Asymmetric Information, Disagreement, and the Valuation of Debt and Equity*

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Abstract

We study debt and equity valuation when investors have private information and may exhibit differences of opinion. Our model generates several predictions that are consistent with empirical evidence but difficult to reconcile with traditional models. Expected debt (equity) returns typically increase (decrease) with default risk, though these relationships reverse for firms close to bankruptcy. Similarly, belief dispersion affects expected equity and debt returns in opposite directions. Firms' capital structures affect their valuations even without classical capital structure frictions (e.g., tax shields, distress costs) – when liquidity is higher in the equity than in the debt market, leverage can raise firm value.

JEL: G10, G12, G14, G32

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1 Introduction

Belief dispersion appears to be *negatively* related to the cross section of stock returns (e.g., Diether, Malloy, and Scherbina (2002)), but *positively* related to the cross section of bond returns (e.g., Güntay and Hackbarth (2010)). Similarly, distress risk has been *positively* linked to expected returns on debt (e.g., Huang and Huang (2012) and Bai, Goldstein, and Yang (2020)), but *negatively* linked to expected returns on equity (e.g., Campbell, Hilscher, and Szilagyi (2008)). Existing theoretical approaches, including noisy rational expectations (RE) and difference of opinions (DO) models, are unable to simultaneously reconcile these stylized facts because they consider settings in which security payoffs are linear in underlying fundamentals, and so ignore the inherent non-linearity in debt and equity payoffs.

To understand how investor information, disagreement and liquidity (noise) trading affect debt and equity returns, we develop a model where investors who have dispersed information about a firm's cash flows trade securities with liquidity, or noise, traders (e.g., as in the noisy RE model of Hellwig (1980)). Our model captures two key features. First, the payoffs to debt and equity depend non-linearly on the firm's underlying cash flows. Second, belief dispersion can arise through a combination of asymmetric information and noise trading (as in noisy RE models) and investor disagreement (as in DO models).[1]

Our analysis shows that the interaction of these features generates a number of predictions that are consistent with empirical evidence but difficult to derive from traditional models. For instance, we show that idiosyncratic distress risk (driven by firm-specific information and noise trading) raises the expected return on investment-grade debt, but lowers the expected return on equity for such firms, in line with existing empirical work. However, this relationship reverses for firms close to bankruptcy, and thus the overall relation between distress risk and equity returns is hump-shaped, consistent with Garlappi, Shu, and Yan (2008).

Moreover, while more noise trading and higher disagreement both lead to higher belief dispersion, we show that these affect expected returns differently: an increase in the former leads to lower equity returns and higher debt returns, while an increase in the latter leads to the opposite effects. As such, our model is able to simultaneously reconcile the negative relation between belief dispersion and equity returns and the positive relation for debt. However, it also suggests that controlling for measures of liquidity trading is important when studying the relation between belief dispersion and expected returns.

[1] We distinguish between "belief dispersion" and "disagreement." Belief dispersion refers to any situation in which investors assign different conditional distributions to firm fundamentals, regardless of the underlying source of such differences. Disagreement refers specifically to situations in which beliefs are dispersed because investors "agree to disagree" about the information content of one another's signals.

Our model also generates novel predictions about how the relation between firm-specific default risk and expected returns varies across firms. We find that this relation weakens, and can even reverse, when disagreement among investors is sufficiently high or when liquidity trading is very low. Moreover, considering the prices of equity and debt jointly, we show that a firm's capital structure affects its valuation even in the absence of traditional frictions such as tax shields of debt or distress costs. Specifically, the optimal choice of leverage depends on the relative amount of liquidity trading in each security. When liquidity trading is higher in equity, the total value of a levered firm can be higher than that of an unlevered firm.

Overview of Model and Mechanism. In our model, privately-informed, risk-averse investors trade the debt and equity of a levered firm alongside liquidity traders.[2] We allow, but do not require, investors to agree to disagree about the quality of others' information and, consequently, dismiss the information in prices. Our model nests two natural benchmarks as special cases: investors may either exhibit rational expectations (RE) and correctly interpret the information in prices, or exhibit pure differences of opinion (DO) and completely ignore price information. A key challenge in characterizing the equilibrium is that the payoffs to levered equity and debt are option-like, and depend non-linearly on the firm's underlying cash flows, and so standard approaches (e.g., Hellwig (1980)) cannot be employed. Instead, we apply recent work on non-linear equilibria by Breon-Drish (2015), and the multi-asset model of Chabakauri, Yuan, and Zachariadis (2022), to characterize an equilibrium in which security prices depend non-linearly on beliefs about fundamentals and liquidity trading.

The option-like nature of the equity and debt payoffs affects how the prices of these securities aggregate investor information and respond to liquidity shocks. To gain intuition, we start with a benchmark setting where liquidity-trader demands in the equity and debt markets are identical. In this case, debt and equity prices each convey the same information signal to investors. We find that equity and debt valuations depend crucially on how investors update from this price signal. Specifically, after controlling for systematic risk, the expected excess return on equity is negative and the expected excess return on debt is positive unless investors are sufficiently dismissive of price information and liquidity-trading volatility is sufficiently low.

These results are driven by how the security prices respond to investors' private information and liquidity-trader demand in equilibrium. We show that security prices can be

[2] In our benchmark analysis, we consider a single-firm model in which the source of systematic risk is the aggregate supply of each security that investors have to hold. In Section 7, we show that our analysis extends naturally to a setting with multiple firms and a systematic risk factor. Our results on "excess" expected returns should be interpreted as predictions about "alphas" from the perspective of an outside econometrician who is controlling for variation in systematic risk exposures of the securities. However, these alphas do not reflect mispricing from the perspective of investors in the model.

expressed as expected security payoffs under a risk-neutral distribution, where the risk-neutral expectation of cash flows is increasing in investors' aggregate cash flow expectations and liquidity-trader demand.

This enables us to provide a simple characterization of expected returns. Since equity payoffs are convex in cash flows, the equity price is a convex function of the risk-neutral cash flow expectation — analogously, the debt price is concave. Building on the intuition from Jensen's inequality, this implies that the expected returns on the securities depend on the difference between the volatility of the risk-neutral cash flow expectation and that of the (physical) cash flow expectation of a typical investor. The convexity in equity payoffs and prices implies that expected excess returns on equity are positive when the risk-neutral expectation is less volatile. The concavity for debt implies that the expected excess returns are positive when the reverse is true.

The relative volatility of the risk-neutral cash flow expectation versus the cash flow expectation of a typical investor depends on asymmetric information, noise-trading volatility, and their interaction with disagreement. On the one hand, because the risk-neutral expectation reflects *aggregate* investor beliefs, it tends to be *less* volatile than the beliefs of an individual investor. On the other hand, because the risk-neutral expectation is also driven by liquidity-trader demand, it tends to be *more* volatile when noise-trading volatility is higher.

We show that the relative impact of these forces depends crucially on disagreement, and specifically how much weight investors give the information in prices when forming their beliefs. When investors interpret the price as being informative (e.g., when they exhibit RE), they put relatively more weight on the common (price) signal. This amplifies the impact of noise-trading volatility and makes the risk-neutral expectation more volatile, which leads to lower equity returns and higher debt returns. In contrast, when investors disagree about the informativeness of others' signals, they dismiss the information in prices, which makes individual expectations more volatile. As a result, when noise-trading volatility is sufficiently low, the first channel dominates, which leads to higher equity returns and lower debt returns.

Implications. The above economic mechanisms have several empirically-relevant implications. First, for firms far from default, we find that higher leverage is associated with lower debt prices, even after controlling for systematic risk. Thus, consistent with the credit-spread puzzle, an increase in firm-specific default risk raises the discount rate on the firm's debt by more than the expected losses conditional on default (e.g., Huang and Huang (2012)). Similarly, equity returns are negatively related to leverage when default probabilities are low (consistent with Campbell et al. (2008)), but increase with leverage for highly distressed firms.[3]

[3]Note this overall hump shape is consistent with the results in Garlappi et al. (2008) and Garlappi and

Second, the above intuition implies that these relations weaken, and can even reverse, when investors are highly dismissive of the information in prices and the volatility of liquidity trading is sufficiently low. As such, our model provides novel predictions on how the relation between expected returns and distress risk varies across firms. Specifically, the expected return on debt is positively related to the interaction between distress risk and liquidity-trading volatility, but negatively related to the interaction between distress risk and disagreement; the predictions for the expected return on equity are reversed.

We then generalize our benchmark model to consider a setting in which liquidity trading in debt and equity markets can have arbitrary correlation. Investors update their beliefs about cash flows from both the debt and equity prices, which generates a spillover across markets: liquidity-trader demand in the equity market increases debt prices and vice versa. Our analysis implies that the correlation between equity and debt prices increases with leverage when the likelihood of default is sufficiently low, consistent with the empirical evidence in Pasquariello and Sandulescu (2021).

In our baseline analysis, because liquidity traders' demands in both markets are identical, their impact on the combined valuation of debt and equity cancel out exactly. As a result, the total value of the levered firm is equal to the value of the unlevered firm i.e., Modigliani and Miller's irrelevance result obtains. However, when liquidity trading differs across the two markets, this is no longer true. Instead, we find that when the volatility of liquidity trading in equity is higher than that in debt, the positive effect on equity prices dominates the negative effect on debt prices and so the total market value of the levered firm is hump-shaped in leverage. This suggests that an interior level of leverage is optimal for the firm, even in the absence of traditional frictions associated with debt financing (e.g., tax shields, distress costs).

Finally, we consider a multi-firm generalization in which there are an arbitrary number of firms, each with a potentially different leverage policy, and whose cash flows are exposed to a systematic risk factor and are subject to firm-specific shocks. We show that the insights of our baseline model apply directly: when investors are informed about the firm-specific component of cash flows and liquidity trading is idiosyncratic across firms, our predictions about expected returns from the baseline model become predictions about expected returns in excess of those in a frictionless economy without private information or liquidity trading (i.e., in which returns are driven only by exposure to the systematic risk factor). This analysis establishes that our key mechanisms survive in a multiple firm setting even after properly controlling for systematic risk, and consequently firm-specific private information and liquidity trade continue to influence expected returns.

Yan (2011).

The rest of the paper is as follows. The next section discusses the related literature and our incremental contribution. Section 3 presents the model, and Section 4 characterizes the equilibrium in the baseline case. Section 5 characterizes how the expected returns on debt and equity depend on the features of the model. Section 6 presents the characterization of the equilibrium when the liquidity-trader demands in the two markets are not identical. Section 7 generalizes the baseline model to a setting with multiple firms and an explicit systematic risk factor. Section 8 presents the empirical implications of the model and discusses existing empirical research that relates to our results. Finally, Section 9 concludes. Unless noted otherwise, proofs are in the Appendix.

2 Related Literature

One contribution of our analysis is to study trade in equity and debt in a setting that allows for both heterogeneity in investor information and differences of opinion. That is, in addition to jointly considering both equity and debt issued by the same firm (as in Chabakauri et al. (2022)), our model differs from the standard noisy rational expectations framework by allowing investors to "agree to disagree". We show that this leads to readily-interpretable closed-form solutions for demands and prices in the two securities and novel predictions on how non-linearity in payoffs affects expected return

Chabakauri et al. (2022) offers the closest model to ours, analyzing private information in a general multi-asset noisy rational expectations framework with CARA investors. When applying their model to study debt and equity prices, their focus is on showing that the informativeness of these prices does not depend on the firm's capital structure (a result that also holds in our model). We complement their work by allowing investors to potentially disregard the information in prices, by analyzing expected debt and equity returns, and by considering a setting with multiple firms and systematic risk factors. Moreover, while Chabakauri et al. (2022) do not focus on the expected returns on debt and equity, they do characterize the relationship between payoff skewness and expected returns when investors exhibit rational expectations. Our results show that the relationship between skewness and prices they document depends on investor disagreement and can reverse when investors dismiss the information in prices and liquidity-trading volatility is low.

Our analysis is related to noisy rational expectations models of debt and equity markets in which non-linearity in security prices plays a key role. To study the credit-spread puzzle, Albagli, Hellwig, and Tsyvinski (2021) consider a setting in which risk-neutral, informed investors have position limits and trade in a bond with binary payoffs, and find that the bond price overweights risk. Similarly, Albagli, Hellwig, and Tsyvinski (2023) argue that in general settings, noisy aggregation of information leads to prices that place excess weight on tail risks. Davis (2017) extends their analysis to consider the firm's issuance decision over time and across markets, in a setting where investors choose how much information to acquire about fundamentals. Back and Crotty (2015) consider the pricing of debt and equity in a continuous-time Kyle model in which a strategic, informed investor can trade in both debt and equity markets, and market making is integrated. They show that the stock-bond correlation depends on the cross-market lambda, and is positive (negative) when the strategic trader is informed about the mean (risk) of firm's assets.

In a single-period Kyle model of debt and equity with segmentation in market making, Pasquariello and Sandulescu (2021) study how changes in leverage affect the sensitivity of debt and equity to firm value, and consequently, affect the intensity of informed speculation in each security. This gives rise to variation in liquidity across debt and equity, and non-monotonicity in the co-movement of their prices. Chaigneau (2022) considers capital structure when investors have information on both upside and downside risks. Finally, Frenkel (2023) considers a Glosten-Milgrom model of debt trading to characterize how negative news for firms that are close to default can trigger more information acquisition, and subsequently, lead to liquidity dry-ups.

We view our analysis as complementary to this earlier work. While these papers largely consider settings in which the price is determined by risk-neutral investors/market makers, investors in our model are risk-averse. Moreover, while these models focus on rational expectations equilibrium, our model allows investors to "agree to disagree" about the informativeness of others' signals, and consequently dismiss the information in prices. We show that this has important implications for how non-linearity in payoffs affects expected returns.[4]

3 Model Setup

We consider a model of trade among informed investors in the spirit of Hellwig (1980), with two modifications: we allow the firm to be levered and for investors to potentially ignore the information in price. We begin with a single-firm model, where our results are most transparent, but illustrate how they extend to a large economy with many firms in Section 7.

[4]Models in which investors "agree to disagree" about others' information include Miller (1977), Morris (1994), Kandel and Pearson (1995), Scheinkman and Xiong (2003), and Banerjee (2011). Our analysis also has implications for settings where investors dismiss the information in prices due to other reasons, including "cursedness" (e.g., Eyster, Rabin, and Vayanos (2018)), costly price information (e.g., Mondria, Vives, and Yang (2022)) and "wishful thinking" (e.g., Banerjee, Davis, and Gondhi (2019)).

Payoffs. Investors trade in the risky debt and equity of a firm alongside a risk-free security. The gross return on the risk-free security is normalized to 1. The firm's total cash flows *per share* (i.e., the sum of its cash flows per share/unit that accrue to equity and debt holders) are $\mathcal{V} \equiv m + \theta$, where m is a constant and $\theta \sim N(0, \sigma_\theta^2)$. The assumption that $\mathcal{V}$ is normally distributed keeps the traders' updating problem simple and transparent, but can be relaxed using the approach of Breon-Drish (2015) and Chabakauri et al. (2022).

The firm has debt with a face value of K per unit, i.e., equity payoffs per share are $V_E = \max(\mathcal{V} - K, 0)$ and debt payoffs per unit are $V_D = \min(\mathcal{V}, K)$, so that $\mathcal{V} = V_E + V_D$. We assume that there are liquidity traders who submit identical demand $z \sim N(0, \sigma_z^2)$ shares / units in both the equity and debt markets. The per capita supply of the firm's securities is $\kappa \geq 0$ (i.e., there are κ units of debt and κ shares outstanding per capita). Note that setting per capita supply equal in the debt and equity securities ensures that changes in the firm's capital structure do not mechanically change the total cash flow paid out to investors. That is, the aggregate cash flow of the two securities $\kappa V_D + \kappa V_E$ always sums to $\kappa \times \mathcal{V}$.

The assumption that the liquidity-trader demands in the debt and equity markets are perfectly correlated is made for expositional clarity in our initial analysis. In Section 6, we explore a setting where liquidity trading in the two markets follows a general bivariate normal distribution, which allows for imperfect correlation and/or different variances across the markets. Note that setting $K \geq 0$ ensures that firms' equity holders always earn non-negative payoffs, consistent with limited liability. However, because the cash flow $\mathcal{V}$ can take on negative values, our baseline analysis allows for potentially negative payoffs to the debt. We extend our model to fully incorporate limited liability in Appendix D in which we consider a setting where the cash flow is bounded below by zero.

Preferences and Information. There is a unit mass of investors indexed by $i \in [0, 1]$. Each investor i is endowed with κ shares of the stock and bond, and exhibits CARA utility with risk-tolerance τ over her terminal wealth W_i. Let $x_{E,i}$ and $x_{D,i}$ denote investor i's demands for the equity and debt, respectively (so that $x_{k,i} - \kappa$ is her net trade in security $k \in \{D, E\}$), and let P_E denote the equity and P_D the debt price per share/unit. The terminal wealth W_i of investor i is therefore:

$$W_i = \kappa(P_D + P_E) + x_{E,i}(V_E - P_E) + x_{D,i}(V_D - P_D).$$

Investor i observes a private signal s_i of the form:

$$s_i = \theta + \varepsilon_i, \tag{1}$$

where the error terms $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ are independent of all other random variables.

Subjective Beliefs. We allow for a flexible specification of subjective beliefs about the private information of others. Following Banerjee (2011), we assume that investor i's beliefs about her own signal are given by (1), but her beliefs about investor j's signal are given by:

$$s_j =_i \rho\,\theta + \sqrt{1-\rho^2}\,\xi_i + \varepsilon_j, \tag{2}$$

where the random variables $\xi_i \sim_i N(0, \sigma_\theta^2)$ and $\varepsilon_j \sim_i N\left(0, \sigma_\varepsilon^2\right)$ are independent of all other random variables and each other. We use a subscript i on expectations, variances, and distributions to refer to investor i's subjective beliefs.

The parameter $\rho \in [0,1]$ captures the **degree of disagreement** across investors.[5] Specifically, when $\rho = 1$, investors agree: all investors share common priors about the joint distribution of fundamentals and signals, and so exhibit rational expectations (as in Hellwig (1980)). In this case, investors fully condition on the information in prices (in addition to their private information) when updating their beliefs about fundamentals. At the other extreme, when $\rho = 0$, investors disagree maximally and exhibit "pure" differences of opinion (as in Miller (1977)): each investor believes no other investor has payoff relevant information, and so prices are not incrementally informative about payoffs. In this case, investors do not place any weight on price information when updating their beliefs about cash flows. Finally, when $\rho \in (0,1)$, investors disagree partially about the informativeness of each other's signals, since each investor believes others' signals are informative, but less so than they actually are. As a result, each investor is partially dismissive of the information in prices when forming beliefs.

