# Choosing to Disagree in Financial Markets


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# How do investors interpret information?

Rational expectations implies learning is efficient

- **Assumes** subjective beliefs agree with objective distribution
- Why? Objective beliefs are *accurate*, forward looking

Overwhelming evidence that people do not behave this way!
e.g., excess predictability, volatility and volume

Behavioral literature explores role of cognitive frictions and biases (e.g., over-confidence, dismissiveness),

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Behavioral literature explores role of cognitive frictions and biases (e.g., over-confidence, dismissiveness), but is silent on **when / why** such distortions arise

# Given a choice, how do investors interpret information?

We allow investors to choose how to interpret information in a standard, Hellwig (1980) setting

- Observe conditionally i.i.d. private signals and (noisy) price
- Well-being *also* depends on anticipation of future outcomes
- Investors choose precision of private / price signals ex-ante

Subjective beliefs trade off:

**Desirability** higher anticipatory utility

versus

**Accuracy** higher experienced utility

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Return predictability, volume, volatility and welfare can be higher under chosen beliefs than under rational expectations

# Related Literature

**Belief Choice** See survey by Benabou and Tirole (2016)

- Caplin and Leahy (2019)'s model of "wishful thinking"
- Brunnermeier and Parker (2005)'s model of "optimal expectations"

**Deviations from Rational Expectations**

- Overconfidence: Odean (1998); Daniel et al. (1998); Daniel, Hirshleifer, and Subrahmanyam (2001); Gervais and Odean (2001)
- Under-weighting price information:
  Difference of opinions (e.g., Banerjee, Kaniel and Kremer, 2009)
  Rational inattention (e.g., Kacperczyk et. al. 2016)
  Cursedness (e.g., Eyster, Vayanos and Rabin, 2018)
  Costly learning from prices (e.g., Vives and Yang, 2018)

# What drives choice of beliefs?
# (a.k.a. motivating motivated beliefs)

# Choice of subjective beliefs depends on overall goal

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**Robust control:** Goal is to optimize against bad (worse-case) scenarios

- Prefer to choose *pessimistic* subjective beliefs
- But, distortion in beliefs $\Rightarrow$ distorted actions, lower ex-post utility

Trade-off: accuracy vs. robustness (down-side protection)

$$\min_{\mu} \max_{a} \mathbb{E}_{\mu}[u(a)] + C(\mu, \mu_0)$$

where $C(\mu, \mu_0)$ is **cost** of choosing beliefs $\mu \neq$ objective beliefs $\mu_0$

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Figure 1: Balcetis & Dunning (2006)

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When desirable, 72% saw "B" and 61% saw "13"

Wishful thinking and motivated reasoning affects the acquisition and interpretation of information in many settings

- Oster, Shoulson & Dorsey (2013): Don't want to learn if at risk for Huntington's even if test is cheap and perfectly predictive
- Ganguly and Tasoff (2016): Pay to avoid getting tested for HSV-1 / HSV-2
- Eli & Rao (2011): People under-react to negative feedback on intelligence / beauty, but respond to good news
- Karlsson, Loewenstein & Seppi (2009): Investors monitor their portfolios more in rising markets
- Babcock and Loewenstein (1997): Randomly assigned "prosecutors" interpret the same evidence to be more consistent with defendant's guilt than assigned "defense attorneys"
- Exley and Kessler (2019): Interpret uninformative signals about ability as favorable

Moreover, expertise / cognitive ability can exacerbate the biases e.g., political bias in Kahan (2013), Kahan, Peters, Dawson & Slovic (2014)

# Model Setup

# Payoffs, Signals and Preferences

There are three dates $t = 0, 1, 2$ and two assets:

- Risk-free asset is normalized to numeraire
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Continuum of investors with CARA ($\gamma$) utility over terminal ($t = 2$) wealth
Normalize initial wealth to $W_0 = 0$ for presentation.

At date $t = 1$, investor $i$

(i) observes private signal $s_i = F + \varepsilon_i$, where $\varepsilon_i \sim \mathcal{N}(0, 1/\tau_e)$ is i.i.d.

