mathrm{var}_i\left[\mathbb{E}_i\left[F-P|s_i,P\right]\right], \tag{30}$$

which in turn, implies the above expression.

[17]This arises because the trading opportunity and the investor's position act as complements - effectively, the utility is convex in the trading opportunity (as captured by the conditional Sharpe ratio), and so an increase in the perceived variance is beneficial.

asset's value which increases anticipated utility. This information effect reduces the volatility of the perceived return on the risky security, a benefit in and of itself, but it also allows the investor to scale up her trading position. Second, the **speculative effect** of believing prices are more informative decreases the perceived variance of the conditional expected return (i.e., $\text{var}_i\left(\mathbb{E}_i\left[F|s_i, P\right] - P\right)$), which lowers anticipated utility. Intuitively, when the price is more informative, it tracks fundamentals more closely and, as a result, the trading opportunity is less profitable. The overall effect of changing the perceived precision of the price signal depends on the relative magnitude of these two effects. As we show in the proof of Lemma 2, the latter effect dominates when $\delta_{z,i}$ is low, while the former effect dominates when $\delta_{z,i}$ is sufficiently high, which is what drives the non-monotonicity in $\delta_{z,i}$.

This is the key distinction between learning from private signals and learning from price information and it drives our equilibrium results below. Learning from either source is informative about fundamentals which naturally increases utility. However, learning from prices also reduces the investor's perception of the potential trading opportunity. We explore how this distinction leads to differences in investors' subjective interpretation of private signals and the information in the price in the next two sections.

Finally, it is important to highlight that the relationship between investors' utility and the conditional variance of the Sharpe ratio is not unique to our specific setting. For instance, the partial equilibrium analysis of Van Nieuwerburgh and Veldkamp (2010) shows that investor utility is increasing in the squared Sharpe ratio under more general preference and payoff assumptions. As such, we expect that the key effects of belief distortion on anticipatory utility (the information and speculative effects) should be qualitatively robust. Our focus is the CARA-normal setting, however, because this allows us to characterize in closed form the (general) equilibrium effects of subjective belief choice.

5 Belief choices and equilibrium characterization

This section presents the characterization of subjective beliefs in equilibrium. We begin by considering a benchmark case where investors only choose their interpretation of private signals, but interpret the information in prices correctly. Section 5.2 then characterizes the properties of a symmetric equilibrium in our model. Section 5.3 establishes the existence of the symmetric equilibrium in which all investors are over-confident about their private information but dismiss the information in prices. Finally, Section 5.4 discusses the non-existence of symmetric equilibria and the existence of asymmetric equilibria.

5.1 Benchmark: Belief choice about private signals

We begin with a benchmark in which investors are forced to have objective beliefs about the price signal (i.e., we assume $\delta_{z,i} = 1$ for all i) but can choose their beliefs about the precision of their private signals. Unsurprisingly, given the intuition laid out above, we find that all investors choose to exhibit over-confidence about their private information in equilibrium.

Proposition 1. *Suppose investors have objective beliefs about the price signal i.e., $\delta_{z,i} = 1$ for all i, and the cost function $C(\delta_{e,i}, \delta_{z,i})$ is well-behaved. Then, there exists a unique symmetric equilibrium in which all agents are over-confident about their private signal.*

With objective beliefs about the informativeness of the price, Lemma 2 implies that an investor's anticipated utility strictly increases in the perceived precision of her private signal. Since the cost of setting $\delta_{e,i} = 1$ is zero (i.e., $C(1,1) = 0$) and the marginal cost of increasing $\delta_{e,i}$ at $\delta_{e,i} = \delta_{z,i} = 1$ is also zero, investors prefer to optimally choose $\delta_{e,i} > 1$ i.e., they optimally choose to be over-confident about her private signal.

Proposition 2. *Suppose investors have objective beliefs about the price signal i.e., $\delta_{z,i} = 1$ for all i, and face the experienced utility penalty, i.e., they solve (10). Then, there exists a unique equilibrium in which the optimal choice of $\delta_{e,i} = \delta_e$ satisfies:*

$$\frac{\left(\tau + \tau_p + \tau_e \delta_e \left(2 - \delta_e\right)\right)^{\frac{3}{2}}}{\left(\tau + \tau_p + \tau_e \delta_e\right)^{\frac{3}{2}}} = 2\left(\delta_e - 1\right). \tag{34}$$

Moreover, the equilibrium overconfidence parameter, δ_e, (i) increases with τ and τ_z, (ii) decreases with risk-aversion γ, and (iii) is U-shaped in τ_e.

Consistent with the intuition laid out above, equation (34) shows that in a symmetric equilibrium, $\delta_{e,i} > 1$ for all agents.[18] What drives the degree of overconfidence? As prior uncertainty falls ($\uparrow \tau$) and as the quality of the information in prices rises ($\uparrow \tau_z, \downarrow \gamma$), both the benefit and cost of being overconfident falls: overconfidence is less distortive of the investor's perceived information advantage. Interestingly, as overconfidence grows, the cost falls more quickly, and so when investors have access to better outside information, overconfidence is higher.[19] While similar logic applies with respect to the quality of the investor's private signal, increasing overconfidence directly distorts how this information is utilized. As a result, for low values of τ_e the benefit of increased overconfidence falls more quickly, which introduces a non-monotonicity in δ_e as τ_e increases.

[18] The LHS of (34) is always positive, which indicates that $\delta_e > 1$.

[19] This can be seen by evaluating the numerator of equation (34).

5.2 Belief choices about private signals and price information

We now turn to the more general setting in which investors can optimally choose their beliefs about both the quality of their private signal as well as the information contained in prices. We begin by characterizing the characteristics of any feasible symmetric equilibrium.

Proposition 3. *Suppose the cost function $C(\delta_{e,i}, \delta_{z,i})$ is well-behaved. In any symmetric equilibrium, all investors are (weakly) over-confident about their private signal (i.e., $\delta_{e,i} \geq 1$ for all i) but choose to under-react to the information in prices (i.e., $\delta_{z,i} < 1$ for all i).*

Thus, in any symmetric equilibrium, investors always choose to over-confident about their private information (as above) but under-react to the information in prices. This is a robust outcome in our setting. Consider the choices $\delta_{e,i}$ and $\delta_{z,i}$ of investor i in a symmetric equilibrium where all other investors choose δ_e and δ_z, respectively. Recall that for a well-behaved cost function, deviations away from rational expectations (i.e., $\delta_{e,i} = 1$ and $\delta_{z,i} = 1$) are penalized i.e., the cost function is decreasing below one and increasing above one. Since anticipated utility is always (weakly) increasing in $\delta_{e,i}$, this immediately implies any symmetric equilibrium features (weak) over-confidence about private information (i.e., $\delta_{e,i} \geq 1$). Intuitively, increasing the perceived precision of private information always increases anticipated utility, and so it is natural that investors choose to be over-confident about their private signals.

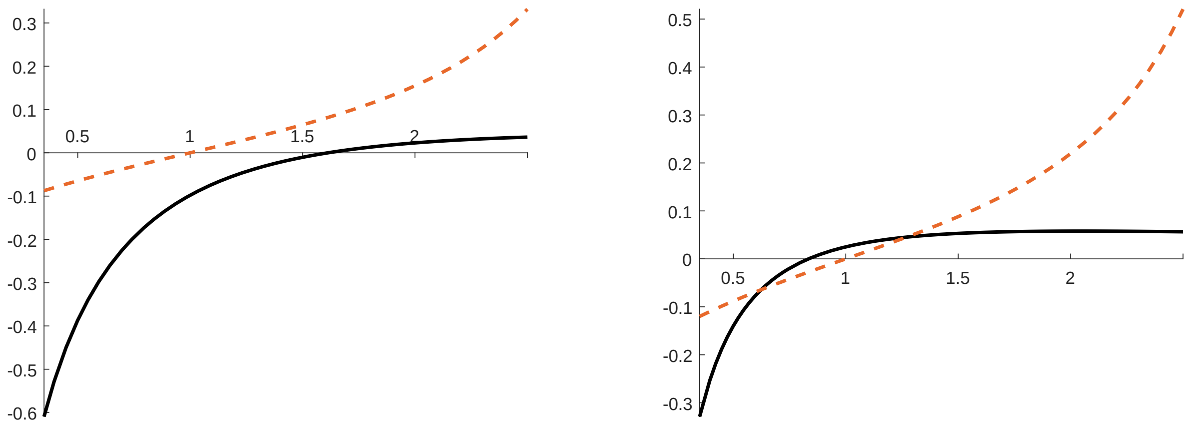
However, as Lemma 2 establishes, anticipated utility is U-shaped in $\delta_{z,i}$. Moreover, as we show in the proof of Proposition 3, when other investors choose δ_z, anticipated utility is decreasing in $\delta_{z,i}$ at $\delta_{z,i} = \delta_z$. This implies the equilibrium choice of $\delta_{z,i}$ cannot be higher than one, since if this were the case, investor i could increase her anticipated utility and decreases her costs by reducing $\delta_{z,i}$, an unambiguously better outcome. Intuitively, in a symmetric equilibrium, investor i has an incentive to decrease the perceived precision of price information relative to the choice of others because by doing so, she improves her ability to speculate on her private information by decreasing the correlation between her conditional valuation (μ_i) and those of other investors (i.e., $\int_j \mu_j dj$), which is reflected in the equilibrium price.

Given the above characterization for arbitrary cost functions, the next subsections characterize conditions for the existence of symmetric equilibria for the experienced utility penalty benchmark.

5.3 Symmetric equilibrium and under-reaction to prices

We begin with a sufficient condition for the existence and uniqueness of symmetric equilibria.

Figure 1: Marginal Anticipated Utility vs. Marginal Cost for the experienced utility penalty
The figure shows marginal anticipated utility (solid black line) and marginal cost function (dashed orange line) as a function of δ_{zi}. The marginal cost function is under the assumption that investors incur the experienced utility penalty (i.e., their beliefs satisfy (10)). Other parameters are: $\tau = \tau_e = \tau_z = \delta_e = \delta_{e,i} = 1$, $\delta_z = 0.5$.



$\gamma = 0.8$ (b) $\gamma = 0.3$

Proposition 4. *Suppose investors incur the experienced utility penalty i.e., their beliefs satisfy (10). There exists a $\bar{\gamma}$ such that for all $\gamma \geq \bar{\gamma}$, there exists a unique, symmetric equilibrium in which all investors ignore the information in prices (i.e., $\delta_{z,i} = 0$ for all i), and correctly interpret their private signals (i.e., $\delta_{e,i} = 1$ for all i) .*

The plot in Figure 1, panel (a), provides a numerical illustration. The figure plots the marginal anticipated utility (solid) and the marginal cost function (dashed) for an investor i as a function of her choice $\delta_{z,i}$. Recall that deviations away from $\delta_{z,i}$ are costly — as a result, the marginal cost for $\delta_{z,i} < 1$ is negative. Moreover, note that Lemma 2 implies that the marginal anticipated utility is negative below a threshold $\bar{\delta}$ (which is a little above 1.5 in the plot). Finally, note that while the marginal anticipated utility when $\delta_{z,i} = 0$ is $-\infty$, the marginal cost in this case is always negative but *finite*. At any alternative symmetric equilibrium the marginal benefit and marginal cost must intersect.[20] A sufficiently high γ ensures that (i) the marginal anticipated utility curve intersects zero at a point to the right of $\delta_{z,i} = 1$, and (ii) the marginal cost curve is sufficiently flat between $\delta_{z,i} = 0$ and $\delta_{z,i} = 1$. This in turn ensures that the two curves never intersect, and the symmetric equilibrium is a corner solution at $\delta_{z,i} = 0$.

Intuitively, when γ is high, price informativeness τ_p is relatively low. In this case, the

[20]Specifically, any other potential maximum lies at every second intersection of the two curves.

speculative effect of learning from prices dominates the information effect, and investors prefer to under-weight the information in prices. When γ is sufficiently high, the price is sufficiently uninformative, and investors optimally choose to ignore the information in prices. Since the marginal anticipated utility does not change with $\delta_{e,i}$ when $\delta_{z,i} = 0$, investors optimally choose to correctly interpret their private information (i.e., $\delta_{e,i} = 1$).

5.4 Asymmetric equilibrium and specialization of beliefs

The next result establishes sufficient conditions to rule out the existence of a symmetric equilibrium.

Proposition 5. *Suppose investors incur the experienced utility penalty, i.e., their beliefs satisfy (10). There exists a $\underline{\gamma}$ such that for all $\gamma < \underline{\gamma}$, there cannot exist a symmetric equilibrium.*

Again, consider the numerical example plotted in Figure 1, panel (b). When γ is sufficiently low, the marginal anticipated utility curve crosses zero below $\delta_{z,i} = 1$. This implies that there is always a (local) maximum, corresponding to the second intersection of the solid and dotted lines. Investor i might prefer to deviate from the corner ($\delta_{z,i} = 0$) to this interior maximum, if her expected anticipated utility, net of cost, is higher.

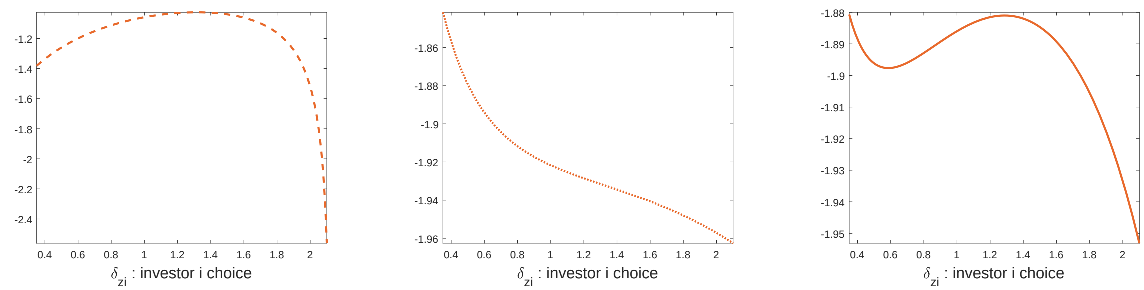
Intuitively, this can occur when γ is sufficiently low because price informativeness is sufficiently high (investors trade more aggressively on their information). Moreover, in any symmetric equilibrium, investors under-react to the information in prices. Together, these imply that an individual investor may have an incentive to deviate and condition *more* aggressively on the information in prices — in such a case, the speculative effect is dominated by the information effect. But such profitable deviations rule out a symmetric equilibrium.

The plots in Figure 2 provide a numerical example. The panels show investor i's anticipated utility, net of costs, as a function of $\delta_{z,i}$, given the behavior of others. In panel (a), all other investors choose $\delta_z = 0$. In this case, investor i has an incentive to deviate by over-weighting the information in prices (i.e., by setting $\delta_{zi} \approx 1.3$). Even though the price is objectively very informative (large information effect), because other investors are ignoring it ($\delta_z = 0$), the speculative effect of overweighting the price is relatively small. In panel (b), we consider an alternative symmetric equilibrium in which all other investors choose $\delta_z > 0$. Now, the speculative effect dominates and investor i strictly prefers to ignore the information in prices. In both cases, a symmetric equilibrium is ruled out because an individual investor has an incentive to deviate from the equilibrium behavior.

Given the non-existence of symmetric equilibria, we numerically explore the existence of asymmetric equilibria in which investors mix between two sets of beliefs. We assume a

Figure 2: Anticipated utility net of costs versus $\delta_{z,i}$

The figure plots the anticipated utility net of costs for investor i as a function of her choice $\delta_{z,i}$. Other parameters are: $\tau = \tau_e = \tau_z = \delta_e = \delta_{e,i} = 1$, $\gamma = 0.3$



(a) $\delta_z = 0$ (b) $\delta_z = 1$ (c) Mixed eqm.

fraction λ optimally chooses $\delta_{e,i} = 1$ and $\delta_{z,i} = 0$, while the remaining fraction $1-\lambda$ optimally chooses $\delta_{e,i} = \delta_e$ and $\delta_{z,i} = \delta_z$. The following result characterizes the existence of such an equilibrium.