The assumption that all investors can trade in both markets is made for tractability, but also serves as a useful benchmark. It allows us to focus on the implications of belief heterogeneity on debt and equity valuations without introducing differences in clienteles, investor information, or risk aversion across these securities. In practice, one might argue that bond markets are more specialized and have less participation than equities. Although we expect the economic mechanisms that we study to operate in richer settings, explicitly accounting for different groups of investors in each security (e.g., investor specialization) is intractable in our framework.[6]

Equilibrium. An equilibrium consists of demands $\{x_{E,i}, x_{D,i}\}_{i\in[0,1]}$ and prices (P_D, P_E)

[5]The assumption that ξ_i has the same distribution as θ ensures that investor i cannot detect the error in her subjective beliefs based on the unconditional mean and variance of others' signals. Note that investor i believes that ξ_i is the common "error" in all other investors' signals. This is analogous to the subjective beliefs of investors in other difference of opinions models (e.g., Scheinkman and Xiong (2003)) and in the "cursed equilibrium" of Eyster et al. (2018).

[6]As Section 6 illustrates, we can allow for differences in the informativeness of debt and equity prices by assuming that liquidity trading in the two markets have different variances.

such that (i) the demands $(x_{D,i}, x_{E,i})$ maximize investor i's expected utility, given her information $\mathcal{F}_i = \sigma(s_i, P_D, P_E)$ and subjective belief formation mechanism described above, and (ii) the equilibrium prices (P_D, P_E) are determined by market clearing

$$\int x_{k,i} di + z = \kappa; \qquad k \in \{D, E\}. \tag{3}$$

4 Analysis

Because the equity and debt securities are effectively options on the underlying cash flows, their payoffs are not normally distributed. As a result, the equilibrium in which prices are linear in the fundamental and liquidity trade, which is common in traditional CARA-Normal rational expectations models, does not exist. Instead, we focus on the following more general class of equilibria, which is a two-asset version of the equilibrium studied in Breon-Drish (2015) and Chabakauri et al. (2022) and allows for non-linear price functions.[7] We emphasize that this class of equilibria is not an approximation to a linear equilibrium; rather, it entertains more general functional forms for asset prices.

Definition 1. *A* generalized linear equilibrium *is one in which there exists an injective function $(P_D(\cdot), P_E(\cdot))$ mapping $\mathbb{R}^2$ into $\mathbb{R}^2$, and linear statistics of the form*

$$s_{p1} = \overline{s} + b_1 z \tag{4}$$

$$s_{p2} = \overline{s} + b_2 z, \tag{5}$$

where $\overline{s} = \int s_j dj$ is the average private signal and b_1, b_2 are endogenous constants such that the equilibrium debt and equity prices are given by $P_D(s_{p1}, s_{p2})$ and $P_E(s_{p1}, s_{p2})$.

The key feature of such an equilibrium is that the information in prices reduces to two linear statistics s_{p1} and s_{p2}, corresponding to the fact that investors observe two prices. This implies that Bayesian updating takes a tractable form as in linear noisy rational expectations models. As we will see, in the case of identical liquidity trading in both markets, in equilibrium, the debt and equity prices convey the same information, and as such, the two price statistics are identical. We refer to the single statistic conveyed by prices as

$$s_{p1} = s_{p2} = s_p \equiv \overline{s} + bz =_i \rho\,\theta + \sqrt{1-\rho^2}\,\xi_i + bz. \tag{6}$$

[7] We follow existing applied work in focusing on equilibria in the generalized linear class. Indeed, in classic CARA-Normal settings (e.g., Grossman and Stiglitz (1980), Hellwig (1980)), the set of generalized linear equilibria is precisely the set of *linear* equilibria. The equilibria we characterize below are unique in the class of generalized linear equilibria.

It is worth noting that the objective distribution of this signal is given by $s_p = \theta + bz$, which coincides with investors' beliefs when $\rho = 1$.

To solve for an equilibrium, we derive investors' demands given these beliefs, apply market clearing, and verify that the resulting price indeed takes the "generalized linear" form from Definition 1.

4.1 Benchmarks

To provide intuition for the equilibrium that arises in the general case, we start by characterizing the equilibrium in two natural benchmarks.

4.1.1 Unlevered firm benchmark

First, consider the case in which the firm issues only equity (i.e., when $K \to -\infty$). In this case, the payoff to equity holders is normally distributed as in traditional models, and so we recover the standard, linear equilibrium. Moreover, since the firm only issues one type of security, investor i infers a single linear statistic from the unlevered equity price that takes the form in (6).

Given this signal, investor i's conditional beliefs about cash flows $\mathcal{V}$ are normal with moments given by

$$\mu_i \equiv \mathbb{E}_i\left[\mathcal{V}|s_i, P_U\right] = m + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{s_p}{\rho\sigma_p^2}\right) \text{ and} \tag{7}$$

$$\sigma_s^2 \equiv \mathbb{V}_i\left[\mathcal{V}|s_i, P_U\right] = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}, \text{ where} \tag{8}$$

$$\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{b^2\sigma_z^2}{\rho^2} \tag{9}$$

and where it is understood that when $\rho = 0$, we take $\frac{1}{\sigma_p^2} = \frac{1}{\rho\sigma_p^2} = 0$ in the above expressions. Standard calculations imply that investor i's optimal demand for the security is given by

$$x_i = \tau\left(\frac{\mu_i - P_U}{\sigma_s^2}\right), \tag{10}$$

and market clearing implies that the equilibrium price is given by:

$$P_U = \int \mu_i di + \frac{\sigma_s^2}{\tau}\left(z - \kappa\right).$$

This implies the following result.

Lemma 1. ***Unlevered firm benchmark.*** *Suppose that the firm only issues equity (i.e., $K \to -\infty$). Then, there is a unique linear equilibrium in which the firm's price satisfies:*

$$P_U(\cdot) = m + \sigma_s^2 \left(\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right) (\overline{s} + bz) - \frac{\kappa}{\tau} \right), \tag{11}$$

where $b = \frac{\sigma_\varepsilon^2}{\tau}$, and $\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} \right)^{-1}$.

Notably, the above equilibrium coincides with the rational expectations equilibrium in Hellwig (1980) when $\rho = 1$. On the other hand, when $\rho = 0$, investors ignore the information in prices (since the weight they put on s_p in (7) is zero).

4.1.2 Risk-neutral, uninformed benchmark

As a second benchmark, consider the setting in which investors are risk neutral (i.e., $\tau \to \infty$) and completely uninformed (i.e., $\sigma_\varepsilon^2 \to \infty$). In this case, the price of each security is given by the unconditional expectation of its payoff i.e.,

$$P_E = \mathbb{E}\left[\max\left(\mathcal{V} - K, 0\right)\right] \text{ and } \quad P_D = \mathbb{E}\left[\min\left(\mathcal{V}, K\right)\right].$$

In what follows, the definition below will be convenient.

Definition 2. *Suppose $x \sim N(\mu_x, \sigma_x^2)$. Let $M_E(\mu_x, \sigma_x^2, K)$ and $M_D(\mu_x, \sigma_x^2, K)$ denote:*

$$M_E(\mu_x, \sigma_x^2, K) \equiv \mathbb{E}\left[\max\left(x - K, 0\right)\right] = \left[1 - \Phi\left(\frac{K - \mu_x}{\sigma_x}\right)\right] \left[\mu_x - K + \sigma_x \frac{\phi\left(\frac{K-\mu_x}{\sigma_x}\right)}{1 - \Phi\left(\frac{K-\mu_x}{\sigma_x}\right)}\right], \tag{12}$$

$$M_D(\mu_x, \sigma_x^2, K) \equiv \mathbb{E}\left[\min\left(x, K\right)\right] = K - \Phi\left(\frac{K - \mu_x}{\sigma_x}\right) \left[K - \mu_x + \sigma_x \frac{\phi\left(\frac{K-\mu_x}{\sigma_x}\right)}{\Phi\left(\frac{K-\mu_x}{\sigma_x}\right)}\right]. \tag{13}$$

It is worth noting that since $\max(x - K, 0)$ is an increasing, convex function of $x - K$, we immediately have that $M_E(\mu_x, \sigma_x^2, K)$ is increasing in μ_x and σ_x^2, but decreasing in K. Similarly, since $\min(x, K) = K + \min(x - K, 0)$ is increasing and concave in x, we have that $M_D(\mu_x, \sigma_x^2, K)$ is increasing in μ_x and K, but decreasing in σ_x^2.

Given the above definition, we can characterize the equilibrium in this benchmark as follows.

Lemma 2. ***Risk-neutral, uninformed benchmark.*** *Suppose that investors are risk neutral and uninformed (i.e., $\tau \to \infty, \sigma_\varepsilon^2 \to \infty$). Then, there is a unique equilibrium in which the firm's equity and debt prices are given by $P_E = M_E\left(m, \sigma_\theta^2, K\right)$ and $P_D = M_D\left(m, \sigma_\theta^2, K\right)$. Moreover, the total value of the firm is given by $P_E + P_D = m$.*

The above results are intuitive. Note that $\Pr(\mathcal{V} < K) = \Phi\left(\frac{K-m}{\sigma_\theta}\right)$ reflects the probability that the firm defaults on its debt. Given this, the price of equity is given by the probability of no default times the conditional expected cash flows, given no default i.e.,

$$P_E = \Pr(\mathcal{V} > K) \times \mathbb{E}[\mathcal{V} - K | \mathcal{V} > K],$$

which corresponds to the expression for M_E in (12), evaluated at the firm's cash flow mean and variance. Similarly, the price of debt is given by the face value of debt, K, minus the probability of default times the loss given default i.e.,

$$P_D = K - \Pr(\mathcal{V} < K) \times \mathbb{E}[K - \mathcal{V} | \mathcal{V} < K],$$

which corresponds to the expression for M_D in (13). Not surprisingly, since investors are uninformed and risk-neutral, the total value of the firm reflects the unconditional expected cash flows. In the following subsection, we show that the equilibrium prices when investors are risk averse and privately informed are natural generalizations of the above expressions.

4.2 Equilibrium

To start, we study investors' demands holding fixed the equity and debt prices. We then show that the firm's equity and debt prices contain the same information as in the unlevered firm benchmark, which lends tractability to our model.

Lemma 3. *Given equity and debt prices P_E and P_D, investors' demands take the form:*

$$\begin{pmatrix} x_{E,i} \\ x_{D,i} \end{pmatrix} = \frac{\tau}{\sigma_s^2} \left[\begin{pmatrix} \mu_i \\ \mu_i \end{pmatrix} - G\begin{pmatrix} P_E \\ P_D \end{pmatrix} \right], \tag{14}$$

for a function $G(\cdot)$ defined in the appendix. As a result, the firm's equity and debt prices contain the same information as in the unlevered firm benchmark, i.e., they depend upon $\{s_i\}$ and z only through the statistic s_p.

The first part of the lemma illustrates that investors' demands are additively separable in investors' beliefs about the firm's total cash flow, $\mu_i = \mathbb{E}_i[\mathcal{V}]$, and the prices P_E, P_D. Moreover, each investor speculates on her beliefs in the same direction in both markets, and exhibits the same trading aggressiveness in the two markets:

$$\frac{\partial x_{E,i}}{\partial \mu_i} = \frac{\partial x_{D,i}}{\partial \mu_i} = \frac{\tau}{\sigma_s^2} = \frac{\text{risk tolerance}}{\text{posterior uncertainty}}. \tag{15}$$

Intuitively, both securities are exposed to the firm's underlying cash flows in the same direction. One might posit that investors would trade more aggressively in the security that is more exposed to a shift in the firm's cash flows. For instance, when the firm's expected cash flows μ_i are large, the debt almost certainly pays off K, and so the equity is considerably more sensitive to a change in μ_i. Thus, one might expect an investor with a positive signal to take a larger position in the equity than the debt. However, while the expected payoffs to trading on private information are greater in the security that is more exposed to θ, so too is the risk, and these two effects precisely offset.[8]

Equation (15) holds regardless of the firm's debt level K, which implies that the firm's capital structure does not influence the investors' trading aggressiveness. In addition, the total supplies of the equity and debt to be absorbed by the investors, $\kappa - z$, are identical. Together, these results imply that the equity and debt prices contain the same information and that this information is the same as in the case where the firm is unlevered, as in Chabakauri et al. (2022). Therefore, investors' expectations and variances of total firm cash flows in equilibrium are identical to those in equations (7) and (8).

Building on these findings, the next proposition characterizes the equilibrium price and investor demands.

Proposition 1. *There exists a generalized linear equilibrium, unique within this class, in which the equity and debt prices satisfy:*

$$P_E = M_E\left(P_U, \sigma_s^2, K\right) \text{ and } P_D = M_D\left(P_U, \sigma_s^2, K\right). \tag{16}$$

Moreover, the total value of the equity and debt is equal to P_U i.e., $P_U = P_E + P_D$, and investors' equilibrium equity and debt demands satisfy:

$$x_{E,i} = x_{D,i} = \tau \frac{\mu_i - \int \mu_j dj}{\sigma_s^2} - z + \kappa. \tag{17}$$

This proposition demonstrates that the firm's equity and debt prices are equal to their expected payoffs under the distribution $\mathcal{V} \sim N(P_U, \sigma_s^2)$. That is, $\mathcal{V} \sim N(P_U, \sigma_s^2)$ is the risk-neutral distribution of cash flows in our equilibrium.[9] Importantly, the mean of this distribution $P_U = \int \mu_i di + \frac{\sigma_s^2}{\tau}(z - \kappa)$ captures both how prices aggregate investors' diverse information signals (via the term $\int \mu_i di$), as well as the risk premium associated with bearing

[8] Note we have verified that this result on demand linearity in private information holds when the firm's cash flow $\mathcal{V}$ follows an arbitrary distribution and investors receive conditionally iid signals about $\mathcal{V}$. It does not depend upon normally distributed cash flows.

[9] Similarly, in the case of an unlevered firm, the price P_U of the unlevered equity is precisely the expectation of the firm's cash flows under this distribution.

the net supply of equity and debt (via the term $\frac{\sigma_s^2}{\tau}(z-\kappa)$).

The total market price of the firm's equity and debt, $P_E + P_D$, is independent of the firm's capital structure and equal to the price were the firm unlevered P_U. This implies the Modigliani-Miller theorem holds in this setting, even though investors have private information.[10] Finally, consistent with the finding in Lemma 3 that investors speculate on their beliefs equally in both markets, their equilibrium demands in the two markets are identical and coincide with their demands in the unlevered firm case.

Intuitively, because the investors have private information about the firm's cash flows, their goal is to obtain exposure the firm's total cash flows (either positive or negative, depending on their signal). Moreover, investors can do this by reconstructing a security that pays off in proportion to the firm's total cash flows by buying *equal amounts* of the outstanding debt and equity securities. Importantly, because the noise trading in the two securities is identical (as is the aggregate supply of each security), both security markets can clear in this case regardless of the firm's capital structure. As a result, each investor's *equilibrium* demands for debt and equity are equal, and the equilibrium price of a claim to the firm's total cash flows (i.e., P_U) is independent of the firm's capital structure and equal to the sum of the debt and equity prices (i.e., $P_E + P_D$).

Because the securities' prices can be expressed as their expected payoffs under a risk-neutral cash flow distribution, they satisfy a number of intuitive features. For instance, any feature that shifts up the unlevered price, P_U, while holding fixed posterior uncertainty σ_s^2, will also cause the prices of the debt and equity to increase. This yields the following result.

Corollary 1. *The firm's equity and debt prices:*

(i) increase with an increase in mean cash flows, m,

(ii) increase with an increase in liquidity-trader demand, z, and

(iii) decrease with an increase in per capita supply, κ.

The firm's equity (debt) price decreases (increases) with an increase in the face value of debt, K. Moreover, the firm's equity (debt) price is convex (concave) in the risk-neutral expectation, P_U.

[10] Our analysis differs from Simsek (2013) along this dimension. Simsek (2013) models a security whose payoff has multiple components and shows that splitting the security into separate securities that allow investors to trade on the individual components changes investors' equilibrium risk exposures. This effect does not arise in our model because investors are informed about total firm cash flows, $\mathcal{V}$, and consequently seek to speculate on this. They can always do so regardless of the firm's capital structure by trading one unit of each of the firm's outstanding securities.

Figure 1: Price Function

This figure plots the equilibrium price of equity, debt, and a claim to the total cash flow of the firm as a function of the risk-neutral mean P_U. The parameters are set to: $\sigma_\theta^2 = \sigma_\varepsilon^2 = \sigma_z^2 = \rho = m = \tau = K = 1; \kappa = 0.1$.

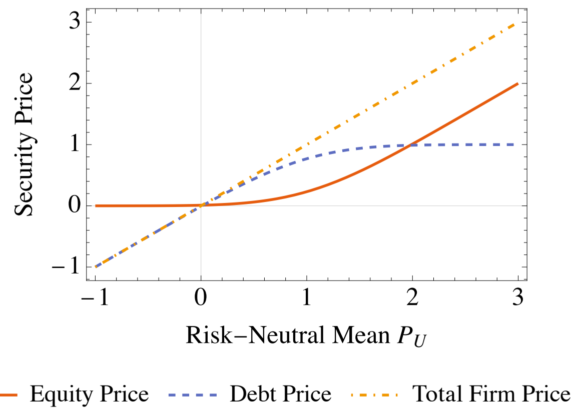


Figure 1 illustrates the equity and debt price functions as a function of the risk-neutral mean P_U. Importantly, note that the equity price is convex in the risk-neutral expectation P_U of cash flows, while the debt price is concave in it. To understand the economic intuition, consider the price of equity — the intuition is precisely reversed in the case of debt. Note that the equity payoff is a convex function of the firm's cash flows. Intuitively, since equity payoffs are unbounded above, but bounded below by zero, this implies that good cash flow news has a larger price impact on equity than bad cash flow news.

