Feedback Effects and Systematic Risk Exposures

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ABSTRACT

We model the "feedback effect" of a firm's stock price on investment in projects exposed to a systematic risk factor, like climate risk. The stock price reflects information about both the project's cash flows and its discount rate. A cash-flow maximizing manager treats discount rate fluctuations as "noise," but a price maximizing manager interprets such variation as information about the project's NPV. This difference qualitatively changes how investment behavior depends on the project's risk exposure. Moreover, traditional objectives (e.g., cash-flow or price maximization) need not maximize welfare because they do not correctly account for hedging and risk-sharing benefits of investment.

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Since Hayek (1945), it has been recognized that prices aggregate information that is dispersed across the economy and convey it to real decision makers. The "feedback effects" literature studies this mechanism in the context of corporate investment, emphasizing how asset prices reflect information about future investment opportunities, and how this information affects the production and investment decisions of firms (see Bond, Edmans, and Goldstein (2012) and Goldstein (2023) for insightful surveys). The existing analysis focuses on the extent to which prices reflect information about future cash flows, and interprets non-cash-flow variation in prices as "noise" that needs to be filtered out by decision makers.

Yet, a fundamental tenet of capital budgeting is that firms' optimal investment decisions should depend not only on projects' expected cash flows, but also on their discount rates. Moreover, a project's discount rate is driven by its loadings on systematic sources of risk, and investors' aggregate preferences over, and exposures to, these risks. While these preferences and exposures are privately known by investors, and so cannot be directly observed by managers, they impact investors' demands and equilibrium asset prices. This suggests that managers must rely primarily on prices to learn about discount rates when making investment decisions.

To study feedback effects when managers learn about discount rates from prices, we develop a model in which a firm's stock price conveys information about future cash flows and about investors' risk exposures. When the manager chooses investment to maximize expected cash flows, she interprets non-cash-flow variation in prices as noise. In contrast, when the manager chooses investment to maximize the future share price, non-cash-flow variation in prices conveys useful information about the project's discount rate.[1] Consequently, she no longer explicitly seeks to filter out such information and instead incorporates the information in prices on *both* cash flows and discount rates when making her investment decisions.

This difference has important implications for how investment in a project depends

[1] As we discuss below, the project's risk exposure is known to the manager and to investors, but the stock price conveys information about the associated factor risk premium to the manager.

on its risk exposure. For a cash-flow maximizing manager, an increase in a project's risk exposure makes the stock price a noisier signal. This makes the manager's investment decision less sensitive to the information in the price. In contrast, for a price maximizing manager, an increase in the project's risk exposure makes the price a more informative signal about the project's discount rate, and so has an opposite effect on the investment decision.

Finally, we show that traditional managerial objectives, like cash-flow or price maximization, do not generally lead to investment decisions that maximize investor welfare. For instance, since the cash-flow maximization objective ignores the impact of the project's systematic risk exposure on welfare, it leads to under-investment in projects that reduce investors' aggregate exposure to, or "hedge," the systematic risk factor.[2] Similarly, price maximization leads to inefficient investment decisions because, while the share price does reflect information about risk exposures through the discount rate, (i) the risk premium in price reflects the disutility that the risk of a *marginal* share of the stock imposes on an investor, while welfare depends on the disutility from bearing the risk of their entire share holdings, and (ii) the price does not reflect that investing in a risk-exposed project makes the stock a better instrument for risk-sharing across investors and, thus, increases welfare.

Model and Intuition. Our analysis applies quite broadly to investment in risky projects when market feedback plays an important role. However, a particularly salient application is to climate-sensitive investment, and so we use it to describe our model's economic forces and predictions. A firm's manager decides whether to invest in a project that is exposed to a systematic, climate-risk factor. The firm's stock is traded by risk-averse investors who are informed about the project's expected cash flows and have heterogeneous exposures to climate risk. The price aggregates not only investors' infor-

[2]In what follows, our terminology explicitly distinguishes between *hedging* and *risk-sharing*. The former refers to investors' desire to buy more (less) of assets that pay out more (less) during adverse systematic factor outcomes. The latter refers to investors' ability to share and reallocate *differential* exposures to systematic risk by trading a risk-exposed security.

mation about cash flows, but also their dispersed exposures towards the project's climate risk exposure, or "greenness." A "green" ("brown") project is defined as one that pays higher (lower) cash flows when climate outcomes are worse, while a "neutral" project's cash flows are uncorrelated with climate outcomes. As such, green projects are negatively exposed to the climate risk factor, while brown projects are positively exposed.[3]

For example, consider a consumer electronics firm deciding whether to invest in electric vehicle (EV) technology, such as batteries or semiconductors. Such a green investment is negatively exposed to climate risk: for instance, shifts in regulatory policy in response to climate change may lead to more favorable treatment of electric vehicles relative to traditional vehicles.[4] Thus, the firm's price and the information it provides to the manager depend, in part, on the fact that such investments are likely to perform better when aggregate climate outcomes are worse.[5]

We compare two objectives for the manager. First, in line with the existing feedback literature, we consider the case in which the manager chooses investment to maximize

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[3]Our definitions of "green" versus "brown" projects are consistent with the labels in the empirical literature (e.g., Engle et al. (2020), Bolton and Kacperczyk (2021)), as we discuss in Section II.A.

[4]Panasonic, historically associated with consumer electronics, is now also a leading manufacturer of rechargeable batteries for electric vehicle companies (e.g., Tesla supplier Panasonic plans additional $4 bln U.S. EV battery plant, Reuters, Aug 26, 2022). Such investments are likely to benefit from regulatory changes that provide tax subsidies to encourage the purchase of electric vehicles, which is an example of climate transition risk. Giglio, Kelly, and Stroebel (2021) distinguish two types of climate risk: transition risk (i.e., "risks to cash flows arising from a possible transition to a low- carbon economy") versus physical risks (i.e., "direct impairment of productive assets resulting from climate change").

[5]More generally, consistent with our model's key assumptions, there is substantial evidence that investors have time-varying exposures to climate risk that affect their demands for green and brown stocks and alter these stocks' discount rates (e.g., Choi, Gao, and Jiang (2020), Bolton and Kacperczyk (2023), Pástor, Stambaugh, and Taylor (2022)). Moreover, consistent with managers responding to the information that prices contain about cash flows and discount rates, empirical evidence shows that firms' investment in climate-exposed projects often responds to changes in their stock prices, even when driven by shocks to investor demand for green exposure rather than cash flow news (e.g., Li et al. (2020), Bai et al. (2021), Briere and Ramelli (2021)).

expected cash flows.[6] In this case, we show that a higher (absolute) exposure to climate risk shocks makes the price a noisier signal about cash flows, which, in turn, makes the manager's investment decision *less* sensitive to the price. As a result, for ex-ante unattractive projects (i.e., projects with negative ex-ante, net expected cash flows), the manager is less likely to invest in green (or brown) projects than in climate-neutral ones.

Second, we consider the case where the manager's objective is to maximize the firm's expected stock price. In this case, she only invests when the stock price is sufficiently high, because this implies that the project's net present value (NPV), conditional on the price information, is positive. In effect, when conditioning on the price, she learns about both investors' cash flow information *and* their aggregate risk exposure, which drives the project's discount rate. In fact, we show that price aggregates these two types of information in an efficient manner from the manager's perspective, in that she makes the same investment decision that she would if she observed them separately.[7]

Once again, when the project has a greater absolute exposure to climate risk, the firm's price is a noisier signal of cash flows. Yet, in stark contrast to cash-flow maximization, this causes her investment decision to become *more* sensitive to the price. This is because the NPV of projects with larger absolute climate exposures are driven to a greater extent by investors' aggregate demand for climate hedges. Hence, prices of more highly climate-exposed projects convey more information to the manager about the discount rate. For an ex-ante unattractive project, this increased sensitivity to discount rate information increases the likelihood that the project will have a positive conditional NPV, and so *increases* the likelihood of investment.

An increase in the project's climate exposure also affects its expected NPV: greener projects provide a hedge against bad climate outcomes and so, all else equal, carry lower

[6] Such objectives are relevant in practice, since the majority of executive compensation plans include bonuses based on earnings or other accounting measures (e.g., Guay, Kepler, and Tsui (2019)).

[7] This establishes an equivalence between our setting, where the manager infers their project's discount rate and cash-flow information from prices, and traditional production-based asset pricing models, where the manager is assumed to exogenously know these two types of information (e.g., Cochrane (1991)).

discount rates. The overall effect of a project's climate exposure on the likelihood of investment trades off the impact of these channels. In fact, when the ex-ante uncertainty over the aggregate demand for a climate hedge is sufficiently high, the effect of climate exposure on the volatility of a project's NPV dominates its effect on its expected NPV. This implies, for example, that the manager may be more likely to invest in brown projects that are ex-ante unattractive than in comparable neutral projects.

Welfare. Differences in managerial objectives also have important implications for investor welfare. We first consider a benchmark in which all investors have identical exposures to the climate risk factor. In this case, maximizing cash flows clearly does not align with maximizing shareholder welfare, because it ignores the impact of investment on investors' aggregate climate exposure – for example, it leads to under-investment in green projects. More surprisingly, we show that the price-maximizing investment rule also does not align with the welfare-maximizing price-contingent investment rule as long as the firm is not arbitrarily small (that is, as long as the investment decision has an effect on aggregate exposures). Intuitively, this is because the price reflects the marginal disutility from bearing the risk of the last outstanding share, but welfare depends on the average disutility from bearing the risk of all outstanding shares. Because the marginal disutility of the last share is higher than the average disutility of all shares, the price-maximizing rule tends to under-invest. Finally, we show that for brown projects, price and cash-flow incentives can be balanced through appropriate weighting to induce the manager to maximize investor welfare, while for green projects, this may not be possible.

We then consider the general setting in which investors have heterogeneous exposures to climate risk. This is a realistic feature: investors' climate exposures differ with age, geography, and adaptability (Giglio, Kelly, and Stroebel (2021)), and, as evidenced by the swath of actively-traded climate-based ETFs, investors appear to use financial markets as a means to hedge and share such risk exposures.[8] For example, an investor who lives

[8]There is ample evidence that investors use financial assets to attempt to hedge and share climate risks – for example, see Ilhan (2020), Krueger, Sautner, and Starks (2020), Ilhan et al. (2023), Giglio, Kelly, and Stroebel (2021) and our discussion in Section II.A. Moreover, total assets under management in sustainability-focused funds roughly doubled from Q4 2019 to Q3 2022, concurrent with over 200

in coastal California is more exposed to climate risk due to rising sea levels, and so has a different demand for green stocks, than an investor who lives in central Kansas.[9] In such settings, a firm's investment in a climate-sensitive project has an additional impact on welfare because it allows investors to use the firm's stock to help share risk: all else equal, both investors are better off when the Kansas investor sells some shares of a firm that invests in green EV projects to the California investor. However, the welfare improvement as a result of this "risk-sharing" channel is not captured by the stock price, which reflects investors' disutility of risk of a marginal share of the stock, and not the heterogeneity in their exposures.

This implies that, even when the per-capita endowment of shares is negligible (so that the investment decision does not affect aggregate risk), both price maximization and cash-flow maximization lead to under-investment relative to welfare maximization. Moreover, while feedback necessarily increases the firm's expected cash flows or share price (depending on the manager's objective), we show that it can decrease investor welfare.[10] Intuitively, without feedback, the manager would always invest in an ex-ante attractive project, but with feedback, she would not invest in such a project if the equilibrium price was sufficiently low. This lower investment increases welfare due to higher valuations, but decreases welfare due to the risk-sharing channel. When investors' exposures to climate risk are sufficiently diverse or per-capita ownership of the firm is sufficiently small, the latter effect dominates and welfare is higher without feedback than with. In such settings, our analysis suggests that providing additional incentives for managers to invest in green projects (e.g., by linking their compensation to climate scores) can increase investor welfare, even though it may lead to lower valuations and lower future profitability.

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sustainability fund launches per year (Morningstar (2022)).

[9] Consistent with this, Ilhan (2020) documents that households with differential exposures to sea level rise have different participation in equity markets, and consequently, different portfolios.

[10] For simplicity, we assume investors do not have any access to other securities that let them share climate risks. However, we expect similar results would arise if the market for trading climate risk shocks is imperfect. As we discuss in Section II.A, this is consistent with the empirical evidence that suggests investors have different exposures to climate risk and find this risk difficult to hedge.

Overview. The rest of the paper is organized as follows. The next section discusses related literature. Section II presents the model and discusses key assumptions. Section III characterizes the equilibrium under cash-flow maximization and price maximization. Section IV presents our results on investor welfare. Section V concludes. Proofs of our results are in Appendix A, and additional analysis is presented in the Internet Appendix.

I. Related Literature

Our paper adds to the literature on feedback effects (see Bond, Edmans, and Goldstein (2012) and Goldstein (2023) for recent surveys and early work by Khanna, Slezak, and Bradley (1994), Subrahmanyam and Titman (2001), and others). In contrast to our setting, much of this literature focuses on economies in which (i) investors are either risk-neutral or the stock price is set by a risk-neutral market maker, (ii) the noise in prices arises due to noise traders with unmodeled utility functions, and (iii) the manager's investment choice maximizes the firm's expected terminal cash flow. As a result, such models are not well suited to study how discount rate variation affects investment decisions or how feedback affects investor welfare.[11] To our knowledge, our paper is the first model of feedback effects in which managers learn not only about cash flows but also about discount rates from prices, even though prior work has alluded to this channel (Diamond (1967)).

Bond, Edmans, and Goldstein (2012) highlight the important distinction between forecasting price efficiency (FPE), which measures how well prices predict future cash-flows, and revelatory price efficiency (RPE), which reflects how useful prices are for real investment decisions. While in many settings, more informative prices lead to better investment decisions, a key takeaway of their analysis is that, in some cases, RPE may be low even when FPE is high. Our analysis provides an instance where the opposite is

[11] See Diamond and Verrecchia (1981), Wang (1994), Schneider (2009), Ganguli and Yang (2009), Manzano and Vives (2011), and Bond and Garcia (2022) for models in which noise is driven by hedging needs as in our model. Existing feedback models with risk-neutral pricing include Dow, Goldstein, and Guembel (2017), Davis and Gondhi (2019), and Goldstein, Schneemeier, and Yang (2020).

true: with price maximization, we show that feedback raises investment efficiency and so RPE is high even through FPE may be low since prices are noisy signals of cash-flows.

The most closely related papers in this literature are Dow and Rahi (2003), Hapnes (2020), and Gervais and Strobl (2021). Dow and Rahi (2003) explore how increases in informed trading affect investment efficiency and risk sharing in a setting where investors are risk averse but prices are set by a risk-neutral market maker. They argue that investment efficiency always improves with more informed trading, but risk sharing may either worsen due to the Hirshleifer (1971) effect, or improve when information decreases uncertainty over the component of the asset's payoffs that are unrelated to the component that investors wish to hedge. Hapnes (2020) characterizes managerial investment behavior and investor information acquisition in a Grossman and Stiglitz (1980)-type model with feedback; however, the analysis does not study the effect of feedback on welfare. Gervais and Strobl (2021) consider the impact of informed, active money management on investment decisions in a setting with feedback. They study how the gross and net performance of the actively managed fund compares with the market portfolio and study how the presence of an informed money manager affects welfare.

We view our analysis as complementary. We focus on how investment in a project affects the risk exposure of a firm's cash flows, which in turn, affects how useful the stock is for hedging. This highlights a novel channel through which feedback affects welfare: intuitively, firms' investment decisions *endogenously* affect the degree of market completeness in the economy.[12] Also, since investors are identically informed in our analysis, the traditional Hirshleifer (1971) effect is turned off, which allows us to clearly distinguish our novel channel from earlier work.[13]

[12]This also distinguishes our analysis from Marín and Rahi (1999), Marín and Rahi (2000), and Eckwert and Zilcha (2003), who consider how exogenous differences in market completeness influence investor welfare.

[13]While the Hirshleifer (1971) effect and our risk-sharing channel both affect the ability of investors to share risk, the two mechanisms are distinct. The Hirshleifer (1971) effect refers to the phenomenon where the introduction of public information destroys risk-sharing opportunities. In contrast, our risk-sharing channel captures the fact that endogenous investment decisions can affect the effective completeness of

Our focus on welfare is also complementary to recent work by Bond and Garcia (2022), who show that while indexing may reduce price efficiency, it improves retail investor welfare due to improvements in risk sharing. Bond and Garcia (2022) also make substantial progress on characterizing welfare in CARA-Normal settings, which we leverage in our derivations. Tension between notions of firm profitability and welfare also appears in Goldstein and Yang (2022), who show that improvements in price informativeness increase producer profits due to better-informed real investment, but may harm welfare by destroying risk-sharing opportunities, similar to the Hirshleifer (1971) effect. Similar to our findings, other papers studying discrete investment choice also emphasize the importance of the firm's "default" investment decision in the absence of feedback.[14] Our analysis complements this earlier work by identifying a novel tension between managerial investment choices and welfare that is driven by how investment affects the ability of investors to use the stock to hedge risk.

Our paper is also related to the growing theoretical literature on ESG investing and climate risk.[15] Our work is most closely related to Pástor, Stambaugh, and Taylor (2021) and Goldstein et al. (2021). Pástor, Stambaugh, and Taylor (2021) show that green assets have lower costs of capital because investors enjoy holding them and they hedge climate risk. Goldstein et al. (2021) consider a model where traditional and green investors are informed about a firm's financial and ESG output, and demonstrate that this can lead to multiple equilibria. Our setting generates distinct predictions for green investment deci-

the market by directly changing the risk exposures of traded securities.

[14]For instance, Dow, Goldstein, and Guembel (2017) show that investors' equilibrium information acquisition hinges on whether the firm defaults to a risky or a riskless project. Davis and Gondhi (2019) show that complementarity in learning depends on both the default investment decision and on the correlation between the investment and assets in place. Goldstein, Schneemeier, and Yang (2020) study information acquisition in a feedback model with multiple sources of uncertainty. They show that investors seek to acquire the same information as management for positive NPV projects, but different information for negative NPV projects.

[15]Additional studies include Heinkel, Kraus, and Zechner (2001), Friedman and Heinle (2016), Chowdhry, Davies, and Waters (2019), Oehmke and Opp (2020), Pedersen, Fitzgibbons, and Pomorski (2021), and Jagannathan, Kim, McDonald, and Xia (2023).

sions and welfare by incorporating the feedback effect and considering green investment's impact on risk-sharing efficiency.

The production-based asset pricing literature beginning with Cochrane (1991) also considers how variation in firms' discount rates affects the relationship between investment, expected cash flows, and expected returns. This work assumes that a manager knows not only her project's risk factor loadings, but also the conditional risk-premia associated with these factors. However, in practice, factor risk-premia depend on dispersed information (e.g., investors' risk exposures and preferences), and so are difficult for managers to observe directly. Instead, prices are a crucial source of information about discount rates. Our analysis explores the implications of such managerial learning on investment decisions and investor welfare.

II. Model

We consider a model of feedback effects where the investment is exposed to a systematic risk. We present the model in the context of climate risk as it is a significant and direct application, but, as we discuss in the conclusion, our analysis has other applications.

Payoffs. There are four dates $t \in \{1, 2, 3, 4\}$ and two securities. The risk-free security is normalized to the numeraire. A share of the risky security is a claim to terminal per-share cash flows V generated by the firm at date four, and trades on dates one and three at prices P_1 and P_3, respectively.

Investors. There is a continuum of investors, indexed by $i \in [0, 1]$, with CARA utility over terminal wealth with risk aversion γ. Investor i has initial endowment of n shares of the risky asset and $z_i = Z + \zeta_i$ units of exposure to a non-tradeable source of income that has payoff of $-\eta_C$, where $Z \sim N\left(\mu_Z, \tau_Z^{-1}\right)$, $\zeta_i \sim N\left(0, \tau_\zeta^{-1}\right)$ and $\eta_C \sim N\left(0, \tau_\eta^{-1}\right)$ are independent of each other and all other random variables.[16] Investor i chooses trades

[16] We let $\tau_{(\cdot)}$ denote the unconditional precision and $\sigma^2_{(\cdot)}$ the unconditional variance of all random variables.

X_{it}, $t \in \{1, 3\}$ to maximize her expected utility over terminal wealth, which is given by

$$W_i = (n + X_{i1} + X_{i3})\, V - X_{i3} P_3 - X_{i1} P_1 - z_i \eta_C. \tag{1}$$

We interpret η_C as climate risk shocks, which reduce investor wealth and, consequently, utility.[17] Furthermore, Z captures investors' aggregate exposure to climate risk shocks, and μ_Z is the average exposure to climate risk. The natural restriction for this interpretation is $\mu_Z > 0$, which implies that shocks to the climate (i.e., positive innovations to η_C) have, in expectation, a negative impact on the average investor. In our analysis, we will focus on this restriction to clearly distinguish between projects that are positively vs. negatively exposed to the climate.

We further require the parameter restriction $1 > \gamma^2 \frac{1}{\tau_\eta} \left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta} \right)$ in order to ensure that the unconditional expected utility is finite. Intuitively, if this condition is violated, the climate payoffs $z_i \eta_C$ are sufficiently uncertain ex-ante that the expected utility diverges to $-\infty$. This is a natural condition that arises when characterizing ex-ante expected utility in any CARA-Normal model in which traders have random endowments and therefore the unconditional distribution of wealth involves a product of normally distributed random variables.[18] We summarize these restrictions in the following assumption, which is maintained throughout our analysis.

ASSUMPTION 1: *(i) The average exposure to climate risk μ_Z is positive, that is, $\mu_Z > 0$. (ii) Uncertainty about overall climate payoffs is sufficiently small that is,* $1 > \gamma^2 \frac{1}{\tau_\eta} \left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta} \right)$.

The firm. The firm generates cash flows per share $A \sim N\left(\mu_A, \tau_A^{-1}\right)$ from assets in place. In addition, the firm's manager decides whether to invest in a new project. The

[17] While, for concreteness, we refer to η_C as a non-tradeable payoff, it is equivalent to interpret it as a non-monetary climate shock to which investors are differentially exposed and that affects their utility directly.

[18] See, for instance, Assumption 1.1 in Rahi (1996), Assumption 1 in Marín and Rahi (1999), eq. (1.2) in Vayanos and Wang (2012), and eq. (8) in Bond and Garcia (2022), among others.

investment decision is binary and denoted by $k \in \{0,1\}$. The firm's cash flow per share, given an investment choice k, equals

$$V(k) = A + k\left(\theta + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I - c\right), \tag{2}$$

where $\theta \sim N\left(\mu_\theta, \tau_\theta^{-1}\right)$ and η_C, $\eta_I \sim N\left(0, \tau_\eta^{-1}\right)$ are independent of each other and other random variables, $\alpha \in [-1,1]$, and $c \geq 0$. The component θ reflects the learnable component of cash flows for the investment opportunity, η_C reflects shocks to the "climate" component of cash flows, and η_I reflects shocks to the "idiosyncratic" component of cash flows. The cost of investment is c, which is assumed to be non-negative.

The parameter α captures the extent to which the project's cash flows are correlated with climate risk shocks. When $\alpha = 0$, the new project's cash flows are uncorrelated with climate risk and so are not useful for hedging – we refer to such projects as "neutral" projects. When $\alpha > 0$, the project's cash flows are *higher* when climate outcomes are worse (η_C is higher), and so we refer to these projects as "green" projects. This increase in cash flows may be due to higher demand for the product (e.g., electric vehicles) or regulatory changes (e.g., higher taxes on greenhouse gas emissions) driven by adverse changes in the climate. Analogously, when $\alpha < 0$, the project's cash flows are *lower* when climate outcomes are worse, and so we refer to these projects as "brown" projects.[19]

Information and timing of events. Figure 1 summarizes the timing of events. At date one, all investors observe θ perfectly. Let $\mathcal{F}_{i1} = \sigma(\theta, z_i, P_1)$ and $\mathcal{F}_{i3} = \sigma(\theta, z_i, P_1, P_3, k)$ denote investor i's information set at the trading stages, with associated expectation, covariance, and variance operators, $\mathbb{E}_{it}[\cdot]$, $\mathbb{C}_{it}[\cdot]$ and $\mathbb{V}_{it}[\cdot]$, respectively. Then, investor i chooses trade X_i to maximize her expected utility:

$$\mathcal{W}_i \equiv \sup_{x\in\mathbb{R}} \mathbb{E}_{i1}\left[-e^{-\gamma W_i}\right]. \tag{3}$$

[19]Note that since positive realizations of η_C shocks increase marginal utility, green projects are *negatively* exposed to climate risk, while brown projects are *positively* exposed.

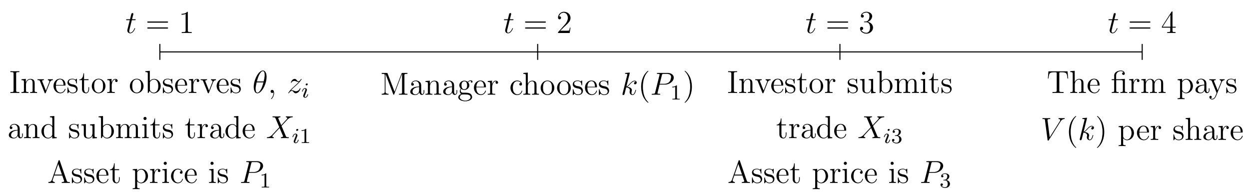

Figure 1. Timeline of events

The date one price is determined by the market clearing condition

$$\int_i X_{i1} di = 0. \tag{4}$$

At date two, the manager chooses investment k given her information. Importantly, the manager does not observe θ directly, but can condition on the information in the stock price P_1. Hence, her information set is $\mathcal{F}_m = \sigma(P_1)$. We consider two natural objectives for the manager. A **cash-flow maximizing** manager chooses investment to maximize her conditional expectation of the terminal cash-flow:

$$k(P_1) = \arg\max_k \mathbb{E}[V|\mathcal{F}_m], \tag{5}$$

while a **price-maximizing** manager chooses investment to maximize her conditional expectation of the date three price:

$$k(P_1) = \arg\max_k \mathbb{E}[P_3|\mathcal{F}_m]. \tag{6}$$

As we discuss below, these objectives lead to different investment rules and differ in their effect on investor welfare.

The date three price is again determined by the market clearing condition (4), evaluated at the $t=3$ trades X_{i3} that maximize investor expected utilities at that date. Note, however, that since the manager's investment decision is perfectly anticipated by investors at date one, and there are no additional shocks or information, we show that *in equilibrium* the date three price is equal to the date one price. At date four, the firm's terminal cash flows per share V are realized and paid to the investors.

Equilibrium. An equilibrium consists of trades $\{X_{i1}, X_{i3}\}$, prices $\{P_1, P_3\}$, and an investment rule $k(P_1)$ such that (i) the trades X_{it} maximize investor i's expected utility, given her information $\mathcal{F}_{it}$ and the investment rule $k(P_1)$, (ii) the investment rule $k(P_1)$ satisfies (5) or (6), and (iii) the equilibrium prices $\{P_1, P_3\}$ are determined by market clearing at dates one and three, respectively.

A. Discussion of Assumptions

The manager's objective. We consider two possible objectives for the manager: cash-flow maximization and price maximization. The former corresponds to the benchmark in the existing feedback effects literature and speaks to the incentives created by compensation linked to earnings and other accounting-based performance metrics. Such incentives are important in practice. As Guay, Kepler, and Tsui (2019) argue, the majority of CEO compensation plans include cash-based bonuses with such features.[20]

We contrast this objective with the case where the manager maximizes the expected share price, which in our setting is equivalent to maximizing the project's risk-adjusted net present value (NPV). This benchmark is consistent with prior work that builds upon the investment CAPM and q-theory of investment, which typically assumes that the firm invests to maximize its market capitalization (e.g., Cochrane (1991), Liu, Whited, and Zhang (2009)). This case speaks to the incentives created by equity compensation.

Considering the benchmarks separately allows us to provide a sharp comparison of the impact of feedback on investment decisions under these different objectives. Moreover, as we discuss further in Section IV, we show that neither objective alone necessarily maximizes welfare, even though in some settings, a combination of the two can be used to do so.

Two trading dates. The manager in our model both learns from the stock price

[20] Li and Wang (2016) and Bettis et al. (2018) provide evidence that the use of accounting performance based compensation has increased in recent years. Also, see De Angelis and Grinstein (2015), Bennett et al. (2017) and the survey by Edmans, Gabaix, and Jenter (2017) for evidence of the prevalence and importance of such incentives more generally.

and may seek to maximize the stock price. This requires a well-defined market price prior to the investment decision, from which the manager can learn, and a well-defined market price after the investment decision, over which we can specify the manager's maximization problem. However, our results are not an artifact of the two-date setting. As will be seen, because the trading dates are otherwise identical, *in equilibrium* the price is identical at both dates. In particular, one may be able to capture similar forces with a single trading date if the manager could simultaneously commit to a real investment schedule $k(P)$ at the same time that investors trade. In this case, one would need to appropriately generalize the noisy RE approach to formalize the notion of an investment schedule $k(P)$ that simultaneously maximizes P while also conditioning on P, and ensures that investors correctly anticipate this when making their trading decisions.

Green and brown projects. There is some disagreement in the literature regarding how different types of stocks' returns correlate with climate outcomes (e.g., see Giglio, Kelly, and Stroebel (2021)). In our model, we simply define green and brown projects as those which perform better and worse when given adverse climate shocks, respectively. As we shall see, green stocks carry a price premium, while brown stocks carry a discount, as a result of their exposure to climate risk. Thus, given the evidence in Bolton and Kacperczyk (2021) and Hsu, Li, and Tsou (2023), green (brown) firms can, for instance, be thought of as those with low (high) emissions. For tractability, we abstract from other sources of systematic risk and only focus on exposure to climate risk. However, we expect our results will be qualitatively similar in a multi-factor model in which the project's discount rate depends on its exposure to the relevant risk factors.

Homogeneous investor information. Since our primary focus is on *managerial* learning from prices, we shut down *investor* learning from prices by assuming that all investors share a common signal about fundamentals. The assumption simplifies the analysis and ensures that the financial market equilibrium does not exhibit multiplicity of the type studied by Ganguli and Yang (2009). Moreover, this assumption ensures that the traditional Hirshleifer (1971) effect does not arise in our setting, in contrast to results from the existing literature (e.g., Marín and Rahi (2000), Dow and Rahi (2003)). Finally, we have confirmed that our main results are qualitatively similar when investors have private signals and learn from the price.