Lemma 3. *An asymmetric equilibrium is characterized by the triple $(\lambda, \delta_e, \delta_z)$ which solve a system of three equations (specified in the Appendix). The equilibrium price is given by: $P = m + \Lambda_{AE}(s_p - m)$, where $s_p = F - \frac{\gamma}{\bar{\delta}_e \tau_e} z$,*

$$\Lambda_{AE} \equiv \frac{\bar{\delta}_e \tau_e + \bar{\delta}_z \tau_{p,AE}}{\tau + \bar{\delta}_e \tau_e + \bar{\delta}_z \tau_{p,AE}}, \qquad \tau_{p,AE} = \frac{\tau_z \tau_e^2 \bar{\delta}_e^2}{\gamma^2}, \tag{35}$$

$\bar{\delta}_e = (\lambda + (1-\lambda)\delta_e)$ and $\bar{\delta}_z = (1-\lambda)\delta_z$. Moreover, $\delta_e, \delta_z \leq 3/2$, and $\Lambda_{AE} < \Lambda_{RE} = \frac{\tau_e + \tau_p}{\tau + \tau_e + \tau_p}$ under some conditions reported in the appendix.

Panel (c) of Figure 2 illustrates an instance of the asymmetric equilibrium. In this case, each investor is indifferent between two (sets of) beliefs. In equilibrium, one set of investors (a fraction $\lambda = 0.95$) ignore the information in prices while the remaining fraction $1-\lambda = 0.05$) overweight the information in prices.

It is well-known that in financial markets investors exhibit a variety of different investment styles. The asymmetric equilibria we analyze suggests that some of this observed heterogeneity in investment styles may be endogenous. For instance, the over-confident, dismissive investors in our model resemble fundamental investors who identify mispriced securities using their private information. On the other hand, the investors who choose to overweight the information found in prices engage in behavior that resembles technical or momentum trading, where investors over-extrapolate from past price changes. Interestingly, that such heterogeneity arises in otherwise ex-ante identical traders implies that the observation of conflicting biases or investment styles does not require that investors exhibit differential ability, preferences or information.

6 Implications for market observables

When investors experience anticipated utility, our model predicts that they will systematically and predictably deviate from rational expectation in their interpretation of both private signals and the information in prices. However, these predictions are not directly testable because the information and beliefs of investors are difficult to observe directly. In this section, we derive implications of our model for observables including volatility, volume, return predictability and price informativeness that distinguish the wishful thinking equilibria from the rational expectations benchmark.

We begin by characterizing how the observables of interest. Since the risk-free security is the numeraire, the (net) return on it is zero. Consequently, the (dollar) return on the risky security is given by $R \equiv F - P$. In our benchmark model, the risky security is in zero net supply and so the unconditional expected return is zero. i.e., $\mathbb{E}[R] = 0$.[21] However, conditional on the price, the expected return can be expressed as:

$$\mathbb{E}\left[F - P \middle| P\right] = m + \theta (P - m), \text{ where } \theta \equiv \frac{\text{cov}(F - P, P)}{\text{var}(P)}. \tag{36}$$

Here θ reflects the degree to which the returns are predictable and, as such, we refer to it as the return predictability coefficient. The unconditional variance in returns is given by

$$\sigma_R^2 = \text{var}(F - P), \tag{37}$$

while the conditional variance in returns is characterized by

$$\text{var}(F - P|P) = \tau_p^{-1}. \tag{38}$$

Note that the conditional variance in returns is inversely related to one measure of price informativeness, as τ_p reflects how precise the price signal is about fundamentals F. Finally, since investors start without an endowment of the risky security, trading volume in our economy can be characterized as

$$\mathcal{V} \equiv \int_i |x_i^*| \, di. \tag{39}$$

Given investor beliefs, the following result describes how these return-volume characteristics

[21] We discuss an extension with non-zero aggregate supply below.

depend on both the endogenous and exogenous parameters of our model.

Lemma 4. *Consider the unique financial market equilibrium described in Lemma 1. Then,*

(i) the unconditional variance in returns is $\sigma_R^2 = \frac{(1-\Lambda)^2}{\tau} + \frac{\Lambda^2\beta^2}{\tau_z}$

(ii) the return predictability coefficient is $\theta = \frac{1}{\Lambda}\left(\frac{1/\tau}{\beta^2/\tau_z+1/\tau} - \Lambda\right)$,

(ii) price informativeness is $\tau_p = \tau_z/\beta^2$, *and*

(iv) expected trading volume is

$$\mathbb{E}\left[\mathcal{V}\right] = \int_i \frac{\omega_i}{\gamma}|\mu_i - P|\, di = \int_i \frac{\omega_i}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{A_i^2}{\tau_e} + \frac{(B_i-\Lambda)^2\beta^2}{\tau_z} + \frac{(A_i+B_i-\Lambda)^2}{\tau}\right)}di, \tag{40}$$

where ω_i, A_i,B_i, β *and* Λ *are defined in (19)-(20) and Lemma 1.*

To provide intuition for the dependence of these equilibrium characteristics on the underlying parameters, we make use of the signal to noise ratio (or Kalman gain) for the price signal s_p, which can be written as

$$\kappa \equiv \frac{\text{var}\left(F\right)}{\text{var}\left(s_p\right)} = \frac{1/\tau}{\beta^2/\tau_z + 1/\tau} = \frac{\tau_p}{\tau+\tau_p}. \tag{41}$$

First, note that an increase in the price sensitivity Λ has two offsetting effects on return volatility. On the one hand, when the price is more sensitive to s_p, it reflects fundamentals more closely, and this decreases volatility (through the $\frac{(1-\Lambda)^2}{\tau}$ term). On the other hand, a higher Λ also implies that the price is more sensitive to shocks to the asset supply, which increases volatility (through the $\frac{\Lambda^2\beta^2}{\tau_z}$ term). The first effect dominates when the price sensitivity Λ is lower than the signal to noise ratio κ (i.e., $\Lambda < \kappa$), while the latter effect dominates when price sensitivity is higher.

Second, note that the return predictability coefficient is positive (i.e., $\theta > 0$) when the signal to noise ratio is higher than the price sensitivity. In this case, prices exhibit drift — a higher price today predicts higher future returns. On the other hand, when the signal to noise ratio is lower than Λ, prices exhibit reversals. Comparing the expression of κ above to Λ in (25), prices cannot exhibit drift unless investors under-react to price information (i.e., $\delta_{z,i} < 1$). Conversely, when investors correctly interpret the precision of price information, the prices exhibit reversals (i.e., $\delta_{z,i} = 1$ implies $\Lambda > \kappa$). In particular, prices exhibit reversals when investors exhibit rational expectations.

Third, price informativeness, τ_p, naturally decreases in the magnitude of β — when investors have less private information, the price is more sensitive to aggregate supply shocks. Finally, note that volume reflects the cross-sectional variation across investor valuations (i.e.,

μ_i), scaled by their posterior variance (i.e., ω_i^{-1}). The variation is driven by three channels: (i) the weight each investor's beliefs place on the noise in their private signals (i.e., the $\frac{1}{\tau_e}$ term), (ii) the relative weight on the noise in prices (i.e., the $\frac{\beta^2}{\tau_z}$ term) and (iii) the relative weight on the true fundamental value (i.e., the $\frac{1}{\tau}$ term). Note that the last term is absent in symmetric equilibria, since $A_i + B_i = \Lambda$ in this case. However, in asymmetric equilibria, this final term reflects the variation in valuations due to asymmetric reaction to private signals and the information in prices (see Section 5.4).

The next result compares the return-volume characteristics of the symmetric and asymmetric equilibria to the rational expectations benchmark.

Proposition 6. *(1) Relative to the rational expectations equilibrium, the* ***symmetric*** *equilibrium characterized in Proposition 4 features: (i) a higher predictability coefficient, (ii) equal price informativeness, and (iii) equal expected volume. Return volatility is higher than in the rational expectations equilibrium iff price informativeness is sufficiently high (i.e., $\tau_p \geq \frac{\sqrt{\tau^2+8\tau\tau_e+8\tau_e^2}}{2} - \frac{\tau}{2}$).*

(2) Relative to the rational expectations equilibrium, the ***asymmetric*** *equilibrium characterized in Lemma 3 features: (i) a higher predictability coefficient, (ii) higher price informativeness, and (iii) higher expected volume.*

Figure 3 provides a numerical illustration of these results. Specifically, the figure plots volatility, predictability, and volume for the rational expectations (dashed) and subjective beliefs equilibria (solid) as a function of risk aversion γ. The kink in the solid lines corresponds to the value of γ at which the subjective beliefs equilibrium switches from the asymmetric equilibrium (low γ) to the symmetric equilibrium (higher γ). Consistent with Proposition 6, predictability and volume are (weakly) higher under subjective beliefs than under rational expectations. Moreover, volatility is higher under subjective beliefs when risk aversion is sufficiently low (i.e., τ_p is sufficiently high), but is higher otherwise.

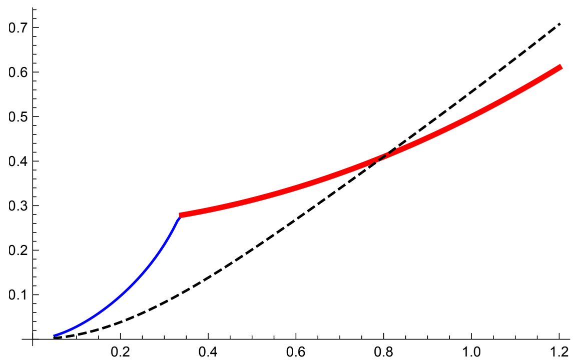
The figure also plots the unconditional expected return from an extension of the model, described in Appendix B.4, in which the aggregate supply of the risky asset is $Z > 0$. While this extension is not as analytically tractable, we can solve it numerically and find that the resulting equilibria are very similar to the case with $Z = 0$. The unconditional expected return is given by:

$$\mathbb{E}[R] = \frac{\gamma}{\int_i \omega_i di} Z, \tag{42}$$

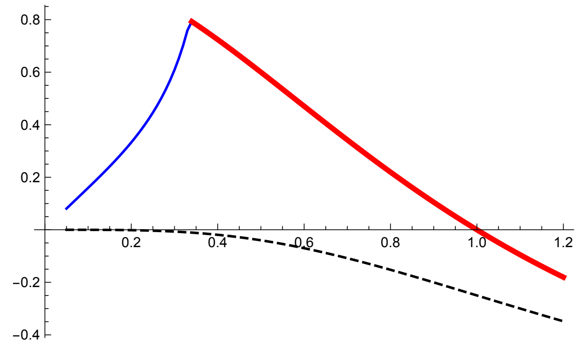
where ω_i is investor i's posterior precision about F (i.e., $\omega_i = (\mathrm{var}_i[F|s_i, s_p])^{-1}$), γ is the coefficient of risk aversion and Z is the aggregate supply of the risky asset. We find that expected returns are higher under the subjective beliefs equilibrium than under the rational

Figure 3: Comparison of return and volume characteristics

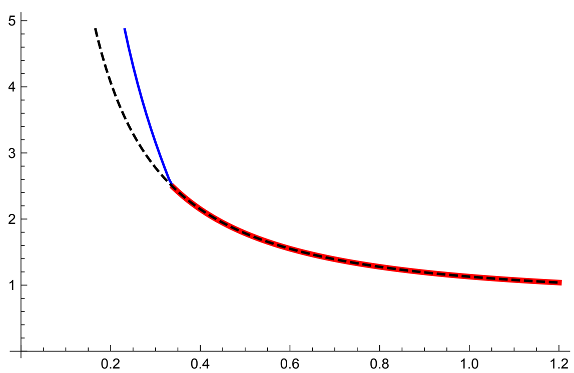
The figure plots return volatility (variance), return predictability, trading volume and unconditional expected return as a function of risk aversion for subjective beliefs (solid line) and rational expectations (dotted line). Other parameters are set to $\tau = \tau_e = \tau_z = 1$, and $Z = 0$. The thin (blue) part of the solid line corresponds to the asymmetric equilibrium, while the thick (red) part corresponds to the symmetric equilibrium.



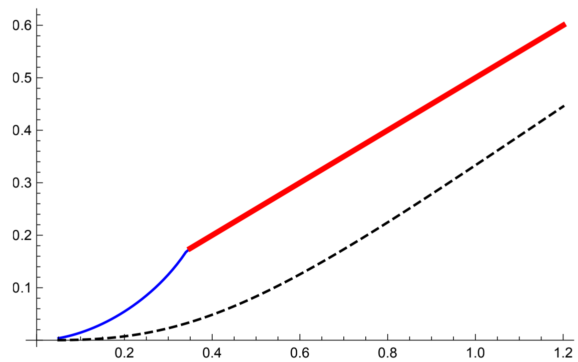
(a) Volatility vs. γ



(b) Predictability θ vs. γ



(c) Volume vs. γ



(d) Expected Return vs. γ ($Z = 1$)

expectations equilibrium. This is intuitive: since investors do not condition on all available information correctly, their posterior precision is lower and, therefore, the expected return is higher.

Perhaps surprisingly, we show that price informativeness and volume are higher than the rational expectations equilibrium for the asymmetric equilibrium only. Intuitively, this is because in the asymmetric equilibrium some investors put more weight on their private signals, i.e., they exhibit strict over-confidence. Moreover, in the asymmetric equilibrium, in addition to volume generated by cross-sectional variation in private signals, there is additional trade between the different groups of investors. In the symmetric equilibrium with the experienced utility penalty, however, investors weight their private information objectively and so the former channel is dormant. However, under more general cost specifications, we do not expect this to be the case. For instance, note that in the benchmark model of Section 5.1 where investors exhibit strict over-confidence (i.e., $\delta_{e,i} > 1$), we have the following result.

Corollary 1. *Relative to the rational expectations equilibrium, the equilibrium characterized in Proposition 1 features: (i) a less negative predictability coefficient, (ii) higher price informativeness, and (iii) higher expected volume.*

We expect that these results obtain more generally: in settings where the symmetric equilibrium features strict over-confidence (as highlighted in Proposition 3), we expect trading volume and price informativeness to be higher than in the rational expectations equilibrium. Overconfidence induces investors to trade more aggressively based on their signals. This results in more informative prices, which is consistent with the empirical evidence discussed in Hirshleifer, Subrahmanyam, and Titman (1994); Kyle and Wang (1997); Odean (1998b); Hirshleifer and Luo (2001). Finally, consistent with the large literature on overconfidence, our model suggests that such behavior by investors can help explain the relatively high trading volume that has been extensively documented empirically.

Overall, our results suggest that return predictability is a key feature that distinguishes the subjective beliefs equilibria from the rational expectations equilibrium. Specifically, the return predictability coefficient is higher with wishful thinking than in the corresponding rational expectations equilibrium: in fact, it can even be positive. In the traditional rational expectations setting with exogenous, transient noise trading (e.g., Hellwig (1980)), returns exhibit reversals. Intuitively, an aggregate demand (supply) shock temporarily pushes the current price up (down, respectively), but since the shock is not persistent, prices revert in the future. In our model, because some investors underweight the information in prices, prices do not adjust to their informationally efficient levels and there is residual (positive) predictability in returns. This mechanism is similar to the one found in Hong and Stein (1999) and Banerjee et al. (2009).

Further, note that return predictability and volatility are positively related in the (asymmetric) subjective beliefs equilibrium (when risk-aversion, γ, is sufficiently low), while they are negatively related in the rational expectations equilibrium. On the other hand, the two are also negatively related in the symmetric subjective beliefs equilibrium, i.e., when risk aversion is sufficiently high. As a result, to the extent that higher risk tolerance is associated with bull markets and economic booms, our model predicts such periods are more likely to feature (i) greater diversity in investment strategies, (ii) a positive relation between return predictability (or time-series momentum) and volatility, and (iii) higher (average) levels of return predictability. In contrast, periods of high market stress, characterized by low risk tolerance and high return volatility, are more likely to be associated with (i) less heterogeneity in investment styles, (ii) a negative relation between predictability and volatility, and

(iii) low, or even negative, return predictability. These predictions appear broadly consistent with the evidence on momentum returns and crashes (e.g., see Moskowitz et al. (2012), Daniel and Moskowitz (2016)).