Similarly, the impact of liquidity trading on prices is also non-linear. When liquidity traders sell the firm's equity, investors must hold larger long positions and demand a drop in price to do so. However, since the equity payoffs are truncated from below (i.e., equity payoffs are positively skewed), the downside from being long is limited, and the price compensation is relatively small.[11] On the other hand, when liquidity traders buy equity, informed investors bear the risk of being short. In this case, their downside is unlimited and so they charge a large increase in the price for bearing the risk.[12]

Since the risk-neutral expectation of cash flows, P_U, is linear in (aggregate) investor beliefs and noise-trading demand, the above implies that the equity price is convex in P_U. Analogously, the debt price is concave in P_U. As we show in the next section, this has

[11] Investors with CARA utility exhibit a preference for (positive) skewness, see e.g., Eeckhoudt and Schlesinger (2006).

[12] This asymmetric risk-compensation effect is absent in traditional models with linear prices because the value is symmetric and unbounded. However, it is analogous to the "skewness effect" discussed in Albagli et al. (2021), Chabakauri et al. (2022), Cianciaruso, Marinovic, and Smith (2022), and Banerjee, Marinovic, and Smith (2022).

implications for the expected returns on debt and equity.

5 Expected Return on Debt and Equity

In our setting the (dollar) return on debt and equity can be expressed as $R_D = V_D - P_D$ and $R_E = V_E - P_E$, respectively. When the security payoff is linear in fundamental shocks and liquidity trade, the expected return typically increases with the per capita supply of the asset. For instance, note that the price of the unlevered firm from Lemma 1 implies that the expected return on unlevered equity is given by

$$\mathbb{E}[R_U] = \mathbb{E}[\mathcal{V} - P_U] = \frac{\sigma_s^2}{\tau}\kappa.$$

This implies that if the per capita supply of the firm is zero (i.e., $\kappa = 0$), so is the expected return, because the firm does not expose investors, on average, to any risk. The firm's price under zero net supply corresponds to the price of the idiosyncratic cash flows of a typical firm in the economy. The reason is that, as the typical firm is a small part of the overall economy, its idiosyncratic cash flows exhibit negligible correlation with the average investor's wealth.

In contrast, when security payoffs are non-linear, this is no longer true. The following result illustrates that in general, the expected return on debt and equity systematically differ from zero, even when the per capita supply of shares is zero. This implies that idiosyncratic risk affects the expected returns on debt and levered equity in our model. We show in Section 7 that this remains true when we extend the model to a multi-firm setting where firm cash flows are driven by idiosyncratic and systematic shocks.

Proposition 2. *Suppose the per capita endowment of shares is zero (i.e., $\kappa = 0$). Then:*

(i) the expected return on equity is positive (i.e., $\mathbb{E}[R_E] > 0$) if and only if $\mathbb{V}[P_U] < \mathbb{V}_i[\mu_i]$.

(ii) the expected return on debt is positive (i.e., $\mathbb{E}[R_D] > 0$) if and only if $\mathbb{V}[P_U] > \mathbb{V}_i[\mu_i]$.

To gain intuition for the above, note that one can express the expected return on equity and debt as

$$\mathbb{E}[R_E] = \mathbb{E}[V_E - P_E] = \mathbb{E}_i\left[M_E(\mu_i, \sigma_s^2, K)\right] - \mathbb{E}\left[M_E(P_U, \sigma_s^2, K)\right] \quad \text{and} \tag{18}$$

$$\mathbb{E}[R_D] = \mathbb{E}[V_D - P_D] = \mathbb{E}_i\left[M_D(\mu_i, \sigma_s^2, K)\right] - \mathbb{E}\left[M_D(P_U, \sigma_s^2, K)\right], \tag{19}$$

respectively.[13] Now, the unconditional means of μ_i and P_U are zero — the assumption that $\kappa = 0$ ensures the latter. Moreover, since $M_E(\cdot)$ is convex in its first argument, the sign of expected equity returns depends on the relative variance of μ_i versus P_U — the expected return on equity is positive when the variance of a typical investor's subjective expectation of cash flows, μ_i, is higher than the variance of the risk-neutral expectation of cash flows, P_U. Similarly, since $M_D(\cdot)$ is concave in its first argument, the expected return on debt is positive when the difference in the two variances is flipped.

To understand the economic intuition underlying these results, it is useful to consider the characterization in the following corollary.

Corollary 2. *Suppose the per capita endowment is zero (i.e., $\kappa = 0$).*

(i) When $\sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$, then for any value of $\rho \in [0, 1]$, the expected return on equity is negative, and the expected return on debt is positive i.e., $\mathbb{E}[R_E] < 0$ and $\mathbb{E}[R_D] > 0$.

(ii) When $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, then there exists $\rho^* \in (0, 1)$ such that the expected return on equity is positive when $\rho < \rho^*$ and negative otherwise, and the expected return on debt is negative when $\rho < \rho^*$ and positive otherwise.

Recall that Proposition 2 establishes that the key quantity that determines the signs of the expected returns is $\mathbb{V}[P_U] - \mathbb{V}_i[\mu_i]$, which reflects the difference between the variance of the risk-neutral expectation of cash flows and the variance of a typical investor's expectation of cash flows. The difference in variances is driven by two countervailing effects. On the one hand, the risk-neutral expectation P_U can be less variable than the expectation μ_i of the typical investor because it reflects the aggregate (or average) valuation (i.e., $\mathbb{V}[\int \mu_j dj] < \mathbb{V}_i[\mu_i]$). In fact, in the limit when the volatility of noise trading approaches zero (i.e., $\sigma_z \to 0$), the risk-neutral expectation P_U perfectly reflects fundamentals, and so is always (weakly) less volatile than the typical investor's beliefs i.e., $\mathbb{V}[P_U] \leq \mathbb{V}_i[\mu_i]$.[14] On the other hand, P_U can be more variable that μ_i because it is more sensitive to liquidity-trading shocks via the "risk compensation" term $\frac{\sigma_s^2}{\tau} z$. For instance, in the benchmark with no private information (i.e., when $1/\sigma_\varepsilon^2 \to 0$), the typical investor's conditional expectation is constant, and so $\mathbb{V}_i[\mu_i] < \mathbb{V}[P_U]$ so long as there is some noise trading.

Importantly, the relative impact of these forces depends on (i) the magnitude of noise-trading volatility relative to the precision of private information, and (ii) the degree of disagreement. When the volatility of noise trading is sufficiently high relative to the precision

[13] The second equality follows from the fact that investors' *unconditional* distribution of cash flows coincides with the objective distribution and so we have $\mathbb{E}[V_E] = \mathbb{E}_i[V_E]$ and $\mathbb{E}[V_D] = \mathbb{E}_i[V_D]$.

[14] The inequality is strict as long as investors do not exhibit rational expectations i.e., for all $\rho < 1$, and their information is noisy i.e., $\sigma_\varepsilon > 0$.

of private information (i.e., $\sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$), the second channel dominates irrespective of the degree of disagreement. As a result, the variance of the risk-neutral expectation is higher than that of a typical investor's expectation (i.e., $\mathbb{V}[P_U] > \mathbb{V}_i[\mu_i]$), and consequently, the expected return on debt (equity) is always positive (negative).

However, when noise-trading volatility is not too high, the relative impact of the two forces depends on the degree of disagreement. Specifically, when disagreement is high (i.e., ρ is low), investors put relatively less weight on the price and more weight on their private information, and as a result, the first channel dominates. This implies the risk-neutral expectation is less volatile than that of a typical investor (i.e., $\mathbb{V}[P_U] < \mathbb{V}_i[\mu_i]$) and consequently, the expected return on debt (equity) is negative (positive). On the other hand, when disagreement is low (i.e., ρ is high), investors put a lot of weight on the price, and so the second channel dominates and the implications for debt and equity returns are reversed.

We next characterize how expected returns on the two securities relate to the model's parameters.

Corollary 3. *Suppose the per capita endowment of shares is zero (i.e., $\kappa = 0$).*

(i) The magnitudes of the expected returns in the debt and equity, $|\mathbb{E}[R_E]|, |\mathbb{E}[R_D]|$, are hump-shaped in K and maximized at $K = m$.

(ii) Expected equity returns decrease and expected debt returns increase with liquidity-trading volatility σ_z.

This corollary follows directly from equations (18) and (19). Together with Jensen's inequality, these equations reveal that two quantities drive the magnitude of expected returns in the securities: the extent of convexity/concavity the security's expected payoff in the expected cash flow $\frac{\partial}{\partial t^2} M_x(t, \sigma_s^2, K)$ and the magnitude of $\mathbb{V}[P_U] - \mathbb{V}_i[\mu_i]$. Part (i) of the corollary follows because, in the two limits in which leverage K is large or small, one of the debt or equity securities is approximately linear, while the other has value close to zero. Thus, in either such limit, the extent of convexity/concavity, $\frac{\partial}{\partial t^2} M_x(t, \sigma_s^2, K)$, is close to zero for both debt and equity. When $K = m$, the firm's expected cash flows lie directly at the point of default ($\mathbb{E}[\mathcal{V}] = K$), when debt and equity payoffs are the most concave/convex. This can also be understood by via the analogy between equity/debt and call/put options: the convexity, or "vega," of option prices tends to be largest for options that are approximately at the money. Part (ii) of the corollary follows since, when liquidity-trading volatility increases, the variance of P_U, and thus $\mathbb{V}[P_U] - \mathbb{V}_i[\mu_i]$, increases, which reduces equity returns and increases debt returns.

Figure 2: Expected Return Comparative Statics

This figure plots expected returns on the equity and debt as a function of the model parameters. The parameters are set to: $\sigma_\theta^2 = \sigma_\varepsilon^2 = \sigma_z^2 = m = \tau = K = 1; \kappa = 0; \rho = 0.5$.

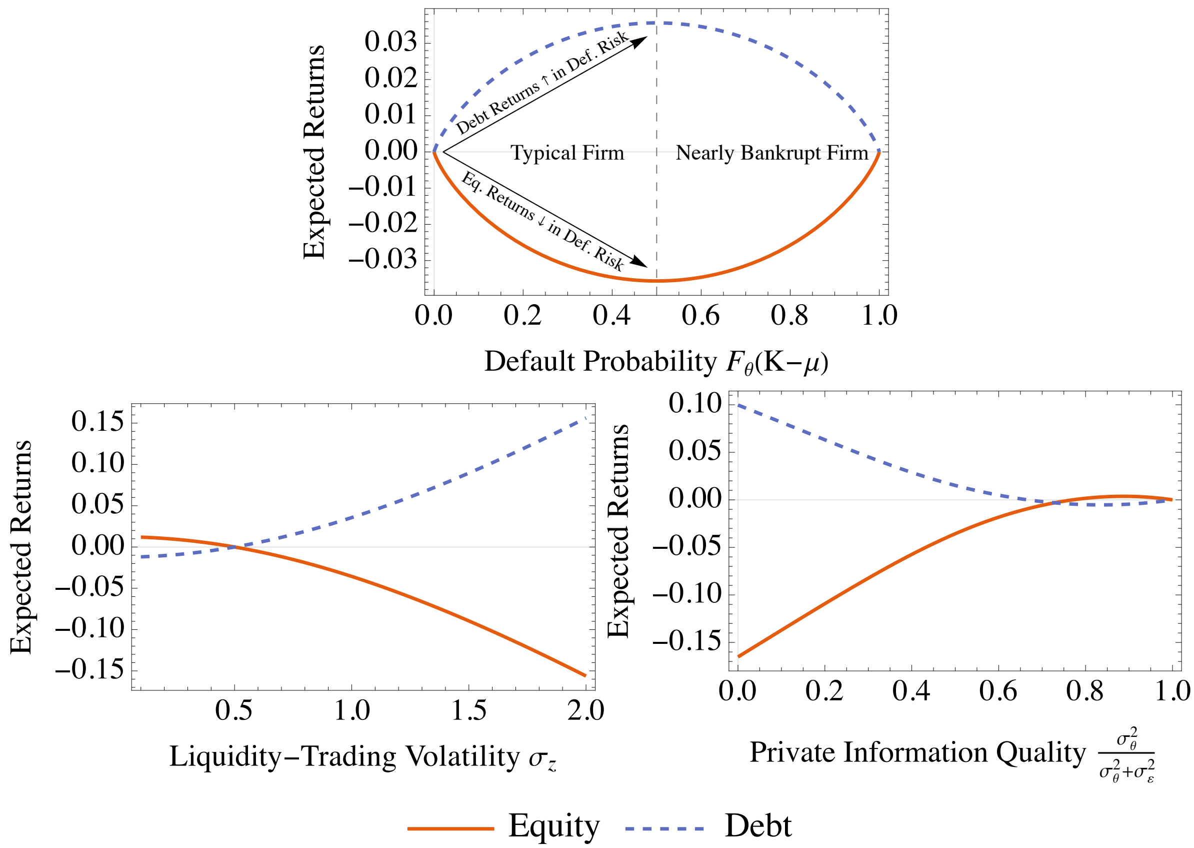


Figure 2 illustrates the corollary. The upper panel demonstrates how expected returns vary with the firm's leverage, in terms of the probability of default.[15] The plot shows that the magnitude of these returns are maximized when the debt level is equal to the firm's expected cash flows, corresponding to a 50% default probability (which corresponds to $K = m$). These results suggest that for the typical firm with default probability well below 50%, expected returns on debt increase, and expected returns on equity fall, with default risk (even when this risk is idiosyncratic). This is consistent with the credit-spread puzzle, i.e., the finding that expected debt returns increase with default risk. We further note that the specific pattern in expected debt returns as a function of default risk — an increasing and then decreasing relationship — is consistent with empirical evidence (e.g., Ang (2014), Chapter 9). The plot also demonstrates that the marginal impact of distress risk on expected debt returns is strongest for firms that have a low degree of default risk.

[15]Both expected returns and default risk depend on μ and K only through $\mu - K$, and so this plot looks identical regardless of whether variation in default risk is driven by K or μ.

The figure further shows that prior uncertainty and private information quality, which we can jointly capture via the signal-to-noise ratio $\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}$, have a non-monotonic impact on expected returns. Intuitively, when investors' private information quality is very high, their beliefs converge towards the true value of the firm, and hence expected returns must converge to zero. When investors' private information quality is very low, they rely on their common priors and so the only force that drives $\mathbb{V}[P_U] - \mathbb{V}_i[\mu_i]$ is liquidity trade. Thus, equity and debt returns are negative and positive, respectively. Finally, for intermediate values of private information quality, investors' beliefs are more dispersed and prices aggregate these diverse beliefs. As discussed above, this reduces $\mathbb{V}[P_U] - \mathbb{V}_i[\mu_i]$, thereby raising equity and lowering debt returns.

6 Extension: Imperfectly-Correlated Liquidity Trade

In this section, we consider a generalization of the baseline setting in which liquidity-traders' demands in the debt and equity markets differ, but are potentially correlated. Specifically, in contrast to the baseline setting described in Section 3, demand shocks $z = (z_D, z_E) \in \mathbb{R}^2$ follow a general bivariate normal distribution $z \sim N(\mathbf{0}, \Sigma_z)$ with Σ_z an arbitrary 2×2 positive definite covariance matrix.[16] As in the baseline setting, we let $x_{D,i}$ and $x_{E,i}$ denote the investor's demand for debt and equity respectively, with $x_i = (x_{D,i}, x_{E,i})$ the vector of demands.

The definition of a generalized linear equilibrium is analogous to that above, but generalized to account for the fact that in this setting the debt and equity prices generally depend on two non-identical linear statistics.

Definition 3. *A "generalized linear equilibrium" is one in which there exists an injective function $P(\cdot,\cdot) = (P_D(\cdot), P_E(\cdot))$ mapping $\mathbb{R}^2$ into $\mathbb{R}^2$ and linear statistics of the form*

$$s_{p1} = \int s_j dj + b_{1D} z_D + b_{1E} z_E$$
$$s_{p2} = \int s_j dj + b_{2D} z_D + b_{2E} z_E$$

such that the equilibrium price vector is

$$P(s_{p1}, s_{p2}) = \begin{pmatrix} P_D(s_{p1}, s_{p2}) \\ P_E(s_{p1}, s_{p2}) \end{pmatrix}.$$

[16] The proofs of all results in this section allow for an arbitrary mean vector $\mu_z \in \mathbb{R}^2$ and allow for a covariance matrix Σ_z that is only positive semi-definite. In the text, we normalize the means to zero and consider only strictly positive definite Σ_z for expositional clarity.

Let $\overline{s} \equiv \int s_j dj$ denote the cross-sectional average signal and let

$$s_p = \mathbf{1}\overline{s} + Bz$$

concisely denote the stacked vector of price-signals, with $\mathbf{1}$ a conformable vector of ones and $B = \begin{pmatrix} b_{1D} & b_{1E} \\ b_{2D} & b_{2E} \end{pmatrix}$ the 2×2 matrix of coefficients on z. In the main text we will focus on the case in which Σ_z is invertible (i.e., strictly positive definite).[17]

Given s_i and the conjectured s_p, investor i's beliefs about the firm cash flow $\mathcal{V}$ are normal with conditional moments

$$\mu_i \equiv \mathbb{E}\left[\mathcal{V}|s_i, s_p\right] = m + \sigma_s^2 \left(\frac{s_i}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}s_p\right), \text{ and} \tag{20}$$

$$\sigma_s^2 \equiv \mathbb{V}\left(\mathcal{V}|s_i, s_p\right) = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1}\mathbf{1}\right)^{-1}, \tag{21}$$

where $\Sigma_p \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2\mathbf{1}\mathbf{1}' + \frac{1}{\rho^2}B\Sigma_z B'$.[18] These are the analogues to Equations (7)-(8) in the benchmark analysis.

We next extend our characterization of the investor's optimal demand in Lemma 3 to this case.

Lemma 4. *Fix any $P = (P_D, P_E) \in \mathbb{R}^2$. The optimal demand of trader i is given by*

$$x_i = \frac{\tau}{\sigma_s^2}\left(\mathbf{1}\mu_i - G\left(P\right)\right),$$

where $G : \mathbb{R}^2 \to \mathbb{R}^2$ is a function defined in the proof.

As before, investor i's optimal demand is additively separable in her beliefs μ_i and the prices, and her trading aggressiveness again remains the same in each security. The equilibrium debt and equity prices follow from imposing market clearing and matching coefficients on the price-signal vector s_p.