Binary investment decision. The manager's investment decision in our model is binary and thus resembles exercising a real option. This discreteness implies that the firm is only useful for hedging when the manager invests, which makes our results stark. However, the economic forces underlying our results, including the nature of the equilibrium and our welfare analysis, carry over to more general investment decisions (subject to $k \geq 0$). For example, under a continuous investment choice, as the firm invests more, its cash flows are increasingly driven by the risk investors seek to hedge, as opposed to the firm's assets in place, which generates similar results to the ones we study.[21]

Divestment decisions. Since the investment decision is binary, one can equivalently apply our analysis to study divestment decisions. For instance, a firm with $k = 1$ and $\alpha < 0$ is a firm with an existing negative climate exposure (e.g., a traditional car manufacturer). In this case, a decision of $k = 0$ corresponds to divesting brown technology, or equivalently, investing in green technology that mitigates the firm's existing exposure (e.g., by transitioning to electric vehicle technology). However, the leading application we have in mind is a neutral firm (i.e., a firm with status quo $k = 0$) deciding whether to invest in a climate-exposed project.

Assets in place. The presence of assets in place is not qualitatively important for our results, but aids tractability by ensuring the firm's cash flows remain uncertain in the absence of investment. However, the assumption that assets in place are uncorrelated with climate risk is made for expositional clarity and can be relaxed.[22]

Aggregate demand for hedging. In our model, the average investor's exposure

[21]An earlier version of the paper considered more general investment decisions and found that the key economic forces that drive our results obtain in this more general setting.

[22]For instance, if A is positively correlated with η_C, one can decompose A as $A = \lambda\eta_C + \varepsilon_A$ for $\lambda > 0$ and $\mathbb{C}(\varepsilon_A, \eta_C) = 0$. In this case, the investment decision still changes the overall exposure of the firm to climate risk (i.e., λ with no investment vs. $\lambda + \alpha$ with investment), and the economic forces underlying our analysis continue to operate.

to climate shocks, and, consequently, their desire for hedging (as captured by Z) is stochastic. This can be interpreted as any feature that causes investors' concern for and desire to hedge climate change to vary over time. For instance, one can capture news that suggests climate change is accelerating as an increase in Z. It is further consistent with the empirical evidence that aggregate demand for climate hedges varies across time and with economic conditions. For instance, Bolton and Kacperczyk (2021) shows that the pricing of carbon-transition risk varies across countries and has risen over time. Moreover, Choi, Gao, and Jiang (2020) show that the price premium applied to green vs. brown stocks varies with weather patterns, and Alekseev et al. (2021) shows that weather patterns influence mutual-fund demand for climate-exposed stocks. As we discuss below, this variation generates changes in the discount rate that the manager applies to the project when making her investment decision.

Market incompleteness and hedging ability. Our model is one of incomplete markets. The firm's investment decision endogenously changes the completeness of the market by allowing investors to trade the climate risk factor (we refer to this as the "risk-sharing" channel; see Section IV). The starkness of this result is a consequence of discrete investment choice, but the economic mechanism arises more generally. Under a continuous investment choice, as the firm invests more, its cash flows are more sensitive to the risk investors seek to hedge vs. the assets in place. All else equal, this makes it less costly for investors to hedge their exposures using the stock, in the sense that they are exposed to less extraneous risk.[23]

A potential concern is that this channel would disappear if markets were complete and investors could trade η_C directly. In practice, markets appear to be far from complete: investors have different exposures to climate risk due to differences in their demographic characteristics and risk preferences (e.g., Ilhan et al. (2023)), and find this risk difficult to hedge (e.g., Pástor, Stambaugh, and Taylor (2021), Giglio, Kelly, and Stroebel (2021),

[23] An earlier version of the paper considered more general investment decisions and found that the key economic forces that drive our results obtain in this more general setting.

Krueger, Sautner, and Starks (2020)).[24] Indeed, Engle et al. (2020) find that a dynamic equity portfolio optimized to hedge climate risk is at most 30% correlated with news on such risk.[25]

Another potential concern is that the investment decision of a single firm will not have a meaningful impact on market completeness. A multi-firm model with discount rate variation and feedback effects is not analytically tractable. However, we expect that the impact of climate investment on market completeness would aggregate across firms and thus continue to be relevant in such a setting. That is, one can interpret our model as that of a representative firm in an industry or sector with correlated shocks to profitability and climate exposures. In practice, we expect that correlated investment choices (e.g., several automakers investing in EV technology) should affect investors' ability to hedge climate risk. Moreover, since stock prices do not fully reflect the risk-sharing benefit of climate-sensitive investment, our observation that managers fail to internalize this welfare externality would continue to hold in a multi-firm economy.

III. Equilibrium

In general, solving for an equilibrium with feedback effects is complicated by the fact that the asset price must simultaneously clear the market, be consistent with manager

[24]As Pástor, Stambaugh, and Taylor (2021) point out "[u]nanticipated climate changes present investors with an additional source of risk, which is non-traded and only partially insurable." Similarly, Giglio, Kelly, and Stroebel (2021) state "... many of the effects of climate change are sufficiently far in the future that neither financial derivatives nor specialized insurance markets are available to directly hedge those long-horizon risks. Instead, investors are largely forced to insure against realizations of climate risk by building hedging portfolios on their own." Finally, based on their survey evidence, Krueger, Sautner, and Starks (2020) state that "... many market participants, including institutional investors, find climate risks difficult to price and hedge, possibly because of their systematic nature, [...] and challenges in finding suitable hedging instruments."

[25]The multidimensional nature of climate risk may also contribute to market incompleteness. Different types of investments may be necessary to hedge the various dimensions of climate risk. For instance, green energy may serve as a hedge of carbon-transition risk, while green real estate may better hedge the potential for sea-level rise.

and investor beliefs, and be consistent with the anticipated real investment decision. We focus on equilibria of the following form. A **threshold equilibrium** is one in which:
(i) the price at both dates depends on the underlying random variables through a linear statistic, $s_p = \theta + \frac{1}{\beta}Z$, where β is an endogenous constant,
(ii) the price takes an identical piecewise linear form at both dates

$$P_3 = P_1 = \begin{cases} A_1 + B_1 s_p & \text{when } s_p > \overline{s} \\ A_0 & \text{when } s_p \leq \overline{s} \end{cases}, \tag{7}$$

where the price coefficients A_0, A_1, and B_1, and the threshold $\bar{s}$ are endogenous, and
(iii) the manager invests in the project if and only if $P_1(s_p) \neq P_1(\bar{s})$, that is, the share price is not equal to the constant no-investment price.

This type of equilibrium has an intuitive structure and several desirable properties. First, the equilibrium price is a generalized linear function of fundamentals: it depends on θ and Z only through a linear statistic $s_p = \theta + \frac{1}{\beta}Z$. Second, there is a price level $P_1(\bar{s})$ that reveals to the manager that the market anticipates she will not invest, and, consistent with this, she indeed finds it optimal not to invest. Thus, the price naturally is piecewise linear in s_p, increasing in s_p when the manager invests, and constant when she does not. These properties ensure the analysis is tractable and facilitate comparison to existing work.

As is common in feedback effects models, in general, there can exist multiple equilibria, each one characterized by a different investment policy. For instance, if the project is ex-ante sufficiently unprofitable, there is an equilibrium in which investors do not trade on their information and the manager relies on her ex-ante optimal choice, which is not to invest. These equilibria are sustained only because the price does not reveal any information when the market expects the manager not to invest. We focus on the equilibrium with the lowest threshold $\bar{s}$, that is, with the most investment. This equilibrium is the natural one as it would be the unique equilibrium if the price always revealed s_p, which would arise, for instance, if the firm's assets in place were correlated with the payoff on the project. This is a common feature of feedback effects models; Dow, Goldstein, and Guembel (2017) follows a similar approach to ours in choosing among equilibria, selecting the most informative equilibrium (see, for example, the discussion immediately following their Lemma 1). See also Dow and Gorton (1997), another feedback setting that generally features multiple equilibria.[26]

In the appendix, we formally solve the model by working backwards. We sketch the approach here. Given an investment decision $k \in \{0,1\}$ at date 2, investor i's beliefs about the asset payoff at $t=3$ are conditionally normal, with

$$\mathbb{E}_{i3}\left[V\left(k\right)\right] = \mu_A + k\left(\theta - c\right), \; \mathbb{C}_{i3}\left(V\left(k\right), \eta_C\right) = \frac{k\alpha}{\tau_\eta}, \text{ and } \mathbb{V}_{i3}\left(V\left(k\right)\right) = \frac{1}{\tau_A} + \frac{k^2}{\tau_\eta}, \tag{8}$$

and hence her optimal trade is

$$X_{i3} = \frac{\mathbb{E}_{i3}\left[V\left(k\right)\right] + \gamma \mathbb{C}_{i3}\left(V\left(k\right), \eta_C\right) z_i - P_3}{\gamma \mathbb{V}_{i3}\left(V\left(k\right)\right)} - (n + X_{i1}). \tag{9}$$

In turn, market clearing implies:

$$P_3 = \mu_A - \frac{\gamma}{\tau_A} n + k\left(\theta - c - \frac{\gamma}{\tau_\eta}\left(n - \alpha Z\right)\right). \tag{10}$$

This immediately implies that, in any equilibrium, regardless of the manager's objective function, we must have $s_p = \theta + \frac{\gamma}{\tau_\eta}\alpha Z$, or equivalently, $\beta = \frac{\tau_\eta}{\gamma\alpha}$.

At date two, the manager chooses whether to invest to maximize her objective, given her information set $\mathcal{F}_m = \sigma(P_1)$. Below, we characterize the equilibrium under cash-flow maximization and price maximization separately. As we will see, these equilibria differ

[26]Note that if investors in our model also learned noisy information from the equilibrium price (e.g., if they received heterogeneous private signals) then there would be a further potential source of non-uniqueness, even holding fixed the manager's investment policy. As shown by Pálvölgyi and Venter (2015), in standard static noisy rational expectations models investor learning from prices generally leads to a continuum of discontinuous equilibria in the financial market. Characterizing such equilibria in a version of our model with heterogeneous information would be an interesting problem for future work but is beyond the scope of the current paper.

only in the threshold price $P_1(\bar{s})$ above which the manager chooses to invest.

A. *Cash-Flow Maximization*

The manager's conditional expectation of cash flows, given s_p, is

$$\mathbb{E}\left[V\left(k\right)|s_p\right] = \mu_A + k\left(\mathbb{E}\left[\theta|s_p\right] - c\right), \text{ where} \tag{11}$$

$$\mathbb{E}\left[\theta|s_p\right] = \frac{\tau_\theta\mu_\theta + \tau_p\left(s_p - \frac{\gamma}{\tau_\eta}\alpha\mu_Z\right)}{\tau_\theta + \tau_p}, \text{ and } \tau_p = \tau_Z\left(\frac{\tau_\eta}{\gamma\alpha}\right)^2. \tag{12}$$

This implies that, if the manager were directly able to observe the signal s_p in all states of the world, her optimal investment rule would be:

$$k = \begin{cases} 1 & \text{when} \quad \mathbb{E}[\theta|s_p] > c \\ 0 & \text{when} \quad \mathbb{E}[\theta|s_p] \leq c. \end{cases} \tag{13}$$

The manager cannot always observe s_p because the price does not vary with s_p when the market expects her not to invest. However, in the threshold equilibrium with the most investment, this creates no additional difficulty for the manager, because the price is a sufficient statistic for s_p in making her investment decision. In this equilibrium, the investment threshold, which we refer to as $\bar{s}_C$, satisfies $\mathbb{E}[\theta|s_p = \bar{s}_C] = c$, so that the manager is precisely indifferent between investing and not investing when $s_p = \bar{s}_C$. Applying (11), we obtain

$$\bar{s}_C = c - \frac{\tau_\theta\left(\mu_\theta - c\right)}{\tau_p} + \frac{\gamma}{\tau_\eta}\alpha\mu_Z. \tag{14}$$

Given the conjectured price function, if the manager observes $P_1 = A_0$, she infers that, with probability 1, $s_p \leq \bar{s}_C$, and chooses not to invest. If she observes any $P_1 \neq A_0$, she infers the realized value of s_p, necessarily strictly greater than $\bar{s}_C$, and so she chooses to invest. Thus, in equilibrium, she is able to implement the same investment rule almost everywhere that she would if she directly observed s_p.

Finally, stepping back to $t = 1$, note that the manager's investment decision is a deterministic function of P_1. Thus, investors can anticipate the manager's investment decision by observing the date one price. In turn, *in equilibrium* investors can perfectly anticipate P_3 and therefore the equilibrium price at $t = 1$ must satisfy $P_1 = P_3$ in order for the market to clear. Following this reasoning, the next proposition characterizes the threshold equilibrium with maximum investment.

PROPOSITION 1: *Suppose the manager maximizes expected cash flows. In the investment-maximizing threshold equilibrium, equilibrium prices are*

$$P_1 = P_3 = \mu_A - \frac{\gamma n}{\tau_A} + k\left(s_p - c - \frac{\gamma}{\tau_\eta} n\right), \tag{15}$$

and the manager's investment decision is

$$k = \mathbf{1}\left\{P_1 \neq \mu_A - \tfrac{\gamma}{\tau_A} n\right\}, \tag{16}$$

where $s_p \equiv \theta + \frac{\gamma\alpha}{\tau_\eta} Z$, $\tau_p \equiv \left(\frac{\tau_\eta}{\gamma\alpha}\right)^2 \tau_Z$, *and* $\bar{s}_C \equiv c - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p} + \frac{\gamma}{\tau_\eta}\alpha\mu_Z$.

It is worth noting that investors' beliefs about the asset payoff remain normal given their information set in all states of the world, since the manager's investment decision is determined by the date one price P_1. This ensures that the equilibrium is tractable.

For a cash-flow maximizing manager, discount rate variation (i.e., shocks to Z) adds noise to the information about θ that is relevant for her investment decision. The above proposition clarifies how the project's greenness affects the manager's inference about cash flows from the price. First, an increase in the project's sensitivity to climate risk (i.e., higher $|\alpha|$) makes the price less informative about cash flows (i.e., it decreases forecasting price efficiency) – this is apparent from the expression for τ_p. Second, since $\mu_Z > 0$, an increase in greenness α leads to a higher threshold $\bar{s}_C$. Intuitively, the manager corrects for the fact that a green project provides a hedge to investors and so, fixing investors' cash flow information θ, tends to have a higher price.

Together, these effects reflect that for a cash-flow maximizing manager, an increase in climate sensitivity makes the price a noisier and more biased signal. As we show in the next subsection, this is no longer the case when the manager chooses investment to maximize the expected price.

B. Price Maximization

We can follow similar steps to derive the equilibrium when the manager maximizes the firm's stock price. Recall that the date three market clearing price can be expressed as

$$P_3 = \begin{cases} \mu_A - \frac{\gamma}{\tau_A} n & \text{when } k = 0 \\ \mu_A - \frac{\gamma}{\tau_A} n + k\left(s_p - c - \frac{\gamma}{\tau_\eta} n\right) & \text{when } k = 1 \end{cases}. \tag{17}$$

This implies that, if the manager observed s_p in all states, she would invest when $s_p > c + \frac{\gamma}{\tau_\eta} n$. Similarly to the cash-flow maximization case, in the equilibrium with maximum investment, the price reveals s_p whenever knowing the value of s_p would lead the manager to invest. Thus, she is able to implement the same investment rule that she would if she could directly observe s_p, and so the investment threshold satisfies:

$$\bar{s} = \bar{s}_P = c + \frac{\gamma}{\tau_\eta} n. \tag{18}$$

Finally, the manager's investment decision is again known given the price at date one, so that no new information arrives between dates one and three, and P_1 and P_3 must be equal. The following proposition formally establishes these results.

PROPOSITION 2: *Suppose the manager maximizes the expected date three price. In the investment maximizing threshold equilibrium, equilibrium prices are*

$$P_1 = P_3 = \mu_A - \frac{\gamma}{\tau_A} n + k\left(s_p - c - \frac{\gamma}{\tau_\eta} n\right), \tag{19}$$

and the manager's investment decision is

$$k = \mathbf{1}\left\{P_1 \neq \mu_A - \tfrac{\gamma}{\tau_A} n\right\} = \mathbf{1}\left\{s_p > \bar{s}_P\right\}, \tag{20}$$

where $s_p = \theta + \frac{\gamma}{\tau_\eta}\alpha Z$*, and* $\bar{s}_P \equiv c + \frac{\gamma}{\tau_\eta}n$.

The manager's optimal investment takes the form of a NPV rule, whereby she invests if and only the statistic

$$NPV \equiv s_p - \bar{s} = \underbrace{\theta - c}_{\text{cash flows}} \underbrace{- \frac{\gamma}{\tau_\eta}(n - \alpha Z)}_{\text{discount rate}} \tag{21}$$

is greater than zero. The first term, $\theta - c$, reflects the expected cash flows from the project, net of investment costs – this captures the "cash-flow news" contained in the price. The second term, $-\frac{\gamma}{\tau_\eta}(n - \alpha Z)$, reflects a discount due to the risk premium investors demand for holding shares of the stock. We refer to this as "discount rate news" because it reflects variation in the project's impact on price that is driven by factors other than its expected cash flows. Consistent with intuition, the discount is higher (the NPV is lower) when the firm is larger (i.e., n is higher) because investors have to bear more aggregate risk. Moreover, the discount is lower (higher) for green (brown, respectively) projects when $Z > 0$.[27] This is because green projects reduce investors' exposure to (negative) climate shocks, while brown projects exacerbate it.

While the cash-flow and discount rate news in prices are not separately observable to the manager, they both factor into her decision of whether to invest because they both influence how the project will impact the date three price. In principle, this implies that the manager must learn about both from the date one price, that is, she must separately compute $\mathbb{E}[\theta|P_1]$ and $\mathbb{E}[Z|P_1]$. However, her inference problem takes a transparent form in our setting because the price signal she conditions on and the objective she intends to maximize put the same (relative) weights on θ and Z. In particular, the equilibrium date one and date three prices put the same weights on θ and Z. This implies that the manager does not need to *separately* update on θ and Z to determine whether investment will lead to a higher price. Instead, she can directly infer the relevant combination $\theta + \frac{\gamma}{\tau_\eta}\alpha Z$ from

[27] It is possible that $Z < 0$ in our model so that brown projects are priced at a premium. However, the probability of this outcome can be made arbitrarily small by setting μ_Z and τ_Z appropriately.

the date one price.[28]

Note that this simplification of the manager's learning problem in the case of price maximization is a derived result, not an assumption. We show in Internet Appendix C that this result extends to the case in which investors are endowed with dispersed, private noisy signals about θ and learn about θ from prices, similar to Hellwig (1980). The reason is that, in this setting, date one and three prices continue to place the same weights on cash flow and discount rate news. However, it need not arise when the date one price puts different relative weights on θ and Z than the manager's objective does. For instance, the simplification does not obtain if the manager maximizes a combination of expected cash flows (or earnings) and expected price.

Similarly, if a public signal about η_C becomes available before trade at date three, but after the date two investment decision, then the relative weights on θ and Z will differ across the two dates. We focus on the simpler specification without a public signal in our model because it is a natural benchmark that transparently illustrates the main economic mechanisms that result from the manager learning about discount rates from the price. In richer settings, we expect similar forces to apply, although the analysis would be less transparent.

The above also clarifies that while feedback plays an important role in the equilibrium, the equilibrium of our specific setting turns out to be identical to one in which the manager directly observes θ and Z. As such, our analysis highlights an important equivalence between a class of feedback-effects model where the manager maximizes the expected price of the firm, and traditional, production-based asset pricing models in which the manager is assumed to exogenously know the profitability and discount rate of the project she is considering. To the extent that, in practice, managers rely on prices to learn about discount rate information, our analysis of the price maximization benchmark provides a micro-foundation for the latter class of models. It is worth noting that since the project's

[28]One may be able to capture similar forces with a single trading date if the manager could simultaneously commit to a real investment schedule $k(P)$ to maximize the equilibrium price P at the same time that investors trade.

discount rate depends on investors' exposures to climate risk, which are privately known and dispersed across investors, it is not clear how the manager could observe this directly without conditioning on prices.

C. Probability of Investment

In this section, we compare how feedback from prices affects investment decisions under the two managerial objectives. We begin by characterizing the likelihood of investment with cash-flow maximization.

PROPOSITION 3: *Suppose the manager maximizes $\mathbb{E}[V|\mathcal{F}_m]$. In equilibrium, the unconditional probability of investment is given by*

$$\Pr\left(s_p > \bar{s}_C\right) = \Phi\left(\frac{\mathbb{E}[s_p] - \bar{s}_C}{\sqrt{\mathbb{V}[s_p]}}\right), \tag{22}$$

where $\mathbb{E}[s_p] = \mu_\theta + \frac{\gamma\alpha}{\tau_\eta}\mu_Z$, $\mathbb{V}[s_p] = \frac{1}{\tau_\theta} + \frac{1}{\tau_p}$, $\tau_p = \left(\frac{\tau_\eta}{\gamma\alpha}\right)^2 \tau_Z$, and $\Phi(\cdot)$ denotes the CDF of a standard normal random variable. The probability of investment:

(i) increases with ex-ante profitability $\mu_\theta - c$;

(ii) does not depend on firm size n or the average climate risk exposure μ_Z;

(iii) increases with τ_θ and $|\alpha|$ and decreases with τ_Z if the project is ex-ante profitable (i.e., $\mu_\theta - c > 0$); and

(iv) decreases with τ_θ and $|\alpha|$ and increases with τ_Z if the project is ex-ante unprofitable (i.e., $\mu_\theta - c < 0$).

Consistent with intuition, the probability of investment increases with the ex-ante profitability $\mu_\theta - c$ of the project. Moreover, since the manager's objective is to maximize expected cash flows, the firm's systematic risk (e.g., n) and investors' aggregate exposure to climate risk (i.e., μ_Z) do not affect the likelihood of investment.

The above also clarifies for a cash-flow maximizing manager, variation in the project's risk premium, as captured by $\frac{\gamma\alpha}{\tau_\eta}Z$, generates noise in her price signal. In fact, a higher exposure to climate risk (i.e., higher $|\alpha|$) serves to make the price a noisier signal about cash flows, and so the manager is more likely to invest in line with her prior beliefs. This implies that for ex-ante profitable projects (i.e., if $\mu_\theta - c > 0$), the manager is more likely to invest in climate-exposed projects than in climate-neutral ones. On the other hand, for unprofitable projects, an increase in climate exposure leads to a decrease in the likelihood of investment.

The above results are largely consistent with traditional feedback effects models in which the manager maximizes expected cash flows, and so treats non-cash-flow variation in prices as noise. As we show next, this is no longer the case when the manager maximizes the share price.

PROPOSITION 4: *Suppose the manager maximizes $\mathbb{E}\left[P_3|\mathcal{F}_m\right]$. In equilibrium, the unconditional probability of investment is given by*

$$\Pr\left(s_p > \bar{s}_P\right) = \Phi\left(\frac{\mathbb{E}\left[s_p\right] - \bar{s}_P}{\sqrt{\mathbb{V}\left[s_p\right]}}\right). \tag{23}$$

The probability of investment:

(i) increases with ex-ante profitability $\mu_\theta - c$;

(ii) decreases with firm size n;

(iii) increases with μ_Z for green firms (i.e., $\alpha > 0$), but decreases with μ_Z for brown firms (i.e., $\alpha < 0$);

(iv) increases with τ_θ and τ_Z if and only if $\mathbb{E}[s_p] - \bar{s}_P = \mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta} > 0$; and

(v) decreases with greenness α if and only if $\left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} - \frac{\tau_\eta \tau_Z}{\gamma\alpha}\frac{1}{\tau_\theta}\mu_Z\right)\operatorname{sgn}(\alpha) > 0$.

Consistent with intuition, the proposition establishes that the probability of investment increases in the expected NPV of the project $\mathbb{E}[s_p] - \bar{s}_P$ and decreases (increases) with the variance of the price signal $\mathbb{V}[s_p]$ when $\mathbb{E}[s_p] - \bar{s}_P > 0$ ($\mathbb{E}[s_p] - \bar{s}_P < 0$, respectively). This directly implies parts (i)-(iv) of the proposition. From equation (21), we know that the expected NPV increases with expected profitability $\mu_\theta - c$, decreases with firm size n, and increases with μ_Z if and only if $\alpha > 0$, which implies (i)-(iii). Similarly, part (iv) follows because an increase in τ_θ or τ_Z leads to a reduction in the variance of the price signal $\mathbb{V}[s_p]$, which leads to more investment when the expected NPV is positive (i.e., $\mathbb{E}[s_p] - \bar{s}_P > 0$), but less investment when it is negative.

Part (v) of the proposition shows that the project's sensitivity to the risk factor, α, has a nuanced impact on the likelihood that the manager invests. An increase in α has two, potentially-offsetting, effects. First, an increase in α increases the expected NPV $\mathbb{E}[s_p] - \bar{s}_P$ because it reduces the on-average discount due to climate risk. Since the manager's objective is to maximize the share price, this implies that all else equal, investment is likelier in green projects than brown projects. We refer to this as the "expected NPV" channel.

Second, an increase in the magnitude of the project's climate exposure $|\alpha|$ increases the variance of the price signal $\mathbb{V}[s_p]$, which in turn makes the conditional NPV of the project more variable. All else equal, this makes it more likely that a project with negative expected NPV will be desirable ex-post (i.e., increases the likelihood that the investment option will be "in the money"), and so increases the likelihood of investment of such a project. Similarly, it reduces the likelihood that a project with positive expected NPV will be ex-post desirable, and so decreases the likelihood of investment in such a project. We refer to this as the "variance of NPV" channel. The overall effect of α depends on the relative magnitude of these two channels.

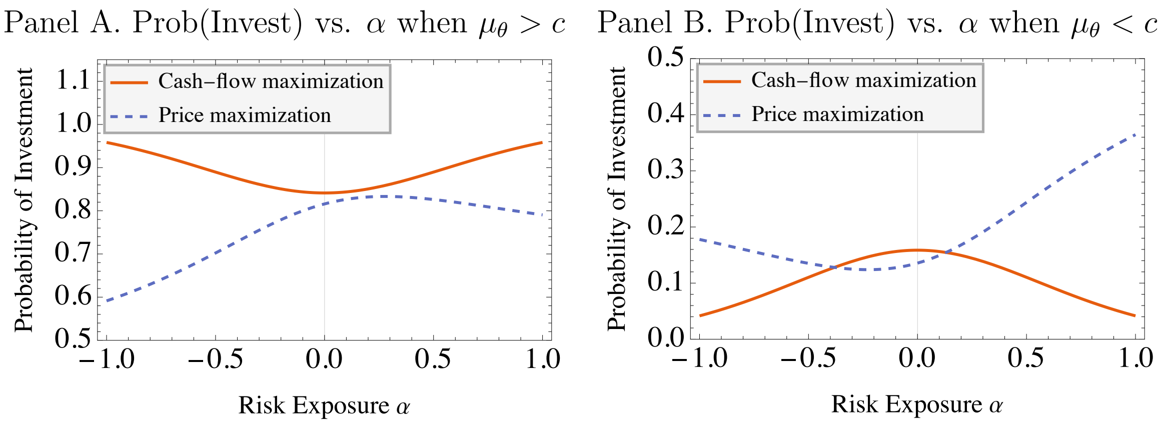

Figure 2. Probability of Investment. This figure compares the probability that the firm invests investment as a function of α and μ_Z under the cash-flow and price-maximization benchmarks. Unless otherwise mentioned, the parameters employed are: $\tau_\theta = \tau_\eta = \tau_A = \gamma = 1$; $\tau_Z = \mu_Z = 0.5$; $n = 0.1$. The left-hand plot depicts results for both projects that have positive and negative ex-ante NPV. In the solid (dashed) lines, we consider price maximization (cash-flow maximization). In the left-hand (right-hand) plots, we set $\mu_\theta - c = 1$ ($\mu_\theta - c = -1$), which implies that the project is ex-ante desirable (undesirable) in both the cash-flow and price-maximization cases, that is, $\forall \alpha \in [-1, 1]$, $\mathbb{E}[s_p] > \bar{s}_P$ and $\mathbb{E}[s_p] > \bar{s}_C$ ($\mathbb{E}[s_p] < \bar{s}_P$ and $\mathbb{E}[s_p] < \bar{s}_C$).

As Figure 2 illustrates, this is in sharp contrast to the case where the manager maximizes cash flows. The figure compares the probability of investment as a function of climate exposure α for the two managerial objectives. Consistent with the above results, for ex-ante profitable projects (i.e., $\mu_\theta > c$) an increase in $|\alpha|$ leads to more investment under cash-flow maximization but can lead to less investment under price maximization – see panel A. In contrast, for ex-ante unprofitable projects (i.e., $\mu_\theta < c$), panel B illustrates that the opposite results hold.

The characterization of the equilibrium thresholds under the two managerial objectives immediately gives us the following result.

COROLLARY 1: *Cash-flow maximization leads to more investment than price maximization (i.e., $\bar{s}_P > \bar{s}_C$) if and only if:*

$$\frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z) > -\frac{\tau_\theta}{\tau_p}\left(\mu_\theta - c\right). \tag{24}$$

In particular, cash-flow maximization leads to "over-investment" relative to price maximization when the project is expected to be highly profitable ex-ante (i.e., $\mu_\theta - c$ is sufficiently high), investors' expected climate exposures are small (i.e., μ_Z is low), or the project is sufficiently brown (i.e., α is small or negative).[29]

More generally, the price and cash-flow maximization benchmarks can be thought of as lying on the opposite end of a spectrum. While we focus on these two extremes to simplify the exposition and clarify the intuition for our results, we expect that a manager's decision reflects a weighted average of both considerations in practice. Our analysis suggests that when the manager focuses more on prices and less on cash flows, she will treat prices as less noisy signals and place more weight on them when investing.

Our results also imply that how shareholders or regulators can better incentivize managers to pursue green investment depends on the ex-ante desirability of the projects. For ex-ante unprofitable projects (i.e., $\mu_\theta < c$), tilting the manager's incentives towards price

[29]Note that when investors are risk-neutral (i.e., $\gamma = 0$), $\bar{s}_P = \bar{s}_C = c$ and so the investment rules coincide.

maximization (e.g., by proving more short-term, price-sensitive compensation) increases the likelihood of investing in green projects. This is likely to apply to speculative investments in green technology, which may be ex-ante unprofitable on a purely cash-flow basis. On the other hand, for ex-ante profitable projects (i.e., $\mu_\theta > c$), making compensation more sensitive to accounting-based measures of expected cash flows (e.g., earnings) tilts the manager towards cash-flow maximization, and consequently, increases investment in green projects.

IV. Welfare

In this section, we explore the relationship between feedback, investment, and investor welfare. We begin by characterizing the channels through which investment affects investor welfare in Section IV.A. In Section IV.B, we consider a special case in which investors have homogeneous climate exposures (i.e., $\tau_\zeta \to \infty$). This allows us to explicitly characterize the welfare-maximizing price-contingent investment rule and compare it to price-maximizing and cash-flow maximizing rules. In Section IV.C, we re-introduce heterogeneity in risk-exposures and show how the manager's use of the information in price may harm investor welfare.