Our analysis suggests that accounting for subjective belief choice has important implications for price informativeness. Intuitively, the subjective beliefs equilibria we analyze are associated with higher price informativeness because investors choose to over-weight their private information — this is immediate from the expression for price informativeness:

$$\tau_p = \frac{\tau_z \tau_e^2 \left(\int_i \delta_{e,i} di\right)^2}{\gamma^2}. \tag{43}$$

The above results also suggest that price informativeness should be higher for the asymmetric equilibrium. As such, our model predicts that, on average, price informativeness should be positively related to risk tolerance and trading volume, but negatively related to volatility. These results appear consistent with the empirical results from Bai et al. (2016), who show that price informativeness has increased since the 1960's, especially for firms with higher turnover, and with Dávila and Parlatore (2019), who document that volatility and price informativeness tend to be negatively correlated. Finally, our model predicts that on average, price informativeness should be positively related to measures of both over-confidence and diversity in investment strategies.

7 Extensions and supplementary analysis

In Appendix B, we provide supplementary analysis and explore several modifications to our benchmark assumptions. We summarize the key results below.

Subjective beliefs about other investors. In our benchmark model, we summarize each investor's subjective belief about the price signal by the parameter $\delta_{z,i}$. Appendix B.1 considers an alternative setting in which investors hold subjective beliefs about both the variance of, and the correlation between, the errors in other investors' signals. Specifically, investor i believes that the error ε_j (for all $j \neq i$) is given by:

$$\varepsilon_j =_i \rho_i \eta + \sqrt{1-\rho_i^2} u_i, \quad \eta, u_i \sim N\left(0, \frac{1}{\pi_i \tau_e}\right), \tag{44}$$

where $\rho_i \in [-1, 1]$ parametrizes the correlation across others' errors and $\pi_i \in [0, \infty]$ parametrizes the precision of their signals. We show that, in equilibrium, such beliefs affect an agent's utility only through their perception of the price signal's informativeness and establish a formal mapping from ρ_i and π_i to $\delta_{z,i}$. In particular, the perceived informativeness of the price signal is given by:

$$\tau_{p,i} = \left(\frac{\rho_i^2}{\pi_i \tau_e} + \frac{\beta^2}{\tau_z} \right)^{-1}. \tag{45}$$

This mapping also allows us to consider a model in which there is no noise in aggregate supply — in particular, we can assume the aggregate supply of the asset is commonly known to be $z = 0$ (or equivalently, that $\tau_z = \infty$). In this case, the price provides a perfectly revealing signal about the fundamental, F, under the objective distribution. However, because investors hold subjective beliefs about the informativeness of others' private signals, they may still perceive the price as noisy. It can be shown that, even in this special case, conditions exist in which investors not only to exhibit over-confidence about private information but choose to dismiss the perfectly revealing price signal.

Subjective beliefs about public signals. In Appendix B.2, we explore an economy in which investors can also choose how to interpret the informativeness of a public (non-price) signal. Specifically, we assume that in addition to observing the price and her private signal, investor i observes a public $s_q = F + \eta$, where $\eta \sim N\left(0, \tau_\eta^{-1}\right)$ and is independent of all other random variables. However, investor i believes that the noise in the public signal is given by

$$\eta \sim_i \mathcal{N}\left(0, \frac{1}{\delta_{\eta,i}\tau_\eta}\right), \tag{46}$$

where $\delta_{\eta,i}$ is chosen by each investor. We show that, similar to the signal obtained from the price, an increase in the perceived precision of the public signal increases anticipated utility through the information channel, but reduces it through the speculative channel. However, in contrast to the information in prices, we show that the informational channel dominates the speculative channel in any symmetric equilibrium. As a result, we find that investors tend to over-weight the information in public, non-price, signals. This suggests that, in some ways, the dismissiveness we find is a unique result of the dual role played by the price in financial markets.

Ex-post belief choice. Our analysis assumes that investors choose the interpretation of their information before observing the realizations of these signals. In Appendix B.3, we consider an alternative specification in which investors first observe the realization of their private signal and the price and then choose how to interpret this information. Unfortunately, solving for the general equilibrium in which all investors choose their beliefs is not feasible in this setting.[22] However, the partial equilibrium analysis of a single investor's interpretation suggests that similar biases can arise even under this alternative timing: when signal

[22]Because the perceived precisions depend on the realizations of signals, the linearity of the market-clearing price is not preserved and "closing the model" is intractable.

realizations deviate sufficiently from the investor's prior beliefs, she chooses to overweight her private signal but underweight the information in prices.

Welfare. In Appendix B.6, we conduct a welfare analysis of our benchmark model. Given that we are in a setting with heterogenous beliefs, the notion of welfare depends on the reference beliefs (see Brunnermeier, Simsek, and Xiong (2014) for a recent discussion and an alternative approach). We use a measure of welfare that is conservative in that it ignores the gain in anticipatory utility that investors experience by distorting their beliefs. Since they choose to deviate from rational expectations, the investors' anticipated utility net of costs must be higher than under the rational expectations equilibrium (according to their subjective beliefs); otherwise, they would optimally choose to hold rational expectations. On the other hand, and unsurprisingly, from the perspective of a social planner who holds objective beliefs and fails to account for investors' anticipated utility, investors' demand for the risky asset is suboptimal.

Our analysis uncovers a number of interesting results. First, we find that the technical traders have higher (objective) expected utility than the fundamental traders because they use the information in prices instead of ignoring them. Second, we show that liquidity traders are always better off because they incur a lower price impact on their trades relative to the rational expectations equilibrium. Surprisingly, we find that overall welfare (where we sum the objective expected utility across informed and liquidity traders) can also be lower in the rational expectations equilibrium. This depends on the aggregate risk tolerance: welfare is lower under the rational expectations equilibrium when risk tolerance is sufficiently low, but higher otherwise.

8 Concluding remarks

In this paper, we analyze how investors who experience anticipated utility optimally choose their subjective beliefs about their informational environment in the context of an otherwise standard competitive trading environment. We show that in any symmetric equilibrium, investors are always (i) weakly overconfident in the quality of their private signal (so that their perceived private information advantage is preserved or amplified), and (ii) discount the quality of the information in prices (so that their perceived trading opportunity is maximized).

Our analysis is stylized for tractability and clarity, but suggests a number of potential extensions for future work. For instance, it would be interesting to characterize the equilibrium of an economy with both RE and "wishful thinking" investors. Given our current analysis, we would still expect "wishful thinking" investors to dismiss the information in prices and over-weight their private information. However, a natural next step would be to explore whether "wishful thinkers" survive in a competitive market when faced with a sufficient measure of rational expectations traders (e.g., as in Kogan, Ross, Wang, and Westerfield (2006)). The current model is not well suited for this analysis because we restrict attention to CARA utility for tractability. However, with other utility functions, we expect the degree and nature of belief distortion to depend on current wealth. For instance with the experienced utility penalty, investors internalize the cost of their wishful thinking which suggests that they would therefore also internalize the impact it has on their likelihood of survival.

More broadly, an important consequence of our analysis is that the choice of subjective beliefs, and the consequent "behavioral bias," depends on the surrounding environment. As such, allowing for subjective belief choice is likely to be a fruitful approach to better understanding why different biases arise in different strategic settings. For example, our comparative statics results suggest that in a richer, dynamic trading environment, the degree of over-confidence and dismissiveness is likely to endogenously vary with economic conditions (e.g., more endogenous disagreement during economic expansions, when aggregate risk tolerance is higher). Our analysis also implies that market participants with different incentives (e.g., fund managers who are incentivized to maximize assets under management or track a benchmark) may choose to respond to public and private information differently. More generally, other strategic settings (e.g., beauty contest or coordination games) are likely to generate different patterns in the way agents distort their beliefs. In future work, we hope to explore the implications of subjective belief choice in such settings.

References

George A Akerlof and William T Dickens. The economic consequences of cognitive dissonance. *The American economic review*, 72(3):307–319, 1982.

Jennie Bai, Thomas Philippon, and Alexi Savov. Have financial markets become more informative? *Journal of Financial Economics*, 122(3):625 – 654, 2016. ISSN 0304-405X.

Snehal Banerjee. Learning from prices and the dispersion in beliefs. *Review of Financial Studies*, 24(9):3025–3068, 2011.

Snehal Banerjee, Ron Kaniel, and Ilan Kremer. Price drift as an outcome of differences in higher-order beliefs. *Review of Financial Studies*, 22(9):3707–3734, 2009.

Roland Bénabou and Jean Tirole. Mindful economics: The production, consumption, and value of beliefs. *Journal of Economic Perspectives*, 30(3):141–64, 2016.

Markus K Brunnermeier and Jonathan A Parker. Optimal expectations. *American Economic Review*, 95(4):1092–1118, 2005.

Markus K Brunnermeier, Christian Gollier, and Jonathan A Parker. Optimal beliefs, asset prices, and the preference for skewed returns. *American Economic Review*, 97(2):159–165, 2007.

Markus K Brunnermeier, Alp Simsek, and Wei Xiong. A welfare criterion for models with distorted beliefs. *The Quarterly Journal of Economics*, 129(4):1753–1797, 2014.

Andrew Caplin and John Leahy. Psychological expected utility theory and anticipatory feelings. *The Quarterly Journal of Economics*, 116(1):55–79, 2001.

Andrew Caplin and John Leahy. Wishful thinking. 2019.

Olivier Coibion and Yuriy Gorodnichenko. What can survey forecasts tell us about information rigidities? *Journal of Political Economy*, 120(1):116–159, 2012.

Kent Daniel and Tobias J Moskowitz. Momentum crashes. *Journal of Financial Economics*, 122(2):221–247, 2016.

Kent Daniel, David Hirshleifer, and Avanidhar Subrahmanyam. Investor psychology and security market under-and overreactions. *the Journal of Finance*, 53(6):1839–1885, 1998.

Kent D Daniel, David Hirshleifer, and Avanidhar Subrahmanyam. Overconfidence, arbitrage, and equilibrium asset pricing. *The Journal of Finance*, 56(3):921–965, 2001.

Eduardo Dávila and Cecilia Parlatore. Volatility and informativeness. Technical report, National Bureau of Economic Research, 2019.

David Eil and Justin M. Rao. The good news-bad news effect: Asymmetric processing of objective information about yourself. *American Economic Journal: Microeconomics*, 3(2): 114–138, 2011.

Kfir Eliaz and Ran Spiegler. Contracting with diversely naive agents. *The Review of Economic Studies*, 73(3):689–714, 2006.

Christine L. Exley and Judd B. Kessler. Motivated errors. *Working Paper*, 2019.

Erik Eyster, Matthew Rabin, and Dimitri Vayanos. Financial markets where traders neglect the informational content of prices. *Journal of Finance*, 74(1):371–399, 2018.

Ananda Ganguly and Joshua Tasoff. Fantasy and dread: The demand for information and the consumption utility of the future. *Management Science*, 63(12):4037–4060, 2017.

Simon Gervais and Terrance Odean. Learning to be overconfident. *The Review of Financial Studies*, 14(1):1–27, 2001.

Robin Greenwood and Andrei Shleifer. Expectations of returns and expected returns. *The Review of Financial Studies*, 27(3):714–746, 2014.

Lars Peter Hansen and Thomas J Sargent. *Robustness*. Princeton university press, 2008.

LarsPeter Hansen and Thomas J Sargent. Robust control and model uncertainty. *American Economic Review*, 91(2):60–66, 2001.

J.M. Harrison and D.M. Kreps. Speculative investor behavior in a stock market with heterogeneous expectations. *The Quarterly Journal of Economics*, 92:323–336, 1978.

Friedrich August Hayek. The use of knowledge in society. *The American economic review*, 35(4):519–530, 1945.

M. F. Hellwig. On the aggregation of information in competitive markets. *Journal of Economic Theory*, 22(3):477—498, 1980.

David Hirshleifer and Guo Ying Luo. On the survival of overconfident traders in a competitive securities market. *Journal of Financial Markets*, 4(1):73–84, 2001.

David Hirshleifer, Avanidhar Subrahmanyam, and Sheridan Titman. Security analysis and trading patterns when some investors receive information before others. *The Journal of Finance*, 49(5):1665–1698, 1994.

Harrison Hong and Jeremy C Stein. A unified theory of underreaction, momentum trading, and overreaction in asset markets. *The Journal of Finance*, 54(6):2143–2184, 1999.

Narasimhan Jegadeesh and Sheridan Titman. Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of finance*, 48(1):65–91, 1993.

Herbert Stanley Jevons. *Essays on economics*. Macmilan, 1905.

Marcin Kacperczyk, Stijn Van Nieuwerburgh, and Laura Veldkamp. A rational theory of mutual funds' attention allocation. *Econometrica*, 84(2):571–626, 2016.

Dan M. Kahan. Ideology, motivated reasoning, and cognitive reflection. *Judgment and Decision Making*, 8(4):407–424, 2013.

Dan M Kahan, Ellen Peters, Erica Cantrell Dawson, and Paul Slovic. Motivated numeracy and enlightened self-government. *Behavioural Public Policy*, 1(1):54–86, 2017.

E. Kandel and N. D. Pearson. Differential interpretation of public signals and trade in speculative markets. *Journal of Political Economy*, 103(4):831—872, Aug 1995.

Niklas Karlsson, George Loewenstein, and Duane Seppi. The ostrich effect: Selective attention to information. *Journal of Risk and Uncertainty*, 38(2):95–115, 2009.

Martin G Kocher, Michal Krawczyk, and Frans van Winden. 'let me dream on!'anticipatory emotions and preference for timing in lotteries. *Journal of Economic Behavior & Organization*, 98:29–40, 2014.

Leonid Kogan, Stephen A Ross, Jiang Wang, and Mark M Westerfield. The price impact and survival of irrational traders. *The Journal of Finance*, 61(1):195–229, 2006.

Albert S Kyle and F Albert Wang. Speculation duopoly with agreement to disagree: Can overconfidence survive the market test? *The Journal of Finance*, 52(5):2073–2090, 1997.

George Loewenstein. Anticipation and the valuation of delayed consumption. *The Economic Journal*, 97(387):666–684, 1987.

Markus M. Mobius, Muriel Niederle, Paul Niehaus, and Tanya Rosenblat. Managing self-confidence. 2014.

Tobias J Moskowitz, Yao Hua Ooi, and Lasse Heje Pedersen. Time series momentum. *Journal of financial economics*, 104(2):228–250, 2012.

T. Odean. Volume, Volatility, Price, and Profit When All Traders Are Above Average. *Journal of Finance*, 53(6):1887–1934, 1998a.

Terrance Odean. Volume, volatility, price, and profit when all traders are above average. *The journal of finance*, 53(6):1887–1934, 1998b.

Terrance Odean. Do investors trade too much? *The American Economic Review*, 89:1279–1298, 1999.

Emily Oster, Ira Shoulson, and E. Ray Dorsey. Limited life expectancy, human capital, and health investments. *American Economic Review*, 103(5):1977–2002, 2013.

J.A. Scheinkman and W. Xiong. Overconfidence and speculative bubbles. *Journal of Political Economy*, 111(6):1183–1220, 2003.

Robert J Shiller. Do stock prices move too much to be justified by subsequent changes in dividends? *The American Economic Review*, 71(3):421–436, 1981.

Stijn Van Nieuwerburgh and Laura Veldkamp. Information acquisition and under-diversification. *Review of Economic Studies*, 77(2):779–805, 2010.

Xavier Vives and Liyan Yang. Costly interpretation of asset prices. 2017.