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Aggregate supply of the asset is $z \sim \mathcal{N}(0, 1/\tau_z)$, so market clearing:

$$\int_i x_i(s_i, P) di = z$$

# Subjective Beliefs

Investor $i$'s subjective beliefs about:

- error in private signal: $\varepsilon_i \sim_i \mathcal{N}\left(0, \frac{1}{\delta_{e,i}\tau_e}\right)$
- aggregate supply shock: $z \sim_i \mathcal{N}\left(0, \frac{1}{\delta_{z,i}\tau_z}\right)$

where $\delta_{e,i}, \delta_{z,i} \in [0, \infty)$ parameterize the degree to which the investor **over**- or **under-estimates** info from $s_i$ and $P$, respectively

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Remarks:

- **Rational Expectations** is a special case: $\delta_{e,i} = \delta_{z,i} = 1$
- Beliefs about supply noise $\Leftrightarrow$ Beliefs about others

# Anticipated Utility

**Each investor adopts her chosen beliefs as her "true" model.**

- At date $t = 1$, optimal demand is

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- At date $t = 0$, anticipated utility is

$$AU(\delta_{e,i}, \delta_{z,i}) = \mathbb{E}_i \left[ \mathbb{E}_i \left[ -e^{-\gamma x_i(s_i, P) \times (F-P)} \middle| s_i, P \right] \right]$$

Anticipated utility is *current utility* derived from expectation of the future.

# Cost of Belief Distortion

Deviations from objective distribution impose a cost $C(\delta_{e,i}, \delta_{z,i})$, so investor $i$ chooses $\delta_{e,i}$ and $\delta_{z,i}$ to maximize:

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**Experienced utility penalty:** The cost reflects objective utility loss from distorted actions i.e.,

$$C(\delta_{e,i}, \delta_{z,i}) \equiv \mathbb{E}\left[-e^{-\gamma x_i(\delta_{e,i}, \delta_{z,i})(F-P)}\right] - \mathbb{E}\left[-e^{-\gamma x_i(1,1)(F-P)}\right]$$

- Similar to Brunnermeier and Parker (2005)'s optimal expectations

**Well-behaved cost function:** $C(\cdot)$ is strictly convex, and

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**Note:** Investor need not "know" objective distribution, convenience / discipline for economist

# Solving the Model

# “Standard" Financial Market Equilibrium

**Lemma:** Given investors' subjective beliefs $\delta_{e,i}$ and $\delta_{z,i}$ $\forall i \in [0,1]$, there always exists a unique, linear equilibrium with

$$P = \Lambda s_p, \text{ where } \Lambda = \frac{\int_i \delta_{e,i}\tau_e + \delta_{z,i}\tau_p di}{\int_i \tau + \delta_{e,i}\tau_e + \delta_{z,i}\tau_p di}, \quad s_p = F + \beta z$$

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Subjective beliefs affect prices through two channels:

- Higher private signal precision $\delta_{e,i}$ increases signal to noise ratio of $s_p$ increases $|\beta|$
- Higher precision of either signal increases price sensitivity to shocks i.e., increases $\Lambda$

# Subjective Beliefs and Anticipated Utility

Anticipated utility increases in the volatility of conditional Sharpe Ratio:

$$AU(\delta_{e,i}, \delta_{z,i}) = -\sqrt{\frac{1}{\text{var}_i(SR_i)}} = -\sqrt{\frac{\text{var}_i[F|s_i, P]}{\text{var}_i[F - P]}}$$

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**Lemma:** Anticipated utility increases in perceived private precision ($\delta_{e,i}$), but is $U$-shaped in perceived price precision ($\delta_{z,i}$)

# Results

# Benchmark: Overconfidence in private information

Suppose investors have **objective** beliefs about **prices** i.e., $\delta_{z,i} = 1$.

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With experienced utility penalty, $\delta_e$ increases with $\tau$ and $\tau_z$, decreases with risk aversion $\gamma$.

**Intuition:** More informative prior or price $\Rightarrow$ less costly to distort $\delta_{e,i}$

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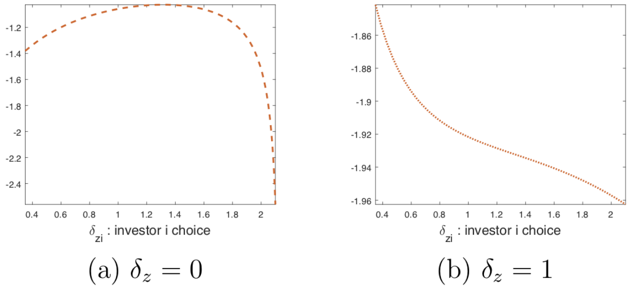


Figure 2: $AU(\cdot) - C(\cdot)$ versus $\delta_{z,i}$

# Dismissiveness in symmetric equilibria

**Theorem:** In **any** symmetric equilibrium, all investors are:
(i) (weakly) over-confident about their private info i.e., $\delta_{e,i} \geq 1$
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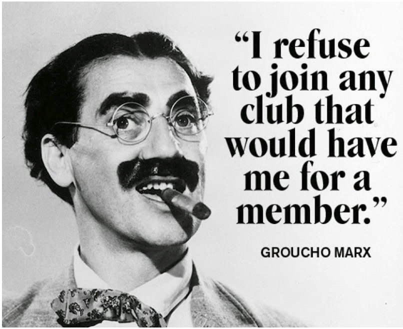
**Intuition:** I refuse to learn from prices when others are doing so.