A. *The Impact of Investment on Welfare*

Existing models of feedback effects focus on the impact that feedback has on a firm's expected cash flows. In many such models, investors are risk neutral so that maximizing expected cash flows aligns with welfare maximization.[30] However, in our model, investor risk aversion implies that investment has multiple, potentially-offsetting effects on investor welfare, due to the riskiness of the project and the stock's usefulness as a hedge.

Because investors are ex-ante symmetric, the ex-ante expected utility of an arbitrary investor is an unambiguous measure of welfare:

$$\mathcal{W} \equiv \mathbb{E}\left[-e^{-\gamma W_i(k(s_p))}\right] \tag{25}$$

[30] Section I discusses notable exceptions, like Dow and Rahi (2003).

$$= \Pr(k=1)\,\mathbb{E}\left[-e^{-\gamma W_i(1)}|k=1\right] + \Pr(k=0)\,\mathbb{E}\left[-e^{-\gamma W_i(0)}|k=0\right], \tag{26}$$

where

$$W_i(k) = \begin{cases} X_i\left(V(1) - P\right) + nV(1) - z_i\eta_C & k=1 \\ nV(0) - z_i\eta_C & k=0 \end{cases}. \tag{27}$$

Proposition IA2 in Internet Appendix A characterizes this expression in closed form. However, to understand the relevant economic forces, it is helpful to study the simpler special case in which investment is *fixed* at arbitrary level k, in which case the model reduces to a standard unconditionally linear-normal form. In this case, we have

$$\mathbb{E}\left[-e^{-\gamma W_i(k)}\right] = -e^{-\gamma\, CE(k)}, \tag{28}$$

where the certainty equivalent $CE(k)$ can be expressed, after grouping terms, as

$$\begin{aligned} CE(k) = {} & \underbrace{\mathbb{E}[V(k)]n}_{\text{Cash flow channel}} \underbrace{-\frac{\gamma}{2}\left(\frac{1}{\tau_A} + k^2\left(\frac{1}{\tau_\theta} + \frac{1-\alpha^2}{\tau_\eta}\right)\right)n^2}_{\text{Non-climate risk channel}} \\ & \underbrace{-\frac{\gamma}{2}\frac{1}{\tau_\eta}\left(\mu_Z - k\alpha n\right)^2}_{\text{Climate risk channel}}(1+\Gamma) - \frac{1}{\gamma}\log(D(k)) \end{aligned} \tag{29}$$

where

$$D(k) = \underbrace{\sqrt{\frac{\frac{1}{\tau_A} + k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A} + k^2\left(\frac{1}{\tau_\eta} + \frac{1}{\beta^2\left(\tau_Z+\tau_\zeta\right)}\right)}}}_{\text{Value of information}}\sqrt{\frac{\Gamma}{\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)}}, \tag{30}$$

and

$$\Gamma(k) \equiv \frac{\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)}{1 - \gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)\left(1 - \underbrace{\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)} \times \left(\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^2}_{\text{Risk-sharing channel}}\right)}. \tag{31}$$

We have explicitly labeled all five terms in these expressions that depend on the investment choice k and will discuss them in turn.

- The **cash flow channel** reflects that investment affects the investor's expected wealth via their ownership of n shares. Investment increases (decreases) welfare through this channel when the project's expected cash flows are positive (negative).

- The **non-climate risk channel** reflects that the investment increases investors' exposure to non-climate risks via the θ and η_I shocks.

- The **climate risk channel** captures the fact that the investment affects investors' aggregate exposure to climate shocks. The average investor's total climate exposure is $\mu_Z - k\alpha n$, reflecting both the direct exposure and the exposure through ownership of the stock. When the direct exposure is sufficiently large (i.e., $\mu_Z > n$), investment in green ($\alpha > 0$) projects mitigates aggregate climate exposure and consequently raises welfare, while investment in brown ($\alpha < 0$) projects amplifies aggregate climate exposure and reduces welfare. This channel is further scaled by the term $1+\Gamma$, which reflects *uncertainty* about the exposure to climate risk. When investors' total exposure to climate risk $Z + \zeta_i$ is constant (i.e., $\tau_Z, \tau_\zeta \to \infty$), we have $\Gamma = 0$. However, when investors face uncertainty about their exposure from either source, $\Gamma > 0$, which amplifies the disutility of climate exposure.

- The **risk-sharing channel** reflects that the project enables investors to share their idiosyncratic exposures to climate risk, ζ_i, by trading the stock. All else equal, investment improves welfare through this channel. By sharing risk, investors reduce the dispersion in their climate exposures, reducing the effect of uncertainty about exposures, Γ.

  The overall amount of risk-sharing reflects both the effectiveness of the stock as a hedging instrument (i.e., the correlation of the stock return with climate risk), and the proportion of climate exposures that are shared (i.e., the proportion of climate exposures that are idiosyncratic, ζ_i):

$$\text{Risk-sharing channel} = \underbrace{\frac{k^2\alpha^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}}_{\substack{\text{Hedging effectiveness of stock}\\ =\text{Corr}^2(V-P,\eta_C|z_i)}} \times \underbrace{\left(\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^2}_{\text{\% shareable climate exposure}}. \tag{32}$$

- The **value of information channel** captures the fact that investors' information about cash flows renders the stock more useful in hedging. Observing θ increases the conditional correlation between the stock's payoff and η_C. Moreover, this effect is only relevant when the project is undertaken, and so disappears when $k = 0$. This takes a familiar form of the ratio of investors' conditional variance of the asset return with and without conditioning on θ.[31]

Importantly, when the manager chooses investment to maximize the expected price, she fails to appropriately account for the impact of her decision on the other components of welfare, as we discuss next.

B. *Homogeneous Risk Exposures*

We begin with a special case of our model in which all investors have homogeneous exposures to climate risk. In this case, we can explicitly characterize the welfare-maximizing price-contingent investment rule, as we show in the following Proposition.[32]

[31]In our model, investors are endowed with information. However, this term still captures the improvement in utility as a result of observing θ relative to being uninformed. Specifically, given fixed k, this ratio can be represented as $\frac{\mathbb{V}(V|\theta,z_i,P)}{\mathbb{V}(V-P|z_i)}$, which reflects the proportional improvement in expected utility from conditioning on θ, z_i, and P vs. z_i alone. The welfare expressions in Bond and Garcia (2022) include a similar term, which they further decompose into a product of the classic value of cash-flow information, $\frac{\mathbb{V}(V|\theta,z_i,P)}{\mathbb{V}(V|z_i,P)}$, and the value of providing vs. demanding liquidity (i.e., using a price-contingent schedule vs. not), $\frac{\mathbb{V}(V|z_i,P)}{\mathbb{V}(V-P|z_i)}$. Because these effects are not a primary focus of our analysis we choose to concisely represent them in a single term.

[32]To streamline the presentation and derivation, we formulate the investment rule as s_p-contingent. However, as in the baseline model, it can be implemented as a price-contingent rule. Intuitively, with probability 1, the equilibrium price reveals s_p when investors anticipate that the investment is undertaken

PROPOSITION 5: *Suppose that investors have identical exposures to climate risk (i.e., $\tau_\zeta \to \infty$). Then, the welfare-maximizing s_p-dependent investment policy is*

$$\arg\max_{k\in\{0,1\}} \mathcal{W}(k; s_p) = \mathbf{1}\{s_p > \bar{s}_W\}, \tag{33}$$

where $\bar{s}_W \equiv c + \frac{1}{2}\frac{\gamma}{\tau_\eta}n$. Moreover,

(i) cash-flow maximization leads to under-investment relative to welfare maximization if and only if

$$\bar{s}_C - \bar{s}_W = \frac{\gamma}{\tau_\eta}\alpha\mu_Z - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p} - \frac{1}{2}\frac{\gamma}{\tau_\eta}n > 0, \tag{34}$$

but over-investment otherwise, and

(ii) price maximization always leads to under-investment relative to welfare maximization, since $\bar{s}_P - \bar{s}_W = \frac{1}{2}\frac{\gamma}{\tau_\eta}n$.

As we show in the appendix, the expressions for welfare simplify considerably when investors have homogeneous exposures to climate risk because there is no risk-sharing trade in equilibrium. Consequently, neither the risk-sharing channel nor the value of information channel are operational.

Proposition 5 clarifies that the cash-flow-maximizing investment rule does not maximize welfare even in this special case. The cash-flow-maximizing rule over-weights expected cash flows, but under-weights both non-climate risk and climate risk, compared to the welfare-maximizing rule. Hence, cash-flow maximization tends to lead to *under-investment* in green projects, but *over-investment* in brown projects that are ex-ante profitable, relative to welfare maximization.

Somewhat surprisingly, we find that the price-maximizing rule also leads to under-investment relative to the welfare-maximizing rule, whenever $n > 0$. The difference between the two thresholds stems from the fact that, while welfare depends on the average

and does not reveal s_p otherwise. This allows one to directly map the s_p-contingent investment rule to an equivalent price-contingent rule in which the manager does not invest if she observes a price realization that anticipates no investment, $P_1 = \mu_A - \frac{\gamma}{\tau_A}n$, and invests otherwise.

risk borne by investors, the price reflects the marginal disutility from the risk of holding the last outstanding share of the firm. Since the marginal disutility from holding the last share is higher than the average disutility from bearing the risk of all shares investors hold, the price-maximizing rule leads to under-investment relative to the welfare-maximizing rule. However, it is worth noting that this difference disappears when the per-capita endowment of shares per firm becomes arbitrarily small (i.e., $n \to 0$).

The point that decisions based on prices may be socially sub-optimal because prices reflect marginal and not average valuations was first raised by Spence (1975) in the context of a monopolist's choice of product quality. He shows that the chosen quantity may be too high or too low, from the perspective of social welfare. The intuition for this result foreshadows ours: quality is chosen based on information contained in a good's price, which reflects the valuation of the marginal consumer, while welfare depends on information about the average consumer.[33]

Proposition 5 also implies that, in this benchmark, one can implement the welfare-maximizing investment rule by inducing the manager to maximize a weighted average of cash flows and the date three price, as summarized by the following result.

PROPOSITION 6: *Suppose that the manager maximizes a weighted average of expected price and expected cash flows:*

$$k(P_1) = \arg\max_k \, \delta \, \mathbb{E}[P_3|P_1] + (1-\delta)\mathbb{E}[V|P_1], \tag{35}$$

where

$$\delta = \frac{\frac{\tau_\theta}{\tau_\theta+\tau_p}\left(\mu_\theta - c\right) - \frac{\tau_p}{\tau_\theta+\tau_p}\frac{\gamma}{\tau_\eta}\left(\alpha\mu_Z - \frac{1}{2}n\right)}{\frac{\tau_\theta}{\tau_\theta+\tau_p}\left(\mu_\theta - c\right) - \frac{\tau_p}{\tau_\theta+\tau_p}\frac{\gamma}{\tau_\eta}\left(\alpha\mu_Z - \frac{1}{2}n\right) + \frac{1}{2}\gamma\frac{1}{\tau_\eta}n}. \tag{36}$$

Then, in the maximum-investment threshold equilibrium, the manager invests if and only

[33]An analogous difference is also highlighted by Levit, Malenko, and Maug (2022) who show that while prices are determined by the valuation of the marginal investor, valuation is determined by the valuation of the average (post-trade) shareholder in their setting. Bernhardt, Liu, and Marquez (2018) highlight a similar difference in the context of takeovers.

if $s_p > \bar{s}_W$*. When the project is unexposed to the climate* ($\alpha = 0$)*, we have that* $\delta = \frac{1}{2}$*. Moreover,* $\delta \in (0,1)$ *if and only if* $\frac{\tau_\theta}{\tau_\theta+\tau_p}(\mu_\theta - c) > \frac{\tau_p}{\tau_\theta+\tau_p}\frac{\gamma}{\tau_\eta}\left(\alpha\mu_Z - \frac{1}{2}n\right)$*.*

Recall that price maximization leads to under-investment relative to welfare maximization, but cash-flow maximization can lead to over-investment for brown ($\alpha < 0$), ex-ante profitable ($\mu_\theta > c$) projects. In such cases, the above result implies that there exists a $\delta \in (0,1)$ such that a weighted-average objective of the form in (35) leads to the manager to follow a welfare-maximizing investment rule. In particular, by incentivizing the manager to maximize a weighted average of expected price and expected cash-flows where δ is set as in (36), investors can ensure that the manager's investment rule maximizes ex-ante welfare. This provides a natural justification for compensation schemes that use a combination of price-based and earnings-based incentives, even though such incentives lead to lower expected stock prices than purely price-based incentives.

However, the above result also implies that such compensation schemes may not be appropriate when the manager is considering whether to invest in green projects. For instance, consider a green project with $\mu_\theta = c$. If $2\alpha\mu_Z > n > \alpha\mu_Z$, Proposition 6 shows that welfare maximization requires the manager to place a negative weight on the price (i.e., $\delta < 0$). On the other hand, if $n < \frac{2\alpha\mu_Z\tau_\eta^2\tau_z}{\alpha^2\gamma^2\tau_\theta+2\tau_\eta^2\tau_z}$, welfare maximization requires the manager to place a negative weight on cash flows (i.e., $\delta > 1$). Intuitively, both price maximization and cash-flow maximization lead to under-investment, so the welfare-maximizing combination puts a *negative* weight on the objective that leads to greater under-investment. However, such negative sensitivity to prices or earnings is difficult to implement in practice. Moreover, traditional compensation schemes that put positive weights on prices and earnings-based incentives might actually lead to lower investor welfare relative to exclusively focusing on one type of objective or the other.

C. Heterogeneous Risk Exposures

The previous discussion illustrates that even when investors have identical climate exposures, neither cash-flow maximization nor price maximization are generally equivalent to welfare maximization. These differences are further amplified when investors have heterogeneous climate exposures.

When the manager maximizes expected cash flows, she does not account for either the non-climate risk or climate risk channels. Heterogeneity in investors' climate exposures amplifies the impact that her neglect of the climate risk channel has on welfare. Intuitively, as can be seen from the expression for the certainty equivalent in (29), this heterogeneity amplifies the disutility that climate risk creates. Specifically, one can show that the amplification factor Γ increases in τ_ζ^{-1}, and so the project's impact on welfare via aggregate climate risk rises with τ_ζ^{-1}.

To gain intuition for the price-maximization case, note that the share price $P(k)$ can be expressed as

$$P(k) = \mathbb{E}_i [V] + \gamma Z \mathbb{C}_i [V, \eta_C] - \gamma n \mathbb{V}_i [V] . \tag{37}$$

This expression reveals that the price reflects the aggregate climate exposure, Z, but does not reflect the diversity in climate exposures (i.e., τ_ζ^{-1}), which determines the gains from sharing climate risk (i.e., the risk-sharing channel). Similarly, the price does not reflect the value of information channel because it does not capture the additional hedging benefit that investors gain from having observed θ in the event that the manager invests (i.e., when $k = 1$). Because each of these channels improves welfare, this implies that a price-maximizing manager tends to under-invest in climate-sensitive projects relative to a welfare-maximizing rule. Finally, to reiterate, heterogeneity in exposures, as captured by τ_ζ^{-1}, amplifies the welfare effect of the climate risk channel. Thus, the price also does not fully account for the climate risk channel, leading to under-investment in green projects, which reduce aggregate climate risk, and over-investment in brown projects, which increase it.

While we are not able to analytically characterize the welfare-maximizing s_p-dependent investment rule in the general heterogeneous exposures case, we can establish that if the firm is arbitrarily small (i.e., $n \to 0$), then the welfare-maximizing rule is to *always* invest. We record this in the following Proposition.

PROPOSITION 7: *Suppose that the share endowment is zero ($n = 0$) and exposures are heterogeneous $\frac{1}{\tau_\zeta} > 0$. Then, the welfare-maximizing s_p-dependent investment policy is to always invest,*

$$\arg\max_{k \in \{0,1\}} \mathcal{W}(k; s_p) = 1. \tag{38}$$

Hence, both cash-flow maximization and price maximization lead to under-investment relative to welfare maximization, in any states in which they lead the manager to not invest.

The intuition for this result is straightforward. When the firm is in zero supply, investment affects welfare only through the "risk-sharing" and "value of information" channels. Moreover, this implies that, irrespective of the information revealed by s_p, investors strictly prefer that the firm takes the project so that the firm's shares are useful for sharing risk. This result further clarifies that traditional managerial incentives can be misaligned relative to welfare maximization, even if the investment decision has no effect on aggregate expected cash flows or aggregate risk, when investors have heterogeneous risk exposures. The effect of investment on risk-sharing can be sufficient to lead investment to be socially sub-optimal.

The misalignment between the manager's objective and investor welfare implies that feedback from prices need not always improve welfare. To formalize this intuition, we compare investor welfare to a benchmark in which the manager ignores the information in price and instead chooses to maximize the *ex-ante expectation* of cash flows or the share price. In this case, the manager invests if and only if the unconditional expectation of the price signal s_p exceeds the corresponding threshold $\bar{s} \in \{\bar{s}_C, \bar{s}_P\}$.

The next proposition characterizes sufficient conditions under which feedback reduces welfare.

PROPOSITION 8: *Suppose the no-feedback investment policy is $k = 1$ (i.e., $\mathbb{E}[s_p] > \bar{s}$ for the relevant threshold $\bar{s} \in \{\bar{s}_C, \bar{s}_P\}$) and the project is exposed to climate risk (i.e., $\alpha \neq 0$). Then, feedback reduces welfare if* *(i) n is sufficiently small, or*

(ii) gains from risk-sharing are sufficiently large (i.e., τ_ζ is sufficiently small).

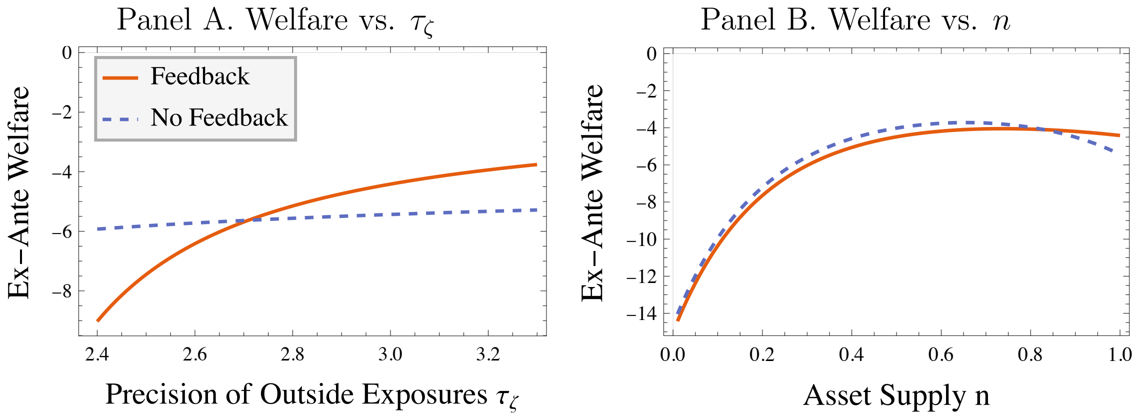

Figure 3. Ex-ante welfare: Feedback vs. no feedback. This figure plots the ex-ante welfare (i.e., ex-ante expected utility) as a function of τ_ζ and n for a project with positive expected NPV. Unless otherwise mentioned, the parameters employed are: $\tau_\theta = 0.5$; $\tau_\zeta = 3$; $\tau_Z = 2$; $\mu_A = 0$; $\tau_A = 5$; $\mu_\theta = c = \tau_\eta = \gamma = \mu_Z = n = \alpha = 1$. These parameters ensure the expected NPV of the project is positive.

Figure 3 illustrates these results for the case where the manager maximizes the price, and the intuition is as follows.[34] When the ex-ante NPV of the project is positive, in the no-feedback benchmark, the manager always invests. In contrast, given feedback, the firm does not invest for $s_p \leq \bar{s}_P$. On the one hand, feedback improves the expected price of the stock, which tends to improve welfare through the cash-flow channel. On the other hand, because it leads to no-investment in some states, feedback reduces the ability of investors to use the stock as a hedge, and so reduces welfare via the risk-sharing channel. It also affects the aggregate exposure to non-climate and climate risk (with the direction depending on the sign of α).

When the per-capita endowment of shares n is small, the cash flow and non-climate risk channels are relatively small. Moreover, the firm's investment decision has a small effect on the aggregate climate exposure, and so the climate risk channel is muted. However, the risk-sharing channel remains important since it is unaffected by n: regardless of the firm's size, its stock remains a useful hedge in the event of investment. Consequently, the risk-sharing channel dominates, and investors are better off with a rule that always

[34] The economic intuition for cash-flow maximization is analogous.

invests, yielding hedging benefits in all states of the world. Analogously, when τ_ζ is small, investors' exposures to the climate are highly diverse, so that the ability to share risk provides them with large welfare gains. Hence, the risk-sharing channel dominates in the limit, once again leading investors to prefer an investment rule that ensures that the asset is always useful for hedging.

It is worth noting that the manager's use of price information *always* increases the firm's expected cash flows (share price) under cash-flow maximization (price maximization): any additional information that she infers from the price can only improve investment efficiency as measured by her objective function. As a result, Proposition 8 implies that an increase in investment efficiency need not align with an improvement in investor welfare.

D. Implications for Managerial Compensation

Our welfare results speak to the recent debate on the effectiveness of the use of climate-risk metrics in executive compensation. On the one hand, there has been a rapid increase in the use of such measures. Edmans (June 27, 2021) cites that "51% of large U.S. companies and 45% of leading U.K. firms use ESG metrics in their incentive plans," and Hill (November 14, 2021) cites a survey conducted by Deloitte in September 2021, which suggests that "24 per cent of companies polled expected to link their long-term incentive plans for executives to net zero or climate measures over the next two years."[35]

On the other hand, there is ample skepticism about the effectiveness of such incentives. In addition to issues around measurement and monitoring of such objectives and the possibility of unintended consequences, Edmans (June 27, 2021) argues that incentivizing ESG performance may not necessarily lead to better financial performance. Instead, he advocates for the use of long-term stock-based compensation, arguing that "[s]ince material ESG factors ultimately improve the long-term stock price, this holds CEOs

[35] More broadly, Edmans, Gosling, and Jenter (2023) find that over 50% of surveyed directors and investors report that offering variable pay to CEO is in part useful to "motivate the CEO to improve outcomes other than long-term shareholder value."

accountable for material ESG issues – even if they aren't directly measurable."

Our analysis suggests that this may not be true because the stock price (even in the long term) does not fully account for the benefit of investing in climate-exposed projects. As such, providing additional incentives based on climate metrics (e.g., bonuses linked to climate targets) can improve overall investor welfare. This is despite the fact that such incentives may decrease stock prices and future profitability on average by leading to inefficient over-investment (from the perspective of a price-maximization or cash-flow maximization objective) in green projects. Yet, when investors have diverse climate risk exposures and find it difficult to hedge these exposures, such incentives improve their ability to hedge risks and, consequently, can improve overall welfare.

V. Conclusions

In this paper, we develop a model of informational feedback effects in which a firm's investment alters its exposure to an aggregate risk, and discuss its application to climate-exposed investment. When a firm invests in a project that is exposed to climate risk, it affects how useful the asset is as a hedge for climate risk. As a result, the firm's stock price reflects information about investors' climate exposures and the project's expected cash flows, which are both relevant to the manager's investment choice. We show that this has novel implications for how a project's greenness affects the likelihood of investment, conditional expected returns and future profitability. Moreover, we show that because the price does not fully reflect the welfare externality generated by investment in climate-sensitive projects, price-maximization tends to lead to under-investment in green projects.

In addition to climate-exposed investments, our model's predictions on investment and managerial incentives apply broadly to investments that are exposed to systematic risks with variable risk premia. For instance, investments that are exposed to commodity prices may serve as inflation hedges and thus may have discount rates that vary with investors' aggregate inflation concerns. Moreover, investments in emerging markets are exposed to aggregate demand in those markets, and so are likely to have discount rates that vary with uncertainty over this demand. Our model's implications for feedback's impact on welfare also apply more generally, whenever the market is incomplete with respect to the investment's risk exposure.

A notable contribution of our analysis is to provide a tractable feedback effects framework with investor risk aversion and priced risk factors. Immediate extensions include generalizations to the structure of cash flows and information. For instance, allowing for both public and private information signals would enable future research to assess the merits of disclosure regarding firms' climate risk exposures. Similarly, introducing multiple dimensions of fundamentals as in Goldstein and Yang (2019) and Goldstein, Schneemeier, and Yang (2020) could enable future work to assess how climate-exposed investments interact with the other risks that firms face. Finally, it may be interesting to consider how dynamics and multiple traded assets influence managers' ability to infer discount rate information from prices.

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A Proofs

A. Proof of Proposition 1

We first establish the existence of the stated equilibrium, and then argue that, among all threshold equilibria, it involves the most investment. Begin by conjecturing an equilibrium of the form posited in the text. That is, suppose that there is a random variable of the form $s_p = \theta + \frac{1}{\beta}Z$ and threshold $\bar{s} \in \mathbb{R}$ such that the asset prices at the two trading dates are identical and take the form

$$P_1 = P_3 = \begin{cases} A_1 + B_1 s_p & s_p > \bar{s} \\ A_0 & s_p \leq \bar{s} \end{cases}. \tag{A1}$$

We can now derive the equilibrium, and confirm the above conjecture, by working backwards. At date $t = 3$, investors can observe the actual investment decision made at $t = 2$. Hence, they perceive the asset payoff as conditionally normally distributed with conditional moments

$$\mathbb{E}_{i3}[V(k)] = \mathbb{E}_{i3}[A + k(\theta + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I - c)] = \mu_A + k(\theta - c) \tag{A2}$$
$$\mathbb{C}_{i3}(V(k), \eta_C) = k\alpha\frac{1}{\tau_\eta} \tag{A3}$$
$$\mathbb{V}_{i3}(V(k)) = \frac{1}{\tau_A} + k^2\frac{1}{\tau_\eta}. \tag{A4}$$

An arbitrary investor i solves the following static optimization problem at this date:

$$\max_{x \in \mathbb{R}} \mathbb{E}_{i3}[-e^{-\gamma W_{i4}}]. \tag{A5}$$

Given her demand x, her terminal wealth W_{i4} is

$$W_{i4} = (n + X_{i1} + x)\,V - xP_3 - X_{i1}P_1 - z_i\eta_C, \tag{A6}$$

where X_{i1}, the trade from the $t = 1$ trading round, is taken as given.

Applying well-known results for CARA utility, this problem leads to a standard mean-variance demand function:

$$X_{i3} = \frac{\mathbb{E}_{i3}[V(k)] + \gamma\mathbb{C}_{i3}(V(k), \eta_C)z_i - P_3}{\gamma\mathbb{V}_{i3}(V(k))} - (n + X_{i1}). \tag{A7}$$

Plugging in for the conditional moments from above and enforcing market clearing yields equilibrium price

$$P_3 = \mu_A + k(\theta - c) + \gamma k \alpha \tfrac{1}{\tau_\eta} Z - \gamma \left(\tfrac{1}{\tau_A} + k^2 \tfrac{1}{\tau_\eta} \right) n \tag{A8}$$

$$= \mu_A - \gamma \tfrac{1}{\tau_A} n + k \left(\theta + \gamma \alpha \tfrac{1}{\tau_\eta} Z - c - \gamma \tfrac{1}{\tau_\eta} n \right), \tag{A9}$$

where the second line collects terms and uses the fact that $k \in \{0, 1\}$ implies $k = k^2$ to simplify. Hence, to be consistent with our initial conjecture, the endogenous signal s_p must have coefficient $\frac{1}{\beta} = \frac{\gamma\alpha}{\tau_\eta}$ on Z. To be consistent with our conjecture, the price coefficients must satisfy

$$A_0 = \mu_A - \gamma \tfrac{1}{\tau_A} n \tag{A10}$$

$$A_1 = \mu_A - \gamma \tfrac{1}{\tau_A} n - c - \gamma \tfrac{1}{\tau_\eta} n \tag{A11}$$

$$B_1 = 1. \tag{A12}$$

Stepping back to $t = 2$, the manager's problem is to solve

$$\max_{k \in \{0,1\}} \mathbb{E}[V(k) | P_1], \tag{A13}$$

where she can condition on the first period price, P_1. The optimal investment is therefore

$$k = \begin{cases} 1 & \mathbb{E}[\theta | P_1] > c \\ 0 & \mathbb{E}[\theta | P_1] \le c \end{cases}. \tag{A14}$$

Now, let $\bar{s}_C$ denote the level of s_p such that the manager would be indifferent to investing and not investing if she observed s_p, that is,

$$\mathbb{E}[\theta | s_p = \bar{s}_C] - c = 0.$$

Because $\mathbb{E}[\theta | s_p] = \mu_\theta + \frac{\tau_p}{\tau_\theta + \tau_p} \left(s_p - \mu_\theta - \frac{1}{\beta} \mu_Z \right)$, with $\tau_p \equiv \beta^2 \tau_Z$, we have:

$$\mathbb{E}[\theta | s_p = \bar{s}_C] - c = 0 \Leftrightarrow \bar{s}_C = \mu_\theta + \frac{\gamma\alpha}{\tau_\eta} \mu_Z - \frac{\tau_\theta + \tau_p}{\tau_p} (\mu_\theta - c)$$

$$= c - \frac{\tau_\theta (\mu_\theta - c)}{\tau_p} + \frac{\gamma}{\tau_\eta} \alpha \mu_Z.$$

We claim that the threshold $\bar{s} = \bar{s}_C$ is consistent with our conjectured equilibrium, that is, the manager invests if and only if $s_p > \bar{s}_C$. Under such a threshold, when the manager observes A_0, she knows s_p lies below $\bar{s}_C$ with probability 1, and so infers that it is suboptimal to invest. In contrast, when she observes $P_1 \neq A_0$, she infers s_p and knows that s_p lies above $\bar{s}_C$, and so chooses to invest. Thus, the investment threshold $\bar{s}_C$ is indeed consistent with conjectured form of equilibrium.

Stepping back to $t = 1$, the problem of an arbitrary investor is

$$\max_{x \in \mathbb{R}} \mathbb{E}_{i1}[-e^{-\gamma W_{i4}}],$$

where her terminal wealth is

$$W_{i4} = (n + x + X_{i3})\, V - X_{i3}P_3 - xP_1 - z_i\eta_C.$$

and where the optimal $t = 3$ demand X_{i3} was derived above. Given the functional form for P_3, the realization of P_3 is perfectly anticipated under the investor's information set $\mathcal{F}_{i1} = \sigma(\theta, z_i, P_1)$. Hence, to rule out arbitrage, the price must satisfy $P_1 = P_3$, and consequently all investors are indifferent to trading at $t = 1$ at this equilibrium price. Thus, we have now shown the equilibrium stated in the proposition exists.