A Proofs

A.1 Proof of Lemma 2

Lemma 1 implies that the price is of the form: $P = m + \Lambda(s_p - m)$. This implies anticipated utility is given by

$$AU(\delta_{e,i}, \delta_{z,i}) = -\sqrt{\frac{\frac{1}{\tau+\delta_{e,i}\tau_e+\delta_{z,i}\tau_p}}{(1-\Lambda)^2\frac{1}{\tau}+\Lambda^2\frac{1}{\delta_{z,i}\tau_p}}}. \tag{47}$$

Note that given other investors' choices, investor i's marginal anticipated utility is

$$\frac{\partial}{\partial\delta_{e,i}}AU = \frac{\tau_e}{2(\tau+\delta_{e,i}\tau_e+\delta_{z,i}\tau_p)} \times \sqrt{\frac{\frac{1}{\tau+\delta_{e,i}\tau_e+\delta_{z,i}\tau_p}}{(1-\Lambda)^2\frac{1}{\tau}+\Lambda^2\frac{1}{\delta_{z,i}\tau_p}}} \geq 0 \tag{48}$$

$$\frac{\partial}{\partial\delta_{z,i}}AU = \frac{(1-\Lambda)^2\delta_{z,i}^2\tau_p^2 - \Lambda^2\tau(\delta_{e,i}\tau_e+\tau)}{2\delta_{z,i}(\Lambda^2\tau+(1-\Lambda)^2\delta_{z,i}\tau_p)(\delta_{e,i}\tau_e+\delta_{z,i}\tau_p+\tau)} \times \sqrt{\frac{\frac{1}{\tau+\delta_{e,i}\tau_e+\delta_{z,i}\tau_p}}{(1-\Lambda)^2\frac{1}{\tau}+\Lambda^2\frac{1}{\delta_{z,i}\tau_p}}} \tag{49}$$

This implies anticipated utility is always increasing in $\delta_{e,i}$, and increasing in $\delta_{z,i}$ when

$$\frac{\delta_{z,i}^2}{\delta_{e,i}\tau_e+\tau} > \frac{\Lambda^2}{(1-\Lambda)^2}\frac{\tau}{\tau_p^2}, \tag{50}$$

i.e., it is initially decreasing and then increasing in $\delta_{z,i}$. Moreover, note that

$$\lim_{\delta_{z,i}\to 0}\frac{\partial}{\partial\delta_{z,i}}AU = -\infty, \quad \lim_{\delta_{z,i}\to 0}\frac{\partial}{\partial\delta_{z,i}}AU = 0 \tag{51}$$

and $\frac{\partial}{\partial\delta_{z,i}}AU$ equals zero at:

$$\delta_{z,i}^* = \frac{1}{\tau_p}\left(\frac{\Lambda}{1-\Lambda}\right)\sqrt{\tau(\delta_{e,i}\tau_e+\tau)} \tag{52}$$

$\square$

A.2 Proof of Proposition 1

The objective of investor i given by

$$\max_{\delta_{ei}} AU_i(\delta_{ei}) - C(\delta_{ei}).$$

Lemma 2 implies that anticipated utility increases with overconfidence parameter δ_{ei}. So, investor tries to balance the benefit of increasing δ_{ei} with the cost of increasing δ_{ei}. The FOC with respect to δ_{ei} is

$$\frac{\tau_e}{2\left(\frac{(1-\Lambda)^2}{\tau_0}+\frac{\Lambda^2}{\tau_p}\right)^{\frac{1}{2}}\left(\tau_e\delta_{ei}+\tau_p+\tau_0\right)^{\frac{3}{2}}}=\frac{\partial C}{\partial \delta_{ei}} \tag{53}$$

and the second order condition is

$$-\frac{3\tau_e^2}{4\left(\frac{(1-\Lambda)^2}{\tau_0}+\frac{\Lambda^2}{\tau_p}\right)^{\frac{1}{2}}\left(\tau_e\delta_{ei}+\tau_p+\tau_0\right)^{\frac{5}{2}}}-\frac{\partial^2 C}{\partial \delta_{ei}^2}<0.$$

Condition (53) implies that the optimal overconfidence parameter is always greater than one i.e., $\delta_e^* \geq 1$. □

A.3 Proof of Proposition 2

Substituting $\delta_{zi}=1$ in equation 56, the cost function in this case is given by

$$C\left(\delta_{ei}\right)=\frac{1}{\sqrt{\left(\frac{(1-\Lambda)^2}{\tau_0}+\frac{\Lambda^2}{\tau_p}\right)\left(\tau_0+\tau_p+\tau_e\delta_{ei}\left(2-\delta_{ei}\right)\right)}}$$

The FOC in the case of experienced utility penalty is given by

$$\frac{\tau_e}{2\left(\frac{(1-\Lambda)^2}{\tau_0}+\frac{\Lambda^2}{\tau_p}\right)^{\frac{1}{2}}\left(\tau_e\delta_{ei}+\tau_p+\tau_0\right)^{\frac{3}{2}}}=\frac{\tau_e\left(\delta_{ei}-1\right)}{\left(\frac{(1-\Lambda)^2}{\tau_0}+\frac{\Lambda^2}{\tau_p}\right)^{\frac{1}{2}}\left[\left(\tau_0+\tau_p+\tau_e\delta_{ei}\left(2-\delta_{ei}\right)\right)\right]^{\frac{3}{2}}} \tag{54}$$

which simplifies to

$$\frac{\left(\tau_0+\tau_p+\tau_e\delta_{ei}\left(2-\delta_{ei}\right)\right)^{\frac{3}{2}}}{\left(\tau_p+\tau_0+\tau_e\delta_{ei}\right)^{\frac{3}{2}}}=2\left(\delta_{ei}-1\right) \tag{55}$$

which establishes the result. □

Lemma 5. *With experienced utility penalty, the cost function is the disutility that the investor incurs under the objective distribution and is given by*

$$C\left(\delta_{ei},\delta_{zi}\right)=\frac{1}{\sqrt{\Lambda^2\left(\delta_{zi}-1\right)^2+var(F-P)\left(\tau+\tau_e\delta_{ei}\left(2-\delta_{ei}\right)+\tau_p\delta_{zi}\left(2-\delta_{zi}\right)\right)}}. \tag{56}$$

Proof. Based on definition 2 and ignoring the second term (which is constant), the cost function is

$$
\begin{aligned}
C\left(\delta_{ei}, \delta_{zi}\right) &= -\mathbb{E}\left[-\gamma \exp\left\{-\gamma x_i^*\left(\delta_{e,i}, \delta_{z,i}\right) \times (F-P) - \gamma W_0\right\}\right] \\
&= \mathbb{E}\left[\gamma \exp\left\{-\omega_i\left(\mu_i - P\right) \times (F-P)\right\}\right]
\end{aligned}
$$

Suppose we have:

$$
\begin{pmatrix} \mu_i - P \\ F - P \end{pmatrix} \sim N\left(\begin{pmatrix} m_i \\ m \end{pmatrix}, \begin{pmatrix} \sigma_{ERi}^2 & \sigma_{ERi,ER} \\ \sigma_{ERi,ER} & \sigma_{ER}^2 \end{pmatrix}\right). \tag{57}
$$

In this case, the cost function is given by

$$
C\left(\delta_{ei}, \delta_{zi}\right) = \sqrt{\frac{\omega_i^{-2}}{\left(\omega_i^{-1} + \sigma_{ERi,ER}\right)^2 - \sigma_{ER}^2 \sigma_{ER,i}^2}}. \tag{58}
$$

Note that

$$
\begin{aligned}
\sigma_{ER,i}^2 &= \operatorname{var}(\mu_i - P) \\
&= \operatorname{var}\left(A_i\left(s_i - m\right) + B_i\left(s_p - m\right) - \Lambda\left(s_p - m\right)\right) \\
&= \frac{\left(A_i + B_i - \Lambda\right)^2}{\tau} + \frac{A_i^2}{\tau_e} + \frac{\left(B_i - \Lambda\right)^2}{\tau_p}
\end{aligned}
$$

$$
\sigma_{ER}^2 = \operatorname{var}(F - P) = \tfrac{(1-\Lambda)^2}{\tau} + \tfrac{\Lambda^2}{\tau_p}
$$

$$
\begin{aligned}
\sigma_{ERi,ER} &= \operatorname{cov}\left(\mu_i - P, F - P\right) \\
&= \operatorname{cov}\left(A_i s_i + B_i s_p - \Lambda s_p, F - \Lambda s_p\right) \\
&= \frac{\left(A_i + B_i - \Lambda\right)\left(1 - \Lambda\right)}{\tau} - \frac{\left(B_i - \Lambda\right)\Lambda}{\tau_p}.
\end{aligned}
$$

Substituting these coefficients into the cost function given in equation 58 and simplifying, we get

$$
C\left(\delta_{ei}, \delta_{zi}\right) = \frac{1}{\sqrt{\Lambda^2\left(\delta_{zi} - 1\right)^2 + \operatorname{var}\left(F - P\right)\left(\tau + \tau_e \delta_{ei}\left(2 - \delta_{ei}\right) + \tau_p \delta_{zi}\left(2 - \delta_{zi}\right)\right)}}.
$$

□

A.4 Proof of Proposition 3

Equation (48) shows that marginal anticipated utility is weakly increasing in $\delta_{e,i}$. As long as $\frac{\partial C(1,\delta_z)}{\partial \delta_{e,i}} = 0$ (which holds under any well-behaved cost function), then the first-order condition in a symmetric equilibrium

$$\frac{\tau_e}{2\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\delta_z\tau_p}\right)^{\frac{1}{2}}\left(\tau_e\delta_e+\delta_z\tau_p+\tau\right)^{\frac{3}{2}}} = \frac{\partial C(1,\delta_z)}{\partial \delta_{ei}} \tag{59}$$

implies that $\delta_e \geq 1$ with $\delta_e > 1$ if $\delta_z \neq 0$. This proves the first half of the proposition.

Lemma 1 implies that in any symmetric equilibrium, we have $\Lambda = \frac{\delta_e\tau_e+\delta_z\tau_p}{\tau+\delta_e\tau_e+\delta_z\tau_p}$. Moreover, note that $\frac{\partial}{\partial \delta_{z,i}} AU = 0$ at

$$\begin{aligned} \bar{\delta}_{z,i} &= \frac{1}{\tau_p}\left(\frac{\Lambda}{1-\Lambda}\right)\sqrt{\tau\left(\delta_e\tau_e+\tau\right)} && (60)\\ &= \sqrt{1+\frac{\tau_e}{\tau}}\left(\delta_z+\tfrac{\tau_e}{\tau_p}\delta_s\right) > \delta_z && (61) \end{aligned}$$

But this implies $\frac{\partial}{\partial \delta_{z,i}} AU\left(\delta_{z,i}=\delta_z\right) < 0$ since $\frac{\partial AU}{\partial \delta_{z,i}} < (>)0$for all $\delta_{z,i} < (>)\bar{\delta}_{z,i}$. Next, note that if $\delta_{z,i} = \delta_z \geq 1$, then $C'(\delta_{z,i}) > 0$. Taken together, this proves that at any proposed symmetric equilibrium where $\delta_z > 1$, investor i has an incentive to deviate. Thus, the only possible symmetric equilibrium is one in which each investor chooses $\delta_{z,i} < 1$. This proves the second half of the proposition.

□

A.5 Proof of Propositions 4 and 5

For an investor incurring the experienced utility penalty, choosing $(\delta_{ei}, \delta_{zi})$ yields anticipated utility and cost given by:

$$\begin{aligned} AU\left(\delta_{e,i},\delta_{z,i}\right) &= -\sqrt{\frac{\frac{1}{\tau+\delta_{e,i}\tau_e+\delta_{z,i}\tau_p}}{\left(1-\Lambda\right)^2\frac{1}{\tau}+\Lambda^2\frac{1}{\delta_{z,i}\tau_p}}} && (62)\\ C\left(\delta_{ei},\delta_{zi}\right) &= \frac{1}{\sqrt{\Lambda^2\left(\delta_{zi}-1\right)^2+\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)\left(\tau+\tau_e\delta_{ei}\left(2-\delta_{ei}\right)+\tau_p\delta_{zi}\left(2-\delta_{zi}\right)\right)}}. && (63) \end{aligned}$$

Let $\kappa \equiv \left(\frac{\Lambda}{1-\Lambda}\right)^2 \frac{\tau}{\tau_p}$. Then,

$$AU\left(\delta_{e,i}, \delta_{z,i}\right) = -\sqrt{\frac{\tau}{(1-\Lambda)^2}}\sqrt{\frac{1}{\left(1+\frac{\kappa}{\delta_{zi}}\right)\left(\tau+\delta_{e,i}\tau_e+\delta_{z,i}\tau_p\right)}} \tag{64}$$

$$C\left(\delta_{ei}, \delta_{zi}\right) = \sqrt{\frac{\tau}{(1-\Lambda)^2}}\sqrt{\frac{1}{\left(1-\delta_{zi}\right)^2\kappa\tau_p+(1+\kappa)\left(\tau+\tau_e\delta_{ei}\left(2-\delta_{ei}\right)+\tau_p\delta_{zi}\left(2-\delta_{zi}\right)\right)}} \tag{65}$$

Suppose all others are playing $\bar{\delta}_e, \bar{\delta}_z$. Then, $\Lambda = \frac{\tau_e\bar{\delta}_e+\tau_p\bar{\delta}_z}{\tau+\tau_e\bar{\delta}_e+\tau_p\bar{\delta}_z}$ and so

$$\kappa = \left(\frac{\Lambda}{1-\Lambda}\right)^2\frac{\tau}{\tau_p} = \frac{\gamma^2\left(\tau_e\bar{\delta}_e+\tau_p\bar{\delta}_z\right)^2}{\tau\tau_e^2\tau_z\bar{\delta}_e^2}. \tag{66}$$

Then, $(1,0)$ is a symmetric equilibrium iff

$$AU(1,0) - C(1,0) > AU\left(\delta_{e,i}, \delta_{z,i}\right) - C\left(\delta_{e,i}, \delta_{z,i}\right) \tag{67}$$

for all $\delta_{e,i}, \delta_{z,i}$, or equivalently,_

$$H \equiv 1 + R - L > 0 \tag{68}$$

where

$$R \equiv \frac{AU\left(\delta_{e,i}, \delta_{z,i}\right)}{-C\left(\delta_{ei}, \delta_{zi}\right)} \tag{69}$$

$$= \sqrt{\frac{\gamma^4\tau\delta_z\tau_z\left(\tau_e\left(\frac{\tau_e\tau_z\bar{\delta}_e^2\left(\bar{\delta}_z^2-\left(\delta_{z,i}-2\right)\delta_{z,i}\right)}{\gamma^2}+2\bar{\delta}_e\bar{\delta}_z-(\delta_{e,i}-2)\delta_{e,i}\right)+\frac{\tau_e\left(\tau_e\tau_z\bar{\delta}_e^2-\gamma^2\left(\delta_{e,i}-2\right)\delta_{e,i}\right)\left(\tau_e\tau_z\bar{\delta}_e\bar{\delta}_z+\gamma^2\right)^2}{\gamma^4\tau\tau_z}+\tau+\frac{\gamma^2}{\tau_z}\right)}{\left(\tau_e^2\delta_{z,i}\tau_z\bar{\delta}_e^2+\gamma^2(\delta_{e,i}\tau_e+\tau)\right)\left(\left(\tau_e\tau_z\bar{\delta}_e\bar{\delta}_z+\gamma^2\right)^2+\gamma^2\tau\delta_{z,i}\tau_z\right)}} \tag{70}$$

$$L \equiv \frac{C(1,0)}{C\left(\delta_{ei}, \delta_{zi}\right)} \tag{71}$$

$$= \sqrt{\frac{\tau_e\left(\frac{\tau_e\tau_z\bar{\delta}_e^2\left(\bar{\delta}_z^2-\left(\delta_{z,i}-2\right)\delta_{z,i}\right)}{\gamma^2}+2\bar{\delta}_e\bar{\delta}_z-(\delta_{e,i}-2)\delta_{e,i}\right)+\frac{\tau_e\left(\tau_e\tau_z\bar{\delta}_e^2-\gamma^2\left(\delta_{e,i}-2\right)\delta_{e,i}\right)\left(\tau_e\tau_z\bar{\delta}_e\bar{\delta}_z+\gamma^2\right)^2}{\gamma^4\tau\tau_z}+\tau+\frac{\gamma^2}{\tau_z}}{(\tau_e+\tau)\left(\frac{(\tau_e\tau_z\bar{\delta}_e\bar{\delta}_z+\gamma^2)^2}{\gamma^2\tau\tau_z}+1\right)+\frac{\tau_e^2\bar{\delta}_e^2(\tau_e\tau_z\bar{\delta}_e\bar{\delta}_z+\gamma^2)^2}{\gamma^4\tau}}} \tag{72}$$

Note that

$$\lim_{\gamma\to\infty} R = \sqrt{\frac{\left(\left(2-\delta_{e,i}\right)\delta_{e,i}\tau_e + \tau\right)\delta_{z,i}}{\delta_{e,i}\tau_e + \tau}}, \quad \lim_{\gamma\to\infty} L = \sqrt{\frac{\left(2-\delta_{e,i}\right)\delta_{e,i}\tau_e + \tau}{\tau_e + \tau}} \tag{73}$$

$$\lim_{\gamma\to\infty} H = 1 + \sqrt{\frac{\left(\left(2-\delta_{e,i}\right)\delta_{e,i}\tau_e + \tau\right)\delta_{z,i}}{\delta_{e,i}\tau_e + \tau}} - \sqrt{\frac{\left(2-\delta_{e,i}\right)\delta_{e,i}\tau_e + \tau}{\tau_e + \tau}} \tag{74}$$

$$\geq 1 + \sqrt{\frac{\left(\left(2-\delta_{e,i}\right)\delta_{e,i}\tau_e + \tau\right)\delta_{z,i}}{\delta_{e,i}\tau_e + \tau}} - \sqrt{\frac{\tau_e + \tau}{\tau_e + \tau}} \geq 0 \tag{75}$$

which implies $(1,0)$ is an equilibrium for γ sufficiently high.