Proposition 3. *There exists a generalized linear equilibrium in the financial market, unique within the generalized linear class. The vector of equilibrium asset prices takes the form*

$$P = g'\left(\mathbf{1}\frac{\int \mu_j \, dj}{\sigma_s^2} - \frac{1}{\tau}\left(\kappa\mathbf{1} - z\right)\right) \tag{22}$$

[17] The case in which Σ_z is singular (e.g., perfectly correlated liquidity trade across both markets, or one of the liquidity trades constant) is considered in the formal derivations in the appendix.

[18] Because Σ_z is assumed positive definite, it follows that $B\Sigma_z B'$ is positive definite. Furthermore, Σ_p, being a sum of a positive definite and positive semidefinite matrix is itself positive definite and therefore invertible, where it is understood that we take $\Sigma_p^{-1} = \mathbf{0}$ and $\Sigma_p^{-1}\frac{1}{\rho} = (\rho\Sigma_p)^{-1} = \mathbf{0}$ in the above expressions when $\rho = 0$.

$$= g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}m + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \mathbf{1}\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \tag{23}$$

where the equilibrium coefficient matrix is $B = \begin{pmatrix} \frac{\sigma_\varepsilon^2}{\tau} & 0 \\ 0 & \frac{\sigma_\varepsilon^2}{\tau} \end{pmatrix}$, *and* $g' : \mathbb{R}^2 \to \mathbb{R}^2$ *is the gradient of a function* $g : \mathbb{R}^2 \to \mathbb{R}$, *both given in closed-form in the Appendix.*

The above result extends the generalized linear equilibrium characterized in Proposition 1. Combining the expression for the optimal demand from Lemma 6 and the equilibrium price in this proposition immediately yields the equilibrium quantity demanded by each investor, which we record in the following corollary.

Corollary 4. *The equilibrium demand of investor* i *is*

$$x_i = \tau\frac{\mu_i - \int \mu_j dj}{\sigma_s^2}\mathbf{1} + \kappa\mathbf{1} - z. \tag{24}$$

This result shows that the speculative portion of each investor's holdings are equal across the debt and equity markets, and, as in our benchmark model, are given by $\tau\frac{\mu_i - \int \mu_j dj}{\sigma_s^2}$. Thus, investors' debt and equity demands differ if and only if the liquidity trade in the debt and equity markets differ.

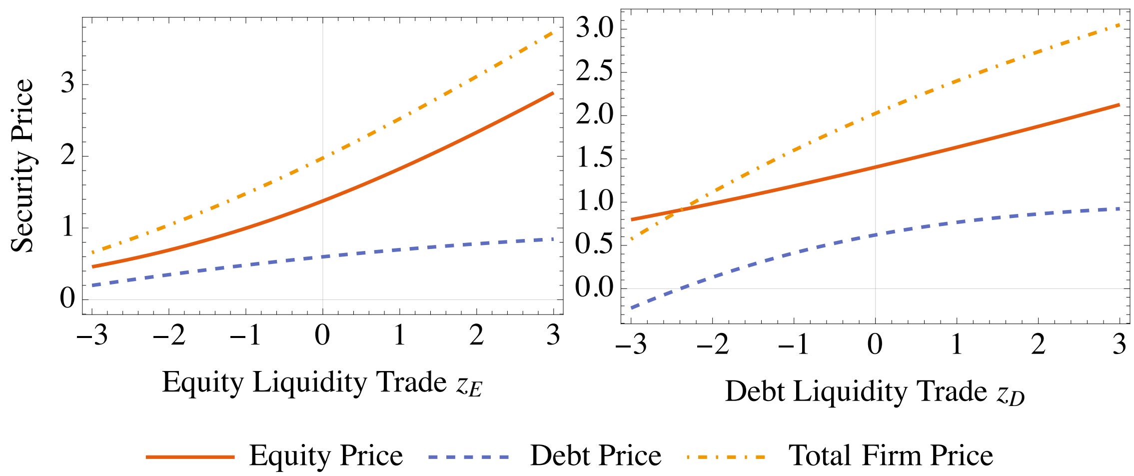
6.1 Cross-Market Demand Spillovers

Figure 3 illustrates how the equity and debt prices respond to liquidity-trader demand in each market. Specifically, liquidity-trader demand for a given security affects not only the price of that security, but also the price of the other security. Intuitively, this is driven by both information and risk effects. Since demand in either security may be perceived as informed, it raises investors' expectations of cash flows, and consequently, the price of both securities. In addition, holding debt (equity) exposes an investor to the risk of the firm's underlying cash flows, which also makes them view the equity (debt) as riskier. Thus, equity demand also raises the price of debt, and vice versa, via investor risk aversion. However, demand for equity has a much stronger effect on the equity price than on the debt price through this risk aversion effect, and vice versa. As a result, the demand spillover between the two markets is incomplete in the sense that z_E has a stronger impact on the equity price than z_D, and z_D has a stronger impact on the debt price than z_E.

As in the baseline model (e.g., see Figure 1), the equity (debt) price is a convex (concave) function of demand in *each* market. This, in turn, implies that our main results regarding expected returns continue to hold in this case. Interestingly, however, the total price of

Figure 3: Cross-Market Demand Spillovers

This figure plots the expected security prices conditional on equity liquidity trade z_E (left panel) and debt liquidity trade z_D (right panel). The parameters are set to: $\sigma_\theta^2 = 1.5^2; \sigma_\varepsilon^2 = \mathbb{V}[z_E] = \mathbb{V}[z_D] = m = \tau = K = 1; \kappa = 0; \mathbb{C}[z_E, z_D] = 0$.



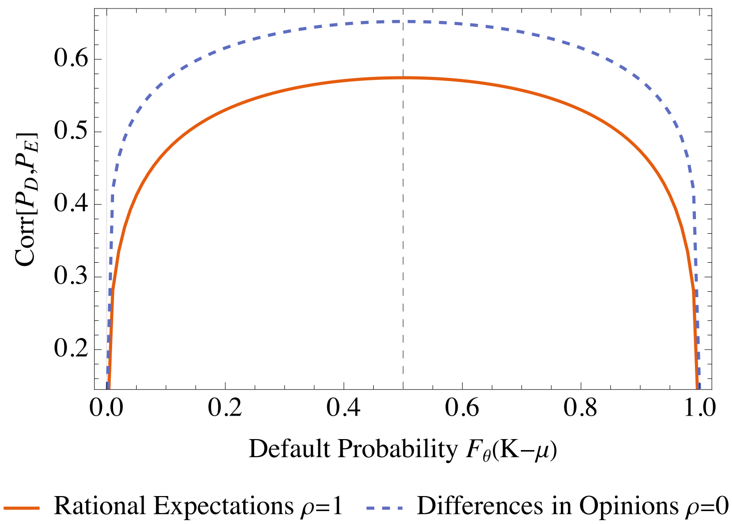
the firm (i.e., $P_E + P_D$) is convex in equity liquidity-trader demand, but concave in debt liquidity-trader demand. As we discuss further in the next subsection, this implies that the Modigliani-Miller theorem no longer holds in this setting. Instead, the sum of the firm's debt and equity prices is greater (lower) than the price of an unlevered firm, on average, when the volatility of equity liquidity trading is higher (lower) than that of debt liquidity trading.

The incomplete spillover of demand shocks across securities also affects the correlation between equity and debt prices. As Figure 4 illustrates, the correlation between debt and equity prices is maximized when the likelihood of default is 50%.[19] Intuitively, when the default probability approaches zero, the payoff to debt is almost risk-free, and so demand shocks in either security have little impact on the debt price but significant impact on the equity price. Similarly, when the probability of default approaches one, the value of equity approaches zero and is relatively insensitive to demand shocks, but the price of debt remains responsive to such shocks. As a result, the correlation in prices approaches zero in both extremes. In contrast, for intermediate levels of distress, both security prices are sensitive to demand shocks, and so price correlation is high.

[19] Pasquariello and Sandulescu (2021) derive a similar result in a Kyle model where risk-neutral market makers specialize in either debt or equity, while investors can trade both securities. As such, the market making in their model is segmented: the price in each market depends only on the order flow in that market. In contrast, markets are integrated in our setting: investors can update their beliefs from equity and debt prices and can trade in both markets.

Figure 4: Leverage and Debt-Equity Price Correlation

This figure plots the correlation between the debt and equity prices as a function of the firm's default risk. The parameters are set to: $m = 3; K = 2; \sigma_\theta^2 = 1; \sigma_\varepsilon^2 = \mathbb{V}[z_E] = \mathbb{V}[z_D] = \tau = 1; \kappa = 0; \mathbb{C}[z_E, z_D] = 0$.



6.2 Capital Structure and Total Firm Valuation

We next show that, in contrast to our baseline specification, the Modigliani-Miller theorem does not hold, i.e., the firm's equity and debt prices do not, in general, sum to the price of the unlevered firm: $\mathbb{E}[P_E + P_D] \neq \mathbb{E}[P_U]$. As a result, the firm's capital structure can meaningfully impact its value, even in the absence of traditional frictions (e.g., tax shields of debt, costs of financial distress).

In Figure 5, we show that the expected price of the debt plus equity relative to the price of the unlevered firm depends upon the relative amount of liquidity trade in the two markets. For instance, the left panel of Figure 5 plots $\mathbb{E}[P_E + P_D]$ as a function of $\sqrt{\mathbb{V}[z_E]}$, holding fixed the volatility of debt liquidity trade (i.e., $\sqrt{\mathbb{V}[z_D]}$). The plot illustrates that the expected value of debt plus equity is higher than the expected value of the unlevered firm (i.e., $\mathbb{E}[P_E + P_D] > \mathbb{E}[P_U] = 1$) if and only if the volatility of equity liquidity trading is higher than that of debt liquidity trading (i.e., $\mathbb{V}[z_E] > \mathbb{V}[z_D]$). Intuitively, this is because liquidity trade in the equity market does not fully spill over into the debt market. As such, the price-increasing effect that equity liquidity-trading volatility has on equity prices tends to raise the overall value of the firm.

This result holds irrespective of whether investors use the information in prices (i.e., whether $\rho = 1$ or $\rho = 0$), but is stronger when investors do not condition on prices (i.e., when $\rho = 0$). This is because, even though the effect of belief dispersion on expected prices in the two securities precisely offset, investors face more uncertainty when they do not condition on prices, and this increases the sensitivity of prices to liquidity trading shocks. The right panel of Figure 5 shows the same result by plotting the expected value of debt plus equity as a function of debt liquidity-trading volatility, holding fixed the equity liquidity-trading volatility.

Figure 5: Violations of Modigliani-Miller

This figure plots the total expected firm price, $\mathbb{E}[P_E + P_D]$, as a function of liquidity trade volatility in the two markets. The parameters are set to: $\sigma_\theta^2 = 1.5^2; \sigma_\varepsilon^2 = \mathbb{V}[z_E] = \mathbb{V}[z_D] = m = \tau = K = 1; \kappa = 0; \mathbb{C}[z_E, z_D] = 0$. Note that $\mathbb{E}[P_U] = m = 1$.

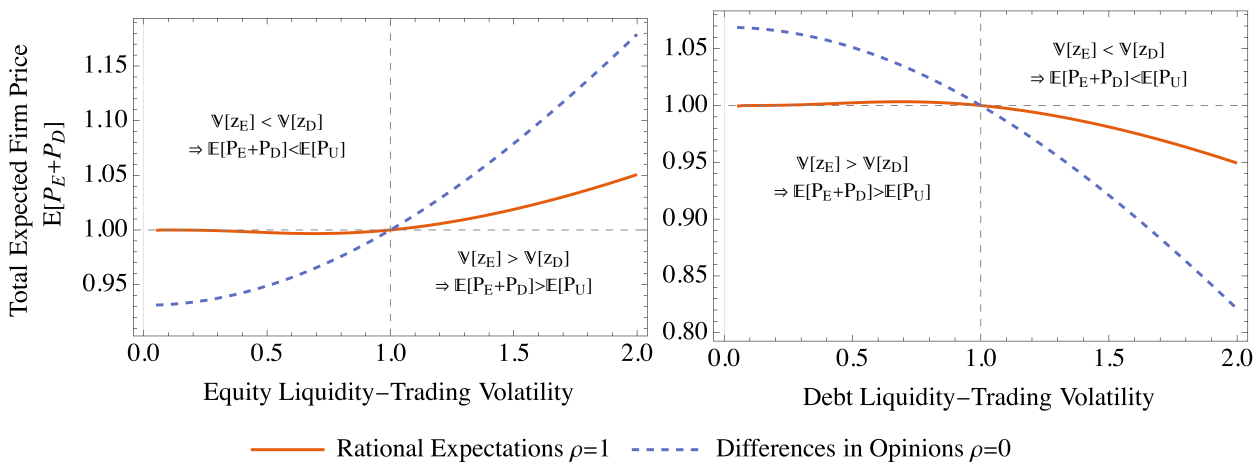


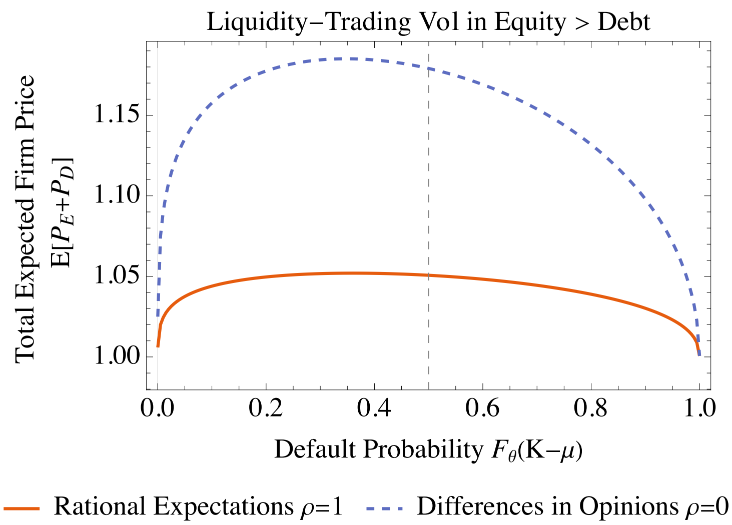
Figure 6 illustrates how the firm's capital structure influences its valuation. When equity liquidity-trading volatility exceeds that in the debt, the value of the levered firm is higher than an unlevered firm $\mathbb{E}[P_E + P_D] > \mathbb{E}[P_U]$. In this case, there is an interior optimal capital structure that maximizes the firm's valuation. Intuitively, an all equity or all debt firm is suboptimal as, in either case, the firm has linear payoffs, and so $\mathbb{E}[P_E + P_D] \to \mathbb{E}[P_U]$. This implies that even in the absence of traditional frictions associated with leverage, heterogeneity in information, beliefs, and liquidity trading across debt and equity markets causes the value of the firm to be maximized at an interior level of debt.

7 Extension: Systematic Risk and Multiple Firms

In this section, we illustrate how our results extend to a multi-firm economy with systematic and idiosyncratic sources of risk. We show that the condition that determines the sign of expected returns in our baseline analysis now determines how the firm's expected returns compare to a benchmark without liquidity and informed trade. Moreover, we verify that our

Figure 6: Optimal Capital Structure

This figure plots the total expected firm price, $\mathbb{E}\left[P_E + P_D\right]$, as a function of the firm's leverage, paramaterized in terms of its ex-ante probability of default, $F_\theta\left(K - m\right)$. The parameters are set to: $\sigma_\theta^2 = 1.5^2; \sigma_\varepsilon^2 = m = \tau = K = 1; \kappa = 0; \mathbb{C}\left[z_E, z_D\right] = 0; \mathbb{V}\left[z_E\right] = 4$; and $\mathbb{V}\left[z_D\right] = 1$. Note that $\mathbb{E}[P_U] = m = 1$.



comparative statics results continue to apply in this setting.

Formally, assume now that investors trade in the securities of N firms. Firm n's total cash flows per share are:

$$\mathcal{V}_n \equiv m_n + \theta_n + \beta_n F.$$

The terms $\theta_n \sim N\left(0, \sigma_\theta^2\right)$ are independent across firms and the term $F \sim N\left(0, \sigma_F^2\right)$ captures a common systematic risk factor. Moreover, m_n denotes firm n's expected cash flows and β_n captures firm n's cash-flow beta.

We assume that the only source of systematic risk in the economy is F and that there is a tradeable factor security with payoff $\mathcal{V}_F = m_F + F$. Investors are endowed with κ shares of the factor security and zero shares of each firm's equity and debt, which corresponds to a large economy where each individual firm is small.[20] Thus, in aggregate, investors have no exposure to θ_n, and so θ_n is a purely idiosyncratic source of risk and would not affect expected returns if the investors were homogeneous.

We allow for arbitrary differences in leverage across firms: firm n has debt and equity

[20]We can obtain similar results by taking the limit of an economy with finite investors and firms as the number of investors and firms grows large. In this case, each firm becomes a small part of the economy and so the per capita supply of each firm approaches zero. However, to avoid the notational burden of having to take limits throughout the analysis, it is more convenient to simply assume that F is the only systematic source of risk.

defined as in the baseline model, where the face value of the debt is K_n. Investor i now observes a private signal s_{in} about the idiosyncratic cash flows of each firm n:

$$s_{in} = \theta_n + \varepsilon_{in}, \tag{25}$$

where the error terms $\varepsilon_{in} \sim N\left(0, \sigma_\varepsilon^2\right)$ are independent of all other random variables. We continue to allow investors to agree to disagree about one another's signals. Specifically, each investor i believes that others' signals are of the form

$$s_{jn} = \rho\theta_n + \sqrt{1-\rho^2}\xi_{in} + \varepsilon_{jn}$$

where $\xi_{in} \sim N(0, \sigma_\theta^2)$ are independent of one another.

Finally, we assume that there are liquidity traders in each firm's stock who seek exposures of $z_n \sim N\left(0, \sigma_z^2\right)$ to the idiosyncratic cash flows of each firm θ_n. Formally, these traders submit demands $z_n \sim N\left(0, \sigma_z^2\right)$ in the debt and equity of each firm and hedge the risk exposure that these demands create by submitting demands of $-\beta_n z_n$ in the factor asset. Thus, market clearing in the factor security requires that:

$$\int_i x_{iF} di - \sum_n \beta_n z_n = \kappa,$$

and market clearing in each individual firm's equity and debt requires that:

$$\int_i x_{inE} di + z_n = 0 \;\text{ and }\; \int_i x_{inD} di + z_n = 0.$$

The assumption that liquidity traders hedge in this fashion enables us to interpret the risk exposures they create as idiosyncratic. One can interpret these traders as allocating a fixed trading budget across stocks, while keeping their total allocation to systematic risk fixed. Specifically, this ensures that liquidity trade in a given stock does not affect the supply of systematic risk F to be borne by the remaining traders, and thus does not shift the price of the factor. Hence, the price variation that such liquidity traders create would not be captured by betas in standard factor models of returns and is purely idiosyncratic. However, this assumption is generally not essential for our equilibrium construction; except in the case in which $\rho = 1$, our model allows as a special case the limit in which there is no liquidity trade.[21]

[21] When $\rho = 1$ and there is no liquidity trade, investors correctly infer that firm n's equity/debt prices perfectly reveal θ_n, and so expected excess returns (i.e., expected returns after accounting for β_n) are zero.