Finally, we argue that this equilibrium maximizes investment over all possible threshold equilibria. Suppose by contradiction that there were an equilibrium with a lower investment threshold $\bar{s} < \bar{s}_C$. Then, the date one price would reveal s_p to the manager for $s_p \in (\bar{s}, \bar{s}_C)$ and the manager would invest for such s_p. However, by the definition of $\bar{s}_C$, investment reduces expected cash flows in this region, and so the manager could improve the expected cash flows by deviating to not investing when she observes s_p in this region. This contradicts the purported existence of an equilibrium with $\bar{s} < \bar{s}_C$ and establishes the claim. □

B. *Proof of Proposition 2*

The proof proceeds similarly to the proof of Proposition 1 above. Again, we begin by conjecturing asset prices in the two trading dates that take the form in (A1). At date $t = 3$, investors can observe the actual investment decision made at $t = 2$. Hence, the date three equilibrium, given the manager's investment choice, follows exactly as in the previous proof: they perceive the asset payoff as conditionally normally distributed with conditional moments as in equations (A2)–(A4), their optimal demands take the form in (A7), the date three price takes the form in (A9), and the endogenous signal s_p again must have coefficient $\frac{1}{\beta} = \frac{\gamma\alpha}{\tau_\eta}$ on Z.

Stepping back to $t = 2$, the manager's problem is now to solve

$$\max_{k \in \{0,1\}} \mathbb{E}[P_3 | P_1], \tag{A15}$$

where she can condition on the first period asset price, P_1. Using the expression for P_3 derived in the first step of the proof, the manager's problem reduces to

$$\max_{k \in \{0,1\}} k\mathbb{E}\left[s_p - c - \gamma \frac{1}{\tau_\eta} n \middle| P_1 \right]. \tag{A16}$$

The optimal investment is therefore

$$k = \begin{cases} 1 & \mathbb{E}[s_p|P_1] > c + \gamma\frac{1}{\tau_\eta}n \\ 0 & \mathbb{E}[s_p|P_1] \leq c + \gamma\frac{1}{\tau_\eta}n \end{cases}. \tag{A17}$$

Note that the threshold $\bar{s}_P$, defined by $\bar{s}_P \equiv c + \gamma\frac{1}{\tau_\eta}n$ is the value such that, if the manager always observed s_p, she would invest if and only if $s_p > \bar{s}_P$. We claim that this threshold is consistent with our conjectured equilibrium. To see this, note that if the manager observes $P_1 = A_0$, she infers $s_p \leq \bar{s}_P$, and so she chooses not to invest. On the other hand, if she observes any $P_1 \neq A_0$, she infers the realized value of s_p, necessarily strictly greater than $\bar{s}_P$ and therefore finds it optimal to invest. Hence, the investment threshold $\bar{s}_P$ is indeed consistent with our initial conjecture.

Stepping back to $t = 1$, as in the prior proof, since the manager's investment decision is a function of P_1, investors can anticipate k given the price. Thus, they can perfectly anticipate the date three price, and, to rule out arbitrage, the price must satisfy $P_1 = P_3$, and consequently all investors are indifferent to trading at $t = 1$ at this equilibrium price. This completes the construction of equilibrium.

This equilibrium maximizes investment over all possible threshold equilibria. Suppose by contradiction that there were an equilibrium with a lower investment threshold $\bar{s} < \bar{s}_P$. Then, the price would reveal s_p to the manager for $s_p \in (\bar{s}, \bar{s}_P)$. Moreover, by the definition of $\bar{s}_P$, investment lowers price on this region, and so the manager prefers to deviate to not investing when observing s_p in this region. □

C. *Proof of Proposition 3*

The probability of investment is given by

$$\Pr(s_p > \bar{s}_C) = 1 - \Phi\left(\frac{\bar{s}_C - \mathbb{E}[s_p]}{\mathbb{V}[s_p]}\right) = \Phi\left(\frac{\mathbb{E}[s_p] - \bar{s}_C}{\sqrt{\mathbb{V}[s_p]}}\right) \tag{A18}$$

$$= \Phi\left(\frac{\tau_\theta\left(\frac{1}{\tau_\theta} + \frac{1}{\tau_p}\right)(\mu_\theta - c)}{\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}}\right) \tag{A19}$$

$$= \Phi\left(\tau_\theta\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}(\mu_\theta - c)\right). \tag{A20}$$

Recalling that $\tau_p = \beta^2\tau_Z = \left(\frac{\tau_\eta}{\gamma\alpha}\right)^2\tau_Z$, direct inspection now immediately yields the claimed results. □

D. *Proof of Proposition 4*

The probability of investment is given by

$$\Pr(s_p > \bar{s}) = 1 - \Phi\left(\frac{\bar{s}_P - \mathbb{E}[s_p]}{\mathbb{V}[s_p]}\right) = \Phi\left(\frac{\mathbb{E}[s_p] - \bar{s}_P}{\sqrt{\mathbb{V}[s_p]}}\right) \tag{A21}$$

$$= \Phi\left(\frac{\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}\right) \tag{A22}$$

This immediately implies probability of investment is increasing in $\mu_\theta - c$, decreasing in n, increasing in μ_Z. Moreover, for any arbitrary parameter b we have, after applying the monotonic transformation $\Phi^{-1}(\cdot)$ and using the definition $NPV = \theta - c - \gamma\frac{1}{\tau_\eta}(n - \alpha Z)$ from the text to condense notation:

$$\frac{\partial}{\partial b}\Pr(s_p > \bar{s}) \propto \frac{\partial}{\partial b}\frac{\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}} \tag{A23}$$

$$= \frac{\partial}{\partial b} \frac{\mathbb{E}[NPV]}{\sqrt{\mathbb{V}(NPV)}} \tag{A24}$$

$$= \frac{\sqrt{\mathbb{V}(NPV)}\,\frac{\partial}{\partial b}\mathbb{E}[NPV] - \mathbb{E}[NPV]\,\frac{\partial}{\partial b}\sqrt{\mathbb{V}(NPV)}}{\mathbb{V}(NPV)} \tag{A25}$$

$$= \frac{|\mathbb{E}[NPV]|}{\sqrt{\mathbb{V}(NPV)}} \left(\frac{\frac{\partial}{\partial b}\mathbb{E}[NPV]}{|\mathbb{E}[NPV]|} - \operatorname{sgn}(\mathbb{E}[NPV]) \frac{\frac{\partial}{\partial b}\sqrt{\mathbb{V}(NPV)}}{\sqrt{\mathbb{V}(NPV)}} \right) \tag{A26}$$

$$= \frac{|\mathbb{E}[NPV]|}{\sqrt{\mathbb{V}(NPV)}} \left(\frac{\frac{\partial}{\partial b}\mathbb{E}[NPV]}{|\mathbb{E}[NPV]|} - \tfrac{1}{2}\operatorname{sgn}(\mathbb{E}[NPV]) \frac{\frac{\partial}{\partial b}\mathbb{V}(NPV)}{\mathbb{V}(NPV)} \right). \tag{A27}$$

For α we have

$$\frac{\partial}{\partial \alpha} \Pr(s_p > \bar{s}) \propto \left(\tfrac{\partial}{\partial \alpha}\mathbb{E}[NPV] - \tfrac{1}{2}\mathbb{E}[NPV] \frac{\frac{\partial}{\partial \alpha}\mathbb{V}(NPV)}{\mathbb{V}(NPV)} \right) \tag{A28}$$

$$= \frac{\gamma\mu_Z}{\tau_\eta} - \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta} \right) \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} \tag{A29}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} \left(\frac{\gamma\mu_Z}{\tau_\eta} \frac{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} - \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta} \right) \right) \tag{A30}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} \left(\frac{\alpha\gamma\mu_Z}{\tau_\eta} \frac{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} - \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta} \right) \right) \tag{A31}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} \left(\frac{\alpha\gamma\mu_Z}{\tau_\eta} \left(1 + \frac{\frac{1}{\tau_\theta}}{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} \right) - \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta} \right) \right) \tag{A32}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} \left(\left(\frac{\frac{1}{\tau_\theta}}{\left(\frac{\alpha\gamma}{\tau_\eta}\right) \frac{1}{\tau_Z}} \right) \mu_Z - \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} \right) \right) \tag{A33}$$

$$= -\frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} - \tfrac{\tau_\eta \tau_Z}{\alpha\gamma} \tfrac{1}{\tau_\theta} \mu_Z \right) \tag{A34}$$

which implies

$$\frac{\partial}{\partial \alpha} \Pr(s_p > \bar{s}) < 0 \Leftrightarrow \operatorname{sgn}(\alpha) \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} - \tfrac{\tau_\eta \tau_Z}{\alpha\gamma} \tfrac{1}{\tau_\theta} \mu_Z \right) > 0. \tag{A35}$$

Moreover, note that because the parameters $\tau \in \{\tau_Z, \tau_\theta\}$ do not enter the expected NPV and increases in these τ strictly decrease the variance of the NPV, we have

$$\frac{\partial}{\partial \tau_Z} \Pr(s_p > \bar{s}), \frac{\partial}{\partial \tau_\theta} \Pr(s_p \geq \bar{s}) \propto -\tfrac{1}{2}\operatorname{sgn}(\mathbb{E}[NPV]) \frac{\frac{\partial}{\partial \tau}\mathbb{V}(NPV)}{\mathbb{V}(NPV)} \tag{A36}$$

$$\propto \operatorname{sgn}(\mathbb{E}[NPV]) \tag{A37}$$

so that the dependence is pinned down by the sign of the expected NPV, which immediately establishes the claimed result. □

E. *Proof of Proposition 5*

To establish the welfare-maximizing rule, note that the s_p-conditional expected utility is a special case of Proposition IA1 in which $\tau_\zeta \to \infty$. In this limit, the functions Q and D that characterize the expected utility $\mathcal{W}(k; s_p) = -D(k; s_p) \exp\{Q(k; s_p)\}$ are

$$\begin{aligned} Q(k; s_p) = &-\gamma \mathbb{E}_p[V] n + \frac{1}{2}\gamma^2 \left(\frac{1}{\tau_A} + k^2 \left(\mathbb{V}_p(\theta) + \frac{1-\alpha^2}{\tau_\eta}\right)\right) n^2 \\ &+ \frac{1}{2}\gamma^2 \frac{1}{\tau_\eta} \left(\mathbb{E}_p[Z] - k\alpha n - \gamma k \mathbb{C}_p(Z, \theta) n\right)^2 (1 + \Gamma(k; s_p)) \end{aligned}$$

and

$$D(k; s_p) \sqrt{\frac{\Gamma(k; s_p)}{\gamma^2 \frac{1}{\tau_\eta} \mathbb{V}_p(Z)}}$$

where

$$\Gamma(k; s_p) = \gamma^2 \frac{1}{\tau_\eta} \mathbb{V}_p(Z) \left(1 - \gamma^2 \frac{1}{\tau_\eta} \mathbb{V}_p(Z)\right)^{-1}.$$

Only the function Q above depends on the investment decision k. Further, grouping terms and plugging in for $\Gamma(k; s_p)$ allows us to write

$$\begin{aligned} Q(k; s_p) = &-\gamma \mathbb{E}_p[V] n + \frac{1}{2}\gamma^2 \left(\frac{1}{\tau_A} + k^2 \left(\mathbb{V}_p(\theta) + \frac{1-\alpha^2}{\tau_\eta}\right)\right) n^2 \\ &+ \frac{1}{2}\gamma^2 \frac{1}{\tau_\eta} \left(\mathbb{E}_p[Z] - (\alpha + \gamma \mathbb{C}_p(Z, \theta)) k n\right)^2 \left(1 - \gamma^2 \frac{1}{\tau_\eta} \mathbb{V}_p(Z)\right)^{-1}. \end{aligned} \tag{A38}$$

Further, because $\frac{1}{\beta} = \frac{\gamma\alpha}{\tau_\eta}$ and $\mathbb{C}_p(\theta, Z) = \mathbb{C}_p(s_p - \frac{1}{\beta}Z, Z) = -\frac{1}{\beta}\mathbb{V}_p(Z)$ we can write

$$\alpha + \gamma \mathbb{C}_p(Z, \theta) = \frac{1}{\beta}\frac{\tau_\eta}{\gamma}\left(1 - \gamma^2 \frac{1}{\tau_\eta}\mathbb{V}_p(Z)\right)$$

and plugging this back in to the last term in eq. (A38) gives

$$\begin{aligned} &\frac{1}{2}\gamma^2 \frac{1}{\tau_\eta}\left(\mathbb{E}_p[Z] - \frac{1}{\beta}\frac{\tau_\eta}{\gamma}\left(1 - \gamma^2 \frac{1}{\tau_\eta}\mathbb{V}_p(Z)\right) kn\right)^2 \left(1 - \gamma^2 \frac{1}{\tau_\eta}\mathbb{V}_p(Z)\right)^{-1} \\ &= \frac{1}{2}\gamma^2 \frac{1}{\tau_\eta}\left(\mathbb{E}_p[Z] - \frac{1}{\beta}\frac{\tau_\eta}{\gamma}\left(1 - \gamma^2 \frac{1}{\tau_\eta}\mathbb{V}_p(Z)\right) kn\right)^2 \left(1 - \gamma^2 \frac{1}{\tau_\eta}\mathbb{V}_p(Z)\right)^{-1} \\ &= \frac{1}{2}\gamma^2 \frac{1}{\tau_\eta}\left(1 - \gamma^2 \frac{1}{\tau_\eta}\mathbb{V}_p(Z)\right)^{-1} \mathbb{E}_p^2[Z] - \gamma k \frac{1}{\beta}\mathbb{E}_p[Z] n \\ &\quad + \frac{1}{2}\left(\frac{1}{\beta}\right)^2 \tau_\eta \left(1 - \gamma^2 \frac{1}{\tau_\eta}\mathbb{V}_p(Z)\right) k^2 n^2 \\ &= \frac{1}{2}\gamma^2 \frac{1}{\tau_\eta}\left(1 - \gamma^2 \frac{1}{\tau_\eta}\mathbb{V}_p(Z)\right)^{-1} \mathbb{E}_p^2[Z] - \gamma k \frac{1}{\beta}\mathbb{E}_p[Z] n \end{aligned}$$

$$+\frac{1}{2}\gamma^2\alpha^2\frac{1}{\tau_\eta}-\frac{1}{2}\gamma^2\left(\frac{1}{\beta}\right)^2\mathbb{V}_p(Z)k^2n^2.$$

Noting that $\mathbb{V}_p(\theta)=\mathbb{V}_p(s_p-\frac{1}{\beta}Z)=\left(\frac{1}{\beta}\right)^2\mathbb{V}_p(Z)$, plugging the most recent expression back in to eq. (A38), and collecting terms yields

$$\begin{aligned}
Q(k;s_p) &= -\gamma\mathbb{E}_p[V]n+\frac{1}{2}\gamma^2\left(\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}\right)n^2 \\
&\quad+\frac{1}{2}\gamma^2\frac{1}{\tau_\eta}\left(1-\gamma^2\frac{1}{\tau_\eta}\mathbb{V}_p(Z)\right)^{-1}\mathbb{E}_p^2[Z]-\gamma k\frac{1}{\beta}\mathbb{E}_p[Z]n \\
&= -\gamma\left(\mu_A+k\mathbb{E}_p\left[\theta+\frac{1}{\beta}Z-c\right]\right)n+\frac{1}{2}\gamma^2\left(\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}\right)n^2 \\
&\quad+\frac{1}{2}\gamma^2\frac{1}{\tau_\eta}\left(1-\gamma^2\frac{1}{\tau_\eta}\mathbb{V}_p(Z)\right)^{-1}\mathbb{E}_p^2[Z]
\end{aligned}$$

where we have used $\mathbb{E}_p[V]=\mathbb{E}_p[A+k(\theta-c)]$ and grouped terms in the last line.

Now, since $s_p=\theta+\frac{1}{\beta}Z$ and we have $k^2=k$ for $k\in\{0,1\}$, it follows from the above that the investment $k\in\{0,1\}$ that maximizes $\mathcal{W}(k;s_p)=-D(k;s_p)\exp\{Q(k;s_p)\}$ is

$$\begin{aligned}
k(s_p) &= \arg\max_{k\in\{0,1\}}\left(\gamma(\mu_A+k\mathbb{E}_p[s_p-c])n-\frac{1}{2}\gamma^2\left(\frac{1}{\tau_A}+k\frac{1}{\tau_\eta}\right)n^2\right) \\
&= \mathbf{1}\left\{s_p-c-\frac{1}{2}\gamma\frac{1}{\tau_\eta}n>0\right\}.
\end{aligned}$$

Defining $\bar{s}_W\equiv c+\frac{1}{2}\frac{\gamma}{\tau_\eta}n$ delivers the investment rule in the Proposition.

Furthermore, using the expressions for the price-maximizing threshold, $\bar{s}=c+\frac{\gamma}{\tau_\eta}n$, from Proposition 2, and the cash-flow-maximizing threshold, $\bar{s}_C=c-\frac{\tau_\theta(\mu_\theta-c)}{\tau_p}+\frac{\gamma}{\tau_\eta}\alpha\mu_Z$, from eq. (14), yields the expressions for $\bar{s}-\bar{s}_W$ and $\bar{s}_C-\bar{s}_W$. The claims about over- and under-investment are immediate given the signs of these expressions. □

F. *Proof of Proposition 6*

Given our expressions for P_1 and P_3 in a threshold equilibrium, we can write

$$\delta\mathbb{E}[P_3|P_1]+(1-\delta)\mathbb{E}[V|P_1] \tag{A39}$$

$$\begin{aligned}
&=\delta\left(\mu_A-\gamma\frac{1}{\tau_A}n+k(s_p-c-\gamma\frac{1}{\tau_\eta}n)\right)+ \\
&\quad(1-\delta)\left[\mu_A+k\left(\mu_\theta-c+\frac{\beta^2\tau_Z}{\tau_\theta+\beta^2\tau_Z}(s_p-\mathbb{E}[s_p])\right)\right].
\end{aligned}\tag{A40}$$

The $k\in\{0,1\}$ that maximizes this expression is

$$k(s_p)=\mathbf{1}\left\{\delta\left(s_p-c-\gamma\frac{1}{\tau_\eta}n\right)+(1-\delta)\left(\mu_\theta-c+\frac{\tau_p}{\tau_\theta+\tau_p}(s_p-\mathbb{E}[s_p])\right)>0\right\} \tag{A41}$$

$$= \mathbf{1}\left\{s_p > \frac{1}{\delta + (1-\delta)\frac{\tau_p}{\tau_\theta+\tau_p}}\left(\delta\left(c+\gamma\frac{1}{\tau_\eta}n\right) + (1-\delta)\left(c-\mu_\theta+\frac{\tau_p}{\tau_\theta+\tau_p}\mathbb{E}[s_p]\right)\right)\right\}. \tag{A42}$$

Setting the threshold in this expression equal to $\bar{s}_W = c + \frac{1}{2}\gamma\frac{1}{\tau_\eta}n$ and solving for δ yields

$$\begin{aligned}
\delta &= \frac{\mu_\theta - c - \frac{\tau_p}{\tau_\theta+\tau_p}\mathbb{E}[s_p] + \frac{\tau_p}{\tau_\theta+\tau_p}\bar{s}_W}{\mu_\theta + \gamma\frac{1}{\tau_\eta}n - \frac{\tau_p}{\tau_\theta+\tau_p}\mathbb{E}[s_p] - \frac{\tau_\theta}{\tau_\theta+\tau_p}\bar{s}_W} \\
&= \frac{\mu_\theta - c - \frac{\tau_p}{\tau_\theta+\tau_p}\left(\mu_\theta + \frac{1}{\beta}\mu_Z\right) + \frac{\tau_p}{\tau_\theta+\tau_p}\left(c+\frac{1}{2}\gamma\frac{1}{\tau_\eta}n\right)}{\mu_\theta + \gamma\frac{1}{\tau_\eta}n - \frac{\tau_p}{\tau_\theta+\tau_p}\left(\mu_\theta+\frac{1}{\beta}\mu_Z\right) - \frac{\tau_\theta}{\tau_\theta+\tau_p}\left(c+\frac{1}{2}\gamma\frac{1}{\tau_\eta}n\right)} \\
&= \frac{\frac{\tau_\theta}{\tau_\theta+\tau_p}(\mu_\theta - c) - \frac{\tau_p}{\tau_\theta+\tau_p}\left(\frac{1}{\beta}\mu_Z - \frac{1}{2}\gamma\frac{1}{\tau_\eta}n\right)}{\frac{\tau_\theta}{\tau_\theta+\tau_p}(\mu_\theta - c) - \frac{\tau_p}{\tau_\theta+\tau_p}\left(\frac{1}{\beta}\mu_Z - \frac{1}{2}\gamma\frac{1}{\tau_\eta}n\right) + \frac{1}{2}\gamma\frac{1}{\tau_\eta}n}.
\end{aligned}$$

After substituting in $\frac{1}{\beta} = \frac{\gamma\alpha}{\tau_\eta}$ and $\tau_p = \beta^2\tau_Z$, this matches the expression in the Proposition. If $\alpha = 0$ then $\frac{1}{\beta} \to 0$ and $\tau_p \to \infty$, and this expression reduces to

$$\delta = \frac{\frac{1}{2}\gamma\frac{1}{\tau_\eta}n}{\frac{1}{2}\gamma\frac{1}{\tau_\eta}n + \frac{1}{2}\gamma\frac{1}{\tau_\eta}n} = \frac{1}{2} \tag{A43}$$

as claimed. More generally, note that one can express $\delta = \frac{\omega}{\omega+\upsilon}$, where $\omega = \frac{\tau_\theta}{\tau_\theta+\tau_p}(\mu_\theta - c) - \frac{\tau_p}{\tau_\theta+\tau_p}\left(\frac{1}{\beta}\mu_Z - \frac{1}{2}\gamma\frac{1}{\tau_\eta}n\right)$ and $\upsilon = \frac{1}{2}\gamma\frac{1}{\tau_\eta}n > 0$. This implies $\delta \in (0,1)$ if and only if $\omega > 0$. □

G. *Proof of Proposition 7*

Consider the conditional welfare expression from Proposition IA1 for an arbitrary investment k and price signal realization s_p. We will show that this expression is strictly increasing in k for $k \in [0,1]$, from which it follows that the welfare-maximizing investment is $k = 1$.

For $n = 0$ the conditional welfare is

$$\mathcal{W}(k; s_p) = -D(k; s_p)\exp\{Q(k; s_p)\} \tag{A44}$$

where

$$Q(k; s_p) = \frac{1}{2}\gamma^2\frac{1}{\tau_\eta}\mathbb{E}_p^2[Z](1+\Gamma(k; s_p))$$

where the determinant term D and the function Γ do not depend on n and are as given in Proposition IA1:

$$D(k; s_p) = \sqrt{\frac{\frac{1}{\tau_A} + k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A} + k^2\left(\mathbb{V}_p(\theta|z_i) + \frac{1}{\tau_\eta}\right)}}\sqrt{\frac{\Gamma(k; s_p)}{\gamma^2\frac{1}{\tau_\eta}\left(\mathbb{V}_p(Z) + \frac{1}{\tau_\zeta}\right)}}$$

$$\Gamma\left(k;s_p\right)=\gamma^2\frac{1}{\tau_\eta}\left(\mathbb{V}_p\left(Z\right)+\frac{1}{\tau_\zeta}\right)\left(1-\gamma^2\frac{1}{\tau_\eta}\left(\mathbb{V}_p\left(Z\right)+\frac{1}{\tau_\zeta}\right)\left(1-\frac{k^2\alpha^2\frac{1}{\tau_\eta}\left(\frac{\frac{1}{\tau_\zeta}}{\mathbb{V}_p(Z)+\frac{1}{\tau_\zeta}}\right)^2}{\frac{1}{\tau_A}+k^2\left(\mathbb{V}_p\left(\theta|z_i\right)+\frac{1}{\tau_\eta}\right)}\right)\right)^{-1}.$$

Because of the negative sign in front of the expression in eq. (A44), to show that conditional expected utility increases in k, it suffices to show that the functions Q and D are decreasing functions of k for $k\in[0,1]$. Moreover, because k enters these expressions only in terms of k^2, it suffices to characterize their behavior as functions of k^2 for $k^2\in[0,1]$.

Consider first the function Γ, which appears in both Q and D. We have

$$\frac{\partial}{\partial k^2}\frac{k^2\alpha^2\frac{1}{\tau_\eta}\left(\frac{\frac{1}{\tau_\zeta}}{\mathbb{V}_p(Z)+\frac{1}{\tau_\zeta}}\right)^2}{\frac{1}{\tau_A}+k^2\left(\mathbb{V}_p\left(\theta|z_i\right)+\frac{1}{\tau_\eta}\right)} \tag{A45}$$

$$=\frac{\frac{1}{\tau_A}\alpha^2\frac{1}{\tau_\eta}\left(\frac{\frac{1}{\tau_\zeta}}{\mathbb{V}_p(Z)+\frac{1}{\tau_\zeta}}\right)^2}{\left(\frac{1}{\tau_A}+k^2\left(\mathbb{V}_p\left(\theta|z_i\right)+\frac{1}{\tau_\eta}\right)\right)^2}, \tag{A46}$$

which is strictly positive when $\frac{1}{\tau_\zeta}>0$, from which it follows that Γ is *decreasing* in k^2 and consequently Q is decreasing in k^2.

Considering D, it remains only to show that the term $\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\mathbb{V}_p(\theta|z_i)+\frac{1}{\tau_\eta}\right)}$ is decreasing since we have already shown that Γ is decreasing. We have

$$\frac{\partial}{\partial k^2}\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\mathbb{V}_p\left(\theta|z_i\right)+\frac{1}{\tau_\eta}\right)}=\frac{-\frac{1}{\tau_A}\mathbb{V}_p(\theta|z_i)}{\left(\frac{1}{\tau_A}+k^2\left(\mathbb{V}_p\left(\theta|z_i\right)+\frac{1}{\tau_\eta}\right)\right)^2}, \tag{A47}$$

which is strictly negative when $\frac{1}{\tau_\zeta}>0$. Hence, we have verified that investors are strictly better off when $k=1$ than $k=0$ for a given price signal realization s_p. Because this holds for all realizations of s_p, it follows that the ex-ante welfare-maximizing policy is to always invest. □

H. *Proof of Proposition 8*

Consider either threshold $\overline{s}\in\{\overline{s}_C,\overline{s}_P\}$. If the unconditional policy is to invest (i.e., $\mathbb{E}[s_p]-\overline{s}>0$), then a manager who does not condition on price optimally invests in all states of the world, leading to 'no feedback' investment $k_{NF}\equiv 1$. Hence, to establish that feedback reduces welfare, it suffices to show that welfare is higher with $k_{NF}=1$ than with the threshold policy $k(s_p)=\mathbf{1}_{\{s_p>\overline{s}\}}$.

The small n limit in the Proposition follows immediately from Proposition 7 and continuity of the expected utility in n since that Proposition establishes that the welfare-maximizing investment policy for $n=0$ is for the manager to always invest. Hence, because feedback causes the manager to *not* invest with strictly positive probability, feedback strictly reduces ex-ante welfare.

The τ_ζ limit is easier to establish using the unconditional welfare expression in Proposition IA2 directly. Establishing the limit as $\tau_\zeta \downarrow$, is equivalent to establishing the limit as $1/\tau_\zeta \uparrow$. In order for unconditional expected utility to exist, we must have $\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}} - \gamma^2\frac{1}{\tau_\eta} > 0 \Leftrightarrow 0 \leq \frac{1}{\tau_\zeta} < \frac{\tau_\eta}{\gamma^2} - \frac{1}{\tau_Z}$. Hence, the relevant limit is $\frac{1}{\tau_\zeta} \uparrow \frac{\tau_\eta}{\gamma^2} - \frac{1}{\tau_Z}$. Using the unconditional welfare from Proposition IA2, welfare under the no-feedback investment level $k_{NF} = 1$ is higher than under the feedback policy $k(s_p) = \mathbf{1}_{\{s_p > \bar{s}\}}$ if and only if

$$\begin{aligned} -D(1)\exp\{Q(1)\} > &-\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\{Q(0)\} \\ &-\left(1-\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)\right)D(1)\exp\{Q(1)\} \\ \Leftrightarrow \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\{Q(0)\} > &\ \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)D(1)\exp\{Q(1)\}. \end{aligned}$$

Hence, to establish the claimed result, it suffices to show

$$\lim_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\{Q(0)\} > \lim_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)D(1)\exp\{Q(1)\}.$$

We will show this by establishing that $\lim_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\{Q(0)\} = \infty$, while $\lim_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)D(1)\exp\{Q(1)\} < \infty$.

Letting $a = \frac{\tau_\eta}{\gamma^2} - \frac{1}{\tau_Z}$ to reduce clutter, note first that

$$\begin{aligned} \lim_{\frac{1}{\tau_\zeta}\uparrow a}\Gamma(k) &= \lim_{\frac{1}{\tau_\zeta}\uparrow a}\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)\left(1-\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)\left(1-\frac{k^2\alpha^2\frac{1}{\tau_\eta}\left(\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\eta}+\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}\right)}\right)\right)^{-1} \\ &= \begin{cases}\infty & k=0\\ \text{Finite} & k=1\end{cases} \end{aligned}$$

where the finite limit in the $k = 1$ case relies on the assumption $\alpha \neq 0$.

It follows that we have

$$\begin{aligned} \lim_{\frac{1}{\tau_\zeta}\uparrow a} D(k) &= \lim_{\frac{1}{\tau_\zeta}\uparrow a}\sqrt{\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\eta}+\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}\right)}}\sqrt{\frac{\Gamma(k)}{\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)}} \\ &= \begin{cases}\infty & k=0\\ \text{Finite} & k=1\end{cases}. \end{aligned}$$

Similarly,

$$\lim_{\frac{1}{\tau_\zeta}\uparrow a} Q(k) = \lim_{\frac{1}{\tau_\zeta}\uparrow a} \left\{ -\gamma \mathbb{E}_p\left[V\right] n + \frac{1}{2}\gamma^2 \left(\frac{1}{\tau_A} + k^2 \left(\mathbb{V}_p\left(\theta\right) + \frac{1-\alpha^2}{\tau_\eta} \right) \right) n^2 \right.$$
$$\left. + \frac{1}{2}\gamma^2 \frac{1}{\tau_\eta} \left(\mathbb{E}_p\left[Z\right] - k\alpha n - \gamma k \mathbb{C}_p\left(Z, \theta\right) n \right)^2 \left(1 + \Gamma\left(k; s_p\right)\right) \right\}$$
$$= \begin{cases} \infty & k = 0 \\ \text{Finite} & k = 1 \end{cases}.$$

Because the function Φ is bounded, together these results imply that

$$\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(1)}{\sqrt{v(1)}} \right) D(1) \exp\left\{Q(1)\right\} < \infty$$

as claimed.