Next, note that,

$$\lim_{\gamma\to 0} R = \lim_{\gamma\to 0} L = \sqrt{\frac{\frac{\tau_e^4\tau_z^2\bar{\delta}_e^4\bar{\delta}_z^2}{\tau}}{\frac{\tau_e^4\tau_z^2\bar{\delta}_e^4\bar{\delta}_z^2}{\tau}}} = 1, \tag{76}$$

so that

$$\lim_{\gamma\to 0} H = 1 + R - L > 0 \tag{77}$$

which implies that for sufficiently low γ, an investor prefers to deviate to $(1,0)$ for any $\bar{\delta}_e, \bar{\delta}_z \neq 0$. Finally, consider an equilibrium in which $\bar{\delta}_e = 1, \bar{\delta}_z = 0$. In this case,

$$\lim_{\gamma\to 0} R = \lim_{\gamma\to 0} \sqrt{2-\delta_z} \tag{78}$$

$$\lim_{\gamma\to 0} L = \lim_{\gamma\to 0} \tfrac{1}{\gamma}\sqrt{\left(2-\delta_z\right)} = \infty \tag{79}$$

which suggests that

$$\lim_{\gamma\to 0} H < 0 \tag{80}$$

and so $(1,0)$ cannot be an equilibrium for γ sufficiently low. However,

$$\lim_{\gamma\to\infty} R = \sqrt{\frac{\left(\left(2-\delta_e\right)\delta_e\tau_e + \tau\right)\delta_z}{\delta_e\tau_e + \tau}}, \quad \lim_{\gamma\to\infty} L = \sqrt{\frac{\left(2-\delta_e\right)\delta_e\tau_e + \tau}{\tau_e + \tau}} \tag{81}$$

which implies $\lim_{\gamma\to\infty} H \geq 0$ as before, and so $(1,0)$ is an equilibrium for γ sufficiently high. $\square$

A.6 Proof of Lemma 3

Suppose λ fraction of investors chose $(\delta_{z1}, \delta_{e1})$ and $(1-\lambda)$ investors chose $(\delta_{z2}, \delta_{e2})$. This implies that price is given by

$$P = m + \Lambda\left(s_p - m\right) \text{ ,where } \Lambda = \frac{\left(\lambda\delta_{e1} + (1-\lambda)\,\delta_{e2}\right)\tau_e + \left(\lambda\delta_{z1} + (1-\lambda)\,\delta_{z2}\right)\tau_p}{\tau + \left(\lambda\delta_{e1} + (1-\lambda)\,\delta_{e2}\right)\tau_e + \left(\lambda\delta_{z1} + (1-\lambda)\,\delta_{z2}\right)\tau_p}.$$

Assume that risk aversion is not sufficiently high, this implies that investor's objective function has a local interior maxima. Investor then evaluates his objective at this interior maxima and the boundary $\delta_{zi} = 0$ and chooses the one where his objective is highest. For the mixed equilibrium to sustain, we need $\delta_{z1} = 0$ (which implies $\delta_{e1} = 1$) and $\delta_{z2} = \delta_z^* \geq 1$ (and let $\delta_{e2} = \delta_e^*$). For this mixed equilibrium, investor has to be indifferent between the two points, which implies that the following conditions have to hold:

$$\begin{aligned}
\frac{\partial AU}{\partial \delta_{ei}}|_{\{\delta_{zi}=\delta_z^*,\delta_{ei}=\delta_e^*\}} &= C'\left(\delta_e^*\right) \\
\frac{\partial AU}{\partial \delta_{zi}}|_{\{\delta_{zi}=\delta_z^*,\delta_{ei}=\delta_e^*\}} &= C'\left(\delta_z^*\right) \\
AU\left(0,1\right) - C\left(0,1\right) &= AU\left(\delta_z^*, \delta_e^*\right) - C\left(\delta_z^*, \delta_e^*\right). \qquad (82)
\end{aligned}$$

The first two conditions are the FOCs for local maxima (δ_z^*, δ_e^*) and the third condition says that investors are indifferent between the local maxima and the corner solution (0,1). These three equations will help us solve for 3 unknowns δ_z^*, δ_e^* and λ. Suppose $\bar{\delta}_e = \bar{\delta}_e = (\lambda + (1-\lambda)\,\delta_e)$ and $\bar{\delta}_z = (1-\lambda)\,\delta_z$ denote the average action of investors. The FOCs can be rewritten as

$$R^3 = \frac{2\left(\delta_z^* - 1\right)}{1 - \left(\delta_e^*\tau_e + \tau\right)\frac{\left(\bar{\delta}_e\tau_e + \bar{\delta}_z\tau_p\right)^2}{\tau_p^2\delta_z^{*2}\tau}} \qquad (83)$$

$$R^3 = \frac{2\left(\delta_e^* - 1\right)}{1 + \frac{\Lambda^2}{\tau_p \mathbf{var}(F-P)}\left(\frac{1}{\delta_z^*} - 1\right)}, \qquad (84)$$

where

$$R^2 = \frac{\frac{\left(\bar{\delta}_e\tau_e + \bar{\delta}_z\tau_p\right)^2}{\tau^2} + \left(\frac{1}{\tau} + \frac{\left(\bar{\delta}_e\tau_e + \bar{\delta}_z\tau_p\right)^2}{\tau^2\tau_p}\right)\left(\tau + \tau_e\delta_e^*\left(2 - \delta_e^*\right)\right) + \frac{\tau_p}{\tau}\delta_z^*\left(2 - \delta_z^*\right)}{\left(\frac{1}{\tau} + \frac{\left(\bar{\delta}_e\tau_e + \bar{\delta}_z\tau_p\right)^2}{\delta_z^*\tau_p\tau^2}\right)\left(\tau + \delta_e^*\tau_e + \delta_z^*\tau_p\right)}.$$

Since any deviations from rational expectations generate higher anticipated utility and lower true utility,$R < 1$. Using this inequality in equation 83 gives us that $\delta_z^* < \frac{3}{2}$. Similarly, using

$R < 1$ in equation 84 gives us that $1 < \delta_e^* < \frac{3}{2}$.

Moreover $\Lambda_{RE} > \lambda_{AE} \iff \tau_e + \frac{\tau_p}{\bar{\delta}_e^2} > \bar{\delta}_e\tau_e + \bar{\delta}_z\tau_p$. Let x denote the ratio of these two i.e.,

$$x = \frac{\bar{\delta}_e\tau_e + \bar{\delta}_z\tau_p}{\tau_e + \frac{\tau_p}{\bar{\delta}_e^2}}.$$

We need to prove that $x < 1$.Take the limit as $\gamma \to 0$,

$$\lim_{\gamma\to 0} R^2 = \lim_{\tau_p\to\infty} \frac{\frac{\left(\tau_e+\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\tau^2} + \left(\frac{1}{\tau} + \frac{\left(\tau_e+\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\tau^2\tau_p}\right)\left(\tau + \tau_e\delta_e^*\left(2-\delta_e^*\right)\right) + \frac{\tau_p}{\tau}\delta_z^*\left(2-\delta_z^*\right)}{\left(\frac{1}{\tau} + \frac{\left(\tau_e+\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\delta_z^*\tau_p\tau^2}\right)\left(\tau + \delta_e^*\tau_e + \delta_z^*\tau_p\right)}$$

$$= \lim_{\tau_p\to\infty} \frac{\frac{\left(\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\tau^2} + \frac{\left(\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\tau^2\tau_p}\left(\tau + \tau_e\delta_e^*\left(2-\delta_e^*\right)\right) + \frac{\tau_p}{\tau}\delta_z^*\left(2-\delta_z^*\right)}{\left(\frac{\delta_z^*\tau_p}{\tau} + \frac{\left(\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\tau^2}\right)}$$

There are 2 cases to consider, Case 1: $\tau_p x \to \infty$, Case 2:$\tau_p x \to constant$. In case 2, it is immediate that $x < 1$ for sufficiently large τ_p i.e., for sufficiently large γ. Next, I will prove that case 1 is not possible in our setting.

Suppose, for now, case 1 is possible. In this case, $\tau_p x \to \infty$ as $\gamma \to 0$. In this case,

$$\lim_{\gamma\to 0} R^2 = \lim_{\tau_p\to\infty} \frac{\frac{\left(\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\tau^2} + \frac{\left(\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\tau^2\tau_p}\left(\tau + \tau_e\delta_e^*\left(2-\delta_e^*\right)\right) + \frac{\tau_p}{\tau}\delta_z^*\left(2-\delta_z^*\right)}{\left(\frac{\delta_z^*\tau_p}{\tau} + \frac{\left(\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\tau^2}\right)}$$

$$= 1$$

and the indifference condition of the investor becomes

$$\lim_{\tau_p=\infty} \frac{\sqrt{\frac{\left(\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\tau^2} + \left(\frac{1}{\tau} + \frac{\left(\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\tau^2\tau_p}\right)\left(\tau + \tau_e\delta_e^*\left(2-\delta_e^*\right)\right) + \frac{\tau_p}{\tau}\delta_z^*\left(2-\delta_z^*\right)}}{\sqrt{\frac{\left(\frac{\tau_p}{\bar{\delta}_e^2}\right)^2 x^2}{\tau^2}}} = 2$$

The LHS of above expression is 1 and hence indifference condition cannot be satisfied. This implies that case 1 is not possible. This implies that $\tau_p x$ tends to a finite constant. This immediately implies that for γ sufficiently low, $x < 1$.

□

A.7 Proof of Proposition 6

A.7.1 Proof of Part 1.

Denote the return characteristics in the rational expectations equilibrium (symmetric equilibrium) by RE (SE, respectively). Note that

$$\theta_{RE} - \theta_{OE} = -\frac{\tau \tau_e^2 \tau_z^2}{\left(\gamma^2 + \tau_e \tau_z\right)\left(\gamma^2 \tau + \tau_e^2 \tau_z\right)} < 0 \tag{85}$$

$$\tau_{p,RE} - \tau_{p,OE} = 0 \tag{86}$$

$$\mathbb{E}\left[\mathcal{V}_{RE}\right] - \mathbb{E}\left[\mathcal{V}_{OE}\right] = \frac{\sqrt{\frac{2}{\pi}}\left(\sqrt{\frac{\gamma^2+\tau_e\tau_z}{\tau_z\left(\gamma^2(\tau_e+\tau)+\tau_e^2\tau_z\right)^2}}\left(\gamma^2\left(\tau_e+\tau\right)+\tau_e^2\tau_z\right) - \gamma^2\left(\tau_e+\tau\right)\sqrt{\frac{\gamma^2+\tau_e\tau_z}{\gamma^4(\tau_e+\tau)^2\tau_z}}\right)}{\gamma} = 0 \tag{87}$$

Finally, note that

$$\sigma_{R,RE}^2 - \sigma_{R,OE}^2 = \frac{\tau\left(\tau_p^2 + \tau\tau_p - 2\tau_e\left(\tau_e + \tau\right)\right)}{\left(\tau_e + \tau\right)^2\left(\tau_e + \tau_p + \tau\right)^2}, \tag{88}$$

which is positive iff $\tau_p > \frac{1}{2}\sqrt{8\tau\tau_e + 8\tau_e^2 + \tau^2} - \frac{\tau}{2}$. □

A.7.2 Proof of Part 2.

Denote the return characteristics in the rational expectations equilibrium (subjective beliefs asymmetric equilibrium) by RE (AE, respectively). Let $\bar{\delta}_e = \lambda + (1-\lambda)\,\delta_e^*$ and $\bar{\delta}_z = (1-\lambda)\,\delta_z^*$ denote the average beliefs about the precision of private signals and price signal. Note that

$$\tau_{p,AE} - \tau_{p,RE} = \frac{\tau_z \tau_e^2}{\gamma^2}\left(\bar{\delta}_e^2 - 1\right) > 0.$$

$$\begin{aligned}
\theta_{AE} - \theta_{RE} &= \frac{\tau_z}{\Lambda_{AE}\left(\beta_{AE}^2\tau + \tau_z\right)} - \frac{\tau_z}{\Lambda_{RE}\left(\beta_{RE}^2\tau + \tau_z\right)} \\
&= \frac{\tau_z}{\Lambda_{AE}\left(\frac{\beta^2}{\bar{\delta}_e^2}\tau + \tau_z\right)}\left(1 - \frac{\Lambda_{AE}}{\Lambda_{RE}}\frac{\beta^2\tau + \bar{\delta}_e^2\tau_z}{\bar{\delta}_e^2\beta^2\tau + \bar{\delta}_e^2\tau_z}\right)
\end{aligned}$$

Since $\bar{\delta}_e > 1$, we have $\frac{\beta^2\tau+\bar{\delta}_e^2\tau_z}{\bar{\delta}_e^2\beta^2\tau+\bar{\delta}_e^2\tau_z} < 1$. Moreover, since $\frac{\Lambda_{AE}}{\Lambda_{RE}} < 1$ by Lemma 3, we have $\theta_{AE} > \theta_{RE}$.

$$\mathbb{E}[\mathcal{V}] = \int_i \frac{\omega_i}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{(A_i+B_i-\Lambda)^2}{\tau}+\frac{A_i^2}{\tau_e}+\frac{(B_i-\Lambda)^2}{\tau_p}\right)}di \tag{89}$$

$$\mathbb{E}[\mathcal{V}_{AE}] = \lambda V_1 + (1-\lambda)V_2, \text{where} \tag{90}$$

$$V_1 \equiv \frac{(\tau_e+\tau)}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{1}{\tau}\left(\frac{\tau_e}{\tau+\tau_e}-\Lambda_{AE}\right)^2+\frac{1}{\tau_e}\left(\frac{\tau_e}{\tau+\tau_e}\right)^2+\frac{1}{\tau_{p,AE}}\Lambda_{AE}^2\right)} \tag{91}$$

$$V_2 \equiv \frac{\tau+\delta_e^*\tau_e+\delta_z^*\tau_{p,AE}}{\gamma}\sqrt{\frac{2}{\pi}\left(\begin{array}{c}\frac{1}{\tau}\left(\frac{\delta_e^*\tau_e+\delta_z^*\tau_{p,AE}}{\tau+\delta_e^*\tau_e+\delta_z^*\tau_{p,AE}}-\Lambda_{AE}\right)^2+\frac{1}{\tau_e}\left(\frac{\delta_e^*\tau_e}{\tau+\delta_e^*\tau_e+\delta_z^*\tau_{p,AE}}\right)^2\\+\frac{1}{\tau_{p,AE}}\left(\frac{\delta_z^*\tau_{p,AE}}{\tau+\delta_e^*\tau_e+\delta_z^*\tau_{p,AE}}-\Lambda_{AE}\right)^2\end{array}\right)} \tag{92}$$