7.1 Equilibrium and Expected Returns

Analogous to our baseline model, we search for an equilibrium in which each firm n's price is a generalized linear function of investors' aggregate signal about firm n's cash flows, $\bar{s}_n \equiv \int_i s_{in} di$, and liquidity trade in firm n's stock, z_n. The next proposition is analogous to our Proposition 1.

Proposition 4. *There exists a generalized linear equilibrium. The price of the factor asset satisfies:*

$$P_F = m_F - \tau^{-1}\sigma_F^2\kappa < 0. \tag{26}$$

Each firm n's equity and debt prices satisfy:

$$P_{nE} = M_E(P_{nU}, \sigma_s^2 + \beta_n^2\sigma_F^2, K_n) \text{ and } P_{nD} = M_D(P_{nU}, \sigma_s^2 + \beta_n^2\sigma_F^2, K_n), \tag{27}$$

where:

$$P_{nU} = m_n + \sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)(\bar{s}_n + bz_n) - \beta_n \mathbb{E}_i[F - P_F], \tag{28}$$

and where b, σ_s^2, and σ_p^2 take the same form as in the baseline model.

This proposition illustrates that the equilibrium prices can again be expressed as the risk-neutral expectation of security payoffs, where the risk-neutral distribution of firm n's cash flows are given by $\mathcal{V}_n \sim N(P_{nU}, \sigma_s^2 + \beta_n^2\sigma_F^2)$. Note that these prices differ from those in the single-asset model in two ways. First, each firm's price includes a risk premium that is proportional to its cash-flow beta, β_n. This enters the debt and equity prices through the unlevered price statistic P_{nU}. Second, the risk-neutral variance now includes a term that is driven by the firm's exposure to systematic risk, $\beta_n^2\sigma_F^2$.

We next characterize each firm's expected price and returns. Recall that in our baseline model, we studied whether expected returns were positive or negative; given that we excluded any source of systematic risk, non-zero expected returns could be attributed to asymmetric information and/or liquidity trade. In this setting, however, there is a systematic risk, and so, even absent these forces, expected returns would be non-zero. To understand how firm-specific asymmetric information and liquidity trade influence expected returns, we compare expected returns to a frictionless benchmark without liquidity and informed trade, i.e., $\sigma_z^2 \to 0$ and $\sigma_\varepsilon^2 \to \infty$. In such a setting returns are driven only by exposures to the systematic factor. Specifically, let $R_{nE} = V_{nE} - P_{nE}$ and $R_{nD} = V_{nD} - P_{nD}$ denote the return on firm n's debt and equity, and let $\bar{R}_{nE}$ and $\bar{R}_{nD}$ denote the corresponding returns in a frictionless economy in which $\sigma_z^2 \to 0$ and $\sigma_\varepsilon^2 \to \infty$. The following result shows that an analog to Proposition 2 obtains in this setting.

Proposition 5. *(i) The expected excess return on equity is positive (i.e.,* $\mathbb{E}[R_{nE} - \bar{R}_{nE}] > 0$*) if and only if* $\mathbb{V}[P_{nU}] < \mathbb{V}_i[\mathbb{E}_i[\theta_n|s_i, s_p]]$.
(ii) The expected excess return on debt is positive (i.e., $\mathbb{E}[R_{nD} - \bar{R}_{nD}] > 0$*) if and only if* $\mathbb{V}[P_{nU}] > \mathbb{V}_i[\mathbb{E}_i[\theta_n|s_i, s_p]]$.

This result follows from a similar argument to that in our baseline model: the sign of the expected excess return follows by applying Jensen's inequality to the difference between the expectation of the price in (27) and the expectation of the price in an economy without informed and liquidity trading (since the unconditional expected cash flow is identical across both economies). The result clarifies that, in our setting, expected returns on debt and equity can vary across firms even after controlling for their risk exposures. Specifically, the sign of the *expected excess return* (or "alpha") on a security depends on the difference between the variance of the risk-neutral expectation of cash flows (i.e., $\mathbb{V}[P_{nU}]$) and the variance of cash flow expectations of a typical investor, just as in the benchmark analysis. The impact of information quality $\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}$, liquidity trade volatility σ_z^2, and disagreement ρ on returns are also similar in this setting. The reason is that the magnitude of expected excess returns is again driven by the variance difference $\mathbb{V}[P_{nU}] - \mathbb{V}_i[\mathbb{E}_i[\theta_n]]$, which takes the same form as in our baseline model.[22]

Importantly, neither of the variances $\mathbb{V}[P_{nU}]$ nor $\mathbb{V}_i[\mathbb{E}_i[\theta_n]]$ depend on the firm's risk-factor loading β_n.[23] As such, the above result clearly establishes how firm-specific, or idiosyncratic, risk — driven by firm-specific information about θ_n, firm-specific liquidity demand z_n from noise traders, and disagreement (i.e., ρ) — affects expected returns on debt and equity, even holding fixed a firm's systematic risk-factor loadings β_n.

8 Empirical Implications

Our model generates predictions on expected debt and equity returns and how these returns vary with financial distress, disagreement, and the intensity of liquidity trade. Existing work proposes several proxies for these constructs that render our predictions directly testable. For instance, considerable work proxies for belief dispersion using volume and analyst forecast dispersion (Diether et al. (2002), Banerjee (2011)), while other research proxies for liquidity-trader volatility based upon the concentration of a firm's ownership and the correlation in the liquidity shocks its owners face (Greenwood and Thesmar (2011), Friberg, Goldstein,

[22]Numerical results indicate that, similar to expected returns in our baseline model, $\mathbb{E}[R_{nD} - \bar{R}_{nD}]$ and $\mathbb{E}[R_{nE} - \bar{R}_{nE}]$ are non-monotonic in default risk.
[23]Note that even though β_n affects the price P_{nU}, it does not affect its variance since the factor risk premium $\mathbb{E}[F - P_F]$ is constant.

and Hankins (2022)). We summarize our model's predictions and their relation to existing empirical work below; several are consistent with existing empirical analyses while others have yet to be tested.

Given that a key feature of our model is the ability of both diversely-informed investors and liquidity traders to take positions in debt, our results apply most clearly to public debt markets. As such, when referencing debt markets, we focus on the literature on public bond markets. We further focus on our predictions for firms that are far from bankruptcy (i.e., with less than 50% probability of default), as such firms represent the vast majority of publicly-traded stocks.

Note our findings speak to expected returns after controlling for systematic risk exposures. Specifically, the predictions are about "alphas" from the perspective of an econometrician who accounts for systematic risk, even though these returns do not reflect any mispricing from the perspective of investors in the model. Note because debt and equity are non-linear securities, expected returns in our model depend non-linearly on systematic risk exposures. Thus, to control for the impact of these risk factors on expected returns, one should apply a technique that controls for potential non-linearities, such as sorting firms into portfolios based on standard risk factors. Indeed, this is common in the existing literature that studies the returns to distressed securities (e.g., Campbell et al. (2008)).

Decomposing the dispersion in beliefs. In our benchmark model, dispersion in beliefs can be captured by the *cross-sectional* variance in investor's conditional expectation of cash flows i.e.,

$$\mathcal{D} \equiv \int_i \left(\mu_i - \int \mu_j dj \right)^2 di = \frac{1}{\sigma_\varepsilon^2 \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} \right)^2}$$

where we recall that $\sigma_p^2 = \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{1}{\rho^2}\sigma_z^2 \left(\frac{\sigma_\varepsilon^2}{\tau} \right)^2$ is investors' perceived variance of the error in the price signal. This implies that dispersion increases with (i) the extent to which investors disagree about the informativeness of others' signals (a decrease in ρ) and (ii) the volatility of liquidity trading (an increase in σ_z), all else equal. However, as clarified by Corollary 2, these have opposing effects on expected returns: holding all else fixed, an increase in liquidity-trading volatility decreases equity returns (increases debt returns), but an increase in disagreement (decrease in ρ) can increase equity returns (decrease debt returns). As such, our analysis emphasizes the importance of distinguishing between belief dispersion driven by *disagreement* (lower ρ) versus *liquidity-trading volatility* (higher σ_z).

Specifically, Corollary 2 suggests the following regressions, where the dependence of expected returns on systematic risk-factor loadings are omitted for brevity and the predicted signs are presented below the coefficients:

$$R_{E,t+1} = \gamma_{0,E} + \underbrace{\gamma_{1,E}}_{>0} Disagreement_t + \underbrace{\gamma_{2,E}}_{<0} LiqTradeVol_t;$$

$$R_{D,t+1} = \gamma_{0,D} + \underbrace{\gamma_{1,D}}_{<0} Disagreement_t + \underbrace{\gamma_{2,D}}_{>0} LiqTradeVol_t.$$

Interpreting the negative relation between belief dispersion and equity (e.g., Diether et al. (2002)) and the positive relation for debt (e.g., Güntay and Hackbarth (2010)) through the lens of these predictions suggests that the cross-sectional variation in belief dispersion used in these papers is driven primarily by variation in liquidity-trading volatility across stocks. However, our analysis recommends that in order to fully account for the relation between belief dispersion and security returns, one should ideally decompose the variation in belief dispersion explicitly into variation driven by liquidity-trading volatility and disagreement.[24]

More generally, our analysis also sheds light on the mixed empirical evidence on the relation between belief dispersion and equity returns. The existing empirical literature finds that the sign and magnitude of this relation varies with the empirical proxy for disagreement, firm size, and time period considered (e.g., Diether et al. (2002), Johnson (2004), Banerjee (2011), Cen, Wei, and Yang (2017), Hou, Xue, and Zhang (2020), Chang, Hsiao, Ljungqvist, and Tseng (2022)). Our model predicts that in samples (or for proxies) where dispersion in beliefs is primarily driven by disagreement, the relation between expected returns and dispersion should be positive. On the other hand, when it is driven by liquidity-trading volatility, the relation should be negative.

Distress risk and expected returns. Several empirical studies examine the relationship between firm-specific distress risk and equity returns, controlling for standard systematic risk exposures. However, such a relationship is difficult to reconcile with traditional models due to the forces of diversification, leading the literature to propose that distress risk is mispriced (e.g., Campbell et al. (2008)). Our analysis suggests that such a relation can arise when investors are diversely informed and prices are noisy, even when they rationally incorporate all the information available to them (i.e., even when $\rho = 1$). More generally, our model predicts that distress risk is *negatively* associated with equity returns among firms far from bankruptcy, which is consistent with the empirical evidence (e.g., Dichev (1998), Campbell et al. (2008), Penman, Richardson, and Tuna (2007), and Caskey, Hughes, and Liu (2012)).

[24] In addition to measures of liquidity-trading volatility discussed above, Banerjee (2011) proposes a proxy of ρ based on the correlation between trading volume and return volatility.

In the debt market, several studies find that default risk has an excessive impact on credit spreads, commonly termed the credit-spread puzzle (e.g., Huang and Huang (2012), Bai et al. (2020)). Our analysis implies that expected debt returns increase with firm-specific distress risk for firms with less than a 50% probability of default is further consistent with this finding. Note while other studies offer explanations in terms of systematic distress risk (e.g., Chen, Hackbarth, and Strebulaev (2022)), our findings are novel in that they speak to firm-specific distress risk. Moreover, our model predicts that the marginal impact of distress risk on expected debt returns is strongest for firms that have a low degree of default risk.

Our model further predicts that the relationship between distress risk and expected stock returns depends on (i) the extent to which investors disagree and (ii) the prevalence of liquidity trade in a firm's stock and bonds. Thus, our analysis motivates the following regressions for firms with less than 50% probability of default, where the dependence on systematic risk-factor loadings and the main effects are omitted for brevity and the predicted signs are presented below the coefficients:

$$R_{E,t+1} = \gamma_{0,E} + \underbrace{\gamma_{1,E}}_{>0} Distress_t \times Disagreement_t + \underbrace{\gamma_{2,E}}_{<0} Distress_t \times LiqTradeVol_t;$$

$$R_{D,t+1} = \gamma_{0,D} + \underbrace{\gamma_{1,D}}_{<0} Distress_t \times Disagreement_t + \underbrace{\gamma_{2,D}}_{>0} Distress_t \times LiqTradeVol_t.$$

These predictions follow from the observation that the expected return on equity (debt) is decreasing (increasing) in the difference in variances, $\mathbb{V}[P_U] - \mathbb{V}_i[\mu_i]$ (see Proposition 2), which increases in liquidity-trading volatility and is greater when investors exhibit DO than when they exhibit RE. Moreover, our analysis suggests that for extremely distressed firms, distress risk has the opposite impact on expected returns. Note that some work finds non-monotonic or positive relationships between distress risk and returns (e.g., Chava and Purnanandam (2010), Garlappi et al. (2008)), and these cross-sectional predictions may be useful in future work to reconcile these mixed results.

Covariance between expected equity and debt returns. A central prediction of our model is that a firm's expected excess equity and debt returns are inversely related. The key intuition is that equity and debt payoffs are convex and concave, respectively. Again, our predictions concern "alphas" for debt and equity, after accounting appropriately for systematic risk exposures. Such exposures likely lead to common sources of variation in expected debt and equity returns that counteract the source of return variation we study. Past empirical evidence showing that several of the factors that predict equity returns do not predict debt returns is consistent with this finding (Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2017), Choi and Kim (2018), Bali, Subrahmanyam, and Wen (2021)).

Co-movement in equity and bond prices. The analysis in Section 6.1 implies that shocks to demand in either security impact both debt and equity prices, and so induce correlation in these securities' prices. Our results are broadly consistent with the evidence in Back and Crotty (2015), who show that while the unconditional correlation between stock and bond returns is low, the correlation in the parts driven by order flow is quite large. Our analysis suggests that the stock-bond correlation is higher when liquidity trading in the two markets is more correlated. Our results are also consistent with the evidence of Pasquariello and Sandulescu (2021), who document that the stock-bond correlation is low when the firm-level default probability is either very high or very low, but higher otherwise.

Capital structure and firm valuation. When liquidity traders' demands in equity and bond markets are not identical, our model implies that the capital structure of the firm affects its total valuation, even in the absence of traditional frictions (e.g., tax shields of debt, distress costs). Since we expect that for most firms, the probability of default is lower than 50% and the volatility of liquidity trading in equity is higher than that in debt, our model predicts that an increase in leverage leads to an increase in firm value. Moreover, the model predicts that, ceteris paribus, the impact of an increase in leverage is larger when investors dismiss the information in prices.

9 Conclusion

We develop a model where privately-informed, risk-averse investors trade alongside liquidity traders in the debt and equity of a firm. We show that the impact of private information on security valuation depends on the firm's likelihood of default, the intensity of liquidity trading in each market, and the extent to which investors learn from prices. Finally, we show that a firm's capital structure can affect its total valuation even in the absence of traditional frictions associated with debt issuance (e.g., tax shields, distress costs).

Our model generates a number of novel empirical predictions about the relation among disagreement, liquidity trading, distress risk, and debt and equity valuation. Moreover, our model serves as a useful benchmark for future theoretical analysis. For instance, it would be interesting to explore the incentives of investors to acquire information (e.g., Davis (2017)) in our setting when the liquidity trading in debt and equity are not identical, as well as to study the effects of segmentation across debt and equity markets. It would also be interesting to study how joint trade in equity and debt influences managers' investment decisions, both through their costs of capital and through managerial learning from debt and equity prices (as explored by Davis and Gondhi (2019)).

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A Proofs

A.1 Proof of Lemmas 1 and 2

These results are limiting cases of Proposition 1 below.

A.2 Proof of Lemma 3

This result is a special case of Lemma 6, where we define $G(P) \equiv \sigma_s^2 \times (g')^{-1}(P)$ to condense notation in the statement of the Lemma.