It remains to show that $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}} \right) D(0) \exp\left\{Q(0)\right\} = \infty$. Considering $\frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}}$, if $1/\beta = 0$,then $\frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}}$ is constant in τ_ζ and we are done. Considering $1/\beta \neq 0$, we have

$$\begin{aligned}
\lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}} &= \lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{m(0)}{\sqrt{v(0)}} \\
&= \lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{\gamma \mathbb{C}(s_p, V(0)) n - \gamma^2 \mathbb{C}(s_p, Z) \frac{1}{\tau_\eta} \mu_Z (1 + \Gamma(0))}{\sqrt{\mathbb{V}(s_p) + \gamma^2 \mathbb{C}^2(s_p, Z) \frac{1}{\tau_\eta} (1 + \Gamma(0))}} \\
&= \lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{-\gamma^2 \frac{1}{\beta} \mathbb{V}(Z) \frac{1}{\tau_\eta} \mu_Z (1 + \Gamma(0))}{\sqrt{\mathbb{V}(s_p) + \gamma^2 \mathbb{C}^2(s_p, Z) \frac{1}{\tau_\eta} (1 + \Gamma(0))}} \\
&= \begin{cases} -\infty & \frac{1}{\beta} > 0 \\ \infty & \frac{1}{\beta} < 0 \end{cases}
\end{aligned}$$

where we use the fact that $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Gamma(0) = \infty$.

If $1/\beta < 0$, the proof is complete, since $Q(0) \to \infty$, $D(0) \to \infty$ and in this case $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(1)}{\sqrt{v(1)}} \right) > 0$, so that $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}} \right) D(0) \exp\left\{Q(0)\right\} = \infty$. If $1/\beta > 0$, then $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(1)}{\sqrt{v(1)}} \right) = 0$, so the limit is still indeterminate. Write $\Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}} \right) D(0) \exp\left\{Q(0)\right\}$ as

$$\Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}} \right) D(0) \exp\left\{Q(0)\right\} = \frac{\Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}} \right)}{\frac{1}{D(0)} \exp\left\{-Q(0)\right\}}$$

and note that the relevant limit ultimately depends on the relative rate at which the various terms grow as $x \equiv \left(1 - \gamma^2 \frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)\right)^{-1}$ approaches ∞ so that we can write

$$\lim_{\frac{1}{\tau_\zeta} \uparrow a} \frac{\Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}}\right)}{\frac{1}{D(0)} \exp\left\{-Q(0)\right\}} = \lim_{x \to \infty} \frac{\Phi\left(-\sqrt{x}\right)}{\frac{1}{\sqrt{x}} \exp\left\{-x\right\}},$$

where we have used the fact that $\frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}}$ and $D(0)$ grow at order $\sqrt{x}$ with x and Q grows at order x. Using L'Hospital's rule yields

$$\begin{aligned}
\lim_{x \to \infty} \frac{\Phi\left(-\sqrt{x}\right)}{\frac{1}{\sqrt{x}} \exp\left\{-x\right\}} &= \lim_{x \to \infty} \frac{-\frac{1}{2} x^{-1/2} \phi\left(-\sqrt{x}\right)}{-\frac{1}{\sqrt{x}} \exp\left\{-x\right\} - \frac{1}{2} x^{-3/2} \exp\left\{-x\right\}} \\
&= \lim_{x \to \infty} \frac{\phi\left(-\sqrt{x}\right)}{2 \exp\left\{-x\right\} + x^{-1} \exp\left\{-x\right\}} \\
&= \lim_{x \to \infty} \frac{\frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2} x\right\}}{2 \exp\left\{-x\right\} + x^{-1} \exp\left\{-x\right\}} \\
&= \infty,
\end{aligned}$$

which establishes $\lim_{\frac{1}{\tau_\zeta} \uparrow a} \frac{\Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}}\right)}{\frac{1}{D(0)} \exp\{-Q(0)\}} = \infty$ and completes the proof. $\square$

Feedback Effects and Systematic Risk Exposures: Internet Appendix[1]

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[1]Citation format: Banerjee, Snehal, Bradyn Breon-Drish, and Kevin Smith, Internet Appendix to "Feedback Effects and Systematic Risk Exposures," Journal of Finance [DOI STRING]. Please note: Wiley is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

A Investor Welfare

In this section, we characterize investors' expected utilities, which is a key step in establishing the welfare results in the text. We make extensive use of matrix notation, and let $\mathcal{V} = \begin{pmatrix} V(k) \\ -\eta_C \end{pmatrix}$ denote the stacked vector of payoffs on the stock and the outside endowment and $\mathcal{Z}_i = \begin{pmatrix} n \\ z_i \end{pmatrix}$ and $\mathcal{Z} = \begin{pmatrix} n \\ Z \end{pmatrix}$ denote the vectors of individual and aggregate endowments. Let I_k denote an identity matrix of dimension k. Let $\mathbf{0}_k$ denote a column vector of zeros of length k and let $\mathbf{0}_{j\times k}$ represent a zero matrix of dimension $j \times k$. We will typically suppress subscripts on these objects where no confusion will result and simply let I or $\mathbf{0}$ represent a conformable identity matrix or zero vector / matrix, respectively. We follow the convention that given random vectors $X \in \mathbb{R}^J$ and $Y \in \mathbb{R}^K$, the cross-covariance matrix $\mathbb{C}(X, Y)$ is the $J \times K$ matrix with $(j,k)^{\text{th}}$ element $\mathbb{C}(X_j, Y_k)$. We denote transposes by $'$ and determinants by $|\cdot|$. Finally, let $\mathcal{F}_p = \sigma(s_p)$ denote the information set given s_p, with associated expectation, variance, and covariance operators $\mathbb{E}_p[\cdot]$, $\mathbb{V}_p(\cdot)$, and $\mathbb{C}_p(\cdot)$, respectively.

The proof of Proposition 2 established that, in equilibrium, we have $P_1 \equiv P_3$ and investors are indifferent to any trading strategy with aggregate trade across the two trading rounds given by

$$X_{i3} + X_{i1} = \frac{\mathbb{E}_{i3}[V(k)] + \gamma \mathbb{C}_{i3}(V(k), \eta_C) z_i - P_3}{\gamma \mathbb{V}_{i3}(V(k))} - n. \tag{IA1}$$

Consequently, the following material proceeds, without loss of generality, under the assumption that $X_{i1} = 0$ and $X_{i3} = \frac{\mathbb{E}_{i3}[V(k)] + \gamma \mathbb{C}_{i3}(V(k),\eta_C) z_i - P_3}{\gamma \mathbb{V}_{i3}(V(k))} - n$. Hence, to reduce notational clutter, we suppress the t dependence of the price function and other equilibrium objects where no confusion will result.

The equilibrium welfare expressions rely, in part, on the following key Lemma, which relates investors' conditional variance at the trading dates to the s_p-conditional moments of returns. This is a variant of the key Lemma 1 in Bond and Garcia (2022), applied to our setting, that represents investor covariances at the trading dates in terms of conditional covariances under the s_p information set.

LEMMA IA1: *Consider an arbitrary investor i. Let $K \equiv \gamma \left(\frac{1}{\gamma} I_2 + \mathbb{C}_p(\mathcal{V}, \mathcal{Z}) \right)$ and define the random vector $\mathcal{V}^* = K^{-1}\mathcal{V} = \begin{pmatrix} V - \gamma \mathbb{C}_p(V,Z)U \\ U \end{pmatrix}$. We have*

$$\mathbb{C}_p(V - P, \mathcal{V}^*) = \mathbb{C}_i(V, \mathcal{V})$$

and

$$\mathbb{C}_p(V - P, \mathcal{Z}_i) = \gamma \mathbb{C}_p(V - P, \mathcal{V}^*) \mathbb{V}_p(\mathcal{Z}).$$

Our first result characterizes the conditional expected utility, given observation of the price-signal s_p. This object is directly useful for characterizing the welfare-maximizing s_p-dependent investment rules and is also used as an intermediate result for deriving the unconditional expected utility.

PROPOSITION IA1: *Consider an arbitrary s_p-dependent investment rule $k(s_p)$, with associated asset value $V = V(k(s_p))$ and pricing rule $P(s_p)$. The conditional expected* *utility of investor i given $\mathcal{F}_p = \sigma(s_p)$ can be written as*

$$\mathcal{W}\left(k; s_p\right) \equiv -D(k; s_p) \exp\left\{Q(k; s_p)\right\} \tag{IA2}$$

where the quadratic form Q is

$$\begin{aligned} Q\left(k; s_p\right) = &-\gamma \mathbb{E}_p\left[V\right] n + \frac{1}{2}\gamma^2 \left(\frac{1}{\tau_A} + k^2\left(\mathbb{V}_p\left(\theta\right) + \frac{1-\alpha^2}{\tau_\eta}\right)\right) n^2 \\ &+ \frac{1}{2}\gamma^2 \frac{1}{\tau_\eta}\left(\mathbb{E}_p\left[Z\right] - k\alpha n - \gamma k \mathbb{C}_p\left(Z, \theta\right) n\right)^2 \left(1 + \Gamma\left(k; s_p\right)\right) \end{aligned}$$

and the determinant term D is

$$D(k; s_p) = \sqrt{\frac{\frac{1}{\tau_A} + k^2 \frac{1}{\tau_\eta}}{\frac{1}{\tau_A} + k^2\left(\mathbb{V}_p\left(\theta | z_i\right) + \frac{1}{\tau_\eta}\right)}} \sqrt{\frac{\Gamma\left(k; s_p\right)}{\gamma^2 \frac{1}{\tau_\eta}\left(\mathbb{V}_p\left(Z\right) + \frac{1}{\tau_\zeta}\right)}}$$

where

$$\Gamma\left(k; s_p\right) = \gamma^2 \frac{1}{\tau_\eta}\left(\mathbb{V}_p\left(Z\right) + \frac{1}{\tau_\zeta}\right)\left(1 - \gamma^2 \frac{1}{\tau_\eta}\left(\mathbb{V}_p\left(Z\right) + \frac{1}{\tau_\zeta}\right)\left(1 - \frac{k^2\alpha^2 \frac{1}{\tau_\eta}\left(\frac{\frac{1}{\tau_\zeta}}{\mathbb{V}_p(Z) + \frac{1}{\tau_\zeta}}\right)^2}{\frac{1}{\tau_A} + k^2\left(\mathbb{V}_p\left(\theta | z_i\right) + \frac{1}{\tau_\eta}\right)}\right)\right)^{-1}$$

Taking the expectation of the s_p-conditional utility with respect to s_p delivers the unconditional welfare, which we record in the following Proposition.

PROPOSITION IA2: *Consider an arbitrary threshold investment rule $k(s_p) = \mathbf{1}_{\{s_p > \bar{s}\}}$. The unconditional expected utility under this investment rule can be written as*

$$\begin{aligned} \mathcal{W} = &-\Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(0)}{\sqrt{v(0)}}\right) D(0) \exp\left\{Q(0)\right\} \\ &- \left(1 - \Phi\left(\frac{\bar{s} - \mathbb{E}[s_p] + m(1)}{\sqrt{v(1)}}\right)\right) D(1) \exp\left\{Q(1)\right\} \end{aligned}$$

where

$$D(k) = \sqrt{\frac{\frac{1}{\tau_A} + k^2 \frac{1}{\tau_\eta}}{\frac{1}{\tau_A} + k^2\left(\frac{1}{\tau_\eta} + \frac{1}{\beta^2\left(\tau_Z + \tau_\zeta\right)}\right)}} \sqrt{\frac{\Gamma\left(k\right)}{\gamma^2 \frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)}},$$

$$\begin{aligned} Q(k) = &-\gamma \mathbb{E}\left[V\left(k\right)\right] n + \frac{1}{2}\gamma^2 \left(\frac{1}{\tau_A} + k^2\left(\frac{1}{\tau_\theta} + \frac{1-\alpha^2}{\tau_\eta}\right)\right) n^2 \\ &+ \frac{1}{2}\gamma^2 \left(\mu_Z - k\alpha n\right)^2 \frac{1}{\tau_\eta}\left(1 + \Gamma\left(k\right)\right) \end{aligned}$$

and

$$m(k) = \gamma \mathbb{C}\left(s_p, V(k)\right) n - \gamma^2 \mathbb{C}\left(s_p, Z\right) \frac{1}{\tau_\eta} \left(\mu_Z - k\alpha n\right) \left(1 + \Gamma\left(k\right)\right)$$

$$v(k) = \mathbb{V}\left(s_p\right) + \gamma^2 \mathbb{C}^2\left(s_p, Z\right) \frac{1}{\tau_\eta} \left(1 + \Gamma\left(k\right)\right),$$

where

$$\Gamma\left(k\right) = \gamma^2 \frac{1}{\tau_\eta} \left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right) \left(1 - \gamma^2 \frac{1}{\tau_\eta} \left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right) \left(1 - \frac{k^2 \alpha^2 \frac{1}{\tau_\eta} \left(\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}}\right)^2}{\frac{1}{\tau_A} + k^2 \left(\frac{1}{\tau_\eta} + \frac{1}{\beta^2 \left(\tau_Z + \tau_\zeta\right)}\right)}\right)\right)^{-1}.$$

A. *Proof of Lemma IA1*

The equilibrium price can be represented as

$$\begin{aligned} P &= \int_j \mathbb{E}_j\left[V\right] dj - \gamma \mathbb{C}_i\left(V, U\right) Z - \gamma \mathbb{V}_i\left(V\right) n \\ &= \int \mathbb{E}_j\left[V\right] dj - \gamma \mathbb{C}_i\left(V, \mathcal{V}\right) \mathcal{Z} \end{aligned}$$

where the second equality concisely writes the risk premium terms using vector notation. Now, we can directly compute

$$\begin{aligned} \mathbb{C}_p\left(V - P, \mathcal{V}\right) &= \mathbb{C}_p\left(V - \int_j \mathbb{E}_j\left[V\right] dj, \mathcal{V}\right) + \mathbb{C}_p\left(\gamma \mathbb{C}_i\left(V, \mathcal{V}\right) \mathcal{Z}, \mathcal{V}\right) \\ &= \int_j \mathbb{C}_p\left(V - \mathbb{E}_j\left[V\right], \mathcal{V}\right) dj + \gamma \mathbb{C}_i\left(V, \mathcal{V}\right) \mathbb{C}_p\left(\mathcal{Z}, \mathcal{V}\right) && \text{(IA3)} \\ &= \mathbb{C}_i\left(V, \mathcal{V}\right) + \gamma \mathbb{C}_i\left(V, \mathcal{V}\right) \mathbb{C}_p\left(\mathcal{Z}, \mathcal{V}\right) && \text{(IA4)} \end{aligned}$$

The second equality uses the linearity of the covariance operator to pull the covariance inside the integral in the first term and pull out the constant vector $\gamma \mathbb{C}_i\left(V, \mathcal{V}\right)$ in the second term. The final equality uses the law of total covariance to conclude $\mathbb{C}_p\left(V - \mathbb{E}_j\left[V\right], \mathcal{V}\right) = \mathbb{C}_j\left(V, \mathcal{V}\right)$ and then uses the fact that the conditional covariances are equal across all investors j, that is, $\mathbb{C}_j\left(V, \mathcal{V}\right) = \mathbb{C}_i\left(V, \mathcal{V}\right)$ for all i, j.

Now, rearranging and grouping terms in eq. (IA3) yields

$$\begin{aligned} \mathbb{C}_p\left(V - P, \mathcal{V}\right) &= \mathbb{C}_i\left(V, \mathcal{V}\right) + \gamma \mathbb{C}_i\left(V, \mathcal{V}\right) \mathbb{C}_p\left(\mathcal{Z}, \mathcal{V}\right) \\ &= \mathbb{C}_i\left(V, \mathcal{V}\right) \gamma \left(\frac{1}{\gamma} I_2 + \mathbb{C}_p\left(\mathcal{Z}, \mathcal{V}\right)\right) \end{aligned}$$

Post-multiplying both sides by the matrix $\left(K'\right)^{-1} \equiv \frac{1}{\gamma}\left(\frac{1}{\gamma} I_2 + \mathbb{C}_p\left(\mathcal{Z}, \mathcal{V}\right)\right)^{-1}$ and using the linearity of the covariance operator to pull the K^{-1} inside the right-hand argument of the $\mathbb{C}_p\left(V - P, \mathcal{V}\right)$ yields the expression in the proposition. Now, considering the $\mathbb{C}_p(V-P,\mathcal{Z}_i)$ covariance, analogous steps yield

$$\begin{aligned}
\mathbb{C}_p(V-P,\mathcal{Z}_i) &= \mathbb{C}_p(V-P,\mathcal{Z}) \\
&= \mathbb{C}_p\left(V-\int_j \mathbb{E}_j[V]\,dj,\mathcal{Z}\right)+\mathbb{C}_p(\gamma\mathbb{C}_i(V,\mathcal{V})\mathcal{Z},\mathcal{Z}) \\
&= \int_j \mathbb{C}_p(V-\mathbb{E}_j[V],\mathcal{Z})\,dj+\gamma\mathbb{C}_i(V,\mathcal{V})\,\mathbb{C}_p(\mathcal{Z},\mathcal{Z}) \\
&= \mathbb{C}_i(V,\mathcal{Z})+\gamma\mathbb{C}_i(V,\mathcal{V})\,\mathbb{V}_p(\mathcal{Z}) \\
&= \gamma\mathbb{C}_p(V-P,\mathcal{V}^*)\,\mathbb{V}_p(\mathcal{Z}).
\end{aligned}$$

where the first equality uses the fact that that the idiosyncratic portion of investor endowments are conditionally uncorrelated with $V-P$ to write the covariance in terms of the aggregate endowment $\mathcal{Z}$, the second equality substitutes in for P, and the third equality uses the linearity of the covariance operator. The next-to-last line uses the law of total variance to write $\mathbb{C}_p(V-\mathbb{E}_j[V],\mathcal{Z})=\mathbb{C}_j(V,\mathcal{Z})$ and then symmetry across all i,j to write this covariance in terms of a generic investor $\mathbb{C}_i$. The final line uses the fact that investors' conditional covariance satisfies $\mathbb{C}_i(V,\mathcal{Z})=0$ (since they either infer Z from the price in the event of investment or the asset payoff is independent, $V=A$, in the event of no investment) and uses $\mathbb{C}_i(V,\mathcal{V})=\mathbb{C}_p(V-P,\mathcal{V}^*)$ from the first part of the Lemma. □

B. *Proof of Proposition IA1*

To compute the conditional expected utility, we will use the law of iterated expectations, first computing the expectation conditional on $\mathcal{F}_{i^+}=\sigma(\{\theta,z_i,s_p\})$, which is the investor information set augmented with s_p, and then computing the conditional expectation of that object given $\mathcal{F}_p$. We emphasize that the initial step is, in principle, not identical to computing the expectation given the investor information set $\mathcal{F}_i$ itself since s_p is only inferred by the investor in states in which investment is positive and the asset price has non-trivial dependence on s_p. However, we begin the proof by establishing that, in fact, the conditional expected utilities given $\mathcal{F}_i$ and $\mathcal{F}_{i^+}$ are identical.

Because the investment decision k is in both $\mathcal{F}_i$ and $\mathcal{F}_{i^+}$ (i.e., k is observed/inferred by investors in equilibrium and is in $\mathcal{F}_{i^+}$ since k is s_p-measurable), all random variables are conditionally Gaussian under both $\mathcal{F}_i$ and $\mathcal{F}_{i^+}$. Hence, to show that the two conditional expected utilities are identical, it suffices to show that the conditional moments are identical. Direct calculation establishes that the conditional means satisfy

$$\mathbb{E}_{i^+}[V]=\begin{cases}\mathbb{E}[A|\theta,z_i,s_p] & k=0\\ \mathbb{E}[A+\theta-c+\alpha\eta_C+\sqrt{1-\alpha^2}\eta_I|\theta,z_i,s_p] & k=1\end{cases} \tag{IA5}$$

$$=\begin{cases}\mu_A & k=0\\ \mu_A+\mathbb{E}[\theta-c|\theta,z_i,s_p] & k=1\end{cases} \tag{IA6}$$

$$=\mathbb{E}_i[V] \tag{IA7}$$

and

$$\mathbb{E}_{i^+}[U]=\mathbb{E}[-\eta_C|\theta,z_i,s_p]=0=\mathbb{E}_i[U]. \tag{IA8}$$

Similarly, the conditional variances and covariances satisfy

$$\mathbb{V}_{i^+}[V] = \begin{cases} \mathbb{V}[A|\theta, z_i, s_p] & k=0 \\ \mathbb{V}[A+\theta-c+\alpha\eta_C+\sqrt{1-\alpha^2}\eta_I|\theta, z_i, s_p] & k=1 \end{cases} \tag{IA9}$$

$$= \begin{cases} \frac{1}{\tau_A} & k=0 \\ \frac{1}{\tau_A}+\frac{1}{\tau_\eta} & k=1 \end{cases} \tag{IA10}$$

$$= \mathbb{V}_i[V], \tag{IA11}$$

$$\mathbb{V}_{i^+}[U] = \mathbb{V}[-\eta_C|\theta, z_i, s_p] = \tfrac{1}{\tau_\eta} = \mathbb{V}_i[U] \tag{IA12}$$

and

$$\mathbb{C}_{i^+}(V,U) = \begin{cases} \mathbb{C}(A, U|\theta, z_i, s_p) & k=0 \\ \mathbb{C}(A+\theta-c+\alpha\eta_C+\sqrt{1-\alpha^2}\eta_I, U|\theta, z_i, s_p) & k=1 \end{cases} \tag{IA13}$$

$$= \begin{cases} 0 & k=0 \\ -\alpha\frac{1}{\tau_\eta} & k=1 \end{cases} \tag{IA14}$$

$$= \mathbb{C}_i(V,U). \tag{IA15}$$

Because the conditional moments are identical under both $\mathcal{F}_i$ and $\mathcal{F}_{i^+}$, it follows that the conditional expected utilities are identical and given by substituting the optimal demand back into the investor objective function in Proposition 2. Straightforward algebra establishes that

$$\mathbb{E}_i[-e^{-\gamma W_i^*}] = -e^{-\gamma n\mathbb{E}_i[V]+\gamma\left(n+z_i\frac{\mathbb{C}_i(V-P,U)}{\mathbb{V}_i(V-P)}\right)\mathbb{E}_i[V-P]-\frac{1}{2}\frac{\mathbb{E}_i^2[V-P]}{\mathbb{V}_i(V-P)}+\frac{1}{2}\gamma^2\mathbb{V}_i(U|V-P)z_i^2} \tag{IA16}$$

where we have used $\mathbb{V}_i\left(U|V-P\right) = \mathbb{V}_i\left(U\right) - \mathbb{C}_i^2\left(U, V-P\right)/\mathbb{V}_i\left(V-P\right)$ to condense the final term in the exponential.

To complete the proof, we need to compute the conditional expectation of this quantity given $\mathcal{F}_p$. Let $h_i = \left(1, \frac{\mathbb{C}_i(V-P,U)}{\mathbb{V}_i(V-P)}\right)$ be the 2×1 vector of conditional regression coefficients of (V,U) on $V-P$ and define the 5×5 block matrix

$$a_i = \begin{pmatrix} \mathbb{V}_i^{-1}(V-P) & \mathbf{0}_{1\times 2} & -\gamma h_i' \\ \mathbf{0}_{2\times 1} & \mathbf{0}_{2\times 2} & \gamma I_2 \\ -\gamma h_i & \gamma I_2 & -\gamma^2\mathbb{V}_i(\mathcal{V}|V-P) \end{pmatrix}. \tag{IA17}$$

Similarly, let

$$Y \equiv \begin{pmatrix} V-P \\ \mathcal{V} \\ \mathcal{Z}_i \end{pmatrix} = \begin{pmatrix} V-P \\ \binom{V}{U} \\ \binom{n}{z_i} \end{pmatrix}$$

denote the conformably partitioned vector of asset returns, payoffs, and endowments. With this notation, we can concisely write the $\mathcal{F}_{i^+}$ expected utility above as

$$\mathbb{E}_i[-e^{-\gamma W_i^*}] = -e^{-\frac{1}{2}\mathbb{E}_i[Y]'a_i\mathbb{E}_i[Y]}. \tag{IA18}$$

Because the investment decision $k(s_p)$ is known given $\mathcal{F}_p$, the random vector $\mathbb{E}_i[Y]$ is conditionally jointly normally distributed given $\mathcal{F}_p$.[2] We can now use standard formulas for expected exponential-quadratic forms of normal random vectors to compute

$$\mathbb{E}_R\left[-e^{-\frac{1}{2}\mathbb{E}_p[Y]'a_i\mathbb{E}_p[Y]}\right] \tag{IA19}$$

$$= -|a_i|^{-1/2}\left|\mathbb{V}_p(\mathbb{E}_i[Y]) + a_i^{-1}\right|^{-1/2}\exp\left\{-\frac{1}{2}\mathbb{E}_p[Y]'\left(\mathbb{V}_p(\mathbb{E}_i[Y]) + a_i^{-1}\right)^{-1}\mathbb{E}_p[Y]\right\} \tag{IA20}$$

where we use the law of iterated expectations to write $\mathbb{E}_p[\mathbb{E}_i[Y]] = \mathbb{E}_p[Y]$. This expression requires that the matrix a_i is invertible. Using standard formulas for determinants of partitioned matrices (e.g., eq. (5) in Henderson and Searle (1981)) we can compute its determinant,

$$|a_i| = \left|\mathbb{V}_i^{-1}(V-P)\right|\left|-\gamma^2 I_2\right| = \gamma^4\left|\mathbb{V}_i^{-1}(V-P)\right| > 0, \tag{IA21}$$

which implies that a_i is invertible. Similarly, using standard formulas for inverses of partitioned matrices (e.g., eq. (8) in Henderson and Searle (1981)) we have

$$a_i^{-1} = \begin{pmatrix} \mathbb{V}_i(V-P) & \mathbb{C}_i(V-P,\mathcal{V}) & \mathbf{0} \\ \mathbb{C}_i(\mathcal{V}, V-P) & \mathbb{V}_i(\mathcal{V}) & \frac{1}{\gamma}I_2 \\ \mathbf{0} & \frac{1}{\gamma}I_2 & \mathbb{V}0 \end{pmatrix} \tag{IA22}$$

where we have again used $\mathbb{V}_i(U|V-P) = \mathbb{V}_i(U) - \mathbb{C}_i(U, V-P)\,\mathbb{V}_i^{-1}(V-P)\,\mathbb{C}_i(V-P, U)$ when simplifying the inverse. It now follows from the law of total variance, noting that $\mathcal{Z}_i$ is $\mathcal{F}_i$-measurable and so $\mathbb{V}_i(\mathcal{Z}_i) = \mathbf{0}$, that

$$\mathbb{V}_p(\mathbb{E}_i[Y]) + a_i^{-1} = \mathbb{V}_p(\mathbb{E}_i[Y]) + \begin{pmatrix} \mathbb{V}_i(V-P) & \mathbb{C}_i(V-P,\mathcal{V}) & \mathbf{0} \\ \mathbb{C}_i(\mathcal{V}, V-P) & \mathbb{V}_i(\mathcal{V}) & \frac{1}{\gamma}I_2 \\ \mathbf{0} & \frac{1}{\gamma}I_2 & \mathbf{0} \end{pmatrix} \tag{IA23}$$

$$= \mathbb{V}_p(Y) + \begin{pmatrix} 0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{1}{\gamma}I_2 \\ \mathbf{0} & \frac{1}{\gamma}I_2 & \mathbf{0} \end{pmatrix} \tag{IA24}$$

$$\equiv \mathbb{V}_p(Y) + \mathcal{I} \tag{IA25}$$

where the final equality defines the 5×5 matrix $\mathcal{I} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix}$.

Putting together everything above, the conditional expected utility given $\mathcal{F}_p$ can be

[2] Note that $\mathbb{E}_i[Y]$ follows a singular normal distribution since n is a constant. That is, the conditional variance matrix of $\mathbb{E}_i[Y]$ is only positive semidefinite. However, defining the random vector in this way causes no difficulties in the derivation below and simplifies the algebra by treating the endowment of shares and the non-tradeable in a unified way.

written

$$\mathbb{E}_p[-e^{-\gamma W_i^*}] = -\frac{1}{\gamma^2}\left|\mathbb{V}_i^{-1}(V-P)\right|^{-1/2}\left|\mathbb{V}_p(Y)+\mathcal{I}\right|^{-1/2}\exp\left\{-\frac{1}{2}\mathbb{E}_p(Y)'\left(\mathbb{V}_p(Y)+\mathcal{I}\right)^{-1}\mathbb{E}_p(Y)\right\}. \tag{IA26}$$

Define

$$D\left(k;s_p\right) \equiv \frac{1}{\gamma^2}\left|\mathbb{V}_i^{-1}(V-P)\right|^{-1/2}\left|\mathbb{V}_p(Y)+\mathcal{I}\right|^{-1/2} \tag{IA27}$$

and

$$Q\left(k;s_p\right) \equiv -\frac{1}{2}\mathbb{E}_p(Y)'\left(\mathbb{V}_p(Y)+\mathcal{I}\right)^{-1}\mathbb{E}_p(Y). \tag{IA28}$$

To complete the proof, we will show how to express D and Q in the form in the Proposition.