Let

$$A(x) = \frac{x\tau_e+(1-x)\delta_e^*\tau_e}{x(\tau+\tau_e)+(1-x)(\tau+\delta_e^*\tau_e+\delta_z^*\tau_{p,AE})} \tag{93}$$

$$B(x) = \frac{x0+(1-x)(\delta_z^*\tau_p)}{x(\tau+\tau_e)+(1-x)(\tau+\delta_e^*\tau_e+\delta_z^*\tau_{p,AE})} \tag{94}$$

$$\omega(x) = x(\tau_e+\tau)+(1-x)(\tau+\delta_e^*\tau_e+\delta_z^*\tau_{p,AE}) \tag{95}$$

$$V(x) = \frac{\omega(x)}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{1}{\tau}(A(x)+B(x)-\Lambda)^2+\frac{1}{\tau_e}A(x)^2+\frac{1}{\tau_p}(B(x)-\Lambda)^2\right)} \tag{96}$$

and

$$\mathbb{E}[\mathcal{V}_{AE}] = \lambda V(1)+(1-\lambda)V(0) \tag{97}$$

Note that

$$V(\lambda) = \frac{\omega(\lambda)}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{1}{\tau_e}A(\lambda)^2+\frac{1}{\tau_{p,RE}}(B(\lambda)-\Lambda)^2\right)} \tag{98}$$

$$= \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{(\lambda\tau_e+(1-\lambda)\delta_e^*\tau_e)^2}{\tau_e}+\frac{((1-\lambda)(\delta_z^*\tau_p)-\lambda(\tau_e+0\tau_p)-(1-\lambda)(\delta_e^*\tau_e+\delta_z^*\tau_p))^2}{\tau_{p,RE}}\right)} \tag{99}$$

$$= \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{(\lambda\tau_e+(1-\lambda)\delta_e^*\tau_e)^2}{\tau_e}+\frac{(\lambda\tau_e+(1-\lambda)\delta_e^*\tau_e)^2}{\tau_{p,RE}}\right)} \tag{100}$$

$$= \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\bar{\delta}_e^2\tau_e+\frac{\tau_e^2}{\tau_p}\right)} \geq \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\tau_e+\frac{\tau_e^2}{\tau_p}\right)} = \mathbb{E}[\mathcal{V}_{RE}] \tag{101}$$

where $\delta_e \equiv \lambda + (1-\lambda)\,\delta_e^*$. It remains to be shown that:

$$\lambda V\,(1) + (1-\lambda)\,V\,(0) \geq V\,(\lambda) \tag{102}$$

Note that

$$V\,(x) = \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\tfrac{1}{\tau}\,(\alpha\,(x) + \beta\,(x) - \Lambda\omega\,(x))^2 + \tfrac{1}{\tau_e}\alpha\,(x)^2 + \tfrac{1}{\tau_p}\,(\beta\,(x) - \Lambda\omega\,(x))^2\right)} \tag{103}$$

where

$$\alpha\,(x) = x\tau_e + (1-x)\,\delta_e^*\tau_e \equiv a_0 + a_1 x \tag{104}$$

$$\beta\,(x) = x0 + (1-x)\,(\delta_z^*\tau_p) \equiv b_0 + b_1 x \tag{105}$$

$$\omega\,(x) = x\,(\tau_e + \tau) + (1-x)\,(\tau + \delta_e^*\tau_e + \delta_z^*\tau_{p,AE}) \equiv w_0 + w_1 x \tag{106}$$

$$\frac{V_{xx}}{V^3} = 4\tfrac{\tau+\tau_e+\tau_p}{\pi^2\gamma^4\tau\tau_e\tau_p}\,(-a_0 b_1 + a_0\Lambda w_1 + a_1 b_0 - a_1\Lambda w_0)^2 > 0 \tag{107}$$

which implies $V\,(x)$ is convex, which implies:

$$\mathbb{E}\,[\mathcal{V}_{AE}] = \lambda V\,(1) + (1-\lambda)\,V\,(0) \geq V\,(\lambda) \geq \mathbb{E}\,[\mathcal{V}_{RE}] \tag{108}$$

This completes the proof. □

A.8 Proof of Corollary 1

Taking derivatives of the return-volume characteristics in Lemma (**??**) with respect to δ_e gives:

$$\frac{\partial\sigma_R^2}{\partial\delta_e} = -\frac{2\gamma^2\tau_e\,(\gamma^6 + \delta_e\tau_e\tau_z\,(3\gamma^4 + \gamma^2\tau_z\,(3\delta_e\tau_e + \tau) + \delta_e^2\tau_e^2\tau_z^2))}{\tau_z\,(\gamma^2\tau + \delta_e\tau_e\,(\gamma^2 + \delta_e\tau_e\tau_z))^3} < 0 \tag{109}$$

$$\frac{\partial\theta}{\partial\delta_e} = \frac{\gamma^4\tau\tau_e\tau_z\,(\gamma^2\,(2\delta_e\tau_e + \tau) + 3\delta_e^2\tau_e^2\tau_z)}{(\gamma^2 + \delta_e\tau_e\tau_z)^2\,(\gamma^2\tau + \delta_e^2\tau_e^2\tau_z)^2} > 0 \tag{110}$$

$$\frac{\partial\tau_p}{\partial\delta_e} = \frac{2\delta_e\tau_e^2\tau_z}{\gamma^2} > 0 \tag{111}$$

$$\frac{\partial\mathbb{E}\,[\mathcal{V}]}{\partial\delta_e} = \frac{\sqrt{\frac{2}{\pi}}\delta_e\tau_e}{\gamma\sqrt{\frac{\gamma^2+\delta_e^2\tau_e\tau_z}{\tau_z}}} > 0 \tag{112}$$

which establishes the result. □

B Extensions

B.1 Beliefs about others

In this section, we provide a micro-foundation for the alternative interpretations of the subjective belief parameter $\delta_{z,i}$. As before, we assume that investor i observes signals

$$s_i = F + \varepsilon_i \tag{113}$$

where

$$F \sim N\left(0, \frac{1}{\tau}\right), \quad \varepsilon_i \sim N\left(0, \frac{1}{\tau_e}\right). \tag{114}$$

In addition, there exists a continuum of liquidity traders with individual demand $e_i = \psi z + \sqrt{1-\psi^2}\nu_i$, so that the aggregate noisy supply is $\psi z \sim N\left(0, \frac{\psi^2}{\tau_z}\right)$.

We allow each agent to form subjective beliefs along several dimensions. Specifically, investor i chooses $\delta_{e,i}, \rho_i, \pi_i$ and ψ_i which transform her beliefs as follows:

$$\varepsilon_i \sim_i N\left(0, \frac{1}{\delta_{e,i}\tau_e}\right) \tag{115}$$

$$\varepsilon_j =_i \rho_i \eta + \sqrt{1-\rho_i^2}u_i, \quad \eta, u_i \sim N\left(0, \frac{1}{\pi_i \tau_e}\right) \tag{116}$$

$$e_i \sim_i \psi_i z + \sqrt{1-\psi_i^2}\nu_i \quad \text{so that total noisy supply is } \ \psi_i z \sim N\left(0, \frac{\psi_i^2}{\tau_z}\right) \tag{117}$$

Note that $\rho_i, \psi_i \in [0,1]$ and $\delta_{e,i}, \pi_i \in [0, \infty]$. Intuitively, equation 116 captures the idea that distort their beliefs about both the amount of noise in others' signals (π_i) but also the average correlation across other investors (ρ_i).

Conjecture that, for investor i, observing the price is equivalent to observing $s_p =_i \bar{s} + \beta_i z = F + \rho_i \eta + \beta \psi_i z$. Then, conditional on s_i and s_p, we have:

$$\mathbb{E}\left[F|s_i, s_p\right] = m + A_i\left(s_i - m\right) + B_i\left(s_p - m\right) \equiv \mu_i \tag{118}$$

$$\omega_i \equiv \frac{1}{\mathrm{var}_i\left[F|s_i, s_p\right]} = \frac{\tau}{1 - A_i - B_i} = \tau + \delta_i \tau_e + \tau_{p,i} \tag{119}$$

where

$$A_i = \frac{\tau_e \delta_i}{\tau_e \delta_i + \tau + \tau_{p,i}}, \quad B_i = \frac{\tau_{p,i}}{\tau_e \delta_i + \tau + \tau_{p,i}} \tag{120}$$

$$\frac{1}{\tau_{p,i}} = \frac{\rho_i^2}{\pi_i \tau_e} + \frac{\beta^2 \psi_i^2}{\tau_z}.$$

An investor's optimal demand is given by

$$x_i = \frac{\mu_i - P}{\gamma\left(\frac{1}{\omega_i}\right)} \tag{121}$$

and so market clearing implies that in a symmetric equilibrium, we have:

$$\begin{aligned} P &= \bar{\mu} - \gamma\left(\frac{1}{\bar{\omega}}\right)\psi_i z && (122)\\ &= m + \left(\bar{A} + \bar{B}\right)(s_p - m) && (123)\\ &= m + \Lambda\left(s_p - m\right) && (124) \end{aligned}$$

so that $\beta = -\frac{\gamma\left(\frac{1}{\bar{\omega}}\right)}{\bar{A}}$. Finally, anticipatory utility is given by

$$\hat{J}_i = -\sqrt{\frac{var_i\left(F\right)}{var_i\left(F - P\right)}} = -\sqrt{\frac{\frac{1}{\omega_i}}{var_i\left(F - \Lambda \bar{s}_p\right)}} = -\sqrt{\frac{\frac{1}{\tau + \delta_i \tau_e + \tau_{p,i}}}{\frac{(1-\Lambda)^2}{\tau_0} + \frac{\Lambda^2}{\tau_{pi}}}}$$

This implies that choices ρ_i, π_i and ψ_i only affect anticipatory utility through $\tau_{p,i}$, where

$$\frac{1}{\tau_{p,i}} = \frac{\rho_i^2}{\pi_i \tau_e} + \frac{\beta^2 \psi_i^2}{\tau_z}.$$

Note that if investor i held rational expectations,

$$\frac{1}{\tau_p} = \frac{\beta^2 \psi^2}{\tau_z}.$$

We want to define δ_{zi} such that $\tau_{pi} = \delta_{zi}\tau_p$, which implies that

$$\delta_{zi} = \frac{\frac{\beta^2 \psi^2}{\tau_z}}{\frac{\rho_i^2}{\pi_i \tau_e} + \frac{\beta^2 \psi_i^2}{\tau_z}}$$

which together with $\rho_i, \psi_i \in [0,1]$ and$\pi_i \in [0,\infty]$ implies that

$$\delta_{zi} \in [0,\infty].$$

More generally, note that since $\hat{J}_i$ only depends on $\tau_{p.i}$, and this is well-defined even when $\tau_z \to \infty$ (i.e., there is no aggregate noise in prices), our analysis for anticipated utility still applies.

B.2 Belief choice about public signals

In this section, we introduce a public signal $s_q = F + \eta$, where $\eta \sim N\left(0, \tau_\eta^{-1}\right)$ and is independent of all other random variables. We allow each investor to choose how to interpret the quality of the information in the public signal. Specifically, we assume that investor i believes that the noise in the public signal is given by

$$\eta \sim_i \mathcal{N}\left(0, \frac{1}{\delta_{\eta,i}\tau_\eta}\right). \tag{125}$$

Given investor i's subjective beliefs $\delta_{e,i}$, $\delta_{z,i}$, and $\delta_{\eta,i}$ and conditional on her observed signals, s_i,s_p and s_q, investor i's posterior subjective beliefs are given by:

$$F|s_i, s_p, s_q \sim_i \mathcal{N}\left(\mu_i, \frac{1}{\omega_i}\right), \text{ where} \tag{126}$$

$$\mu_i \equiv \mathbb{E}_i\left[F|s_i, s_p\right] = m + A_i\left(s_i - m\right) + B_i\left(s_p - m\right) + C_i\left(s_q - m\right), \tag{127}$$

$$\omega_i \equiv \frac{1}{\text{var}_i\left[F|s_i, s_p\right]} = \frac{\tau}{1 - A_i - B_i - C_i}, \text{ and} \tag{128}$$

$$A_i \equiv \frac{\delta_{e,i}\tau_e}{\tau+\delta_{e,i}\tau_e+\delta_{z,i}\tau_p+\delta_{\eta,i}\tau_\eta}, B_i \equiv \frac{\delta_{z,i}\tau_p}{\tau+\delta_{e,i}\tau_e+\delta_{z,i}\tau_p+\delta_{\eta,i}\tau_\eta} \; C_i \equiv \frac{\delta_{\eta,i}\tau_\eta}{\tau+\delta_{e,i}\tau_e+\delta_{z,i}\tau_p+\delta_{\eta,i}\tau_\eta} \tag{129}$$

Similar to the benchmark model, the price can be written

$$P = m + \Lambda\left(s_p - m\right) + C\left(s_q - m\right),$$
$$\text{where } \Lambda = \frac{\int_i \delta_{e,i}\tau_e+\delta_{z,i}\tau_p di}{\int_i \tau+\delta_{e,i}\tau_e+\delta_{z,i}\tau_p+\delta_{\eta,i}\tau_\eta di}, \quad C = \frac{\int_i \delta_{\eta,i}\tau_\eta di}{\int_i \tau+\delta_{e,i}\tau_e+\delta_{z,i}\tau_p+\delta_{\eta,i}\tau_\eta di},$$

and where $s_p = F + \beta z$, $\tau_p = \tau_z/\beta^2$, and $\beta = -\frac{\gamma}{\tau_e \int_i \delta_{e,i} di}$. Given this equilibrium price and investor i's subjective beliefs ($\delta_{e,i}$, $\delta_{z,i}$, and $\delta_{\eta,i}$), her anticipated utility is

$$AU\left(\delta_{e,i}, \delta_{z,i}, \delta_{\eta,i}\right) = -\sqrt{\frac{\text{var}_i\left[F|s_i, P, s_q\right]}{\text{var}_i\left[F - P\right]}}. \tag{130}$$

This expression closely matches the expression in the benchmark model, found in (31). The numerator captures the information channel, updated to reflect the investor's beliefs about the quality of the public signal. The denominator captures the speculative channel. Using this, we can update Lemma 2 to reflect the inclusion of the public signal.

Lemma 6. *Anticipated utility is always*

(i) (weakly) increasing in $\delta_{e,i}$: it is strictly increasing as long as $\delta_{z,i} > 0$, and

(ii) non-monotonic in $\delta_{z,i}$: there exists some $\bar{\delta}_z$ such that for all $\delta_{z,i} < \bar{\delta}_z$ anticipated utility is decreasing in $\delta_{z,i}$ while for all $\delta_{z,i} > \bar{\delta}_z$ it is increasing.

When all other investors ignore public information (i.e., $\delta_{\eta,-i} = 0$), then investor i's anticipated utility is strictly increasing in $\delta_{\eta,i}$: Otherwise (when $\delta_{\eta,-i} > 0$), anticipated utility is non-monotonic in $\delta_{\eta,i}$: there exists some $\bar{\delta}_\eta$ such that for all $\delta_{\eta,i} < \bar{\delta}_\eta$ anticipated utility is decreasing in $\delta_{\eta,i}$ while for all $\delta_{\eta,i} > \bar{\delta}_\eta$ it is increasing.

As with beliefs about price information (i.e., $\delta_{z,i}$), anticipated utility is generically non-monotonic in the investor's perception of its informativeness (i.e., $\delta_{\eta,i}$). Moreover, this non-monotonicity is driven by the same channels. There is an (i) information channel, in which learning from the public signal reduces the conditional variance $\text{var}_i [F - P|s_i, P, s_q]$ and a (ii) speculative channel, in which a more informative public signal increases the precision of other investors' beliefs, lowering potential speculative trading gains. Note, however, that when all other investors ignore the information in a public signal, it becomes effectively private for investor i — in this case, the information effect dominates because the speculative effect is zero, and anticipated utility is strictly increasing in $\delta_{\eta,i}$. [23]

Finally, we can characterize the equilibrium optimal beliefs which arise with any well-behaved cost function.

Proposition 7. *Suppose the cost function $C(\delta_{e,i}, \delta_{z,i}, \delta_{\eta,i})$ is well-behaved. In any symmetric equilibrium, all investors are (weakly) over-confident about their private signal (i.e., $\delta_{e,i} \geq 1$ for all i), choose to under-react to the information in prices (i.e., $\delta_{z,i} < 1$ for all i), but choose to over-react to the information in the public signal (i.e., $\delta_{\eta,i} > 1$ for all i).*

Though all investors observe the price and the public signal, they respond to the information in each source very differently: in any symmetric equilibrium, investors under-react to information in price, but overreact to the public signal. At any proposed symmetric equilibrium, we show that an investor's anticipated utility increases when she believes that the public signal is more informative. As a result, the equilibrium choice of $\delta_{\eta,i}$ cannot be lower than one: for any proposed equilibrium with $\delta_{\eta,i} < 1$, an investor can increase her anticipated utility and lower her costs by believing the price is more informative. This logic is similar to that which follows Proposition 3 explaining under-reaction to prices, but with the relative effect of the two channels flipped. Intuitively, believing that the price is more informative has a direct impact on the speculative opportunity, while believing that the public signal is more informative alters the investor's perceived trading gains indirectly through the actions of other investors. This indirect channel is always dominated by the information channel, and so investors choose to overreact to the public signal but not to prices.