# Risk aversion and symmetric equilibria

Note that in a symmetric equilibrium, the price is

$$P = \bar{\mathbb{E}}_i[F \big| s_i, P] - \gamma \text{var}_i[F \big| s_i, P] z$$

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**Theorem:** With exp. utility penalty, there exist cutoffs $\underline{\gamma} < \bar{\gamma}$ such that

(i) For $\gamma \geq \bar{\gamma}$, there exists a unique, symmetric equilibria in which all investors **ignore** price information and correctly interpret private information (i.e., $\delta_{z,i} = 0$ and $\delta_{e,i} = 1$).

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(ii) For $\gamma \leq \underline{\gamma}$, there does **not** exist a symmetric equilibrium.

**Intuition:** When prices are sufficiently uninformative ($\gamma \geq \bar{\gamma}$), ignoring prices is not *too* costly, so symmetric equilibrium can be sustained

More generally, we have $\delta_{z,i} < 1$ and $\delta_{e,i} > 1$

# Risk tolerance and asymmetric equilibria

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There exist **asymmetric** equilibria characterized by $(\lambda, \delta_e, \delta_z)$ where

(i) fraction $\lambda$ optimally chooses $\delta_{e,i} = 1$ and $\delta_{z,i} = 0$

(ii) fraction $1 - \lambda$ optimally chooses $\delta_e, \delta_z > 1$

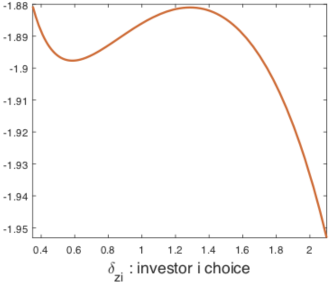


Figure 3: $AU(\cdot) - C(\cdot)$ versus $\delta_{z,i}$

# Implications of Asymmetric Equilibria

Observed heterogeneity in investment styles arise endogenously:

- **Fundamental traders** who find *mispriced securities* using their private info, but *dismiss* the information in prices
- **Technical traders** use price trends, reminiscent of overweighting price information

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Model predicts such heterogeneity arises when risk tolerance and price informativeness are high:

- in more developed financial markets
- in larger (more widely held) assets

# Return and Volume Characteristics

We compare predictions a number of return-volume observables

Focus for the talk:

(i) Price informativeness

(ii) Return predictability

(iii) Return volatility

# (1) Price informativeness is higher under subjective beliefs

Recall that $P = \Lambda s_p$, where $s_p = F + \beta z$

Price informativeness is precision of price signal $s_p$ i.e.,

$$\tau_p = \tau_z / \beta^2, \quad \text{where} \quad \beta = -\frac{\gamma}{\tau_e \int_i \delta_{e,i} di}$$

Relative to RE, investors (weakly) overweight private information

$\Rightarrow$ Lower $\beta^2$

$\Rightarrow$ Higher signal to noise ratio

This has implications for real / allocative efficiency in a richer environment

# (2) Return predictability is higher and can be positive

Return predictability $\theta = \frac{\text{cov}(F-P,P)}{\text{var}(P)}$ regress return on lagged return

With no noise, investors with

- **RE:** Condition on prices correctly $\Rightarrow$ No predictability
- **Wishful thinking:** Under-react to prices $\Rightarrow$ Excess predictability

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Adding noise generates reversals (negative correlation)

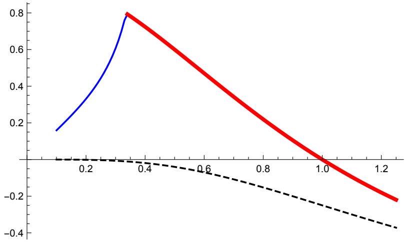


Figure 4: Predictability ($\theta$) versus Risk aversion ($\gamma$)