B.1. Deriving an expression for the quadratic form $Q\left(k;s_p\right)$

We first tackle the quadratic form $Q\left(k;s_p\right)$. Define the matrix $K \equiv \gamma\left(\frac{1}{\gamma}I_2+\mathbb{C}_p\left(\mathcal{V},\mathcal{Z}\right)\right)$ and let

$$\mathcal{V}^* \equiv K^{-1}\mathcal{V} = \begin{pmatrix} V-\gamma\mathbb{C}_p(V,Z)U \\ U \end{pmatrix}.$$

Consider the change of variables $Y \mapsto Y^* = \left(\frac{1}{\gamma}\mathbb{V}_p^{-1}\left(V-P\right)\left(V-P-\gamma\mathbb{C}_p\left(V-P,\mathcal{V}^*\right)\mathcal{Z}_i\right),\mathcal{V}^*,\mathcal{Z}_i\right)$ obtained by premultiplying Y by the matrix

$$M \equiv \begin{pmatrix} \frac{1}{\gamma}\mathbb{V}_p^{-1}(V-P) & \mathbf{0} & -\mathbb{V}_p^{-1}(V-P)\mathbb{C}_p(V-P,\mathcal{V}^*) \\ \mathbf{0} & K^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_2 \end{pmatrix}. \tag{IA29}$$

This allows us to write the quadratic form as

$$\begin{aligned} &-\frac{1}{2}\mathbb{E}_p[MY]'\left(M\begin{pmatrix} \mathbb{V}_p(V-P) & \mathbb{C}_p(V-P,\mathcal{V}) & \mathbb{C}_p(V-P,\mathcal{Z}_i) \\ \mathbb{C}_p(\mathcal{V},V-P) & \mathbb{V}_p(\mathcal{V}) & \frac{1}{\gamma}I_2+\mathbb{C}_p(\mathcal{V},\mathcal{Z}) \\ \mathbb{C}_p(\mathcal{Z}_i,V-P) & \frac{1}{\gamma}I_2+\mathbb{C}_p(\mathcal{Z},\mathcal{V}) & \mathbb{V}_p(\mathcal{Z}_i) \end{pmatrix}M'\right)^{-1}\mathbb{E}_p[MY] \\ &= -\frac{1}{2}\mathbb{E}_p[Y^*]'\begin{pmatrix} A_0 & \mathbb{C}_p(V-P,\mathcal{V}^*) & B_0 \\ \mathbb{C}_p(\mathcal{V}^*,V-P) & \mathbb{V}_p(\mathcal{V}^*) & \frac{1}{\gamma}I_2 \\ B_0' & \frac{1}{\gamma}I_2 & \mathbb{V}_p(\mathcal{Z}_i) \end{pmatrix}^{-1}\mathbb{E}_p[Y^*] \end{aligned} \tag{IA30}$$

where we have pre-multiplied each Y by $I = M^{-1}M$ and pulled the M^{-1}'s inside the inner inverse matrix and where we define

$$\begin{aligned} A_0 &= \frac{1}{\gamma}\mathbb{V}_p^{-1}\left(V-P\right) - \frac{1}{\gamma}\mathbb{V}_p^{-1}\left(V-P\right)\mathbb{C}_p\left(V-P,\mathcal{V}^*\right)\mathbb{C}_p\left(\mathcal{Z}_i,V-P\right)\mathbb{V}_p^{-1}\left(V-P\right) \\ &\quad - \frac{1}{\gamma}\mathbb{V}_p^{-1}\left(V-P\right)\mathbb{C}_p\left(V-P,\mathcal{Z}_i\right)\mathbb{C}_p\left(\mathcal{V}^*,V-P\right)\mathbb{V}_p^{-1}\left(V-P\right) \\ &\quad + \mathbb{V}_p^{-1}\left(V-P\right)\mathbb{C}\left(V-P,\mathcal{V}^*\right)\mathbb{V}\left(\mathcal{Z}_i\right)\mathbb{C}_p\left(\mathcal{V}^*,V-P\right)\mathbb{V}_p^{-1}\left(V-P\right) \\ &= \frac{1}{\gamma}\mathbb{V}_p^{-1}\left(V-P\right) - \frac{1}{\gamma}\mathbb{V}_p^{-1}\left(V-P\right)\mathbb{C}_p\left(V-P,U\right)\mathbb{C}_p\left(z_i,V-P\right)\mathbb{V}_p^{-1}\left(V-P\right) \end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{\gamma}\mathbb{V}_p^{-1}(V-P)\,\mathbb{C}_p(V-P,z_i)\,\mathbb{C}_p(U,V-P)\,\mathbb{V}_p^{-1}(V-P)\\
&+\mathbb{V}_p^{-1}(V-P)\,\mathbb{C}(V-P,U)\,\mathbb{V}(z_i)\,\mathbb{C}_p(U,V-P)\,\mathbb{V}_p^{-1}(V-P)\\
B_0 &= \mathbb{V}_p^{-1}(V-P)\left(\frac{1}{\gamma}\mathbb{C}_p(V-P,\mathcal{Z}_i)-\mathbb{C}_p(V-P,\mathcal{V}^*)\,\mathbb{V}(\mathcal{Z}_i)\right).\\
&= \begin{pmatrix} 0 & \mathbb{V}_p^{-1}(V-P)\left(\frac{1}{\gamma}\mathbb{C}_p(V-P,z_i)-\mathbb{C}_p(V-P,U)\mathbb{V}(z_i)\right)\end{pmatrix}
\end{aligned}
$$

where the second equality in each definition uses the fact that the second element of $\mathcal{Z}_i$ is identically equal to n and so the associated (co)variances are all zero to write the expressions purely in terms of the nontradeable endowment z_i.

Furthermore, using these expressions and grouping terms appropriately in eq. (IA30) it is tedious but straightforward to verify that that the matrix inside the inverse in eq. (IA30) can be written as

$$
\begin{aligned}
&\begin{pmatrix} \frac{1}{\gamma^2}\mathbb{V}_p^{-1}(V-P)\left(\mathbb{V}_p(V-P)-\mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,V-P)\right)\mathbb{V}_p^{-1}(V-P) & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbb{V}_p(\mathcal{V}^*) & \frac{1}{\gamma}I_2\\ \mathbf{0} & \frac{1}{\gamma}I_2 & \mathbf{0}\end{pmatrix}\\
&\quad+\begin{pmatrix}\mathbb{V}_p^{-1}(V-P)\left(\frac{1}{\gamma}\mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i)-\mathbb{C}_p(V-P,U)\right)\\ \mathbf{0}\\ \binom{0}{1}\end{pmatrix}\mathbb{V}_p(z_i)\begin{pmatrix}\mathbb{V}_p^{-1}(V-P)\left(\frac{1}{\gamma}\mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i)-\mathbb{C}_p(V-P,U)\right)\\ \mathbf{0}\\ \binom{0}{1}\end{pmatrix}'\\
&=\begin{pmatrix} \frac{1}{\gamma^2}\mathbb{V}_p^{-1}(V-P)\mathbb{V}_p(V-P|z_i)\mathbb{V}_p^{-1}(V-P) & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbb{V}_p(\mathcal{V}^*) & \frac{1}{\gamma}I_2\\ \mathbf{0} & \frac{1}{\gamma}I_2 & \mathbf{0}\end{pmatrix}\\
&\quad+\begin{pmatrix}\mathbb{V}_p^{-1}(V-P)\left(\frac{1}{\gamma}\mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i)-\mathbb{C}_p(V-P,U)\right)\\ \mathbf{0}\\ \binom{0}{1}\end{pmatrix}\mathbb{V}_p(z_i)\begin{pmatrix}\mathbb{V}_p^{-1}(V-P)\left(\frac{1}{\gamma}\mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i)-\mathbb{C}_p(V-P,U)\right)\\ \mathbf{0}\\ \binom{0}{1}\end{pmatrix}'
\end{aligned}
$$

where the equality uses $\mathbb{V}_p(V-P|z_i) = \mathbb{V}_p(V-P) - \mathbb{C}_p(V-P,z_i)\,\mathbb{V}_p^{-1}(z_i)\,\mathbb{C}_p(z_i,V-P)$ to condense notation.

We can now invert this matrix using eq. (16) in Henderson and Searle (1981) with[3]

$$
\begin{aligned}
A &= \begin{pmatrix} \frac{1}{\gamma^2}\mathbb{V}_p^{-1}(V-P)\mathbb{V}_p(V-P|z_i)\mathbb{V}_p^{-1}(V-P) & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbb{V}_p(\mathcal{V}^*) & \frac{1}{\gamma}I_2\\ \mathbf{0} & \frac{1}{\gamma}I_2 & \mathbf{0}\end{pmatrix}\\
C &= D' = \begin{pmatrix}\mathbb{V}_p^{-1}(V-P)\left(\frac{1}{\gamma}\mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i)-\mathbb{C}_p(V-P,U)\right)\\ \mathbf{0}\\ \binom{0}{1}\end{pmatrix}\\
B &= \mathbb{V}_p(z_i)
\end{aligned}
$$

to conclude

$$
(A+CBD)^{-1} = A^{-1} - A^{-1}C\left(B^{-1}+DA^{-1}C\right)^{-1}DA^{-1}. \tag{IA31}
$$

To pin down the individual terms in eq. (IA31), note that direct calculation using stan-

[3] The expression in Henderson and Searle (1981) uses U and V to denote the matrices that we label C and D here. We use different notation to prevent any confusion with the asset payoffs V and U.

dard methods for inverting a partitioned matrix yields

$$A^{-1} = \begin{pmatrix} \gamma^2 \mathbb{V}_p(V-P)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{V}_p(V-P) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \gamma I_2 \\ \mathbf{0} & \gamma I_2 & -\gamma^2 \mathbb{V}_p(\mathcal{V}^*) \end{pmatrix}. \tag{IA32}$$

And further plugging into the second term in eq. (IA31) and grouping terms yields

$$\begin{aligned} & - A^{-1}C\left(B^{-1}+DA^{-1}C\right)^{-1}DA^{-1} \\ & = -\begin{pmatrix} \gamma^2 \mathbb{V}_p(V-P)\mathbb{V}_p^{-1}(V-P|z_i)\left(\frac{1}{\gamma}\mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i)-\mathbb{C}_p(V-P,U)\right) \\ \gamma\binom{0}{1} \\ -\gamma^2\mathbb{V}_p(\mathcal{V}^*)\binom{0}{1} \end{pmatrix} && \text{(IA33)} \\ & \quad \times \left(\mathbb{V}_p^{-1}(z_i)-\gamma^2\mathbb{V}_p(U)+\gamma^2\left(\tfrac{1}{\gamma}\mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i)-\mathbb{C}_p(V-P,U)\right)'\mathbb{V}_p^{-1}(V-P|z_i)\left(\tfrac{1}{\gamma}\mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i)-\mathbb{C}_p(V-P,U)\right)\right)^{-1} && \text{(IA34)} \\ & \quad \times \begin{pmatrix} \gamma^2 \mathbb{V}_p(V-P)\mathbb{V}_p^{-1}(V-P|z_i)\left(\frac{1}{\gamma}\mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i)-\mathbb{C}_p(V-P,U)\right) \\ \binom{0}{\gamma} \\ -\gamma^2\mathbb{V}_p(\mathcal{V}^*)\binom{0}{1} \end{pmatrix}' && \text{(IA35)} \\ & = -\begin{pmatrix} \gamma^2 \mathbb{V}_p(V-P)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P,U)\left(\mathbb{V}_p(Z)\mathbb{V}_p^{-1}(z_i)-1\right) \\ \gamma\binom{0}{1} \\ -\gamma^2\mathbb{V}_p(\mathcal{V}^*)\binom{0}{1} \end{pmatrix} \\ & \quad \times \left(\mathbb{V}_p^{-1}(z_i)-\gamma^2\mathbb{V}_p(U)+\gamma^2\frac{\mathbb{C}_p^2(V-P,U)}{\mathbb{V}_p(V-P|z_i)}\left(\mathbb{V}_p(Z)\mathbb{V}_p^{-1}(z_i)-1\right)^2\right)^{-1} \\ & \quad \times \begin{pmatrix} \gamma^2 \mathbb{V}_p(V-P)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P,U)\left(\mathbb{V}_p(Z)\mathbb{V}_p^{-1}(z_i)-1\right) \\ \binom{0}{\gamma} \\ -\gamma^2\mathbb{V}_p(\mathcal{V}^*)\binom{0}{1} \end{pmatrix}' && \text{(IA36)} \end{aligned}$$

where the second equality substitutes in for the equilibrium value of $\mathbb{C}_p(V-P,z_i)$ from Lemma (IA1) and collects terms. Eqs. (IA32) and (IA36) together give us the inverse in eq. (IA30), which we need to pre- and post-muliply by $\mathbb{E}[Y^*]$ and its transpose.

We have

$$\mathbb{E}[Y^*] = \mathbb{E}\left[\begin{pmatrix} \frac{1}{\gamma}\mathbb{V}_p^{-1}(V-P)(V-P-\gamma\mathbb{C}_p(V-P,\mathcal{V}^*)\mathcal{Z}_i) \\ \mathcal{V}^* \\ \mathcal{Z}_i \end{pmatrix}\right] = \begin{pmatrix} 0 \\ \binom{0}{\mathbb{E}_p[V]} \\ \binom{n}{\mathbb{E}_p[Z]} \end{pmatrix}$$

where the second equality has used the result from Lemma (IA1), which implies $\mathbb{E}_p[V-P] = \gamma\mathbb{C}_i(V,\mathcal{V})\mathbb{E}_p[\mathcal{Z}] = \gamma\mathbb{C}_p(V-P,\mathcal{V}^*)\mathbb{E}_p[\mathcal{Z}]$, to conclude that the first element of Y^* has conditional expectation zero.

Putting things together, we have established that (IA28) can be written

$$\begin{aligned} & -\frac{1}{2}\mathbb{E}_p[Y^*]'\left(A^{-1}-A^{-1}C\left(B^{-1}+DA^{-1}C\right)^{-1}DA^{-1}\right)\mathbb{E}_p[Y^*] \\ & = -\gamma n\mathbb{E}_p[V]+\frac{1}{2}\gamma^2\binom{n}{\mathbb{E}_p[Z]}'\mathbb{V}_p(\mathcal{V}^*)\binom{n}{\mathbb{E}_p[Z]} \\ & + \frac{1}{2}\gamma^2\binom{n}{\mathbb{E}_p[Z]}'\mathbb{V}_p(\mathcal{V}^*)\binom{0}{1}\left(\mathbb{V}_p^{-1}(z_i)-\gamma^2\mathbb{V}_p(U)+\gamma^2\frac{\mathbb{C}_p^2(V-P,U)}{\mathbb{V}_p(V-P|z_i)}\left(\mathbb{V}_p(Z)\mathbb{V}_p^{-1}(z_i)-1\right)^2\right)^{-1}\gamma^2\binom{0}{1}'\mathbb{V}_p(\mathcal{V}^*)\binom{n}{\mathbb{E}_p[Z]}. \end{aligned} \tag{IA37}$$

Finally, using the definition $\mathcal{V}^* = K^{-1}\mathcal{V} = \begin{pmatrix} 1 & -\gamma\mathbb{C}_p(V,Z) \\ 0 & 1 \end{pmatrix}$ we can write

$$\mathbb{V}_p(\mathcal{V}^*) = \begin{pmatrix} 1 & -\gamma\mathbb{C}_p(V,Z) \\ 0 & 1 \end{pmatrix}\mathbb{V}(\mathcal{V})\begin{pmatrix} 1 & 0 \\ -\gamma\mathbb{C}_p(Z,V) & 1 \end{pmatrix}$$

which implies

$$\left(\begin{smallmatrix} n \\ \mathbb{E}_p[Z] \end{smallmatrix}\right)' \mathbb{V}_p\left(\mathcal{V}^*\right)\left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right) = \left(\begin{smallmatrix} n \\ \mathbb{E}_p[Z]-\gamma\mathbb{C}_p(V,Z)n \end{smallmatrix}\right)' \mathbb{V}\left(\mathcal{V}\right)\left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right) = \left(\mathbb{E}_p\left[Z\right] - k\alpha n - \gamma\mathbb{C}_p\left(Z,V\right)n\right)\frac{1}{\tau_\eta}$$

and

$$\begin{aligned}
&\left(\begin{smallmatrix} n \\ \mathbb{E}_p[Z] \end{smallmatrix}\right)' \mathbb{V}_p\left(\mathcal{V}^*\right)\left(\begin{smallmatrix} n \\ \mathbb{E}_p[Z] \end{smallmatrix}\right) \\
&= \left(\begin{smallmatrix} n \\ \mathbb{E}_p[Z] \end{smallmatrix}\right)' \left(\begin{smallmatrix} 1 & -\gamma\mathbb{C}_p(V,Z) \\ 0 & 1 \end{smallmatrix}\right)\mathbb{V}_p\left(\mathcal{V}\right)\left(\begin{smallmatrix} 1 & 0 \\ -\gamma\mathbb{C}_p(Z,V) & 1 \end{smallmatrix}\right)\left(\begin{smallmatrix} n \\ \mathbb{E}_p[Z] \end{smallmatrix}\right) \\
&= \left(\begin{smallmatrix} n \\ \mathbb{E}_p[Z]-\gamma\mathbb{C}_p(V,Z)n \end{smallmatrix}\right)' \left(\begin{smallmatrix} \frac{1}{\tau_A}+k^2\left(\mathbb{V}_p(\theta)+\frac{1}{\tau_\eta}\right) & -k\alpha\frac{1}{\tau_\eta} \\ -k\alpha\frac{1}{\tau_\eta} & \frac{1}{\tau_\eta} \end{smallmatrix}\right)\left(\begin{smallmatrix} n \\ \mathbb{E}_p[Z]-\gamma\mathbb{C}_p(V,Z)n \end{smallmatrix}\right) \\
&= \left(\frac{1}{\tau_A} + k^2\left(\mathbb{V}_p\left(\theta\right) + \frac{1-\alpha^2}{\tau_\eta}\right)\right)n^2 + \left(\mathbb{E}_p\left[Z\right] - k\alpha n - \gamma\mathbb{C}_p\left(Z,V\right)n\right)^2\frac{1}{\tau_\eta}.
\end{aligned}$$

Plugging back in to eq. (IA37), using $\mathbb{V}_p\left(U\right) = \frac{1}{\tau_\eta}$, $\mathbb{C}_p^2\left(V-P,U\right) = -\alpha k\frac{1}{\tau_\eta}$,$\mathbb{V}_p\left(z_i\right) = \mathbb{V}_p\left(Z\right) + \frac{1}{\tau_\zeta}$ and $\mathbb{V}_p\left(V-P|z_i\right) = \mathbb{V}_p\left(V|z_i\right) = \frac{1}{\tau_A} + k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)} + \frac{1}{\tau_\eta}\right)$, and grouping terms yields

$$\begin{aligned}
Q\left(k;s_p\right) &= -\gamma n\mathbb{E}_p\left[V\right] + \frac{1}{2}\gamma^2\left(\tfrac{1}{\tau_A}+k^2\left(\mathbb{V}_p(\theta)+\tfrac{1-\alpha^2}{\tau_\eta}\right)\right)n^2 + \frac{1}{2}(\mathbb{E}_p[Z]-k\alpha n-\gamma\mathbb{C}_p(Z,V)n)^2\tfrac{1}{\tau_\eta} \\
&\quad + \frac{1}{2}\left(\gamma^2\right)^2 (\mathbb{E}_p[Z]-k\alpha n-\gamma\mathbb{C}_p(Z,V)n)^2\left(\tfrac{1}{\tau_\eta}\right)^2\left(\frac{1}{\mathbb{V}_p(Z)+\frac{1}{\tau_\zeta}}-\gamma^2\frac{1}{\tau_\eta}+\gamma^2\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\left(\frac{\frac{1}{\tau_\zeta}}{\mathbb{V}_p(Z)+\frac{1}{\tau_\zeta}}\right)^2\right)^{-1} \\
&= -\gamma n\mathbb{E}_p\left[V\right] + \frac{1}{2}\gamma^2\left(\frac{1}{\tau_A} + k^2\left(\mathbb{V}_p\left(\theta\right) + \frac{1-\alpha^2}{\tau_\eta}\right)\right)n^2 \\
&\qquad + \frac{1}{2}\gamma^2\frac{1}{\tau_\eta}\left(\mathbb{E}_p\left[Z\right] - k\alpha n - \gamma\mathbb{C}_p\left(Z,V\right)n\right)^2\left(1+\Gamma\left(k;s_p\right)\right)
\end{aligned}$$

where we define

$$\begin{aligned}
\Gamma\left(k;s_p\right) &= \gamma^2\tfrac{1}{\tau_\eta}\left(\frac{1}{\mathbb{V}_p(Z)+\frac{1}{\tau_\zeta}}-\gamma^2\frac{1}{\tau_\eta}+\gamma^2\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\left(\frac{\frac{1}{\tau_\zeta}}{\mathbb{V}_p(Z)+\frac{1}{\tau_\zeta}}\right)^2\right)^{-1} \\
&= \gamma^2\tfrac{1}{\tau_\eta}\left(\mathbb{V}_p(Z)+\tfrac{1}{\tau_\zeta}\right)\left(1-\gamma^2\tfrac{1}{\tau_\eta}\left(\mathbb{V}_p(Z)+\tfrac{1}{\tau_\zeta}\right)\left(1-\frac{k^2\alpha^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\left(\frac{\frac{1}{\tau_\zeta}}{\mathbb{V}_p(Z)+\frac{1}{\tau_\zeta}}\right)^2\right)\right)^{-1}.
\end{aligned}$$

This now matches the expression in the statement of the Proposition after substituting in $\mathbb{C}_p(Z,V) = k\mathbb{C}_p(Z,\theta)$.

B.2. Deriving an expression for the determinant $D\left(k;s_p\right)$

Consider now the determinant term defined in eq. (IA27)

$$D\left(k;s_p\right) = \frac{1}{\gamma^2}\left|\mathbb{V}_i^{-1}(V-P)\right|^{-1/2}\left|\mathbb{V}_p(Y)+\mathcal{I}\right|^{-1/2}$$

Using the same transformation described by the matrix in eq. (IA29), we can write

$$\left|\mathbb{V}_p(Y)+\mathcal{I}\right| \tag{IA38}$$

$$= \left|M^{-1}\right| \left|M\left(\mathbb{V}_p(Y)+\mathcal{I}\right)M'\right| \left|(M')^{-1}\right| \tag{IA39}$$
$$= \left|\gamma\mathbb{V}_p\left(V-P\right)\right|^2 |K|^2 |A+CBD| \tag{IA40}$$
$$= \gamma^2 \left|\mathbb{V}_p\left(V-P\right)\right|^2 |A+CBD| \tag{IA41}$$
$$= \gamma^2 \left|\mathbb{V}_p\left(V-P\right)\right|^2 |A|\,|B| \left|B^{-1}+DA^{-1}C\right| \tag{IA42}$$
$$= \gamma^2 \left|\mathbb{V}_p\left(V-P\right)\right|^2 \frac{1}{\gamma^6} \left|\mathbb{V}_p^{-1}\left(V-P\right)\mathbb{V}_p\left(V-P|z_i\right)\mathbb{V}_p^{-1}\left(V-P\right)\right| \left|\mathbb{V}_p\left(z_i\right)\right| \tag{IA43}$$
$$\times \left| \frac{1}{\mathbb{V}_p\left(Z\right)+\frac{1}{\tau_\varsigma}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2 \frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\varsigma)}+\frac{1}{\tau_\eta}\right)} \left(\frac{\frac{1}{\tau_\varsigma}}{\mathbb{V}_p\left(Z\right)+\frac{1}{\tau_\varsigma}}\right)^2 \right| \tag{IA44}$$
$$= \frac{1}{\gamma^4} \left|\mathbb{V}_p\left(V-P|z_i\right)\right| \left|\mathbb{V}_p\left(z_i\right)\right| \left(\frac{1}{\mathbb{V}_p(Z)+\frac{1}{\tau_\varsigma}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2 \frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\varsigma)}+\frac{1}{\tau_\eta}\right)} \left(\frac{\frac{1}{\tau_\varsigma}}{\mathbb{V}_p(Z)+\frac{1}{\tau_\varsigma}}\right)^2 \right) \tag{IA45}$$

where the first equality pre- and post-multiplies the matrix by $I = M^{-1}M$ and uses the multiplicative property of the determinant, the second equality uses the fact that $|M| = |\gamma\mathbb{V}_p\left(V-P\right)|\,|K|$ and plugs in for $M\left(\mathbb{V}_p(Y)+\mathcal{I}\right)M'$ using the decomposition defined earlier in eq. (IA31), and the third equality uses the fact that $|K| = 1$. The fourth equality uses the multiplicative property of determinants and Sylvester's determinant identity to write $|A+CBD| = |A|\,|I+A^{-1}CBD| = |A|\,|I+DA^{-1}CB| = |A|\,|B|\,|B^{-1}+DA^{-1}C|$. The fifth equality computes the determinants of A and B and substitutes in for $B^{-1} + VA^{-1}U$, which was computed as part of the proof in Section (B.1) above. The final line cancels terms and uses the fact that the $B^{-1}+VA^{-1}U$ term is a scalar and so is identical to its determinant

We can now plug back into eq. (IA27) to yield the overall determinant term

$$D(k;s_p) = \sqrt{\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_p(V-P|z_i)}}\sqrt{\frac{1}{\mathbb{V}_p(z_i)}} \tag{IA46}$$
$$\times \left(\frac{1}{\mathbb{V}_p\left(Z\right)+\frac{1}{\tau_\varsigma}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2 \frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\varsigma)}+\frac{1}{\tau_\eta}\right)} \left(\frac{\frac{1}{\tau_\varsigma}}{\mathbb{V}_p\left(Z\right)+\frac{1}{\tau_\varsigma}}\right)^2 \right)^{-1/2} \tag{IA47}$$
$$= \sqrt{\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\varsigma)}+\frac{1}{\tau_\eta}\right)}}\sqrt{\frac{1}{\mathbb{V}_p\left(Z\right)+\frac{1}{\tau_\varsigma}}} \tag{IA48}$$
$$\times \left(\frac{1}{\mathbb{V}_p\left(Z\right)+\frac{1}{\tau_\varsigma}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2 \frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\varsigma)}+\frac{1}{\tau_\eta}\right)} \left(\frac{\frac{1}{\tau_\varsigma}}{\mathbb{V}_p\left(Z\right)+\frac{1}{\tau_\varsigma}}\right)^2 \right)^{-1/2} \tag{IA49}$$

which matches the expression in the statement of the Proposition after grouping terms in the last terms appropriately in order to express in terms of $\Gamma\left(k,s_p\right)$.

C. *Proof of Proposition IA2*

Using the law of iterated expectations, the unconditional expected utility can be represented as the unconditional expectation of the s_p-conditional expected utility from Proposition IA1, evaluated at the investment rule $k(s_p) = \mathbf{1}_{\{s_p > \bar{s}\}}$:

$$\mathcal{W} = \mathbb{E}\left[\mathcal{W}(s_p)\right] \tag{IA50}$$
$$= \mathbb{P}(s_p > \bar{s})\mathbb{E}[\mathcal{W}(s_p)|s_p > \bar{s}] + \mathbb{P}(s_p \le \bar{s})\mathbb{E}[\mathcal{W}(s_p)|s_p \le \bar{s}]. \tag{IA51}$$

From eq. IA26 in the proof of IA1, we have that the conditional expected utilities are of the form

$$\mathcal{W}(s_p) = -\frac{1}{\gamma^2}\left|\mathbb{V}_i^{-1}\left(V\left(k\right) - P\right)\right|^{-1/2}\left|\mathbb{V}_p\left(Y(k)\right) + \mathcal{I}\right|^{-1/2} \tag{IA52}$$
$$\times \exp\left\{-\frac{1}{2}\mathbb{E}_p\left[Y(k)\right]\left(\mathbb{V}_p(Y(k)) + \mathcal{I}\right)^{-1}\mathbb{E}_p\left[Y(k)\right]\right\} \tag{IA53}$$

where the asset payoff $V = V(k)$ and price function $P = P(k)$ are those associated with the particular investment decision $k \in \{0, 1\}$, and where the vector

$$Y(k) = \begin{pmatrix} V(k) - P \\ \mathcal{V} \\ \mathcal{Z}_i \end{pmatrix}, \tag{IA54}$$

with $\mathcal{V} = \begin{pmatrix} V(k) \\ -\eta_C \end{pmatrix}$ and $\mathcal{Z}_i = \begin{pmatrix} n \\ z_i \end{pmatrix}$ the sub-vectors of the tradeable and non-tradeable payoffs and the endowments, respectively.

To evaluate the expected utility, eq. (IA51), it is straightforward to calculate the probabilities of the two regions. It remains to to calculate the conditional expectation of $\mathcal{W}(s_p)$ given $s_p > \bar{s}$ and $s_p \le \bar{s}$. Given that the determinant terms in eq. (IA52) are constant within each region, this reduces to computing the expectation of the exponential term. To proceed, note that using standard normal-normal updating we can write

$$\mathbb{E}_p[Y(k)] = \mathbb{E}[Y(k)] + \mathbb{C}(Y(k), s_p)\mathbb{V}^{-1}(s_p)(s_p - \mathbb{E}[s_p]). \tag{IA55}$$

Hence, the expression in the exponential in eq. (IA52) can be written as

$$-\frac{1}{2}\mathbb{E}_p[Y(k)]'(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{E}_p[Y(k)] \tag{IA56}$$
$$= -\frac{1}{2}\mathbb{E}[Y(k)]'(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{E}[Y(k)] \tag{IA57}$$
$$- \mathbb{E}[Y(k)]'(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{C}(Y(k), s_p)\mathbb{V}^{-1}(s_p)(s_p - \mathbb{E}[s_p]) \tag{IA58}$$
$$- \tfrac{1}{2}(s_p - \mathbb{E}[s_p])\mathbb{V}^{-1}(s_p)\mathbb{C}(s_p, Y(k))(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{C}(Y(k), s_p)\mathbb{V}^{-1}(s_p)(s_p - \mathbb{E}[s_p]), \tag{IA59}$$

which can be written as a quadratic form $d + a'X + X'AX$ with $X = s_p - \mathbb{E}[s_p]$ and

$$d = -\frac{1}{2}\mathbb{E}[Y(k)]'(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{E}[Y(k)] \tag{IA60}$$
$$a = -\mathbb{V}^{-1}(s_p)\mathbb{C}(s_p, Y(k))(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{E}[Y(k)] \tag{IA61}$$

$$A = -\tfrac{1}{2}\mathbb{V}^{-1}(s_p)\mathbb{C}(s_p, Y(k))(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{C}(Y(k), s_p)\mathbb{V}^{-1}(s_p). \tag{IA62}$$

We can now compute the two conditional expectations, corresponding to the investment and no-investment regions, using Lemma IA2 from Appendix D, which provides a closed-form expression for the expected exponential-quadratic of a truncated normally distributed random variable.

A large amount of tedious algebra, directly analogous to that in the proof of the conditional welfare expression in Proposition IA1 but with expectations and (co)variances computed under the unconditional rather than $\mathcal{F}_p$-conditional distribution, delivers the expression in the Proposition. From inspection of the $\Gamma(k)$ term in the given expression, the maintained parameter restriction $1 - \gamma^2 \frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)^{-1} > 0$ ensures that the expected utility is finite. □

D. *Exponential-quadratic form of truncated normal random vector*

The following result computes the unconditional expectation of an exponential-quadratic form of a truncated normal random vector, which is used to characterize the unconditional welfare under the equilibrium investment rule.

LEMMA IA2: *Suppose $X \in \mathbb{R}^n$ is distributed $N(\mu, \Sigma)$ with positive definite variance matrix Σ. Consider the quadratic form $d + a'X + X'AX$, for conformable d, a and symmetric A. Let $\mathcal{C} \subseteq \mathbb{R}^n$ be an arbitrary measurable set.*

Suppose further that A is such that $\Sigma^{-1} - 2A$ is positive definite and define the composite parameters

$$\hat{\mu} = (I - 2\Sigma A)^{-1}(\mu + \Sigma a) \tag{IA63}$$
$$\hat{\Sigma} = (\Sigma^{-1} - 2A)^{-1} \tag{IA64}$$
$$\widehat{\mathcal{C}} = \left\{B^{-1}(x - \hat{\mu}) : x \in \mathcal{C}\right\}, \tag{IA65}$$

where B is the unique positive definite matrix square root of $\hat{\Sigma}$ (i.e., $\hat{\Sigma} = BB$).