[23] This is similar to the effect found in Myatt and Wallace (2012), whereby acquiring more information from a public source is private information unless all other investors' condition on the same "public" information.

B.3 Ex-post belief choice

In the baseline model, we assumed that investor i chooses subjective beliefs $(\delta_{ei}, \delta_{zi})$ before he/she observes the realization of the signals. In this section, we relax this assumption and assume that investor i can choose the subjective beliefs after she observes the realization of the signals. The anticipated utility, conditional on P and s_i is given by

$$AU_i\left(\delta_{e,i}, \delta_{z,i}; s_i, P\right) = -\exp\left\{-\frac{1}{2}\omega_i\left(\mu_i - P\right)^2\right\}.$$

Assuming that the cost function is well-behaved as defined in 1, the investor's objective function is to choose $\delta_{e,i}, \delta_{z,i}$ by maximizing the anticipated utility, net of costs i.e.,

$$\max_{\delta_{e,i},\delta_{z,i}} AU_i\left(\delta_{e,i}, \delta_{z,i}; s_i, P\right) - C_{obj}\left(\delta_{e,i}, \delta_{z,i}\right).$$

Note that

$$\begin{aligned} \tfrac{\partial}{\partial\delta_{ei}}AU_i &\propto \left(\mu_i - P\right) \times \left[2\tau\left(s_i - m\right) + \omega_i\left(\mu_i - P\right)\right] \\ &\propto \left(s_i + \frac{B_i - \Lambda}{A_i}s_p\right)\left(s_i + \frac{B_i - \Lambda}{A_i + 2\frac{\tau}{\omega_i}}s_p\right) > 0 \iff s_i > \bar{s} \text{ or } s_i < \underline{s}. \end{aligned}$$

This implies that anticipated utility increases in $\delta_{e,i}$ iff the private signal is either too high or too low. This implies that for extreme realizations of investors' private signals, the investor will be have incentive to be overconfident and for low realizations of investors private signals, the investors have an incentive to be under-confident. This is consistent with the results found in Ortoleva and Snowberg (2015) that overconfidence leads to ideological extremes.

Similarly,

$$\begin{aligned} \tfrac{\partial}{\partial\delta_{zi}}AU_i &\propto \left(\mu_i - P\right) \times \left\{2\tau\left(s_p - m\right) + \omega_i\left(\mu_i - P\right)\right\} \\ &\propto \left(s_i + \frac{B_i - \Lambda}{A_i}s_p\right)\left(\left(B_i\omega_i + 2\tau\right)s_p + \omega_i\left(A_i - \Lambda\right)s_i\right) \\ &< 0 \iff s_i > \bar{S} \text{ or } s_i < \bar{S}. \end{aligned}$$

This again implies that for extreme realizations of private signals, an investor has an incentive to dismiss others' information. Moreover, for low absolute realizations of private signals, investors have an incentive to put more weight on the information found in the price.

Taken together, these results imply that for extreme realizations of private signals, investors will overweight private information and underweight price information i.e., the baseline results in the paper are robust for sufficiently extreme realizations of private signals. On the other hand, for moderate realizations of private signals, investors choose to underweight private information and overweight price information.

B.4 Positive aggregate supply and expected returns

We consider an extension to the main model in which the aggregate supply of the risky asset is given by $Z+z$, where $Z>0$ is a constant, and $z\sim N\left(0,1/\tau_z\right)$. Denote the conditional (subjective) beliefs of investor by $F|s_i,s_p\sim_i\mathcal{N}\left(\mu_i,v_i\right)$. Then, one can show that anticipated utility can be expressed as:

$$AU_i\left(\delta_{e,i},\delta_{z,i}\right)=\mathbb{E}_i\left[-\exp\left\{-\frac{\left(\mathbb{E}_i\left[F|s_i,s_p\right]-P\right)^2}{2\mathrm{var}_i\left[F|s_i,s_p\right]}\right\}\right] \tag{131}$$

$$=-\sqrt{\frac{\mathrm{var}_i\left[F|s_i,s_p\right]}{\mathrm{var}_i\left[F-P\right]}}\exp\left\{-\frac{\mathbb{E}_i\left[F-P\right]^2}{2\mathrm{var}_i\left[F-P\right]}\right\} \tag{132}$$

and the cost of distorting beliefs under the experienced utility penalty can be expressed as:

$$C\left(\delta_{e,i},\delta_{z,i}\right)=\frac{\exp\left\{\frac{1}{2}\frac{\mathrm{var}(\mathbb{E}[\mu_i-P](F-P)-\mathbb{E}[F-P](\mu_i-P))-2\mathbb{E}[F-P]\mathbb{E}[\mu_i-P]v_i}{(v_i+\mathrm{cov}(\mu_i-P,F-P))^2-\mathrm{var}(\mu_i-P)\mathrm{var}(F-P)}\right\}}{\sqrt{\left(1+\frac{\mathrm{cov}(\mu_i-P,F-P)}{v_i}\right)^2-\frac{\mathrm{var}(\mu_i-P)\mathrm{var}(F-P)}{v_i^2}}} \tag{133}$$

Moreover, standard calculations imply that the equilibrium price is given by:

$$P=m+\Lambda\left(F+\beta z-m\right)-\Gamma Z \tag{134}$$

where $\Lambda = \frac{\int_i \delta_{e,i}\tau_e + \delta_{z,i}\tau_p di}{\int_i \tau + \delta_{e,i}\tau_e + \delta_{z,i}\tau_p di}$ and $\Gamma = \frac{\gamma}{\int_i \omega_i di} = \frac{\gamma}{\int_i \tau + \delta_{e,i}\tau_e + \delta_{z,i}\tau_p di}$. This implies:

$$\begin{aligned}
\text{var}_i \left[F | s_i, s_p\right] &= \frac{1 - A_i - B_i}{\tau} \equiv v_i && (135)\\
\text{var}_i \left[F - P\right] &= \frac{(1-\Lambda)^2}{\tau} + \frac{\Lambda^2}{\delta_{z,i}\tau_p} && (136)\\
\mathbb{E}_i \left[F - P\right] &= \Gamma Z && (137)\\
\mathbb{E}_i \left[\mu_i - P\right] &= \Gamma Z && (138)\\
\text{var}\left(F - \mu_i\right) &= \text{var}\left(F - m - A_i\left(F + \varepsilon_i - m\right) + B_i\left(F + \beta z - m\right)\right) && (139)\\
&= \frac{(1 - A_i - B_i)^2}{\tau} + \frac{A_i^2}{\tau_e} + \frac{B_i^2}{\tau_p} && (140)\\
\text{var}\left(F - P\right) &= \frac{(1-\Lambda)^2}{\tau} + \frac{\Lambda^2}{\tau_p} && (141)\\
\text{var}\left(\mu_i - P\right) &= \frac{(A_i + B_i - \Lambda)^2}{\tau} + \frac{A_i^2}{\tau_e} + \frac{(B_i - \Lambda)^2}{\tau_p} && (142)\\
\text{cov}\left(\mu_i - P, F - P\right) &= \frac{(A_i + B_i - \Lambda)(1-\Lambda)}{\tau} - \frac{(B_i - \Lambda)\Lambda}{\tau_p}. && (143)
\end{aligned}$$

While analytically characterizing the equilibrium in this case is difficult, we can numerically solve for the equilibrium as before. Specifically, we first search for optimal choice of $(\delta_{ei}, \delta_{zi})$ under the assumption that others are choosing $(\delta_{ei} = 1, \delta_{zi} = 0)$. If there does not exist a symmetric equilibrium, then we search for an asymmetric equilibrium of the type characterized in Lemma 3. We find that the equilibria for $Z > 0$ are qualitatively similar to those for $Z = 0$.

B.5 Bounded variance

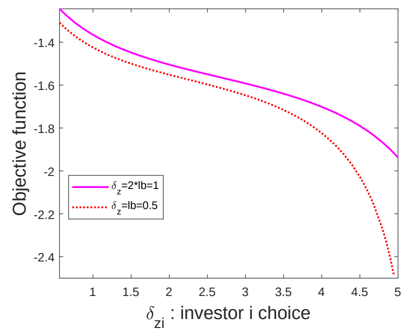
In the main text of the paper, we allow $\delta_{z,i}$ to be any non-negative, real number. In Appendix B.1, we motivate this choice through a model of beliefs about others' signals. As a result, an agent's belief about the variance of the price is also unbounded. In what follows, we numerically establish the robustness of our results in a setting where this variance has an upper bound.

To see how such a bound might arise, we extend the setting introduced in the previous section. If agents are constrained in their beliefs so that $\pi_i = 1$, then

$$\delta_{zi} \in \left[\frac{\frac{\beta^2\psi^2}{\tau_z}}{\frac{1}{\tau_e} + \frac{\beta^2}{\tau_z}}, \infty\right]$$

Figure 4: Anticipated utility net of costs versus δ_{zi}

The figure plots the anticipated utility net of costs for investor i as a function of her choice of δ_{zi}. Other parameters are: $\tau = \tau_e = 1; \tau_z = 1; \gamma = 2$.



where the lower bound $\frac{\frac{\beta^2\psi^2}{\tau_z}}{\frac{1}{\tau_e}+\frac{\beta^2}{\tau_z}} < \psi^2 < 1$. We numerically verify that the baseline model's main results still arise:

1. When risk aversion is sufficiently high, there exists a symmetric equilibrium in which all investors choose to set δ_{zi} at the lower bound i.e., $\delta_{zi} = \frac{\frac{\beta^2\psi^2}{\tau_z}}{\frac{1}{\tau_e}+\frac{\beta^2}{\tau_z}} < 1$. In the baseline model, this is equivalent to choosing $\delta_{z,i} = 0$ for all agents.

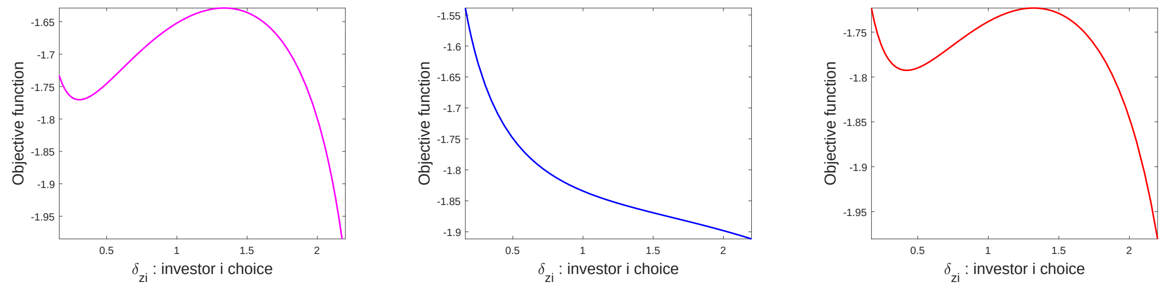
2. When risk aversion is sufficiently low, a symmetric equilibrium does not exist and investors endogenously separate into two groups: for the first group, $\delta_{z,i} = \frac{\frac{\beta^2\psi^2}{\tau_z}}{\frac{1}{\tau_e}+\frac{\beta^2}{\tau_z}} < 1$ and for the second group $\delta_{zi,2} > 1$.

The following plots confirm these predictions. Figure 4 shows investor i's objective as a function of $\delta_{z,i}$,for a given δ_z, chosen by all other agents in the economy, and under the assumption that risk aversion is "high". The lower bound of $\delta_{z,i}$ for the parameters chosen is 0.5. When all investors chose $\delta_z = 1$ (solid line), investor i prefers to deviate and choose $\delta_{z,i} = 0.5$. When all other investors choose $\delta_z = 0.5$,investor i also chooses $\delta_{zi} = 0.5$. Hence $\delta_{zi} = 0.5 \; \forall i$ is a symmetric equilibrium.

Figure 5 shows investor i's objective in the case of low risk aversion. For the parameters chosen, $\delta_z \in [0.16, \infty]$. In panel (a), all other investors choose $\delta_z = 0.16$. In this case, investor i has an incentive to deviate by over-weighting the information in prices (i.e., by setting $\delta_{z,i} \approx 1.3$). Even though the price is objectively very informative (the information channel), because other investors are placing low weight on it ($\delta_z = 0.16$), the speculative effect of overweighting the price is relatively small. In panel (b), we consider an alternative symmetric equilibrium in which all other investors choose $\delta_z = 1.3$. Now, the speculative effect dominates and investor i strictly prefers to underweight the information in prices (i.e., $\delta_{zi} = 0.16$). In both cases, a symmetric equilibrium is ruled out because an individual

Figure 5: Anticipated utility net of costs versus $\delta_{z,i}$

The figure plots the anticipated utility net of costs for investor i as a function of her choice $\delta_{z,i}$. Other parameters are: $\tau = \tau_e = \tau_z = \delta_e = \delta_{e,i} = 1$, $\gamma = 0.3$



(a) $\delta_z = 0.2$ (b) $\delta_z = 1.3$ (c) Mixed eqm.

investor has an incentive to deviate from the equilibrium behavior. Panel (c) of Figure 5 illustrates an instance of the asymmetric equilibrium. In this case, each investor is indifferent between two (sets of) beliefs.

B.6 Welfare

In this section, we explore the welfare implications of allowing investors to choose their beliefs optimally. Since we are in a setting with heterogeneous priors, we begin by noting that welfare for the informed investors depends on the reference beliefs used (see Brunnermeier et al. (2014) for a recent discussion and a proposed welfare criterion in related settings). From the perspective of the investors' subjective beliefs, expected utility is higher when they deviate from rational expectations. However, from the perspective of a social planner who is restricted to hold objective beliefs, expected utility for informed investors is strictly lower when they deviate from rational expectations - their demand for the risky asset is suboptimal given their information sets.

For our welfare analysis below, we use the objective distribution as the reference beliefs and define expected utility for an informed investor as

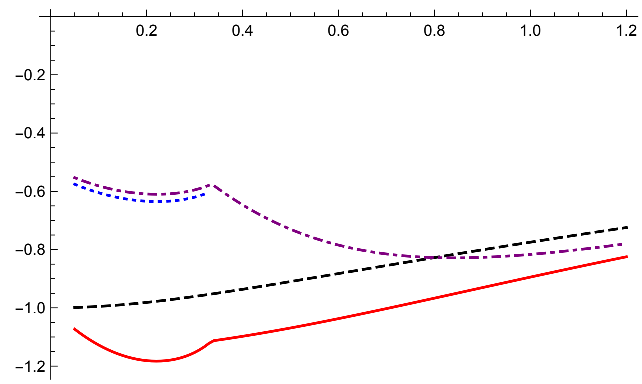
$$U_i \equiv \mathbb{E}\left[-\exp\left\{-\gamma x_i^*\left(\delta_{e,i}^*, \delta_{z,i}^*\right) \times (F - P) - \gamma W_0\right\}\right], \tag{144}$$

where $x_i^*\left(\delta_{e,i}^*, \delta_{z,i}^*\right)$ is her optimal demand under her optimally chosen beliefs $\delta_{e,i}^*, \delta_{z,i}^*$. Importantly, this is a conservative measure of expected utility as it only accounts for the costs of deviating from rational expectations and does not include any of the gains from anticipated utility.