A.3 Proof of Proposition 1

The existence of a generalized linear equilibrium is a special case of Proposition 6 and the representation of the equilibrium demands is a special case of Corollary 4. It remains to show that the expression for the equilibrium price from Proposition 6 can be represented in terms of the M_E and M_D functions and that the equilibrium debt and equity prices sum to P_U. From Proposition 6, we have that the equilibrium price vector satisfies

$$\begin{aligned} P &= g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}m + \mathbf{1}\sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \\ &= g'\left(\mathbf{1}\frac{P_U(\theta, z)}{\sigma_s^2}\right), \end{aligned}$$

where the second line uses the definition of P_U (from Lemma 1) to simplify the argument of the gradient g' and where the function $g : \mathbb{R}^2 \to \mathbb{R}$ satisfies

$$g\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \log\left(\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1 - y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)\right)\right).$$

Computing the two partial derivatives that make up the gradient g' yields

$$\begin{aligned} \frac{\partial g}{\partial y_1} &= \left(\sigma_s^2 y_1 - \sigma_s \frac{\phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}{\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}\right) \frac{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)} \\ &\quad + K \frac{\exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}; \\ \frac{\partial g}{\partial y_2} &= \left(\sigma_s^2 y_2 + \sigma_s \frac{\phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)}{1 - \Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)} - K\right) \frac{\exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}. \end{aligned}$$

Evaluating these expressions at $y_1 = y_2 = \frac{P_U}{\sigma_s^2}$ gives the debt and equity prices, respectively:

$$\begin{aligned} P_D = \frac{\partial g}{\partial y_1}\bigg|_{y_1=y_2=\frac{P_U}{\sigma_s^2}} &= \left(P_U - \sigma_s \frac{\phi\left(\frac{K-P_U}{\sigma_s}\right)}{\Phi\left(\frac{K-P_U}{\sigma_s}\right)}\right)\Phi\left(\frac{K-P_U}{\sigma_s}\right) + K\left(1-\Phi\left(\frac{K-P_U}{\sigma_s}\right)\right) \\ &= M_D(P_U, \sigma_s^2, K), \end{aligned}$$

and

$$\begin{aligned} P_E = \frac{\partial g}{\partial y_2}\bigg|_{y_1=y_2=\frac{P_U}{\sigma_s^2}} &= \left(P_U + \sigma_s \frac{\phi\left(\frac{K-P_U}{\sigma_s}\right)}{1-\Phi\left(\frac{K-P_U}{\sigma_s}\right)} - K\right)\left(1-\Phi\left(\frac{K-P_U}{\sigma_s}\right)\right) \\ &= M_E(P_U, \sigma_s^2, K), \end{aligned}$$

as claimed. Adding the expressions above immediately yields that the overall firm value is:

$$P_D + P_E = M_E\left(P_U, \sigma_s^2, K\right) + M_D\left(P_U, \sigma_s^2, K\right) = P_U.$$

A.4 Proof of Corollary 1

It is straightforward to verify that $M_E(x, \cdot, \cdot)$ and $M_D(x, \cdot, \cdot)$ increase in x. Hence, results (i)-(iii) follow from the fact that, as can be seen in Lemma 1, P_U increases in θ and z and decreases in κ. To verify that the equity and debt prices decrease and increase in K, respectively, note:

$$\begin{aligned} \frac{\partial}{\partial K} M_D(P_U, \sigma_s^2, K) &= -\frac{\partial}{\partial K} M_E(P_U, \sigma_s^2, K) \\ &= 1 - \Phi\left(\frac{K-P_U}{\sigma_s}\right) - \frac{K-P_U}{\sigma_s}\phi\left(\frac{K-P_U}{\sigma_s}\right) - \phi'\left(\frac{K-P_U}{\sigma_s}\right) \\ &= 1 - \Phi\left(\frac{K-P_U}{\sigma_s}\right) > 0. \end{aligned}$$

A.5 Proof of Proposition 2

Observe that P_U is unconditionally normally distributed with mean

$$\begin{aligned} \mathbb{E}\left[P_U\right] &= \mathbb{E}\left[\int \mu_j dj + \frac{\sigma_s^2}{\tau}(z-\kappa)\right] \\ &= \int \mathbb{E}\left[\mu_j\right] dj - \frac{\sigma_s^2}{\tau}\kappa \\ &= m - \frac{\sigma_s^2}{\tau}\kappa. \end{aligned} \tag{29}$$

Thus, we have:

$$\begin{aligned}
\mathbb{E}\left[P_E\left(P_U\right)\right] &= \mathbb{E}\left\{\mathbb{E}\left[\max\left(x-K,0\right)|x\sim N\left(P_U,\sigma_s^2\right)\right]\right\} \\
&= \mathbb{E}\left\{\mathbb{E}\left[\max\left(x+P_U-K,0\right)|x\sim N\left(0,\sigma_s^2\right)\right]\right\} \\
&= \mathbb{E}\left[\max\left(x+y-K,0\right)|x\sim N\left(0,\sigma_s^2\right),y\sim N\left(\mathbb{E}\left[P_U\right],\mathbb{V}\left[P_U\right]\right)\right] \\
&= \mathbb{E}\left[\max\left(x-K,0\right)|x\sim N\left(m-\frac{\sigma_s^2}{\tau}\kappa,\sigma_s^2+\mathbb{V}\left[P_U\right]\right)\right] \\
&= M_E\left(m-\frac{\sigma_s^2}{\tau}\kappa,\Omega,K\right).
\end{aligned}$$

where we have defined $\Omega \equiv \sigma_s^2 + \mathbb{V}[P_U] = \sigma_\theta^2 - \mathbb{V}_i[\mu_i] + \mathbb{V}[P_U]$, using the law of total variance to express $\sigma_s^2 = \sigma_\theta^2 - \mathbb{V}_i[\mu_i]$ in the second equality. The debt result follows analogously.

We next show how the equity payoffs compare to equity expected cash flows; the result for debt follows analogously. Observe that, as $\kappa \to 0$, the expected equity price approaches $M_E(m,\Omega,K)$. Now, the expected equity payoff equals:

$$\mathbb{E}\left[\max\left(\theta-K,0\right)\right] = M_E\left(m,\sigma_\theta^2,K\right).$$

Thus, equity earns negative expected returns if and only if $M_E(m,\Omega,K) - M_E(m,\sigma_\theta^2,K) > 0$ and earns positive expected returns if and only if $M_E(m,\Omega,K) - M_E(m,\sigma_\theta^2,K) < 0$. Now, note that the derivative of M_E with respect to its second argument is:

$$\begin{aligned}
\frac{\partial M_E\left(m,x,K\right)}{\partial x} &= \frac{\partial}{\partial x}\left\{x^{\frac{1}{2}}\phi\left(x^{-\frac{1}{2}}\left(K-m\right)\right) - \left[1-\Phi\left(x^{-\frac{1}{2}}\left(K-m\right)\right)\right]\left(K-m\right)\right\} \\
&= \frac{1}{2}x^{-\frac{1}{2}}\phi\left(x^{-\frac{1}{2}}\left(K-m\right)\right) > 0.
\end{aligned}$$

Thus, we have that:

$$M_E\left(m,\Omega,K\right) - M_E\left(m,\sigma_\theta^2,K\right) \gtrless 0 \Leftrightarrow \Omega - \sigma_\theta^2 \gtrless 0 \Leftrightarrow \mathbb{V}[P_U] - \mathbb{V}_i[\mu_i] \gtrless 0,$$

which completes the proof of statements (i) and (ii) in the proposition.

A.6 Proof of Corollary 2

As in the proof of Proposition 2, let $\Omega \equiv \sigma_s^2 + \mathbb{V}[P_U]$. From Proposition 2, to sign expected returns, it suffices to determine the sign of $\Omega - \sigma_\theta^2 = \mathbb{V}(P_U) + \sigma_s^2 - \sigma_\theta^2$ in terms of the deep parameters of the model.

We can write the unconditional variance of P_U as

$$\begin{aligned}\mathbb{V}(P_U) &= \left(\sigma_s^2\right)^2 \mathbb{V}\left(\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)(\overline{s}+bz)\right)\\ &= \left(\sigma_s^2\right)^2\left(\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2\sigma_\theta^2+\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2 b^2\sigma_z^2\right)\\ &= \left(\sigma_s^2\right)^2\left(\left(\frac{1}{\sigma_s^2}+\frac{1}{\rho\sigma_p^2}-\frac{1}{\sigma_p^2}-\frac{1}{\sigma_\theta^2}\right)^2\sigma_\theta^2+\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2 b^2\sigma_z^2\right)\\ &= \sigma_\theta^2-\sigma_s^2\\ &\quad+\left(\sigma_s^2\right)^2\left(\sigma_\theta^2\left(\frac{1}{\rho\sigma_p^2}+\frac{1}{\sigma_\varepsilon^2}\right)^2-\sigma_\theta^2\left(\frac{1}{\sigma_p^2}+\frac{1}{\sigma_\varepsilon^2}\right)^2+\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2 b^2\sigma_z^2-\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)\right)\end{aligned}$$

where the third equality adds and subtracts $\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_p^2}$ inside the first term in the large parentheses and uses the definition of $\frac{1}{\sigma_s^2}=\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}$ to simplify. The final equality does algebraic manipulations and collects terms. Hence, the sign of $\Omega-\sigma_\theta^2$ is pinned down by the sign of

$$\sigma_\theta^2\left(\frac{1}{\rho\sigma_p^2}+\frac{1}{\sigma_\varepsilon^2}\right)^2-\sigma_\theta^2\left(\frac{1}{\sigma_p^2}+\frac{1}{\sigma_\varepsilon^2}\right)^2+\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2 b^2\sigma_z^2-\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right) \tag{30}$$

The limits of the expression in eq. (30) as ρ tends to zero and one, being careful to account for the fact that $\sigma_p^2=\frac{1-\rho^2}{\rho^2}\sigma_\theta^2+\frac{b^2\sigma_z^2}{\rho^2}$ itself depends on ρ, are

$$\lim_{\rho\to 0}\left(\sigma_\theta^2\left(\frac{1}{\rho\sigma_p^2}+\frac{1}{\sigma_\varepsilon^2}\right)^2-\sigma_\theta^2\left(\frac{1}{\sigma_p^2}+\frac{1}{\sigma_\varepsilon^2}\right)^2+\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2 b^2\sigma_z^2-\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)\right) \tag{31}$$

$$=\frac{1}{\sigma_\varepsilon^2}\left(\frac{b^2\sigma_z^2}{\sigma_\varepsilon^2}-1\right) \tag{32}$$

$$\lim_{\rho\to 1}\left(\sigma_\theta^2\left(\frac{1}{\rho\sigma_p^2}+\frac{1}{\sigma_\varepsilon^2}\right)^2-\sigma_\theta^2\left(\frac{1}{\sigma_p^2}+\frac{1}{\sigma_\varepsilon^2}\right)^2+\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2 b^2\sigma_z^2-\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)\right) \tag{33}$$

$$=\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{b^2\sigma_z^2}\right)\frac{b^2\sigma_z^2}{\sigma_\varepsilon^2} \tag{34}$$

Furthermore, the expression in eq. (30) is strictly increasing in ρ since

$$\frac{\partial}{\partial\rho}\left(\sigma_\theta^2\left(\frac{1}{\rho\sigma_p^2}+\frac{1}{\sigma_\varepsilon^2}\right)^2-\sigma_\theta^2\left(\frac{1}{\sigma_p^2}+\frac{1}{\sigma_\varepsilon^2}\right)^2+\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2 b^2\sigma_z^2-\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)\right) \tag{35}$$

$$=2\frac{\left(\sigma_\theta^2+b^2\sigma_z^2\right)\left((1-\rho)^2\sigma_\theta^2+b^2\sigma_z^2\right)}{\sigma_\varepsilon^2\left((1-\rho^2)\sigma_\theta^2+b^2\sigma_z^2\right)^2}>0. \tag{36}$$

It follows that for $\sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2} \Leftrightarrow \frac{b^2\sigma_z^2}{\sigma_\varepsilon^2} > 1$ we have $\Omega - \sigma_\theta^2 > 0$ for all $\rho \in [0,1]$. On the other hand, for $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2} \Leftrightarrow \frac{b^2\sigma_z^2}{\sigma_\varepsilon^2} < 1$ the above implies that there exists some ρ^* such that for all $\rho < \rho^*$ we have $\Omega - \sigma_\theta^2 < 0$ while for all $\rho > \rho^*$ we have $\Omega - \sigma_\theta^2 > 0$.

A.7 Proof of Corollary 3

Part (i) We consider equity returns; the proof for debt returns is analogous. We have

$$\begin{aligned}
\operatorname{sgn}\left(\frac{\partial}{\partial K}\left|\mathbb{E}[R_E]\right|\right) &= \operatorname{sgn}\left(\operatorname{sgn}\left(\mathbb{E}[R_E]\right)\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) \\
&= \operatorname{sgn}(\mathbb{E}[R_E])\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) \\
&= -\operatorname{sgn}\left(\Omega - \sigma_\theta^2\right)\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right).
\end{aligned}$$

where, again, $\Omega = \sigma_s^2 + \mathbb{V}[P_U]$ is as defined in the proof of Proposition 2. Differentiating the expected return with respect to K yields

$$\begin{aligned}
\frac{\partial}{\partial K}\mathbb{E}[R_E] &= \frac{\partial}{\partial K}\left(M_E(m, \sigma_\theta^2, K) - M_E(m, \Omega, K)\right) \\
&= \frac{\partial}{\partial K}\int_\Omega^{\sigma_\theta^2}\frac{\partial}{\partial x}M_E(m, x, K)dx \\
&= \int_\Omega^{\sigma_\theta^2}\frac{\partial^2}{\partial K\partial x}M_E(m, x, K)dx,
\end{aligned}$$

where the second equality uses the fundamental theorem of calculus to express the difference in the M_E function an integral. Computing the cross-partial derivative of M_E yields

$$\frac{\partial^2}{\partial K\partial x}M_E(m, x, K) = \frac{\partial}{\partial K}\frac{1}{2}\frac{1}{\sqrt{x}}\phi\left(\frac{K-m}{\sqrt{x}}\right) = \frac{1}{2}\frac{1}{x}\phi'\left(\frac{K-m}{\sqrt{x}}\right) = \frac{1}{2}\frac{1}{x}\frac{m-K}{\sqrt{x}}\phi\left(\frac{K-m}{\sqrt{x}}\right).$$

Hence, for $K < m$, we have

$$\begin{aligned}
\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) &= \operatorname{sgn}\left(\int_\Omega^{\sigma_\theta^2}\underbrace{\frac{\partial^2}{\partial K\partial x}M_E(m, x, K)}_{>0}dx\right) \\
&= -\operatorname{sgn}(\Omega - \sigma_\theta^2),
\end{aligned}$$

and therefore

$$\operatorname{sgn}\left(\frac{\partial}{\partial K}\left|\mathbb{E}[R_E]\right|\right) = \operatorname{sgn}^2\left(\Omega - \sigma_\theta^2\right) > 0.$$

On the other hand for $K > m$,

$$
\begin{aligned}
\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) &= \operatorname{sgn}\left(\int_{\Omega}^{\sigma_\theta^2} \underbrace{\frac{\partial^2}{\partial K \partial x} M_E(m,x,K)}_{<0} \, dx\right) \\
&= \operatorname{sgn}(\Omega - \sigma_\theta^2),
\end{aligned}
$$

and therefore

$$
\operatorname{sgn}\left(\frac{\partial}{\partial K}\left|\mathbb{E}[R_E]\right|\right) = -\operatorname{sgn}^2\left(\Omega - \sigma_\theta^2\right) < 0.
$$

Because $|\mathbb{E}[R_E]|$ is strictly increasing in K for $K < m$ and strictly decreasing in K for $K > m$, it follows that $|\mathbb{E}[R_E]|$ is hump-shaped in K and achieves its maximum at $K = m$.

Part (ii) Consider debt returns. We have that:

$$
\begin{aligned}
\frac{\partial \mathbb{E}\left[R_D\right]}{\partial \sigma_z} &= \frac{\partial M_D\left(m, \sigma_\theta^2, K\right)}{\partial \sigma_z} - \frac{\partial M_D\left(m, \Omega, K\right)}{\partial \sigma_z} \\
&= -\frac{\partial M_D\left(m, \Omega, K\right)}{\partial \Omega}\frac{\partial \Omega}{\partial \sigma_z} \propto \frac{\partial \Omega}{\partial \sigma_z}.
\end{aligned}
$$

Similarly, for equity returns, we obtain $\frac{\partial \mathbb{E}[R_E]}{\partial \sigma_z} \propto -\frac{\partial \Omega}{\partial \sigma_z}$. Now,

$$
\frac{\partial \Omega}{\partial \sigma_z} = \frac{2\sigma_\theta^4 \sigma_z \sigma_\varepsilon^4 \begin{pmatrix} \tau^4 \sigma_\theta^2 \sigma_z^2 \sigma_\varepsilon^4 \left(3\left(1-\rho^2\right)^2 \sigma_\theta^4 + \left(3 - \rho^3(\rho+2)\right)\sigma_\theta^2 \sigma_\varepsilon^2 + 2\rho^3 \sigma_\varepsilon^4\right) \\ +(1-\rho)\tau^6 \sigma_\theta^4 \left((1-\rho)^2(\rho+1)^3 \sigma_\theta^4 + (1-\rho)(\rho+1)\left(2\rho^2+\rho+1\right)\sigma_\theta^2 \sigma_\varepsilon^2 + 2\rho^3 \sigma_\varepsilon^4\right) \\ 3\tau^2 \sigma_\theta^2 \sigma_z^4 \sigma_\varepsilon^8 \left(\sigma_\varepsilon^2 + \left(1-\rho^2\right)\sigma_\theta^2\right) + \sigma_z^6 \sigma_\varepsilon^{12}\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right) \end{pmatrix}}{\tau^2\left(\sigma_z^2 \sigma_\varepsilon^4\left(\sigma_\theta^2+\sigma_\varepsilon^2\right) + \tau^2 \sigma_\theta^2\left(\sigma_\varepsilon^2 + \left(1-\rho^2\right)\sigma_\theta^2\right)\right)^3} > 0.
$$

A.8 Proof of Lemma 4

This is a special case of Lemma 6, where we define the function $G(P) = \sigma_s^2 \times (g')^{-1}(P)$ in the text.

A.9 Proof of Proposition 3

This is a special case of Proposition 6 in which $\mu_z = (0,0)$ and Σ_z is positive definite.

A.10 Proof of Proposition 4

The existence of equilibrium and the specific expressions for the security prices follow from Proposition 7 after specializing the vector notation to isolate individual assets. In the vector representation of prices as risk-neutral expected payoffs, the risk-neutral mean for a given firm n's cash flow (or for the factor cash flow) is the unlevered price, P_{nU} (or P_F for the factor) given in Lemma 7. Similarly, the risk-neutral variance for a given cash flow is the n^{th} (or $(N+1)^{\text{st}}$, in the case of the factor) diagonal element of the cash flow variance matrix $\Gamma = \left(\begin{smallmatrix} \sigma_s^2 I + \sigma_F^2 \beta\beta' & \sigma_F^2 \beta \\ \sigma_F^2 \beta' & \sigma_F^2 \end{smallmatrix}\right)$. Finally, we can write $\frac{1}{\tau}\sigma_F^2 \kappa = m_F - P_F = \mathbb{E}_i[F] - P_F$, which allows one to express the final term in eq. (28) in the form of the cash flow beta times the factor expected return. Substituting in these values yields the expression in the Proposition.