Then, we have

$$\mathbb{E}\left[\exp\{d + a'X + X'AX\}\middle| X \in \mathcal{C}\right] \tag{IA66}$$
$$= \frac{\int_{\widehat{\mathcal{C}}}\phi(y)dy}{\int_{\mathcal{C}}\phi(y)dy}\frac{1}{|I - 2\Sigma A|^{1/2}}\exp\left\{d - \tfrac{1}{2}\mu'\Sigma^{-1}\mu + \tfrac{1}{2}(\mu + \Sigma a)'\Sigma^{-1}(\Sigma^{-1} - 2A)^{-1}\Sigma^{-1}(\mu + \Sigma a)\right\}. \tag{IA67}$$

Proof of Lemma IA2 Writing the expectation explicitly as an integral, we have

$$\mathbb{E}\left[\exp\{d + a'X + X'AX\}\middle| X \in \mathcal{C}\right] \tag{IA68}$$
$$= \int_{\mathbb{R}^n} \exp\{d + a'x + x'Ax\}\,\mathbb{P}(X \in dx | X \in \mathcal{C}) \tag{IA69}$$
$$= \int_{\mathcal{C}} \exp\{d + a'x + x'Ax\}\,\frac{\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}}\exp\left\{-\frac{1}{2}(x - \mu)'\Sigma^{-1}(x - \mu)\right\}}{\mathbb{P}(X \in \mathcal{C})}dx. \tag{IA70}$$

By completing the square, we can group the terms in the exponentials as

$$d + a'x + x'Ax - \tfrac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu) \tag{IA71}$$
$$= d - \tfrac{1}{2}\mu'\Sigma^{-1}\mu + \tfrac{1}{2}\left[(\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a)\right]'(\Sigma^{-1}-2A)\left[(\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a)\right] \tag{IA72}$$
$$-\tfrac{1}{2}\left(x-(\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a)\right)'(\Sigma^{-1}-2A)\left(x-(\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a)\right) \tag{IA73}$$
$$= d - \tfrac{1}{2}\mu'\Sigma^{-1}\mu + \tfrac{1}{2}(\mu+\Sigma a)'\Sigma^{-1}(\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a) \tag{IA74}$$
$$-\tfrac{1}{2}\left(x-(I-2\Sigma A)^{-1}(\mu+\Sigma a)\right)'(\Sigma^{-1}-2A)\left(x-(I-2\Sigma A)^{-1}(\mu+\Sigma a)\right). \tag{IA75}$$

where we have used the fact that $\Sigma^{-1}-2A$ being positive definite implies that it is invertible and also implies that $I-2\Sigma A$ is invertible. To see the second claim, compute the determinant, using the fact that positive definite matrices have strictly positive determinants, $|I-2\Sigma A| = |\Sigma\Sigma^{-1}-2\Sigma A| = |\Sigma|\,|\Sigma^{-1}-2A| > 0$.

So, plugging back in to eq. (IA70) yields

$$= \int_{\mathcal{C}} \exp\{d+a'x+x'Ax\}\frac{\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}}\exp\left\{-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu)\right\}}{\mathbb{P}(X\in\mathcal{C})}dx \tag{IA76}$$
$$= \exp\left\{d-\tfrac{1}{2}\mu'\Sigma^{-1}\mu+\tfrac{1}{2}(\mu+\Sigma a)'\Sigma^{-1}(\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a)\right\} \tag{IA77}$$
$$\times \frac{1}{\mathbb{P}(X\in\mathcal{C})}\int_{\mathcal{C}}\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}}\exp\left\{-\tfrac{1}{2}\left(x-(I-2\Sigma A)^{-1}(\mu+\Sigma a)\right)'(\Sigma^{-1}-2A)\left(x-(I-2\Sigma A)^{-1}(\mu+\Sigma a)\right)\right\}.dx \tag{IA78}$$

Let

$$\hat{\mu} = (I-2\Sigma A)^{-1}(\mu+\Sigma a) \tag{IA79}$$
$$\hat{\Sigma} = (\Sigma^{-1}-2A)^{-1} = (I-2\Sigma A)^{-1}\Sigma \tag{IA80}$$
$$\hat{\mathcal{C}} = \left\{B^{-1}\left(x-(I-2\Sigma A)^{-1}(\mu+\Sigma a)\right) : x\in\mathcal{C}\right\} \tag{IA81}$$

where B is the unique positive definite $n\times n$ matrix square root of the positive definite matrix $\hat{\Sigma}$ (i.e., $\hat{\Sigma}=BB$). Using these definitions, we can further express the integral in eq. (IA78) as

$$\int_{\mathcal{C}}\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}}\exp\left\{-\tfrac{1}{2}(x-\hat{\mu})'\hat{\Sigma}^{-1}(x-\hat{\mu})\right\}dx \tag{IA82}$$
$$= \int_{\hat{\mathcal{C}}}\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}}\frac{1}{|B^{-1}|}\exp\left\{-\tfrac{1}{2}y'y\right\}dy \tag{IA83}$$
$$= \frac{1}{|I-2\Sigma A|^{1/2}}\int_{\hat{\mathcal{C}}}\phi(y)dy \tag{IA84}$$

where the first equality changes variables $y = B^{-1}(x-(I-2\Sigma A)^{-1}(\mu+\Sigma a))$ and the final line uses $|B^{-1}| = |B|^{-1} = \left(|BB|^{1/2}\right)^{-1} = \left|\hat{\Sigma}\right|^{-1/2} = |(I-2\Sigma A)^{-1}\Sigma|^{-1/2} = |\Sigma|^{-1/2}|I-2\Sigma A|^{1/2}$ and simplifies notation using the n-dimensional standard normal cdf $\phi(x) = \frac{1}{(2\pi)^{n/2}}\exp\left\{-\frac{1}{2}y'y\right\}$. Recall also that it was shown above that $|I-2\Sigma A|>0$, as required for the $|I-2\Sigma A|^{1/2}$ in this expression to be well-defined. Plugging this expression for the integral back into eq. (IA78) delivers the expression in the Lemma. □

B Expected Returns and Profitability

In this Appendix, we characterize how expected returns and profitability, conditional on investment, differ across the two investment rules. These results highlight how differences in mangerial objectives and, consequently, what managers learn from prices, have important implications.

A. Expected return conditional on investment

The impact of investment on expected returns is widely studied empirically, and existing evidence typically focuses on the average relationship between these variables in the cross-section. Our results highlight that the relation can depend critically on the manager's objective and the nature of projects that firms invest in.

We begin by characterizing the conditional expected return under cash-flow maximization.

PROPOSITION IA1: *Suppose the manager maximizes expected cash flows. In the investment-maximizing equilibrium, the expected return conditional on no investment is*

$$\mathbb{E}\left[V - P|k=0\right] = \frac{\gamma n}{\tau_A}, \tag{IA1}$$

and the expected return conditional on investment is given by

$$\mathbb{E}\left[V - P|k=1\right] = \gamma n\left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma\alpha}{\tau_\eta}\mathbb{E}[Z|s_p > \bar{s}_C]. \tag{IA2}$$

Conditional on investment, the expected return:
(i) increases in $\mu_\theta - c$*, and* n*,*
(ii) decreases in μ_Z *for* $\alpha > 0$ *and increases in* μ_Z *for* $\alpha < 0$*, and,*
(iii) may either increase or decrease in α*, even when* $\alpha > 0$ *and* $\mu_Z > 0$*.*

In contrast, the following result characterizes the conditional expected return under price maximization.

PROPOSITION IA2: *Suppose the manager maximizes the date three price. In the investment-maximizing equilibrium, the expected return conditional on no investment is*

$$\mathbb{E}\left[V - P_3|k=0\right] = \frac{\gamma n}{\tau_A}, \tag{IA3}$$

and the expected return conditional on investment is given by

$$\mathbb{E}\left[V - P_3|k=1\right] = \gamma n\left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma\alpha}{\tau_\eta}\mathbb{E}\left[Z|s_p > \bar{s}_P\right]. \tag{IA4}$$

Conditional on investment, the expected return:
(i) increases in $\mu_\theta - c$*,* τ_Z*, and* n*,*
(ii) decreases in μ_Z *for* $\alpha > 0$ *and increases in* μ_Z *for* $\alpha < 0$*, and,*
(iii) decreases in α *if* $\mu_Z > 0$ *and* $\alpha \geq 0$*.*

When the manager does not invest, the expected return reflects the standard risk premium that investors demand for owning the stock that is, $\frac{\gamma}{\tau_A}n$. Conditional on investment, the expected return is driven by two components: (i) the standard unconditional risk premium $\gamma n\left(\frac{1}{\tau_A}+\frac{1}{\tau_\eta}\right)$, and (ii) the investors' expected aggregate exposure to climate risk conditional on investment: $-\frac{\gamma\alpha}{\tau_\eta}\mathbb{E}\left[Z|s_p>\bar{s}\right]$. Intuitively, as previously discussed, $-\frac{\gamma\alpha}{\tau_\eta}Z$ captures the portion of the project's discount rate that is driven by its exposure to climate risk, and so $-\frac{\gamma\alpha}{\tau_\eta}\mathbb{E}\left[Z|s_p>\bar{s}\right]$ captures the expectation of this discount rate conditional on investment.

To understand the determinants of this expectation, note that we can write:

$$-\frac{\gamma\alpha}{\tau_\eta}\mathbb{E}\left[Z|s_p>\bar{s}\right]=\underbrace{-\frac{\gamma\alpha}{\tau_\eta}\mu_Z}_{\text{avg. risk exposure}}-\overbrace{\left(\frac{\gamma\alpha}{\tau_\eta}\right)^2\frac{\frac{1}{\tau_Z}}{\sqrt{\mathbb{V}\left[s_p\right]}}H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right)}^{\text{adjustment for manager's investment strategy}}, \tag{IA5}$$

where $H(\cdot)=\frac{\phi(\cdot)}{1-\Phi(\cdot)}$ is the hazard ratio of the standard normal distribution. This reveals two channels through which investment relates to expected returns. First, investment unconditionally raises the magnitude of the covariance between the firm's cash flows and the climate risk shock, which leads to the portion of the risk premium driven by the average risk exposure term μ_Z. Second, the manager invests only when the price-signal s_p is sufficiently high. Note that, all else equal, the price signal tends to be high when the climate portion of the discount rate, $-\frac{\gamma\alpha}{\tau_\eta}Z$, is low, regardless of whether the manager internalizes the discount rate (as with price maximization) or not (as with cash-flow maximization). Hence, all else equal investment in the project is associated with lower discount rates and therefore lower expected returns. Consequently, conditional on investment, the discount rate is, in expectation, below its unconditional mean; this channel is captured by the hazard-rate term above.

This decomposition provides intuition for the above proposition. An increase in profitability $\mu_\theta-c$ increases the likelihood of investment and, therefore, increases the expected return via the second term in (IA5). Similarly, when the risk factor generates less variation in the discount rate (i.e., τ_Z is greater), $\mathbb{V}\left[s_p\right]$ tends to fall, which raises expected returns. While an increase in firm size n tends to lower the likelihood of investment and so decreases expected return via the second term in (IA5), this channel is dominated by the direct effect of the standard risk premium channel (i.e., $\gamma n\left(\frac{1}{\tau_A}+\frac{1}{\tau_\eta}\right)$), and so expected returns increase with n.

Moreover, it is immediate that expression (IA5) decreases in μ_Z when $\alpha>0$, but increases otherwise. This is intuitive: when the average exposure to climate risk increases, expected returns for green projects decrease while those for brown projects increase. Notably, the effect of project greenness (α) depends on the manager's objective, which is a consequence of how the manager responds to non-cash-flow variation in prices in the two cases.

Consider a firm with a green project, $\alpha>0$. Since $\mu_Z>0$, an increase in α leads to a direct decrease in the unconditional risk premium term (since $\mu_Z>0$). The increase in α also increases the variance of the price signal $\mathbb{V}[s_p]$. In the case of a price-maximizing manager, because the non-cash-flow variation in the price signal contains useful discount rate information, the manager's investment decision becomes more sensitive and consequently investment is associated even more strongly with lower realizations discount rate portion of the price signal. This works in the same direction as the unconditional risk premium effect and generates an unambiguously negative relation between α and conditional expected returns.

On the other hand, for a cash-flow-maximizing manager with a green project, increases in α are still associated with decreases in the unconditional risk premium (assuming $\mu_Z > 0$). However, because a cash-flow maximizing manager seeks to filter out non-cash-flow variation in the price signal, her the manager's investment decision becomes less sensitive which weakens the association between investment and the discount rate component of the price signal. These two effects work in opposite directions and consequently the overall relation between α and conditional expected returns is ambiguous and depends on the relative strength of the two forces.

B. *Future profitability*

Standard models of feedback effects typically imply that more informative prices lead to more profitable investment decisions. A number of empirical papers find evidence consistent with this prediction (e.g., Chen, Goldstein, and Jiang (2007)). Not surprisingly, we show that this result arises naturally when the manager maximizes cash flows, as summarized below.

PROPOSITION IA3: *Let $\Delta V \equiv \mathbb{E}[V|k=1] - \mathbb{E}[V|k=0]$ denote the change in expected cash flows due to investment. Suppose the manager maximizes expected cash flows. In the investment-maximizing equilibrium, ΔV is always positive.*

However, this is no longer always the case when the manager maximizes the stock price.

PROPOSITION IA4: *Let $\Delta V \equiv \mathbb{E}[V|k=1] - \mathbb{E}[V|k=0]$ denote the change in expected cash flows due to investment. Suppose the manager maximizes the date three price. In the investment-maximizing equilibrium,*

(i) ΔV is always positive when $\alpha = 0$.

(ii) ΔV is negative when $\alpha \neq 0$ and $\mu_\theta - c$ is sufficiently negative.

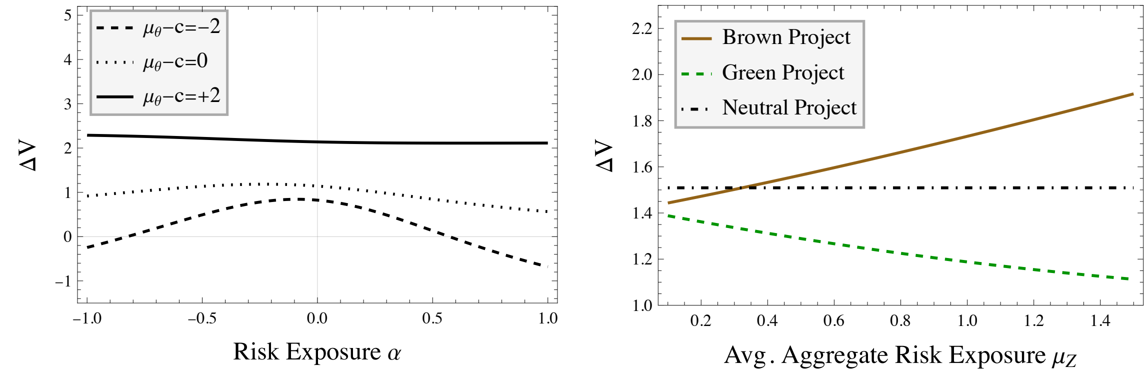
(iii) ΔV is increasing in $\mu_\theta - c$ and n, decreasing in μ_Z when $\alpha > 0$, and increasing in μ_Z when $\alpha < 0$.

Figure B.1 provides an illustration of the latter result. As before, the result follows from the observation that the manager's investment decision depends not only on the project's profitability, but also its discount rate. Part (i) of the above proposition corresponds to the standard intuition from the existing literature – when the project does not have a climate risk exposure, feedback-based investment increases the firm's expected profitability. However, part (ii) implies that when the project has a climate risk exposure, the manager may still invest even when it has low or even negative expected profitability because it is sufficiently valuable to investors as a climate risk hedge. This occurs when the project's ex-ante profitability is low. In this case, the manager invests in the project because of good discount rate news, as opposed to cash flow news. That is, the price signal is more likely to be sufficiently high for investment ($s_p > \bar{s}$) because of high hedging benefits (driven by a realization of Z) as opposed to high future cash flows (θ).

Consistent with intuition, the expected change in future cash flows as a result of investment increases with the project's expected profitability. Moreover, because the

Figure B.1. Change in Expected Cash Flows Due to Investment

This figure plots the impact of the investment on the firm's expected cash flows as a function of α and μ_Z. Unless otherwise mentioned, the parameters employed are: $\tau_\theta = \tau_\eta = c = \tau_A = \tau_Z = \mu_Z = \gamma = 1$; $n = 0.5$. Panel (b) focuses on the case in which $\mu_\theta - c = 1$.



(a) ΔV vs. α (b) ΔV vs. μ_Z

threshold for investment increases with n, conditional on investment, expected future cash flows increase with n. Finally, the dependence on μ_Z follows from the fact that holding cash flows fixed, the expected pricing of the project increases with μ_Z for green projects, but decreases with μ_Z for brown projects. As a result, conditional on investment, expected cash flows are decreasing in μ_Z for green projects, but increasing in μ_Z for brown projects.

C. *Proofs of above results*

C.1. Proof of Proposition IA1

Using the expression for the equilibrium asset price in eq. (15), the expression for the conditional expected return given no investment is straightforward. The expression in the case of investment follows from using the asset price in eq. (15) to write:

$$\mathbb{E}\left[V - P|k = 1\right] = \mathbb{E}\left[V - P|s_p > \bar{s}_C\right] \tag{IA6}$$

$$= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \mathbb{E}\left[\frac{\gamma\alpha}{\tau_\eta} Z \middle| s_p > \bar{s}_C\right] \tag{IA7}$$

$$= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \mathbb{E}\left[\mathbb{E}\left[\frac{\gamma\alpha}{\tau_\eta} Z \middle| s_p\right] \middle| s_p > \bar{s}_C\right] \tag{IA8}$$

$$= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \mathbb{E}\left[\frac{\gamma\alpha}{\tau_\eta}\mu_Z + \frac{\frac{1}{\tau_p}}{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}\left(s_p - \mathbb{E}\left[s_p\right]\right) \middle| s_p > \bar{s}_C\right] \tag{IA9}$$

$$= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma\alpha}{\tau_\eta}\mu_Z - \frac{\frac{1}{\tau_p}}{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}\sqrt{\mathbb{V}[s_p]} H\left(\frac{\bar{s}_C - \mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right) \tag{IA10}$$

$$= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma\alpha}{\tau_\eta}\mu_Z - \frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}} H\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}\left(\mu_\theta - c\right)\right). \tag{IA11}$$

This immediately implies that $\mathbb{E}[V-P|k=1]$ is increasing in $\mu_\theta - c$ and n, and is decreasing in μ_Z for $\alpha > 0$ and increasing in μ_Z for $\alpha < 0$. Now, considering α, we have

$$\begin{aligned}
&\frac{\partial}{\partial\alpha}\mathbb{E}[V-P|k=1] \\
&= -\frac{\gamma}{\tau_\eta}\mu_Z - \left(\frac{\partial}{\partial\alpha}\frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}\right)H(\cdot) - \frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}\frac{\partial}{\partial\alpha}H\left(-\tau_\theta\sqrt{\tfrac{1}{\tau_\theta}+\tfrac{1}{\tau_p}}(\mu_\theta-c)\right) \\
&= -\frac{\gamma}{\tau_\eta}\mu_Z - \left(\frac{\partial}{\partial\tau_p}\frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}\right)\left(\frac{\partial}{\partial\alpha}\tau_p\right)H(\cdot) \\
&\quad + \frac{\tau_\theta(\mu_\theta-c)\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}{\tau_p\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_p}\right)}\left(\frac{\partial}{\partial\tau_p}\sqrt{\tfrac{1}{\tau_\theta}+\tfrac{1}{\tau_p}}\right)\left(\frac{\partial}{\partial\alpha}\tau_p\right)H'\left(-\tau_\theta\sqrt{\tfrac{1}{\tau_\theta}+\tfrac{1}{\tau_p}}(\mu_\theta-c)\right) \\
&= -\frac{\gamma}{\tau_\eta}\mu_Z - \left(\frac{\partial}{\partial\alpha}\tau_p\right)\left(\left(\frac{\partial}{\partial\tau_p}\frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}\right)H(\cdot) - \frac{\tau_\theta(\mu_\theta-c)\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}{\tau_p\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_p}\right)}\left(\frac{\partial}{\partial\tau_p}\sqrt{\tfrac{1}{\tau_\theta}+\tfrac{1}{\tau_p}}\right)H'(\cdot)\right) \\
&= -\frac{\gamma}{\tau_\eta}\mu_Z - \left(\frac{\partial}{\partial\alpha}\tau_p\right)\left(\left(-\frac{1}{\tau_p}\frac{\frac{\partial}{\partial\tau_p}\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}} - \frac{1}{\tau_p^2}\frac{1}{\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}\right)H(\cdot) + \frac{-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}b(\mu_\theta-c)}{\tau_p\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_p}\right)}\left(\frac{\partial}{\partial\tau_p}\sqrt{\tfrac{1}{\tau_\theta}+\tfrac{1}{\tau_p}}\right)H'(\cdot)\right) \\
&= -\frac{\gamma}{\tau_\eta}\mu_Z + \left(\frac{\partial}{\partial\alpha}\tau_p\right)\frac{1}{\tau_p^2}\frac{1}{\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}H(\cdot) \\
&\quad + \left(\frac{\partial}{\partial\alpha}\tau_p\right)\left(\frac{\partial}{\partial\tau_p}\sqrt{\tfrac{1}{\tau_\theta}+\tfrac{1}{\tau_p}}\right)\frac{1}{\tau_p\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_p}\right)}\left(H(\cdot) - \left(-\tau_\theta\sqrt{\tfrac{1}{\tau_\theta}+\tfrac{1}{\tau_p}}(\mu_\theta-c)\right)H'(\cdot)\right).
\end{aligned}$$

The first term is clearly negative for $\mu_Z > 0$. For $\alpha > 0$, the second term is negative, but the final term is positive. □

C.2. Statement and Proof of Lemma IA1

The following Lemma will be useful for proving some of the results from the body of the paper.

LEMMA IA1: *Define*

$$\Gamma = \mathbb{E}[s_p|s_p > \bar{s}]. \tag{IA12}$$

We have

$$\Gamma = \mathbb{E}[s_p] + \sqrt{\mathbb{V}[s_p]}H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right), \tag{IA13}$$

where $H(x) = \frac{\phi(x)}{1-\Phi(x)}$ *is the hazard ratio for the standard normal distribution. Moreover,* Γ *is increasing with* μ_θ*,* c*,* μ_Z *and* n*, and is increasing in* α *for* $\alpha \geq 0$ *if* $\mu_Z > 0$*.*

The expression for Γ follows from standard results for the expectation of a truncated normal random variable. To derive the comparative statics results, note that by plugging in the explicit expressions for the threshold $\bar{s}$ and the moments of s_p, we can express Γ as

$$\Gamma = \mu_\theta + \frac{\alpha\gamma\mu_Z}{\tau_\eta} + \sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} H\left(-\frac{\mu_\theta - c + \frac{\alpha\gamma\mu_Z}{\tau_\eta} - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right). \tag{IA14}$$

Note that $H(x) > 0$ and $H'(x) \in (0,1)$. This immediately implies that Γ is increasing in μ_θ, c, μ_Z, n. To prove the claim for α, let $A \equiv \mathbb{E}[s_p] = \mu_\theta + \frac{\alpha\gamma\mu_Z}{\tau_\eta}$ and $B \equiv \sqrt{\mathbb{V}[s_p]} = \sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}$. Then,

$$\Gamma = A + BH\left(\frac{\bar{s} - A}{B}\right) \tag{IA15}$$

$$\Leftrightarrow \frac{\partial}{\partial\alpha}\Gamma = \frac{\partial}{\partial\alpha}A + \left(\frac{\partial}{\partial\alpha}B\right) H\left(\frac{\bar{s} - A}{B}\right) + BH'\left(\frac{\bar{s} - A}{B}\right) \times \left(\frac{-B\frac{\partial}{\partial\alpha}A - (\bar{s} - A)\frac{\partial}{\partial\alpha}B}{B^2}\right) \tag{IA16}$$

$$= \left(\frac{\partial}{\partial\alpha}A\right)\left(1 - H'\left(\frac{\bar{s} - A}{B}\right)\right) + \left(\frac{\partial}{\partial\alpha}B\right)\left(H\left(\frac{\bar{s} - A}{B}\right) - \left(\frac{\bar{s} - A}{B}\right) H'\left(\frac{\bar{s} - A}{B}\right)\right). \tag{IA17}$$

Note that $H'(x) \in (0,1)$ and $H(x) - xH'(x) > 0$, and that

$$\frac{\partial}{\partial\alpha}A = \frac{\gamma\mu_Z}{\tau_\eta} \tag{IA18}$$

$$\frac{\partial}{\partial\alpha}B = \frac{1}{2}\frac{2\alpha\left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}, \tag{IA19}$$

which are (weakly) positive if $\mu_Z \geq 0$ and $\alpha \geq 0$, respectively. We conclude that $\frac{\partial}{\partial\alpha}\Gamma > 0$ for $\mu_Z > 0$ and $\alpha \geq 0$. □

C.3. Proof of Proposition IA2

Using the expression for the equilibrium asset price in eq. (19), the expressions for conditional expected return given no investment and investment are straightforward.

To derive the comparative statics results for the expected return conditional on investment, note that we can write:

$$\mathbb{E}[V - P_3|k=1] = \mathbb{E}[V - P_3|s_p > \bar{s}] \tag{IA20}$$

$$= \gamma n\left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma\alpha}{\tau_\eta}\mathbb{E}[Z|s_p > \bar{s}] \tag{IA21}$$

$$= \gamma n\left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \mathbb{E}[s_p - \theta|s_p > \bar{s}] \tag{IA22}$$

$$= \gamma n\left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \mathbb{E}\left[s_p - \left(\mu_\theta + \frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}\left(s_p - \mu_\theta - \frac{\gamma\alpha}{\tau_\eta}\mu_Z\right)\right)\middle| s_p > \bar{s}\right] \tag{IA23}$$

$$= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) + \frac{\frac{1}{\tau_p}}{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}} \mu_\theta - \frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}} \frac{\gamma\alpha}{\tau_\eta} \mu_Z - \frac{\frac{1}{\tau_p}}{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}} \Gamma, \tag{IA24}$$

where the third equality uses the law of iterated expectations to write $\mathbb{E}[\theta|s_p > \bar{s}] = \mathbb{E}[\mathbb{E}[\theta|s_p]|s_p > \bar{s}]$, and where the last line collects terms and uses where $\Gamma = \mathbb{E}[s_p|s_p > \bar{s}]$ as in Lemma IA1.

Now, plugging in for Γ from Lemma IA1 and grouping terms further yields

$$\mathbb{E}[V - P_3|k = 1] = \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma\alpha}{\tau_\eta} \left(\mu_Z + \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)\frac{1}{\tau_Z}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} H\left(-\frac{\mu_\theta - c + \frac{\alpha\gamma\mu_Z}{\tau_\eta} - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right)\right). \tag{IA25}$$

This immediately implies $\mathbb{E}[V - P_3|k = 1]$ is increasing in $\mu_\theta - c$. Further, it is decreasing in μ_Z for $\alpha > 0$ and increasing for $\alpha < 0$ since

$$\frac{\partial}{\partial \mu_Z} \mathbb{E}[V - P_3|k = 1] = -\frac{\gamma\alpha}{\tau_\eta} \left(1 - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} H'\left(-\frac{\mu_\theta - c + \frac{\alpha\gamma\mu_Z}{\tau_\eta} - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right)\right) < 0 \tag{IA26}$$

because $0 < H' < 1$.

Now, consider n and note that

$$\frac{\partial}{\partial n} \mathbb{E}[V - P_3|k = 1] = \gamma \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} \frac{\gamma}{\tau_\eta} H'\left(-\frac{\mu_\theta - c + \frac{\alpha\gamma\mu_Z}{\tau_\eta} - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right) \tag{IA27}$$

$$= \gamma \frac{1}{\tau_A} + \gamma \frac{1}{\tau_\eta} \left(1 - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} H'\left(-\frac{\mu_\theta - c + \frac{\alpha\gamma\mu_Z}{\tau_\eta} - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right)\right) \tag{IA28}$$

$$\geq 0 \tag{IA29}$$

again because $0 < H' < 1$.

Considering α, following the proof of Lemma IA1, let $A \equiv \mathbb{E}[s_p] = \mu_\theta + \frac{\alpha\gamma\mu_Z}{\tau_\eta}$ and $B \equiv \sqrt{\mathbb{V}[s_p]} = \sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}$. We can write

$$\frac{\partial}{\partial \alpha} \mathbb{E}[V - P_3|k = 1] = \frac{\partial}{\partial \alpha} \left(-\frac{\gamma\alpha}{\tau_\eta} \left(\mu_Z + \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)\frac{1}{\tau_Z}}{B} H\left(\frac{\bar{s} - A}{B}\right)\right)\right) \tag{IA30}$$

$$= \frac{\partial}{\partial\alpha}\left(\left(-\frac{\gamma\alpha}{\tau_\eta}\mu_Z - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}} BH\left(\frac{\overline{s}-A}{B}\right)\right)\right) \tag{IA31}$$

$$= -\frac{\gamma}{\tau_\eta}\mu_Z - \frac{\partial}{\partial\alpha}\left(\frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\right) BH\left(\frac{\overline{s}-A}{B}\right) \tag{IA32}$$

$$- \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}} \frac{\partial}{\partial\alpha}\left(BH\left(\frac{\overline{s}-A}{B}\right)\right). \tag{IA33}$$

We clearly have $-\frac{\partial}{\partial\alpha}\left(\frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\right) BH\left(\frac{\overline{s}-A}{B}\right) < 0$ as long as $\alpha > 0$, so it remains to establish that the remaining terms are, collectively, negative. Note that we can express $-\frac{\gamma}{\tau_\eta}\mu_Z = -\frac{\partial}{\partial\alpha}A$, so we want to sign

$$-\frac{\partial}{\partial\alpha}A - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}} \frac{\partial}{\partial\alpha}\left(BH\left(\frac{\overline{s}-A}{B}\right)\right). \tag{IA34}$$

Note that we have

$$\begin{aligned}\frac{\partial}{\partial\alpha}BH\left(\frac{\overline{s}-A}{B}\right) &= \left(\frac{\partial}{\partial\alpha}B\right)H\left(\frac{\overline{s}-A}{B}\right) + BH'\left(\frac{\overline{s}-A}{B}\right)\frac{\partial}{\partial\alpha}\frac{\overline{s}-A}{B} \\ &= \left(\frac{\partial}{\partial\alpha}B\right)H\left(\frac{\overline{s}-A}{B}\right) + BH'\left(\frac{\overline{s}-A}{B}\right)\left[\frac{-B\frac{\partial}{\partial\alpha}A-(\overline{s}-A)\frac{\partial}{\partial\alpha}B}{B^2}\right] \\ &= \left(\frac{\partial}{\partial\alpha}B\right)\left(H\left(\frac{\overline{s}-A}{B}\right) - \frac{\overline{s}-A}{B}H'\left(\frac{\overline{s}-A}{B}\right)\right) - \left(\frac{\partial}{\partial\alpha}A\right)H'\left(\frac{\overline{s}-A}{B}\right)\end{aligned}$$

and plugging in to eq. (IA34) now yields

$$-\frac{\partial}{\partial\alpha}A - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\left(\frac{\partial}{\partial\alpha}B\right)\left(H\left(\frac{\overline{s}-A}{B}\right) - \frac{\overline{s}-A}{B}H'\left(\frac{\overline{s}-A}{B}\right)\right) - \left(\frac{\partial}{\partial\alpha}A\right)H'\left(\frac{\overline{s}-A}{B}\right)\right) \tag{IA35}$$

$$= -\left(1 - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}H'\right)\frac{\partial}{\partial\alpha}A - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(H\left(\frac{\overline{s}-A}{B}\right) - \frac{\overline{s}-A}{B}H'\left(\frac{\overline{s}-A}{B}\right)\right)\left(\frac{\partial}{\partial\alpha}B\right). \tag{IA36}$$

We know from the proof of Lemma IA1 that $\frac{\partial}{\partial\alpha}A > 0$ if $\mu_Z > 0$ and $\frac{\partial}{\partial\alpha}B \geq 0$ for $\alpha \geq 0$. Furthermore, we always have $H' \in (0,1)$ and $H(x) - xH'(x) \geq 0$. It follows therefore that the expression in eq. (IA34) is negative and hence that $\frac{\partial}{\partial\alpha}\mathbb{E}[V - P_3|k=1] < 0$ for $\mu_Z > 0$ and $\alpha \geq 0$.