# (3) Return volatility can be higher or lower than RE

$$\sigma_R^2 \equiv \text{var}(F - P) = (1 - \Lambda)^2 \text{var}(F) + \Lambda^2 \beta^2 \text{var}(z)$$

Compared to RE, wishful thinking implies:

- Prices are (weakly) more informative lower $\beta^2$
- Prices are less sensitive to shocks lower $\Lambda$

Overall effect depends on the relative strength of the two

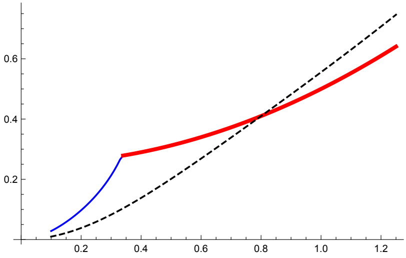


Figure 5: Volatility ($\sigma_R$) versus Risk aversion ($\gamma$)

# Expected Utility of Investors

How do subjective beliefs affect **investor** utility?

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Technical traders are better off than Fundamental traders

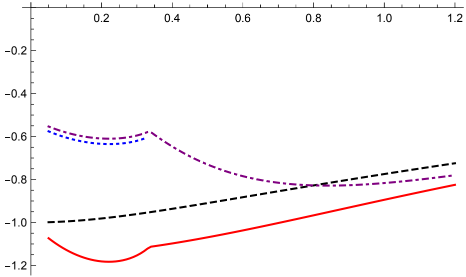


Figure 6: RE (dashed), Technical (dotted), Fundamental (solid), RE in Subj. Eqm (dot-dashed)

# Expected Utility of Liquidity Traders

How do investor's subjective beliefs affect **other participants?**

"Utility" for liquidity traders:

$$U_z(z) \equiv \mathbb{E}\left[-e^{-\gamma_z(W_0 - z(F-P))}\right]$$

Compared to RE, liquidity traders face:

- more risk exposure since prices track fundamentals less closely
- lower price impact

since $\Lambda$ is lower

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$$U_z(z) \equiv \mathbb{E}\left[-e^{-\gamma_z(W_0 - z(F-P))}\right]$$

Compared to RE, liquidity traders face:

- more risk exposure since prices track fundamentals less closely
- lower price impact

since $\Lambda$ is lower

Liquidity trader utility is higher when the second effect dominates

# Welfare is higher subjective beliefs when $\gamma$ is high

**Theorem:** Suppose $\gamma_z \leq \gamma$. Then,

(i) liquidity traders **always** have higher expected utility under symmetric equilibrium that in RE

(ii) when $\Lambda_{AE} < \Lambda_{RE}$, liquidity traders have higher expected utility under the asymmetric equilibrium than in RE

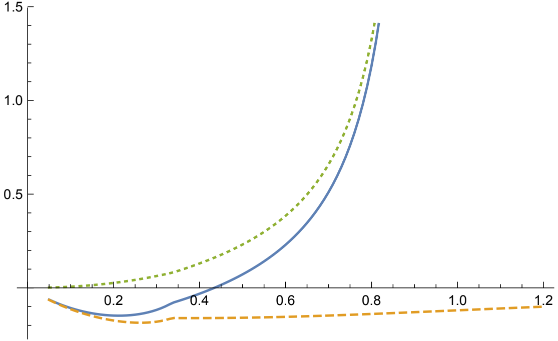


Figure 7: $U - U_{RE}$ vs $\gamma$: Investors (dashed), Noise traders (dotted), Sum (solid)

# Extensions: Public signals and Ex-post belief choice

**Public Signals:** Consider signals of the type $s = F + \eta$

- Tradeoff between information effect vs. speculative effect
- Anticipated utility is $U$-shaped in perceived precision
- But, in any symmetric equilibrium, investors **overweight** public signal

**Ex-post belief choice:** Choose perceived precision *after* observing signal

- Not tractable to solve for general equilibrium prices are not linear
- Taking others actions as fixed, partial equilibrium analysis suggest results are robust:

  When private signal realizations are sufficiently far from priors, investors are over-confident in their private info, but dismissive of prices

# Conclusions and Future Work

Subjective belief choice tells us **when** investors exhibit biases:

- Naturally gives rise to over-confidence and dismissiveness
- Can generate **endogenous** differences in behavior

Fruitful approach to explore how different biases arise in different settings

- How do results change in settings with strategic complementarity (e.g., coordination games)
- How do incentives affect the interpretation of information (e.g., fund managers, entrepreneurs)

# Land or Sea?

# Land or Sea?

# Land or Sea?

66.7% horse vs. 72.7% seal