A.11 Proof of Proposition 5

Note, analogously to the representation of unconditional expected prices in Proposition 2 we can write firm n's unconditional expected equity and debt prices are:

$$\begin{aligned}
\mathbb{E}\left[P_{nE}\right] &= M_E\left(\mathbb{E}[P_{nU}], \mathbb{V}[P_{nU}] + \sigma_s^2 + \beta_n^2 \sigma_F^2, K_n\right) \\
&= M_E\left(m_n - \beta_n \mathbb{E}[F - P_F], \mathbb{V}[P_{nU}] + \sigma_s^2 + \beta_n^2 \sigma_F^2, K_n\right); \\
\mathbb{E}\left[P_{nD}\right] &= M_D\left(\mathbb{E}[P_{nU}], \mathbb{V}[P_{nU}] + \sigma_s^2 + \beta_n^2 \sigma_F^2, K_n\right) \\
&= M_D\left(m_n - \beta_n \mathbb{E}[F - P_F], \mathbb{V}[P_{nU}] + \sigma_s^2 + \beta_n^2 \sigma_F^2, K_n\right).
\end{aligned}$$

Next, note that in the no information, no liquidity trade benchmark, we still have that $\mathbb{E}[P_{nU}] = m_n - \beta_n \mathbb{E}[F - P_F]$ but in that case the variance of the unlevered price is zero, $\mathbb{V}\left[P_{nU}\right] = 0$ and investors' conditional cash flow variance is equal to the prior variance $\sigma_s^2 = \sigma_\theta^2$. Thus, given the convexity of M_E and concavity of M_D, we have that debt (equity) returns are higher (lower) than under the benchmark if and only if $\mathbb{V}\left[P_{nU}\right] + \sigma_s^2 + \beta_n^2 \sigma_F^2 < \sigma_\theta^2 + \beta_n^2 \sigma_F^2 \Leftrightarrow \mathbb{V}\left[P_{nU}\right] < \sigma_\theta^2 - \sigma_s^2 = \mathbb{V}_i[\mathbb{E}_i[\theta_n | s_i, s_p]]$.

B Equilibrium with arbitrary, correlated liquidity trading

In this section, we characterize the equilibrium in the fully-general version of the model in which liquidity trading $z = (z_D, z_E)$ follows a general bivariate normal distribution $N(\mu_z, \Sigma_z)$ where $\mu_z \in \mathbb{R}$ is an arbitrary vector of means, and Σ_z is an arbitrary positive semi-definite covariance matrix. As in the text, we consider equilibria of the "generalized linear" form specified in Definition 3 where the endogenous price statistics take the form

$$s_p = \mathbf{1}\bar{s} + B\left(z - \mu_z\right).$$

with $B = \left(\begin{smallmatrix} b_{1D} & b_{1E} \\ b_{2D} & b_{2E} \end{smallmatrix}\right)$ the 2×2 matrix of coefficients to be determined.

We begin by characterizing an arbitrary investor i's conditional distribution of the vector of debt and equity payoffs, $V = (V_D, V_E)$, given arbitrary $N(\mu_i, \sigma_s^2)$ beliefs about the underlying firm cash flow $\mathcal{V}$.

Lemma 5. *Suppose that $\mathcal{V}$ is conditionally normally distributed with mean μ_i and variance σ_s^2. Then the vector $V = (\min(\mathcal{V}, K), \max(\mathcal{V} - K, 0))$ follows a bivariate exponential family with moment-generating function (MGF) that is finite for any $u \in \mathbb{R}^2$, and is given explicitly by*

$$\mathbb{E}_i[\exp\{u'V\}] = \exp\left\{g\left(u + \mathbf{1}\frac{\mu_i}{\sigma_s^2}\right) - g\left(\mathbf{1}\frac{\mu_i}{\sigma_s^2}\right)\right\}$$

where the function $g : \mathbb{R}^2 \to \mathbb{R}$ is defined as

$$g\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \log\left(\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1 - y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)\right)\right). \tag{37}$$

Proof. (Lemma 5) The claim about finiteness follows immediately once we have proven that the MGF takes the given form since, by inspection, the function g is finite on all of $\mathbb{R}^2$. The claim that the distribution is an exponential family also follows immediately from the functional form (see e.g., Sampson (1975), Hoffmann and Schmidt (1982)). To establish the expression for the MGF, write

$$\begin{aligned}
&\mathbb{E}_i[\exp\{u'V\}] \\
&= \int_{-\infty}^{\infty} \exp\{u_1 \min\{t, K\} + u_2 \max\{t - K, 0\}\}\, dF_\theta(t|s_i, s_p) \\
&= \int_{-\infty}^{K} \exp\{u_1 t\}\, dF_\theta(t|s_i, s_p) + \int_{K}^{\infty} \exp\{u_1 K + u_2(t - K)\}\, dF_\theta(t|s_i, s_p) \\
&= \exp\left\{\mu_i u_1 + \frac{1}{2}\sigma_s^2 u_1^2\right\}\Phi\left(\frac{K - \mu_i - \sigma_s^2 u_1}{\sigma_s}\right) \\
&\quad + \exp\left\{(u_1 - u_2)K + \mu_i u_2 + \frac{1}{2}\sigma_s^2 u_2^2\right\}\left(1 - \Phi\left(\frac{K - \mu_i - \sigma_s^2 u_2}{\sigma_s}\right)\right) \\
&= \exp\left\{\frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_1\right)^2 - \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}\right)^2\right\}\Phi\left(\frac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_1\right)}{\sigma_s}\right) \\
&\quad + \exp\left\{(u_1 - u_2)K + \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_2\right)^2 - \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}\right)^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_2\right)}{\sigma_s}\right)\right)
\end{aligned}$$

$$= \left(\exp\left\{ \tfrac{1}{2}\sigma_s^2 \left(\tfrac{\mu_i}{\sigma_s^2} + u_1 \right)^2 \right\} \Phi\left(\frac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_1\right)}{\sigma_s} \right) \right.$$

$$\left. + \exp\left\{ (u_1 - u_2) K + \tfrac{1}{2}\sigma_s^2 \left(\tfrac{\mu_i}{\sigma_s^2} + u_2 \right)^2 \right\} \left(1 - \Phi\left(\frac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_2\right)}{\sigma_s} \right) \right) \right) \exp\left\{ -\tfrac{1}{2}\sigma_s^2 \left(\tfrac{\mu_i}{\sigma_s^2} \right)^2 \right\}.$$

Taking the logarithm, this expression is identical to that in the Lemma after recognizing that g as defined in the statement of the Lemma satisfies $g\left(\begin{smallmatrix} y \\ y \end{smallmatrix}\right) = \frac{1}{2}\sigma_s^2 y^2$ when both arguments are identical. □

With trader beliefs pinned down, we next characterize the optimal demand.

Lemma 6. *Fix any $P = (P_D, P_E)$ in set of no-arbitrage prices $\{(p_D, p_E) : p_D < K, p_E > 0\}$. There is a unique optimal demand for trader i, given by*

$$x_i = \tau \left(\mathbf{1} \frac{\mu_i}{\sigma_s^2} - (g')^{-1} (P) \right)$$

where $(g')^{-1} : \mathbb{R}^2 \to \mathbb{R}^2$ is the inverse of the gradient $g'\left(\begin{smallmatrix} y_1 \\ y_2 \end{smallmatrix}\right) \equiv \begin{pmatrix} \frac{\partial}{\partial y_1} g \\ \frac{\partial}{\partial y_2} g \end{pmatrix}$.

Proof. (Lemma 6) From Lemma 5, we can compute the trader's conditional expected utility given an arbitrary demand x_i as

$$\mathbb{E}_i \left[-\exp\left\{ -\frac{1}{\tau} x_i' (V - P) \right\} | s_i, s_p \right] = -\exp\left\{ \frac{1}{\tau} x_i' P + g\left(\mathbf{1}\frac{\mu_i}{\sigma_s^2} - \frac{1}{\tau} x_i \right) - g\left(\mathbf{1} \frac{\mu_i}{\sigma_s^2} \right) \right\}.$$

Letting $g' = \begin{pmatrix} \frac{\partial}{\partial y_1} g \\ \frac{\partial}{\partial y_2} g \end{pmatrix}$ denote the gradient of g, the FOC is

$$0 = g'\left(\mathbf{1} \frac{\mu_i}{\sigma_s^2} - \frac{1}{\tau} x_i \right) - P. \tag{38}$$

Note that the Hessian matrix $g'' \equiv \begin{pmatrix} \frac{\partial^2}{\partial y_1^2} g & \frac{\partial^2}{\partial y_1 \partial y_2} g \\ \frac{\partial^2}{\partial y_1 \partial y_2} g & \frac{\partial^2}{\partial y_2^2} g \end{pmatrix}$ is necessarily positive definite, owing to the fact that it is the matrix of 2nd derivatives of the cumulant generating function of V, which is strictly convex. It follows that the optimum, if it exists, is unique and the FOC in (38) is sufficient to characterize it. Hence, it suffices to show that there exists a demand $x_i \in \mathbb{R}^2$ that satisfies eq. (38).

Due to the positive-definiteness of g'' it follows that the gradient g' is injective and therefore invertible on its range. Hence, if we can establish that the range is the set of no-arbitrage prices $\{(p_D, p_E) : p_D < K, p_E > 0\}$, the existence and characterization of the optimal demand will follow immediately from rearranging the FOC in eq. (38).

Let $S = \{(v_D, v_E) : v_D < K, v_E = 0\} \cup \{(v_D, v_E) : v_D = K, v_E > 0\}$ denote the support of the payoff vector (V_D, V_E). We claim that the range of g' is the closed convex hull of S, which is precisely the set of no-arbitrage prices. This follows from the following. First, because the CGF g is defined on all of $\mathbb{R}^2$, and furthermore because $\mathbb{R}^2$ is open, the exponential family described by the CGF is necessarily "regular" as defined by Barndorff-Nielsen (2014). It follows from Theorem 8.2 of Barndorff-Nielsen (2014) that the exponential family is "steep" and therefore from Theorem 9.2 in Barndorff-Nielsen (2014) that the gradient g' maps $\mathbb{R}^2$ onto the interior of the closed convex hull of the support S. This set, $\operatorname{int}\operatorname{conv}(S) = \{(x, y) : x < K, y > 0\}$, is the set of candidate prices in which the debt price is less than the face value K and the equity price is greater than zero, which is precisely the set of prices that do not admit arbitrage. □

Proposition 6. *There exists an equilibrium in the financial market. The vector of equilibrium asset prices takes the form*

$$P = g'\left(\mathbf{1}\frac{\int \mu_j \, dj}{\sigma_s^2} - \frac{1}{\tau}\left(\kappa\mathbf{1} - z\right)\right). \tag{39}$$

where the function $g' : \mathbb{R}^2 \to \mathbb{R}^2$ is given in closed-form in eqs. (45)–(46) the proof. This equilibrium is unique within the generalized linear class.

1. *If Σ_z is invertible, then the equilibrium price vector is*

$$P = g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}m + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \mathbf{1}\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \tag{40}$$

where the equilibrium price signals coefficient matrix is diagonal $B = \begin{pmatrix} \frac{\sigma_\varepsilon^2}{\tau} & 0 \\ 0 & \frac{\sigma_\varepsilon^2}{\tau} \end{pmatrix}$

2. *If Σ_z is singular and of the form $\Sigma_z = \mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., liquidity trade is identical in the two markets, $z_E = z_D = \zeta$ for $\zeta \sim N(0, \sigma_z^2)$), then the equilibrium price vector is*

$$P = g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}m + \mathbf{1}\sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \tag{41}$$

where $s_p = \overline{s} + b\zeta$ is one-dimensional, $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{b^2}{\rho^2}\sigma_z^2$ with $b = \frac{\sigma_\varepsilon^2}{\tau}$, and $\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}$.

3. *If $\rho < 1$, and Σ_z is singular and not of the form $\Sigma_z = \mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., liquidity trade is perfectly positively correlated but with different variances, or is perfectly negatively correlated, or at least one of the z_j is constant), then the equilibrium price vector is*

given by

$$P = g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}m + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\mathbf{1}a'\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \tag{42}$$

where $B = \begin{pmatrix} \frac{\sigma_\varepsilon^2}{\tau} & 0 \\ 0 & \frac{\sigma_\varepsilon^2}{\tau} \end{pmatrix}$, $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2$, $\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}$, *and the vector* $a \in \mathbb{R}^2$ *is defined in the proof.*

4. *If* $\rho = 1$, *and* Σ_z *is singular and not of the form* $\Sigma_z = \mathbf{1}\mathbf{1}'\sigma_z^2$, *then there exists a fully-revealing equilibrium in which* $P_D = \min\{\mathcal{V}, K\}$ *and* $P_E = \max\{\mathcal{V} - K, 0\}$.

Proof. (Proposition 6) Using the expression for trader demand from Lemma 6, the market clearing condition yields

$$\begin{aligned}
&\int x_j dj + z = \mathbf{1}\kappa \\
\Leftrightarrow &\int x_j dj + z - \mu_z = \mathbf{1}\kappa - \mu_z \\
\Leftrightarrow &\tau\left(\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} - (g')^{-1}(P)\right) + z - \mu_z = \mathbf{1}\kappa - \mu_z \\
\Leftrightarrow &P = g'\left(\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}(z - \mu_z) - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z)\right).
\end{aligned} \tag{43}$$

Because the vector of the liquidity trade z enters explicitly multiplied only by a scalar, we can conclude that in any equilibrium it suffices to consider only diagonal coefficient matrices B with identical elements on the diagonal. That is, $B = bI$ for $b \in \mathbb{R}$ still to be determined.

A closed-form expression for the gradient $g'\binom{y_1}{y_2}$ follows from computing the partial derivatives of the function g as defined in Lemma 5:

$$\frac{\partial g}{\partial y_1} = \left(\sigma_s^2 y_1 - \sigma_s \frac{\phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}{\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}\right) \frac{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)} \tag{44}$$

$$+ K\frac{\exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)} \tag{45}$$

$$\frac{\partial g}{\partial y_2} = \left(\sigma_s^2 y_2 + \sigma_s \frac{\phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)}{1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)} - K\right) \frac{\exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}. \tag{46}$$

To complete the proof and derive the explicit expressions in the Proposition, it is convenient to separately consider the cases of positive definite Σ_z and singular Σ_z.

If Σ_z is positive definite, then we can write the conditional moments explicitly as

$$\mu_i = \mathbb{E}\left[\mathcal{V}|s_i, s_p\right] = m + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}s_p\right), \text{ and} \tag{47}$$

$$\sigma_s^2 = \mathbb{V}\left(\mathcal{V}|s_i, s_p\right) = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1}\mathbf{1}\right)^{-1} \tag{48}$$

where $\Sigma_p \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2\mathbf{1}\mathbf{1}' + \frac{1}{\rho^2}B\Sigma_z B'$. Because Σ_z is assumed positive definite, it follows that $B\Sigma_z B'$ is positive definite. Furthermore, Σ_p, being a sum of a positive definite and positive semidefinite matrix is itself positive definite and therefore invertible, where it is understood that we take $\Sigma_p^{-1} = \mathbf{0}$ and $\Sigma_p^{-1}\frac{1}{\rho} = (\rho\Sigma_p)^{-1} = \mathbf{0}$ in the above expressions when $\rho \to 0$.

Substituting the explicit expression for μ_i in the argument of g' in eq. (43) and grouping terms yields

$$\begin{aligned}
&\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}(z-\mu_z) - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z)\\
&= \mathbf{1}\frac{1}{\sigma_s^2}\left(m + \sigma_s^2\frac{1}{\sigma_\varepsilon^2}\bar{s} + \sigma_s^2\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}s_p\right) + \frac{1}{\tau}(z-\mu_z) - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z)\\
&= \frac{1}{\sigma_s^2}\left(\mathbf{1}m + \sigma_s^2\left(\mathbf{1}\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}s_p + \frac{1}{\sigma_\varepsilon^2}\left(\mathbf{1}\bar{s} + \frac{\sigma_\varepsilon^2}{\tau}(z-\mu_z)\right)\right) - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right).
\end{aligned}$$

Matching coefficients on the initial conjecture $s_p = \mathbf{1}\bar{s} + B(z-\mu_z)$, with $B = bI$ as derived above, requires $b = \frac{\sigma_\varepsilon^2}{\tau}$. The previous expression now simplifies to

$$\frac{1}{\sigma_s^2}\left(\mathbf{1}m + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \mathbf{1}\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}\right)s_p - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right)$$

which, upon plugging back into g', matches the expression in the Proposition. Because there is a unique coefficient matrix B that satisfies the initial conjecture, this equilibrium price function is the unique one within the generalized linear class.

If Σ_z is singular, then the matrix Σ_p that appears above is not invertible and the above expressions for beliefs do not apply directly.[25] Intuitively, in this case there is only a single shock to liquidity trading and so the vector of price-signals s_p collapse to an informationally-equivalent one-dimensional signal.

If Σ_z is of the form $\mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., liquidity trade is perfectly positively correlated, with identical variance in both markets, as in the baseline model), then the price statistics themselves

[25]The cases can be handled in a unified way by re-representing the above expressions for the conditional moments in forms involving pseudo-inverses of Σ_p. However, to avoid tedious technical complications, we choose to treat the case of singular Σ_z separately. Details of the unified treatment are available on request.

are necessarily identical across both markets (i.e., $s_{p1} = s_{p2}$). Abusing notation to let $s_p \in \mathbb{R}$ denote this common price statistic and $\zeta = z_D - \mu_{zD} = z_E - \mu_{zE} \in \mathbb{R}$ denote the common liquidity trade shock realization, the expressions for the conditional moments become

$$\mu_i = \mathbb{E}\left[\mathcal{V}|s_i, s_p\right] = m + \sigma_s^2 \left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} s_p\right), \text{ and} \tag{49}$$

$$\sigma_s^2 = \mathbb{V}\left(\mathcal{V}|s_i, s_p\right) = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1} \tag{50}$$

where $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{1}{\rho^2} b^2 \sigma_z^2$ and it is understood that we take $\frac{1}{\sigma_p^2} = 0$ and $\frac{1}{\rho\sigma_p^2} = 0$ in the above expressions when $\rho = 0$.

Substituting this explicit expression for μ_i in the argument of g' in eq. (43) (recalling that $\zeta \in \mathbb{R}$ denotes the common liquidity trade realization in this case) and grouping terms yields

$$\begin{aligned}
&\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}\mathbf{1}\zeta - \frac{1}{\tau}\left(\mathbf{1}\kappa - \mu_z\right) \\
&= \mathbf{1}\frac{1}{\sigma_s^2}\left(m + \sigma_s^2 \frac{1}{\sigma_\varepsilon^2}\overline{s} + \sigma_s^2 \frac{1}{\rho\sigma_p^2} s_p\right) + \mathbf{1}\frac{1}{\tau}\zeta - \frac{1}{\tau}\left(\mathbf{1}\kappa - \mu_z\right) \\
&= \frac{1}{\sigma_s^2}\left(\mathbf{1}m + \mathbf{1}\sigma_s^2\left(\frac{1}{\rho\sigma_p^2} s_p + \frac{1}{\sigma_\varepsilon^2}\left(\overline{s} + \frac{\sigma_\varepsilon^2}{\tau}\zeta\right)\right) - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right).
\end{aligned}$$

Matching coefficients on the initial conjecture $s_p = \overline{s} + b(z - \mu_z)$, with $B = bI$ as derived above, requires $b = \frac{\sigma_\varepsilon^2}{\tau}$. The previous expression now simplifies to

$$\frac{1}{\sigma_s^2}\left(\mathbf{1}m + \mathbf{1}\sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right) s_p - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right)$$

which, upon plugging back into g', matches the expression in the Proposition.