Finally, consider τ_θ. It will be more convenient to study dependence on the variance

$1/\tau_\theta$. We have

$$\begin{aligned}
\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\mathbb{E}\left[V-P_3|k=1\right] &= -\frac{\gamma\alpha}{\tau_\eta}\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\mathbb{E}\left[Z|s_p\geq\bar{s}\right] \\
&= -\frac{\gamma\alpha}{\tau_\eta}\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\left(\mu_Z+\frac{\frac{1}{\beta}\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\frac{1}{\beta^2}\frac{1}{\tau_Z}}\left(\mathbb{E}\left[s_p|s_p\geq\bar{s}\right]-\mathbb{E}\left[s_p\right]\right)\right) \\
&= -\frac{\gamma\alpha}{\tau_\eta}\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\left(\mu_Z+\frac{\frac{1}{\beta}\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\frac{1}{\beta^2}\frac{1}{\tau_Z}}\sqrt{\mathbb{V}\left[s_p\right]}H\left(\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right)\right) \\
&= -\left(\frac{\gamma\alpha}{\tau_\eta}\right)^2\frac{1}{\tau_Z}\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\left(\frac{1}{\sqrt{\mathbb{V}[s_p]}}H\left(\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right)\right).
\end{aligned}$$

Hence, the derivative of expected returns with respect to $1/\tau_\theta$ has the opposite sign of $\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\left(\frac{1}{\sqrt{\mathbb{V}[s_p]}}H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)\right)$. Applying the monotonic transformation $\log(\cdot)$ to $\frac{1}{\sqrt{\mathbb{V}[s_p]}}H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)$ and differentiating yields

$$\begin{aligned}
\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\log\left(\frac{1}{\sqrt{\mathbb{V}[s_p]}}H\left(\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right)\right) &= -\frac{1}{2}\frac{1}{\mathbb{V}[s_p]}+\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\log\left(H\left(\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right)\right) \\
&= -\frac{1}{2}\frac{1}{\mathbb{V}[s_p]}+\frac{H'\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}{H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}} \\
&= -\frac{1}{2}\frac{1}{\mathbb{V}[s_p]}-\frac{1}{2}\frac{1}{\mathbb{V}[s_p]}\frac{H'\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}{H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}} \\
&= -\frac{1}{2}\frac{1}{\mathbb{V}[s_p]}\left(1+\frac{H'\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}{H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right).
\end{aligned}$$

Let $K<0$ be the unique root of the function $1+\frac{H'(K)}{H(K)}K$ and note that this function crosses zero from below as its argument increases, so that it is strictly negative for points below K and strictly positive for points above K.

We conclude that $\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\log\left(\frac{\frac{1}{\beta}\frac{1}{\tau_Z}}{\sqrt{\mathbb{V}[s_p]}}H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)\right)>0$ if and only if $\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}<K$. From this, it follows that $\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\mathbb{E}\left[V-P_3|k=1\right]<0$ and hence $\frac{\partial}{\partial\tau_\theta}\mathbb{E}\left[V-P_3|k=1\right]>0$. $\square$

C.4. Proof of Proposition IA3

We have

$$
\begin{aligned}
\Delta V &= \mathbb{E}\left[V|k=1\right] - \mathbb{E}\left[V|k=0\right] && \text{(IA37)}\\
&= \mathbb{E}\left[V|s_p > \bar{s}_C\right] - \mathbb{E}\left[V|s_p \leq \bar{s}_C\right] && \text{(IA38)}\\
&= \mathbb{E}\left[A + \theta + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I - c|s_p > \bar{s}_C\right] - \mathbb{E}\left[A|s_p \leq \bar{s}_C\right] && \text{(IA39)}\\
&= \mathbb{E}\left[\theta - c|s_p > \bar{s}_C\right] && \text{(IA40)}\\
&= \mathbb{E}\left[\mu_\theta + \frac{\tau_\theta^{-1}}{\mathbb{V}\left[s_p\right]}\left(s_p - \mathbb{E}\left[s_p\right]\right)\middle| s_p > \bar{s}_C\right] - c && \text{(IA41)}\\
&= \mu_\theta - c + \frac{\tau_\theta^{-1}}{\mathbb{V}\left[s_p\right]}\sqrt{\mathbb{V}\left[s_p\right]}H\left(\frac{\bar{s}_C - \mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right) && \text{(IA42)}\\
&= \mu_\theta - c + \frac{1}{\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}H\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}\left(\mu_\theta - c\right)\right) && \text{(IA43)}\\
&= \frac{1}{\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}\left(\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}\left(\mu_\theta - c\right) + H\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}\left(\mu_\theta - c\right)\right)\right) && \text{(IA44)}\\
&> 0, && \text{(IA45)}
\end{aligned}
$$

since $x + H\left(-x\right) > 0$ for all x. □

C.5. Proof of Proposition IA4

$$
\begin{aligned}
\Delta V &= \mathbb{E}\left[V|k=1\right] - \mathbb{E}\left[V|k=0\right] && \text{(IA46)}\\
&= \mathbb{E}\left[V|s_p > \bar{s}_P\right] - \mathbb{E}\left[V|s_p \leq \bar{s}_P\right] && \text{(IA47)}\\
&= \mathbb{E}\left[\theta - c|s_p > \bar{s}_P\right] && \text{(IA48)}\\
&= \mathbb{E}\left[\mu_\theta + \frac{\tau_\theta^{-1}}{\mathbb{V}\left[s_p\right]}\left(s_p - \mathbb{E}\left[s_p\right]\right)\middle| s_p > \bar{s}_P\right] - c && \text{(IA49)}\\
&= \mu_\theta - c + \frac{\tau_\theta^{-1}}{\mathbb{V}\left[s_p\right]}\sqrt{\mathbb{V}\left[s_p\right]}H\left(\frac{\bar{s}_P - \mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right) && \text{(IA50)}\\
&= \mu_\theta - c + \frac{\frac{1}{\tau_\theta}}{\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}H\left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}\left(n - \alpha\mu_Z\right)}{\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}\right) && \text{(IA51)}
\end{aligned}
$$

where the fourth equality follows from the law of iterated expectations after conditioning on s_p, and the fifth equality substitutes in for $\mathbb{E}[s_p|s_p > \bar{s}_P]$ from Lemma IA1.

When $\alpha = 0$, we have

$$
\Delta V = \mu_\theta - c + \frac{1}{\sqrt{\tau_\theta}}H\left(-\sqrt{\tau_\theta}\left(\mu_\theta - c - \frac{\gamma}{\tau_\eta}n\right)\right) \qquad \text{(IA52)}
$$

$$= \frac{1}{\sqrt{\tau_\theta}} \left(\sqrt{\tau_\theta} \left(\mu_\theta - c \right) + H \left(-\sqrt{\tau_\theta} \left(\mu_\theta - c \right) + \sqrt{\tau_\theta} \frac{\gamma}{\tau_\eta} n \right) \right) \tag{IA53}$$

$$\geq \frac{1}{\sqrt{\tau_\theta}} \left(\sqrt{\tau_\theta} \left(\mu_\theta - c \right) + H \left(-\sqrt{\tau_\theta} \left(\mu_\theta - c \right) \right) \right) \tag{IA54}$$

$$> 0 \tag{IA55}$$

since $x + H(-x) > 0$ for all x.

Next, supposing that $\alpha \neq 0$, consider the behavior of ΔV as $\mu_\theta - c$ becomes arbitrarily negative. We have

$$\lim_{\mu_\theta - c \to -\infty} \Delta V \tag{IA56}$$

$$= \lim_{\mu_\theta - c \to -\infty} \left(\mu_\theta - c + \frac{\frac{1}{\tau_\theta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta} \right)^2 \frac{1}{\tau_Z}}} H \left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z)}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta} \right)^2 \frac{1}{\tau_Z}}} \right) \right) \tag{IA57}$$

$$= \lim_{\mu_\theta - c \to -\infty} (\mu_\theta - c) \left(1 + \frac{\frac{1}{\tau_\theta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta} \right)^2 \frac{1}{\tau_Z}}} \frac{H \left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z)}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta} \right)^2 \frac{1}{\tau_Z}}} \right)}{\mu_\theta - c} \right) \tag{IA58}$$

$$= \lim_{\mu_\theta - c \to -\infty} (\mu_\theta - c) \left(1 - \frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta} \right)^2 \frac{1}{\tau_Z}} \right) \tag{IA59}$$

$$= \frac{\left(\frac{\alpha\gamma}{\tau_\eta} \right)^2 \frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta} \right)^2 \frac{1}{\tau_Z}} \lim_{\mu_\theta - c \to -\infty} (\mu_\theta - c) \tag{IA60}$$

$$= -\infty \tag{IA61}$$

where the next-to-last equality follows from continuity and the fact that $\lim_{x \to -\infty} \frac{H(-a(x-b))}{x} = -\lim_{x \to \infty} \frac{H(a(x+b))}{x} = -a$ for any $a > 0, b \in \mathbb{R}$. Hence, if $\alpha \neq 0$, then for $\mu_\theta - c$ sufficiently negative, we have $\Delta V < 0$.

Further, differentiating yields

$$\frac{\partial}{\partial (\mu_\theta - c)} \Delta V = 1 - \frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta} + \left(\frac{\gamma\alpha}{\tau_\eta} \right)^2 \frac{1}{\tau_z}} H' \left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z)}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta} \right)^2 \frac{1}{\tau_Z}}} \right) \tag{IA62}$$

and since $H'(x) \in (0, 1)$, we have $\frac{\partial}{\partial(\mu_\theta - c)} \Delta V \in (0, 1)$.

Moreover,

$$\frac{\partial}{\partial \mu_Z} \Delta V = -\frac{\gamma\alpha}{\tau_\eta} \frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta} + \left(\frac{\gamma\alpha}{\tau_\eta} \right)^2 \frac{1}{\tau_z}} H' \left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z)}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta} \right)^2 \frac{1}{\tau_Z}}} \right), \tag{IA63}$$

which is negative for $\alpha > 0$ and positive for $\alpha < 0$.

Finally,

$$\frac{\partial}{\partial n}\Delta V = \frac{\gamma}{\tau_\eta}\frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta}+\left(\frac{\gamma\alpha}{\tau_\eta}\right)^2\frac{1}{\tau_z}}H'\left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}\left(n-\alpha\mu_Z\right)}{\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}\right) > 0. \tag{IA64}$$

□

C Dispersed Investor Information about Project Fundamentals

This Appendix sets up and solves a version of the model in the paper in which traders have heterogeneous information about the project fundamental θ and the manager seeks to maximize the date-3 stock price. Our key result is that the manager's optimal investment decision remains equivalent to that in an otherwise-identical setting in which she is endowed with perfect knowledge of the aggregate of investor signals $\theta + \omega$ (where we allow for, but do not require, a common error ω) and the aggregate climate exposure Z.

A. Assumptions

We retain the following features from our baseline model. There are four dates $t \in \{1, 2, 3, 4\}$ and two securities, the firm and the risk-free asset. Investors have CARA utility over $t = 4$ wealth with risk aversion γ and are endowed with n shares of the firm and $z_i = Z + \zeta_i$ units of a nontradeable asset with payoff $-\eta_C$. The random variables are distributed as $Z \sim N\left(\mu_Z, \tau_Z^{-1}\right)$, $\zeta_i \sim N\left(0, \tau_\zeta^{-1}\right)$, and $\eta_C \sim N\left(0, \tau_\eta^{-1}\right)$, and are mutually independent. The firm's cash flow per share is

$$V(k) = A + k\left(\theta + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I - c\right)$$

where $\theta \sim N\left(\mu_\theta, \tau_\theta^{-1}\right)$ and $\eta_I \sim N\left(0, \tau_\eta^{-1}\right)$ are independent of one another and all other previously defined random variables.

The key difference with the existing setting in the paper is the information structure. Assume now that at date 1 each trader observes an imperfect private signal regarding θ:

$$s_i = \underbrace{\theta + \omega}_{\equiv S} + \varepsilon_i$$

where $\omega \sim N\left(0, \tau_\omega^{-1}\right)$ is a common error and $\varepsilon_i \sim N\left(0, \tau_\varepsilon^{-1}\right)$ are iid idiosyncratic errors. Let $\tau_S \equiv \frac{1}{\mathbb{V}(S)} = \left(\frac{1}{\tau_\theta} + \frac{1}{\tau_\omega}\right)^{-1}$ concisely denote the prior precision of S. Note our analysis allows, as a special case, the limit in which there is no common error in investors' signals ($\tau_\omega \to \infty$).

We continue to define a threshold equilibrium as in our baseline model, with one difference: we now focus on equilibria in which the price statistic s_p depends on the common error in investors' signals ω, and in particular, takes the form $s_p = S + \frac{1}{\beta}Z$, where β is an endogenous constant. As is standard in noisy rational expectations models, we allow investors to update their beliefs about θ (and Z) from prices.

The following proposition characterizes threshold equilibria, analogous to the equilibrium that we study in our baseline model. Note that, as in, e.g., Ganguli and Yang (2009), there can exist multiple equilibria corresponding to different values of the price-signal coefficient β. We focus on equilibria that maximize investment for a given solution β.

PROPOSITION IA1: *There exists a threshold equilibrium. In this equilibrium, prices are:*

$$P_1 = P_3 = \mu_A - \gamma \frac{1}{\tau_A} n + k \left[\mu_\theta - c + \frac{\tau_\omega}{\tau_\theta + \tau_\omega} \frac{\tau_\varepsilon + \beta^2 (\tau_Z + \tau_\zeta)}{\tau_S + \tau_\varepsilon + \beta^2 (\tau_Z + \tau_\zeta)} \left(s_p - \left(\mu_\theta + \frac{1}{\beta} \mu_Z \right) \right) + \frac{\gamma\alpha}{\tau_\eta} \mu_Z - \gamma \left(\left(\frac{\tau_\omega}{\tau_\theta + \tau_\omega} \right)^2 \frac{1}{\tau_S + \tau_\varepsilon + \beta^2 (\tau_Z + \tau_\zeta)} + \frac{1}{\tau_\theta + \tau_\omega} + \frac{1}{\tau_\eta} \right) n \right]$$

where the price-signal coefficient β satisfies the cubic equation

$$0 = \frac{\gamma\alpha}{\tau_\eta} (\tau_Z + \tau_\zeta) \beta^3 - \tau_\zeta \frac{\tau_\omega}{\tau_\theta + \tau_\omega} \beta^2 + \frac{\gamma\alpha}{\tau_\eta} (\tau_S + \tau_\varepsilon) \beta - \frac{\tau_\omega}{\tau_\theta + \tau_\omega} \tau_\varepsilon$$

and the investment-maximizing threshold $\bar{s}_P$ for any given solution β satisfies

$$0 = \mu_\theta - c + \frac{\tau_\omega}{\tau_\theta + \tau_\omega} \frac{\tau_\varepsilon + \beta^2 (\tau_Z + \tau_\zeta)}{\tau_S + \tau_\varepsilon + \beta^2 (\tau_Z + \tau_\zeta)} \left(\bar{s}_P - \left(\mu_\theta + \frac{1}{\beta} \mu_Z \right) \right) + \frac{\gamma\alpha}{\tau_\eta} \mu_Z - \gamma \left(\left(\frac{\tau_\omega}{\tau_\theta + \tau_\omega} \right)^2 \frac{1}{\tau_S + \tau_\varepsilon + \beta^2 (\tau_Z + \tau_\zeta)} + \frac{1}{\tau_\theta + \tau_\omega} + \frac{1}{\tau_\eta} \right) n.$$

The manager's investment decision is

$$k = \mathbf{1} \left\{ P_1 > \mu_A - \gamma \frac{1}{\tau_A} n \right\},$$

and is identical to the case in which she separately observes S and Z.

As in the baseline model, price aggregates investors' information in an efficient manner from the manager's perspective, in that she takes the same decision that she would if she could separately observe S and Z. To see this, note that the the impact of the manager's investment on her objective function, P_3, depends on S and Z only through the price signal s_p, so that:

$$\mathbb{E}[P_3 | S, Z] = \mathbb{E}[P_3 | P_1].$$

Intuitively, this result again follows because the price signal the manager conditions on and the objective she intends to maximize put the same (relative) weights on S and Z.

B. *Equilibrium*

We again search for a threshold equilibrium, which is defined as in Definition 1 in the paper. Let $s_p = S + \frac{1}{\beta} Z$ denote the endogenous price statistic and $\bar{s}_P$ the endogenous investment threshold.

B.1. $t = 3$ trading date

At the last trading date, information sets are $\mathcal{F}_{i3} = \sigma(s_i, z_i, P_1, P_3, k)$. Hence, investor i perceives the asset payoff as conditionally normally distributed with conditional mean

$$
\begin{aligned}
&\mathbb{E}_{i3}[V(k)] \\
&= \mu_A + k\mathbb{E}_{i3}[\theta - c] \\
&= \mu_A + k(\mathbb{E}_{i3}[\mathbb{E}[\theta|S]] - c) \\
&= \mu_A + k\left(\mu_\theta + \frac{\tau_\omega}{\tau_\theta + \tau_\omega}\mathbb{E}_{i3}[S - \mu_\theta] - c\right) \\
&= \mu_A + k\left(\mu_\theta - c + \frac{\tau_\omega}{\tau_\theta+\tau_\omega}\left(\mathbb{E}[S-\mu_\theta|z_i] + \mathbb{C}\left(S, \begin{pmatrix} s_i \\ s_p \end{pmatrix} \middle| z_i\right)\mathbb{V}^{-1}\left(\begin{matrix} s_i \\ s_p \end{matrix} \middle| z_i\right)\begin{pmatrix} s_i - \mathbb{E}[s_i|z_i] \\ s_p - \mathbb{E}[s_p|z_i] \end{pmatrix}\right)\right) \\
&= \mu_A + k\left(\mu_\theta - c + \frac{\tau_\omega}{\tau_\theta+\tau_\omega}\begin{pmatrix} \frac{1}{\tau_S} \\ \frac{1}{\tau_S} \end{pmatrix}'\begin{pmatrix} \frac{1}{\tau_S}+\frac{1}{\tau_\varepsilon} & \frac{1}{\tau_S} \\ \frac{1}{\tau_S} & \frac{1}{\tau_S}+\frac{1}{\beta^2}\mathbb{V}(Z|z_i) \end{pmatrix}^{-1}\begin{pmatrix} s_i - \mu_\theta \\ s_p - \left(\mu_\theta + \frac{1}{\beta}\mathbb{E}[Z|z_i]\right) \end{pmatrix}\right) \\
&= \mu_A + k\left(\mu_\theta - c + \frac{\tau_\omega}{\tau_\theta+\tau_\omega}\begin{pmatrix} \frac{1}{\tau_S} \\ \frac{1}{\tau_S} \end{pmatrix}'\begin{pmatrix} \frac{1}{\tau_S}+\frac{1}{\tau_\varepsilon} & \frac{1}{\tau_S} \\ \frac{1}{\tau_S} & \frac{1}{\tau_S}+\frac{1}{\beta^2\left(\tau_Z+\tau_\zeta\right)} \end{pmatrix}^{-1}\begin{pmatrix} s_i - \mu_\theta \\ s_p - \left(\mu_\theta + \frac{1}{\beta}\left(\mu_Z + \frac{\tau_\zeta}{\tau_Z+\tau_\zeta}(z_i - \mu_Z)\right)\right) \end{pmatrix}\right) \\
&= \mu_A + k\left(\mu_\theta - c + \frac{\tau_\omega}{\tau_\theta+\tau_\omega}\left(\frac{\tau_\varepsilon}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}(s_i-\mu_\theta) + \frac{\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\left(s_p - \left(\mu_\theta + \frac{1}{\beta}\mu_Z\right)\right) - \frac{\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\frac{1}{\beta}\frac{\tau_\zeta}{\tau_Z+\tau_\zeta}(z_i-\mu_Z)\right)\right)
\end{aligned}
$$

and conditional (co)variances (where we have suppressed the i subscripts since these quantities are identical across traders)

$$
\begin{aligned}
\mathbb{V}_3(V(k)) &= \frac{1}{\tau_A} + k^2\mathbb{V}_3\left(\theta + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I - c\right) \\
&= \frac{1}{\tau_A} + k^2\left(\mathbb{V}_3(\theta) + \frac{1}{\tau_\eta}\right) \\
&= \frac{1}{\tau_A} + k^2\left(\mathbb{V}_3(\mathbb{E}[\theta|S]) + \mathbb{V}(\theta|S) + \frac{1}{\tau_\eta}\right) \\
&= \frac{1}{\tau_A} + k^2\left(\left(\frac{\tau_\omega}{\tau_\theta+\tau_\omega}\right)^2\frac{1}{\tau_S+\tau_\varepsilon+\beta^2(\tau_Z+\tau_\zeta)} + \frac{1}{\tau_\theta+\tau_\omega} + \frac{1}{\tau_\eta}\right) \\
\mathbb{C}_3(V(k), \eta_C) &= \frac{k\alpha}{\tau_\eta}.
\end{aligned}
$$

It follows that the optimal trade is

$$
X_{i3} = \frac{\mathbb{E}_{i3}[V(k)] + \gamma\mathbb{C}_{i3}(V(k), \eta_C)z_i - P_3}{\gamma\mathbb{V}_{i3}(V(k))} - (n + X_{i1}).
$$

Now, market clearing yields

$$
\begin{aligned}
&\int X_{i3}di = 0 \\
&\Leftrightarrow n = \int\frac{\mathbb{E}_{i3}[V(k)] + \gamma\mathbb{C}_{i3}(V(k), \eta_C)z_i - P_3}{\gamma\mathbb{V}_{i3}(V(k))}di \\
&P_3 = \mu_A + k\left(\mu_\theta - c + \frac{\tau_\omega}{\tau_\theta+\tau_\omega}\left(\frac{\tau_\varepsilon}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}(S-\mu_\theta) - \frac{\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\frac{1}{\beta}\frac{\tau_\zeta}{\tau_Z+\tau_\zeta}(Z-\mu_Z)\right)\right)
\end{aligned}
$$

$$+\tfrac{\tau_\omega}{\tau_\theta+\tau_\omega}\tfrac{\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\left(s_p-\left(\mu_\theta+\tfrac{1}{\beta}\mu_Z\right)\right)\Bigg)+k\tfrac{\gamma\alpha}{\tau_\eta}Z-\gamma\mathbb{V}_3(V(k))n$$

$$=\mu_A+k\Bigg(\mu_\theta-c+\tfrac{\tau_\omega}{\tau_\theta+\tau_\omega}\Bigg(\tfrac{\tau_\varepsilon}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}(S-\mu_\theta)-\tfrac{\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\tfrac{1}{\beta}\tfrac{\tau_\zeta}{\tau_Z+\tau_\zeta}(Z-\mu_Z)\Bigg)+\tfrac{\gamma\alpha}{\tau_\eta}Z$$

$$+\tfrac{\tau_\omega}{\tau_\theta+\tau_\omega}\tfrac{\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\left(s_p-\left(\mu_\theta+\tfrac{1}{\beta}\mu_Z\right)\right)\Bigg)-\gamma\mathbb{V}_3(V(k))n$$

$$=\mu_A+k\left(\mu_\theta-c+\tfrac{\tau_\omega}{\tau_\theta+\tau_\omega}\tfrac{\tau_\varepsilon}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\left(S-\mu_\theta+\frac{\tfrac{\gamma\alpha}{\tau_\eta}-\tfrac{\tau_\omega}{\tau_\theta+\tau_\omega}\tfrac{\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\tfrac{1}{\beta}\tfrac{\tau_\zeta}{\tau_Z+\tau_\zeta}}{\tfrac{\tau_\omega}{\tau_\theta+\tau_\omega}\tfrac{\tau_\varepsilon}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}}(Z-\mu_Z)\right)\right.$$

$$\left.+\tfrac{\tau_\omega}{\tau_\theta+\tau_\omega}\tfrac{\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\left(s_p-\left(\mu_\theta+\tfrac{1}{\beta}\mu_Z\right)\right)+\tfrac{\gamma\alpha}{\tau_\eta}\mu_Z\right)-\gamma\mathbb{V}_3(V(k))n.$$

Equating the coefficient on Z with the initial conjecture $\frac{1}{\beta}$ yields a polynomial equation in β

$$\frac{1}{\beta}=\frac{\tfrac{\gamma\alpha}{\tau_\eta}-\tfrac{\tau_\omega}{\tau_\theta+\tau_\omega}\tfrac{\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\tfrac{1}{\beta}\tfrac{\tau_\zeta}{\tau_Z+\tau_\zeta}}{\tfrac{\tau_\omega}{\tau_\theta+\tau_\omega}\tfrac{\tau_\varepsilon}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}}.$$

After substituting this back into the price function and plugging in the explicit expression for $\mathbb{V}_3\left(V(k)\right)$ we can write the price concisely as

$$P_3=\mu_A+k\left(\mu_\theta-c+\frac{\tau_\omega}{\tau_\theta+\tau_\omega}\frac{\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\left(s_p-\left(\mu_\theta+\frac{1}{\beta}\mu_Z\right)\right)+\frac{\gamma\alpha}{\tau_\eta}\mu_Z\right)$$
$$-\gamma\left(\frac{1}{\tau_A}+k^2\left(\left(\frac{\tau_\omega}{\tau_\theta+\tau_\omega}\right)^2\frac{1}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}+\frac{1}{\tau_\theta+\tau_\omega}+\frac{1}{\tau_\eta}\right)\right)n. \quad \text{(IA1)}$$

Note that after manipulating and grouping terms, the polynomial for β is a cubic

$$0=\frac{\gamma\alpha}{\tau_\eta}\left(\tau_Z+\tau_\zeta\right)\beta^3-\tau_\zeta\frac{\tau_\omega}{\tau_\theta+\tau_\omega}\beta^2+\frac{\gamma\alpha}{\tau_\eta}\left(\tau_S+\tau_\varepsilon\right)\beta-\frac{\tau_\omega}{\tau_\theta+\tau_\omega}\tau_\varepsilon. \quad \text{(IA2)}$$

By Descartes' rule of signs, this equation has at least one real root and up to three real roots. All real roots are positive if $\alpha>0$ and are negative if $\alpha<0$.

B.2. $t=2$ investment decision

As in the baseline model, the price-maximizing manager's investment problem is

$$\max_{k\in\{0,1\}}\mathbb{E}\left[P_3|P_1\right].$$

Given the conjectured piecewise linear equilibrium, the optimal investment rule takes a threshold form, $k^*\equiv\mathbf{1}\left\{P_1>\mu_A-\gamma\frac{1}{\tau_A}n\right\}$, which is equivalent to a threshold rule in s_p, $\mathbf{1}\left\{s_p>\overline{s}_P\right\}$. Applying the same arguments as in our main proofs, fixing a solution β to (IA2), the unique investment-maximizing threshold $\overline{s}_P$ equates the firm's price conditional on investment and no-investment. Appealing to equation (IA1), this implies that $\overline{s}_P$ solves

$$\mu_A-\gamma\frac{1}{\tau_A}n=\mu_A+\left(\mu_\theta-c+\frac{\tau_\omega}{\tau_\theta+\tau_\omega}\frac{\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\left(\overline{s}_P-\left(\mu_\theta+\frac{1}{\beta}\mu_Z\right)\right)+\frac{\gamma\alpha}{\tau_\eta}\mu_Z\right)$$

$$
\begin{aligned}
&-\gamma\left(\frac{1}{\tau_A}+\left(\frac{\tau_\omega}{\tau_\theta+\tau_\omega}\right)^2\frac{1}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}+\frac{1}{\tau_\theta+\tau_\omega}+\frac{1}{\tau_\eta}\right)n \\
\Leftrightarrow 0 = \mu_\theta - c &+ \frac{\tau_\omega}{\tau_\theta+\tau_\omega}\frac{\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}\left(\overline{s}_P-\left(\mu_\theta+\frac{1}{\beta}\mu_Z\right)\right)+\frac{\gamma\alpha}{\tau_\eta}\mu_Z \\
&-\gamma\left(\left(\frac{\tau_\omega}{\tau_\theta+\tau_\omega}\right)^2\frac{1}{\tau_S+\tau_\varepsilon+\beta^2\left(\tau_Z+\tau_\zeta\right)}+\frac{1}{\tau_\theta+\tau_\omega}+\frac{1}{\tau_\eta}\right)n. \qquad \text{(IA3)}
\end{aligned}
$$

This can be rearranged to arrive at a unique, explicit solution for $\overline{s}_P$. Following analogous arguments to those in the proofs in our baseline model, given the form of P_1, in equilibrium, this investment rule can be implemented as the stated price-contingent rule.

B.3. $t = 1$ trading date

As in the baseline model, in the conjectured equilibrium, a trader at $t = 1$ can observe $s_p \times \mathbf{1}(s_p > \overline{s}_P)$. This enables her to perfectly anticipate both the manager's investment rule and the date-3 price P_3. Thus we must have $P_1 = P_3$ in equilibrium, or the market could not clear.