Figure 6 provides an illustration of the relative levels of expected utility across both the

Figure 6: Expected utility with subjective beliefs versus γ

The figure plots the expected utility for investors across the subjective beliefs and rational expectations equilibria as a function of risk aversion γ. The dashed line plots the expected utility of informed investors in the rational expectations equilibrium (i.e., $U_{i,RE}$), the solid line plots the expected utility (under the objective distribution) for the investors who dismiss price information, the dotted line plots the expected utility for investors who overweight price information (when the asymmetric equilibrium exists), and the dot-dashed line plots the expected utility of a hypothetical, rational expectations investor in the subjective beliefs equilibrium. Other parameters are set to $\tau = \tau_e = \tau_z = 1$, and $\gamma_z = 0.75$.



rational expectations and subjective beliefs equilibria. Specifically, the dashed line plots the expected utility of an investor in the rational expectations equilibrium. The solid and dotted lines plot the (objective) expected utility of the fundamental (dismissive) and technical (momentum) investors in the subjective beliefs equilibrium, respectively. Finally, the dot-dashed line plots the expected utility of a hypothetical, rational expectations investor in the subjective beliefs equilibrium. Not surprisingly, a rational expectations investor in the subjective beliefs equilibrium (dot-dashed) experiences a higher expected utility than either the fundamental (solid) or technical (dotted) investor — she is optimally using all the information available to her and exploiting the behavior of the other investors and the noise traders. However, it is interesting to note that (i) in the asymmetric equilibrium, the fundamental investors are significantly worse off than the technical traders, and (ii) investors in the rational expectations equilibrium (dashed) may be worse off than (technical) investors in the subjective beliefs equilibrium. The first result follows from the fact that the technical traders use both private information and the price information, and even though they overweight both signals, this is more efficient than ignoring the information in prices. The second result follows from the observation that speculative opportunities are objectively lower in the rational expectations equilibrium than in the subjective beliefs equilibrium, since all other investors are efficiently conditioning on the information available to them. As a result, even though the technical investors are using information sub-optimally, they are better off by exploiting the others in a subjective beliefs equilibrium than an individual investor is in a rational expectations equilibrium.

Next, we consider the effect of informed investors' deviations from rational expectations on the welfare of liquidity (or noise) traders. Recall that the aggregate supply, z, is noisy. Suppose this reflects the sale of the risky asset by a liquidity trader, who has CARA utility with risk aversion γ_z and is endowed with initial wealth W_0. Then, her expected utility is given by

$$U_z \equiv \mathbb{E}\left[-\exp\left\{-\gamma_z\left(-z\right) \times \left(F - P\right) - \gamma_z W_0\right\}\right]. \tag{145}$$

The following result compares the expected utility of liquidity traders and the overall welfare.

Proposition 8. *In equilibrium, the expected utility of a liquidity trader is given by:*

$$U_z = -\sqrt{\frac{\tau_z}{\tau_z + 2\gamma_z\left(\beta\Lambda - \frac{1}{2\tau}\gamma_z\left(1-\Lambda\right)^2\right)}}\exp\left\{-\gamma_z W_0\right\} \tag{146}$$

Suppose $\gamma_z \leq \gamma$. Then:

(i) Liquidity traders have higher expected utility in the symmetric equilibrium than in the rational expectations equilibrium.

(ii) In any asymmetric equilibrium in which $\Lambda_{AE} < \Lambda_{RE}$, liquidity traders have higher expected utility in the asymmetric equilibrium than in the rational expectations equilibrium.

(iii) There exists $\underline{\gamma} \geq 0$ such that for all $\gamma \geq \underline{\gamma}$, total welfare is higher under the subjective beliefs equilibrium than under the rational expectations equilibrium.

Expected utility for a liquidity trader depends on the equilibrium parameters through a term

$$\beta\Lambda - \tfrac{1}{2\tau}\gamma_z\left(1-\Lambda\right)^2, \tag{147}$$

where $\beta = -\frac{\gamma}{\tau_e \delta_e} < 0$. A liquidity trader's utility is driven by two components. The first component ($\beta\Lambda$) reflects her disutility from price impact — for instance, a larger sale (higher z) pushes prices downward, which reduces her proceeds. The second term ($-\frac{1}{2\tau}\gamma_z\left(1-\Lambda\right)^2$) reflects a standard risk-aversion channel — when prices are less informative about fundamentals, the liquidity trader faces more uncertainty about her payoff, which reduces utility.[24]

[24]It is important to note that expected utility is finite only when

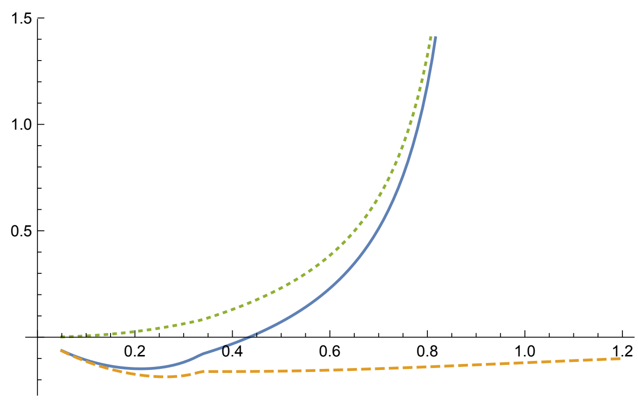
$$\tau_z + 2\gamma_z\left(\beta\Lambda - \tfrac{1}{2\tau}\gamma_z\left(1-\Lambda\right)^2\right) > 0. \tag{148}$$

Intuitively, if the combined disutility from the price impact and risk aversion terms are too large, the liquidity trader's expected utility from being forced to trade z units approaches negative infinity — she would rather exit the market and not trade if she could. This highlights a limitation of assuming that liquidity traders submit price insensitive orders. An alternative approach would be to model liquidity shocks as hedging

Recall that price sensitivity, Λ, is higher when investors exhibit rational expectations: $\Lambda_{RE} > \Lambda_{SE}, \Lambda_{AE}$. This has offsetting effects on the liquidity trader's utility. On the one hand, a lower Λ implies that the price is less sensitive to her trade and so utility increases through the price impact channel. On the other hand, a lower Λ implies prices track fundamentals less closely which increases the risk in the liquidity trader's payoff. As we show in the proof of Proposition 8, the price impact effect always dominates the risk-aversion effect if the risk aversion of the investors is weakly higher than that of the liquidity traders (i.e., $\gamma_z \leq \gamma$). In this case, liquidity traders are always better off when informed investors choose to deviate from rational expectations.

Figure 7: Difference in expected utility $U_{OE} - U_{RE}$ versus γ

The figure plots the difference in utility across the subjective beliefs and rational expectations equilibria as a function of γ. The dashed line plots the difference in expected utility of informed investors (under the objective distribution) (i.e., $U_{i,OE} - U_{i,RE}$), the dotted line plots the difference in expected utility for the noise traders (i.e., $U_{z,OE} - U_{z,RE}$), and the solid line plots the difference in utility across both groups (i.e., $U_{i,OE} + U_{z,OE} - (U_{i,RE} + U_{z,RE})$). Other parameters are set to $\tau = \tau_e = \tau_z = 1$, and $\gamma_z = 0.75$.



ΔU for investors (dashed), noise traders (dotted), both (solid)

Note that $\gamma_z \leq \gamma$ is a sufficient condition, but it is not necessary for liquidity traders to be better off under the subjective beliefs equilibrium. Figure 7 plots the difference in expected utility between the subjective beliefs equilibrium and the rational expectations equilibrium as a function of investor risk aversion γ for each group separately, and for both groups as a whole. The plot illustrates that for this set of parameters, liquidity traders are always better off under subjective beliefs — the dotted line is always above zero — irrespective of whether informed investors are more or less risk averse than them. In particular, note that noise trader risk aversion γ_z is fixed at 0.75, but informed investor risk aversion γ ranges

demands for the informed investors. However, this makes the analysis less tractable and the intuition for the results in the rest of the paper less clear.

from 0.1 to 1.2. Not surprisingly, under the objective distribution, the informed investors are worse off under the subjective beliefs equilibrium — the dashed line is always below zero. The solid line in Figure 7 illustrates the aggregate welfare ranking in Proposition 8. Specifically, aggregate welfare appears to be higher in the rational expectations equilibrium when informed investor risk aversion is low, but higher under subjective beliefs when risk aversion is high.

Our results suggest that while deviations from rational expectations are arguably costly for informed investors, they may make liquidity traders better off. Moreover, our welfare results do not account for changes in the real (allocative) efficiency. Since price informativeness is higher when informed investors deviate from rational expectations, the real efficiency in the economy can also be higher under such deviations if investment / allocative decisions are made on the information in prices.

B.6.1 Proof of Proposition 8

The utility of noise traders is

$$\begin{aligned}
U_z &= -E\left(\gamma_z \exp\{+\gamma_z z\left(F-P\right)\}\right) \\
&= -E\left(\gamma_z \exp\{\gamma_z z F\left(1-\Lambda\right)-\gamma\Lambda\beta z^2\}\right) \\
&= -E\left(\gamma_z \exp\left\{\left(\frac{\gamma_z^2\left(1-\Lambda\right)^2}{2\tau}-\gamma_z\Lambda\beta\right)z^2\right\}\right) \\
&= -\gamma_z \frac{1}{\sqrt{1-2\frac{1}{\tau_z}\left(\left(\frac{\gamma_z^2(1-\Lambda)^2}{2\tau}-\gamma_z\Lambda\beta\right)\right)}} \\
&= -\gamma_z \sqrt{\frac{\tau_z}{\tau_z-\frac{\gamma_z^2(1-\Lambda)^2}{\tau}+2\gamma_z\Lambda\beta}}
\end{aligned}$$

where we used the fact that $E\left(e^{a\epsilon^2}\right)=\frac{1}{\sqrt{1-2a\sigma_\epsilon^2}}$. This implies that utility of noise traders is monotonically decreasing in $\frac{\gamma_z(1-\Lambda)^2}{2\tau}-\Lambda\beta$.

(i) Rational expectations vs. Symmetric equilibrium: In this case, $\bar{\bar{\delta}}_e=1$ and

$$\Lambda_{SE}=\frac{\tau_e}{\tau+\tau_e}, \quad \Lambda_{RE}=\frac{\tau_e+\tau_p}{\tau+\tau_e+\tau_p} \tag{149}$$

so

$$U_{SE} - U_{RE} > 0 \tag{150}$$

$$\Leftrightarrow \frac{\gamma\tau\tau_p}{\tau_e(\tau_e+\tau)(\tau_e+\tau_p+\tau)} > \frac{\tau\gamma_z}{2}\frac{\tau_p(2\tau_e+\tau_p+2\tau)}{(\tau_e+\tau)^2(\tau_e+\tau_p+\tau)^2} \tag{151}$$

$$\Leftrightarrow \frac{\gamma}{\gamma_z} > \frac{\tau_e}{\tau+\tau_e}\frac{2(\tau_e+\tau)+\tau_p}{2(\tau_e+\tau_p+\tau)} \tag{152}$$

which implies if $\gamma \geq \gamma_z$, then $U_{SE} > U_{RE}$.

(ii) Rational Expectations vs. Asymmetric Equilibrium: In this case, $\bar{\delta}_e \geq 1$ and

$$\Lambda_{RE} = \frac{\tau_e+\tau_p}{\tau+\tau_e+\tau_p}, \quad \Lambda_{AE} = \frac{\delta_e\tau_e+\delta_z\tau_p}{\tau+\delta_e\tau_e+\delta_z\tau_p} \tag{153}$$

so that

$$U_{AE} - U_{RE} > 0 \tag{154}$$

$$\Leftrightarrow \gamma\left(\frac{\Lambda_{RE}}{\tau_e} - \frac{\Lambda_{AE}}{\tau_e\delta_e}\right) > \gamma_z\left(\frac{(1-\Lambda_{AE})^2}{2\tau} - \frac{(1-\Lambda_{RE})^2}{2\tau}\right) \tag{155}$$

$$\Leftrightarrow \frac{\gamma}{\tau_e}\left(\Lambda_{RE} - \frac{\Lambda_{AE}}{\delta_e}\right) > \frac{\gamma_z}{2\tau}\left(\Lambda_{RE}-\Lambda_{AE}\right)\left(2-\left(\Lambda_{AE}+\Lambda_{RE}\right)\right) \tag{156}$$

Note that $\delta_e \geq 1$, so it is sufficient to establish:

$$\frac{\gamma}{\tau_e}\left(\Lambda_{RE}-\Lambda_{AE}\right) > \frac{\gamma_z}{2\tau}\left(\Lambda_{RE}-\Lambda_{AE}\right)\left(2-\left(\Lambda_{AE}+\Lambda_{RE}\right)\right) \tag{157}$$

When $\Lambda_{RE} > \Lambda_{AE}$, the above is equivalent to:

$$\frac{\gamma}{\gamma_z} > \frac{\tau_e}{2}\left(\frac{1}{\tau+\tau_e+\tau_p} + \frac{1}{\tau+\delta_e\tau_e+\delta_z\tau_p}\right) \tag{158}$$

which is always true if $\gamma \geq \gamma_z$.

(iii) Total welfare is given by:

$$W\left(\delta_e,\delta_z\right) = -\frac{1}{\sqrt{\Lambda^2(\delta_z-1)^2+\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e\delta_e(2-\delta_e)+\tau_p\delta_z(2-\delta_z))}} - \sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda)^2}{\tau} + 2\gamma_z\Lambda\beta}} \tag{159}$$

Moreover, for the rational expectations equilibrium, we have $\delta_e = \delta_z = 1$, so that

$$W_{RE} = -\frac{1}{\sqrt{\left(\frac{(1-\Lambda_{RE})^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p)}} - \sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau} + 2\gamma_z\Lambda_{RE}\beta_{RE}}} \tag{160}$$

This implies that the difference in welfare is:

$$W_{OE} - W_{RE} = \begin{array}{c} \frac{1}{\sqrt{\left(\frac{(1-\Lambda_{RE})^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p)}} + \sqrt{\frac{\tau_z}{\tau_z-\frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau}+2\gamma_z\Lambda_{RE}\beta_{RE}}} \\ -\frac{1}{\sqrt{\Lambda_{OE}^2(\delta_z-1)^2+\left(\frac{(1-\Lambda_{OE})^2}{\tau}+\frac{\Lambda_{OE}^2}{\tau_p}\right)(\tau+\tau_e\delta_e(2-\delta_e)+\tau_p\delta_z(2-\delta_z))}} \\ -\sqrt{\frac{\tau_z}{\tau_z-\frac{\gamma_z^2(1-\Lambda_{OE})^2}{\tau}+2\gamma_z\Lambda_{OE}\beta_{OE}}} \end{array} \tag{161}$$

Above, we have established that when $\gamma_z \leq \gamma$ and $\Lambda_{OE} < \Lambda_{RE}$, we have:

$$U_{z,OE} = -\sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma^2(1-\Lambda_{OE})^2}{\tau} + 2\gamma\Lambda_{OE}\beta_{OE}}} > -\sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau} + 2\gamma_z\Lambda_{RE}\beta_{RE}}} = U_{z,RE} \tag{162}$$

$$\Leftrightarrow \tau_z - \frac{\gamma_z^2\left(1-\Lambda_{OE}\right)^2}{\tau} + 2\gamma_z\Lambda_{OE}\beta_{OE} \geq \tau_z - \frac{\gamma_z^2\left(1-\Lambda_{RE}\right)^2}{\tau} + 2\gamma_z\Lambda_{RE}\beta > 0 \tag{163}$$

Let

$$\bar{\gamma} \equiv \frac{\tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau}}{\frac{2\gamma_z\Lambda_{RE}}{\tau_e}} = \frac{\tau_e\left(\tau_z(\tau_e+\tau_p+\tau)^2 - \tau\gamma_z^2\right)}{2\gamma_z(\tau_e+\tau_p)(\tau_e+\tau_p+\tau)}. \tag{164}$$

Note that

$$\lim_{\gamma\uparrow\bar{\gamma}} \sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau} + 2\gamma_z\Lambda_{RE}\beta_{RE}}} = \infty, \tag{165}$$

but

$$\lim_{\gamma\uparrow\bar{\gamma}} \begin{array}{c} \frac{1}{\sqrt{\left(\frac{(1-\Lambda_{RE})^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p)}} - \sqrt{\frac{\tau_z}{\tau_z-\frac{\gamma_z^2(1-\Lambda_{OE})^2}{\tau}+2\gamma_z\Lambda_{OE}\beta_{OE}}} \\ -\frac{1}{\sqrt{\Lambda_{OE}^2(\delta_z-1)^2+\left(\frac{(1-\Lambda_{OE})^2}{\tau}+\frac{\Lambda_{OE}^2}{\tau_p}\right)(\tau+\tau_e\delta_e(2-\delta_e)+\tau_p\delta_z(2-\delta_z))}} \end{array} \geq -c \tag{166}$$

for some $c \leq \infty$. This implies

$$\lim_{\gamma\rightarrow\bar{\gamma}} W_{OE} - W_{RE} > 0, \tag{167}$$

or equivalently, $\exists\underline{\gamma} \leq \bar{\gamma}$, such that for all $\gamma > \underline{\gamma}$, $W_{OE} > W_{ER}$. □