If Σ_z is singular but not of the form $\mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., the liquidity trade is perfectly positively correlated but has different variances in the two markets, or is perfectly negatively correlated, or is constant in at least one of the markets), then price statistics $s_p = (s_{p1}, s_{p2})$ can be combined to solve for $\overline{s}$. That is, there exists a vector $a \in \mathbb{R}^2$ such that $\overline{s} = a' s_p$.[26] Hence, the conditional moments for trader i are

$$\mu_i = \mathbb{E}\left[\mathcal{V}|s_i, s_p\right] = m + \sigma_s^2 \left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} a' s_p\right), \text{ and}$$

[26] It can be shown that $a = \left(\frac{\mathbb{V}(z_E)}{\mathbb{V}(z_E) - \mathbb{C}(z_D, z_E)} \;\; \frac{\mathbb{V}(z_D)}{\mathbb{V}(z_D) - \mathbb{C}(z_D, z_E)}\right)$ when the correlation is ± 1, which is finite given the form of Σ_z under consideration in this case. If $\mathbb{V}(z_D) = 0$ or $\mathbb{V}(z_E) = 0$, one can take $a = (1, 0)$ or $a = (0, 1)$, respectively.

$$\sigma_s^2 = \mathbb{V}\left(\mathcal{V}|s_i, s_p\right) = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1},$$

where $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2$ is strictly positive since $\rho < 1$ and it is understood that we take $\frac{1}{\sigma_p^2} = 0$ and $\frac{1}{\rho\sigma_p^2} = 0$ in the above expressions when $\rho = 0$.

Substituting this explicit expression for μ_i in the argument of g' in eq. (43) and grouping terms yields

$$\begin{aligned}
&\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}\left(z - \mu_z\right) - \frac{1}{\tau}\left(\mathbf{1}\kappa - \mu_z\right) \\
&= \mathbf{1}\frac{1}{\sigma_s^2}\left(m + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}a's_p\right)\right) + \frac{1}{\tau}\left(z - \mu_z\right) - \frac{1}{\tau}\left(\mathbf{1}\kappa - \mu_z\right) \\
&= \frac{1}{\sigma_s^2}\left(\mathbf{1}m + \sigma_s^2\left(\mathbf{1}\frac{1}{\rho\sigma_p^2}a's_p + \frac{1}{\sigma_\varepsilon^2}\left(\mathbf{1}\overline{s} + \frac{\sigma_\varepsilon^2}{\tau}(z - \mu_z)\right)\right) - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right).
\end{aligned}$$

Matching coefficients on the initial conjecture $s_p = \overline{s} + B(z - \mu_z)$, with $B = bI$ as derived above, requires $b = \frac{\sigma_\varepsilon^2}{\tau}$. The previous expression now simplifies to

$$\frac{1}{\sigma_s^2}\left(\mathbf{1}m + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\mathbf{1}a'\right)s_p - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right) \tag{51}$$

which, upon plugging back into g', matches the expression in the Proposition. Because there is a unique matrix B satisfying the initial conjecture, we again have that this price is unique within the generalized linear class.

Finally, if $\rho = 1$ in the previous case, then in equilibrium traders can directly infer $\theta = \overline{s} = a's_p$ from the vector of asset prices. Because payoffs are riskless given observation of θ, the equilibrium prices must then be $P_D = \min\{m + \theta, K\}$ and $P_E = \max\{m + \theta - K, 0\}$ to preclude arbitrage. This set of prices is not of the posited generalized linear form, but it is now easily confirmed that such fully-revealing prices constitute an equilibrium. □

C Equilibrium with multiple firms

In this section we characterize the equilibrium in a multi-asset version of the model in which there are N firms, each exposed to a systematic risk factor, and each with a potentially different amount of debt outstanding. Specifically, there are $N \geq 1$ firms, indexed by n. Firm n's total cash flow per unit/share is

$$\mathcal{V}_n = m_n + \theta_n + \beta_n F$$

where m_n and β_n are constants. The idiosyncratic shocks $\theta_n \sim N(0, \sigma_\theta^2)$ are independent across firms, and $F \sim N(0, \sigma_F^2)$ is a systematic factor.

There is a factor asset with payoff $\mathcal{V}_F = m_F + F$ that is directly tradeable and is in supply $\kappa \geq 0$. Each firm has both debt and equity outstanding, all in zero net supply, so that firms are "small" relative to the overall economy. We allow for arbitrary differences in leverage across firms, with K_n denoting the face value of debt for firm n. To condense notation, let $\psi_n(\mathcal{V}_n) = \min\{\mathcal{V}_n, K_n\}$ denote the payoff function for the debt of firm n. As in the baseline model, we $V_{nD} = \psi_n(\mathcal{V}_n)$ and $V_{nE} = \mathcal{V}_n - \psi_n(\mathcal{V}_n)$ denote the debt and equity payoffs as random variables, and for completeness we note that the factor asset payoff is always simply equal to its underlying cash flow $V_F = \mathcal{V}_F$. We let P_{nD} and P_{nE} denote the endogenous prices of each firm n's debt and equity and let P_F denote the endogenous price of the factor asset.

Investors receive private signals about each firm's cash flow

$$s_{in} = \theta_n + \varepsilon_{in}$$

where $\varepsilon_{in} \sim N(0, \sigma_\varepsilon^2)$ are mutually independent. As in the main model, we continue to allow investors to agree to disagree about the information content of one another's signals. Specifically, each investor i believes that other investors' signals are of the form

$$s_{jn} = \rho\theta_n + \sqrt{1-\rho^2}\xi_{in} + \varepsilon_{jn}$$

where $\xi_{in} \sim N(0, \sigma_\theta^2)$ are mutually independent.

Finally, we assume that there are exogenous liquidity traders who trade to gain exposures to the idiosyncratic portion of each firm's cash flow θ_n. In particular, for a given firm n, liquidity traders submit demands $z_n \sim N(0, \sigma_z^2)$ in the debt and equity of firm n and simultaneously submit demand $-\beta_n z_n$ in the factor asset. To ensure that liquidity demand is truly idiosyncratic, we continue to assume that the z_n are independent across firms n. We let $z_F = -\sum_{n=1}^N \beta_n z_n$ concisely denote the total liquidity demand in the factor asset.

In the derivation, it will be convenient to use vector/matrix notation. We let objects without subscripts denote vectors of firm-level objects (i.e., vectors of all firms/assets excluding the factor asset). For instance, $\mathcal{V} = (\mathcal{V}_1, \ldots, \mathcal{V}_N)$ is the vector of firm cash flows, with $m = (m_1, \ldots, m_N)'$ the vector of expected cash flows, $\beta = (\beta_1, \ldots, \beta_N)'$ the vector of cash-flow betas, etc. We let objects with $\vec{\cdot}$ denote vectors augmented with the factor asset in the last slot. For instance, $\vec{\mathcal{V}} = (\mathcal{V}_1, \ldots, \mathcal{V}_N, \mathcal{V}_F)$ is the overall vector of cash flows, with $\vec{m} = (m_1, \ldots, m_N, m_F)'$ the vector of expected cash flows, and $\vec{\beta} = (\beta_1, \ldots, \beta_N, 1)'$ the vector of cash flow betas. In the expressions below, we let I_k denote a $k \times k$ identity matrix, $\mathbf{1}_k$ a k-vector of ones, $\mathbf{0}_k$ a k-vector of zeros, and $\mathbf{0}_{j\times k}$ a $j \times k$ matrix of zeros. To eliminate notational clutter we will sometimes drop the subscripts that explicitly label the dimensions of these objects — it should be understood that they are conformable with the expressions in which they appear (e.g., it will be clear from the context what is the appropriate dimension of a given identity matrix I, whether $\mathbf{0}$ represents a vector or a matrix, etc.).

C.1 Beliefs

As in the main model in the text, we consider equilibria of the generalized linear form, in which there is an N-dimensional vector of price statistics that can be represented in form

$$s_p = \overline{s} + Bz$$

where $\overline{s} \equiv (\overline{s}_1, \ldots, \overline{s}_N) \equiv \left(\int s_{j1} dj, \ldots, \int s_{jN} dj\right)$ is the vector of aggregate signals, and where B is a nonsingular $N \times N$ matrix to be determined, with generic entry b_{mn} in row m and column n.

We begin by characterizing trader beliefs in an arbitrary generalized linear equilibrium of the posited form. Under this conjecture, the beliefs in both levered and unlevered versions of the economy remain conditionally normal though, in principle, they may be associated with different conditional moments since the matrix B could differ across these economies. Using standard updating formulas, the conditional distribution of $\vec{\mathcal{V}}$ for any trader i is conditionally normal with conditional mean vector and variance matrix given by

$$\vec{\mu}_i \equiv \mathbb{E}_i\left[\vec{\mathcal{V}}\right] = \left(\begin{smallmatrix} \mu_i \\ m_f \end{smallmatrix}\right)$$
$$\mu_i \equiv \mathbb{E}_i\left[\mathcal{V}\right] = m + \mathbb{V}_i\left(\theta\right)\left(\frac{1}{\sigma_\varepsilon^2} s_i + \Sigma_p^{-1}\frac{1}{\rho} s_p\right)$$
$$\Gamma \equiv \mathbb{V}_i\left(\vec{\mathcal{V}}\middle| s_i, s_p\right) = \left(\begin{smallmatrix} \mathbb{V}_i(\theta)+\sigma_F^2\beta\beta' & \sigma_F^2\beta \\ \sigma_F^2\beta' & \sigma_F^2 \end{smallmatrix}\right)$$
$$\mathbb{V}_i\left(\theta\right) = \left(\left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2}\right) I_N + \Sigma_p^{-1}\right)^{-1}$$

where $\Sigma_p \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 I + \frac{1}{\rho^2}\sigma_z^2 BB'$. To condense notation, it is understood that we set Σ_p^{-1} and $\Sigma_p^{-1}\frac{1}{\rho}$ to $\mathbf{0}$ in the above expressions when $\rho = 0$, since traders optimally place zero weight on prices when updating in this case. Note further that for $0 < \rho \leq 1$, the inverse Σ_p^{-1} is always well-defined. Because $\frac{1-\rho^2}{\rho^2}\sigma_\theta^2 I$ is positive semidefinite (positive definite if $\rho < 1$) and because B is conjectured to be nonsingular and therefore BB' is positive definite, it follows that Σ_p is the sum of a positive semi-definite and positive definite matrix. Hence, Σ_p is itself positive definite and therefore invertible.

C.2 Unlevered economy

We begin by characterizing the equilibrium in the case that all firms are unlevered. As in the baseline model, the equilibrium price vector in the unlevered economy plays a key role in the representation of the equilibrium prices in the levered case. Let P_{nU} denote the unlevered equity price for firm n, with $P_U = (P_{1U}, \ldots, P_{NU})$ the vector of such prices, and $\vec{P}_U = (P_{1U}, \ldots, P_{NU}, P_F)$ this vector augmented with the factor asset price.

Lemma 7. *Suppose that under each investor's information set, $\vec{\mathcal{V}}$ is conditionally normally distributed with mean $\vec{\mu}_i$ and variance matrix Γ. Then, there is a* ***linear*** *equilibrium in which the vector of equilibrium prices has the representation*

$$\vec{P}_U = \int \vec{\mu}_i di - \frac{1}{\tau}\Gamma\left(\left(\begin{smallmatrix}\mathbf{0}\\ \kappa\end{smallmatrix}\right) - \vec{z}\right).$$

This equilibrium price vector can be written in terms of the underlying random variables and parameters as

$$\vec{P}_U = \begin{pmatrix} m + \mathbb{V}_i(\theta)\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\frac{1}{\rho}\right)s_p - \beta\frac{1}{\tau}\sigma_F^2\kappa \\ m_F - \frac{1}{\tau}\sigma_F^2\kappa \end{pmatrix}$$

where the equilibrium price signal coefficient matrix is diagonal, $B = \frac{\sigma_\varepsilon^2}{\tau}I$, and $\sigma_p^2 = \left(\frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{1}{\rho^2}\sigma_z^2\left(\frac{\sigma_\varepsilon^2}{\tau}\right)^2\right)$. Consequently, (i) the conditional variance matrix of idiosyncratic cash flow shocks is diagonal:

$$\mathbb{V}_i(\theta) = \sigma_s^2 I \tag{52}$$

with $\sigma_s^2 \equiv \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}$, and (ii) the overall conditional variance matrix of cash flows is

$$\Gamma = \begin{pmatrix} \sigma_s^2 I + \sigma_F^2\beta\beta' & \sigma_F^2\beta \\ \sigma_F^2\beta' & \sigma_F^2 \end{pmatrix}. \tag{53}$$

Proof. We begin by solving the partial equilibrium demand problem for an arbitrary trader. Let x_{in} denote investor i's demand for the unlevered equity of firm n, with $x_i = (x_{i1}, \ldots, x_{iN})'$ and $\vec{x}_i = (x_{i1}, \ldots, x_{iN}, x_{iF})'$. Given the conditional normality of $\vec{\mathcal{V}}$, investor i's expected utility at the trading stage is

$$\mathbb{E}_i\left[-\exp\left\{-\frac{1}{\tau}\left(\vec{x}_i'\left(\vec{\mathcal{V}} - \vec{P}_U\right) + \kappa P_F\right)\right\}\middle| s_i, s_p\right] = -\exp\left\{-\frac{1}{\tau}\left(\vec{x}_i'\left(\vec{\mu}_i - \vec{P}_U\right) + \kappa P_F\right) + \frac{1}{2}\frac{1}{\tau^2}\vec{x}_i'\Gamma\vec{x}_i\right\}.$$

Maximizing over $\vec{x}_i$ yields demand function

$$\vec{x}_i = \tau\Gamma^{-1}\left(\vec{\mu}_i - \vec{P}_U\right).$$

Aggregating across traders and enforcing the market-clearing condition yields

$$\begin{aligned}\left(\begin{smallmatrix}\mathbf{0}\\ \kappa\end{smallmatrix}\right) &= \int \tau\Gamma^{-1}\left(\vec{\mu}_i - \vec{P}_U\right) di + \vec{z}\\ \Rightarrow \vec{P}_U &= \int \vec{\mu}_i di - \frac{1}{\tau}\Gamma\left(\left(\begin{smallmatrix}\mathbf{0}\\ \kappa\end{smallmatrix}\right) - \vec{z}\right),\end{aligned} \tag{54}$$

which matches the first representation in the Lemma.

To further characterize the equilibrium B and write the prices in the second form in the lemma, note that, using the belief expressions from Section C.1, we have

$$\int \vec{\mu}_i di = \left(\begin{smallmatrix} m+\mathbb{V}_i(\theta)\left(\frac{1}{\sigma_\varepsilon^2}\bar{s}+\Sigma_p^{-1}\frac{1}{\rho}s_p\right)\\ m_F\end{smallmatrix}\right)$$

and

$$\begin{aligned}&\Gamma\left(\left(\begin{smallmatrix}\mathbf{0}\\ \kappa\end{smallmatrix}\right) - \vec{z}\right)\\ &= \left(\begin{smallmatrix}\mathbb{V}_i(\theta)+\sigma_F^2\beta\beta' & \sigma_F^2\beta\\ \sigma_F^2\beta' & \sigma_F^2\end{smallmatrix}\right)\left(\left(\begin{smallmatrix}\mathbf{0}\\ \kappa\end{smallmatrix}\right) - \left(\begin{smallmatrix}z\\ -\beta' z\end{smallmatrix}\right)\right)\\ &= \left(\begin{smallmatrix}\beta\sigma_F^2\kappa\\ \sigma_F^2\kappa\end{smallmatrix}\right) - \left(\begin{smallmatrix}\mathbb{V}_i(\theta)z\\ 0\end{smallmatrix}\right).\end{aligned}$$

Substituting back in to eq. (54) and collecting terms yields

$$\begin{aligned}\Rightarrow \vec{P}_U &= \left(\begin{smallmatrix} m+\mathbb{V}_i(\theta)\left(\frac{1}{\sigma_\varepsilon^2}\bar{s}+\Sigma_p^{-1}\frac{1}{\rho}s_p\right)\\ m_F\end{smallmatrix}\right) - \left(\begin{smallmatrix}\frac{1}{\tau}\beta\sigma_F^2\kappa\\ \frac{1}{\tau}\sigma_F^2\kappa\end{smallmatrix}\right) + \left(\begin{smallmatrix}\frac{1}{\tau}\mathbb{V}_i(\theta)z\\ 0\end{smallmatrix}\right)\\ &= \left(\begin{smallmatrix} m+\mathbb{V}_i(\theta)\left(\frac{1}{\sigma_\varepsilon^2}\bar{s}+\Sigma_p^{-1}\frac{1}{\rho}s_p+\frac{1}{\tau}z\right)-\beta\frac{1}{\tau}\sigma_F^2\kappa\\ m_F-\frac{1}{\tau}\sigma_F^2\kappa\end{smallmatrix}\right)\\ &= \left(\begin{smallmatrix} m+\mathbb{V}_i(\theta)\left(\frac{1}{\sigma_\varepsilon^2}\left(\bar{s}+\frac{\sigma_\varepsilon^2}{\tau}z\right)+\Sigma_p^{-1}\frac{1}{\rho}s_p\right)-\beta\frac{1}{\tau}\sigma_F^2\kappa\\ m_F-\frac{1}{\tau}\sigma_F^2\kappa\end{smallmatrix}\right).\end{aligned}$$

Grouping terms, to satisfy the initial conjecture about s_p, we must have that the equilibrium coefficient matrix B satisfies

$$B = \frac{\sigma_\varepsilon^2}{\tau}I,$$

which is diagonal. Hence,

$$\vec{P}_U = \begin{pmatrix} m+\mathbb{V}_i(\theta)\left(\frac{1}{\sigma_\varepsilon^2}+\Sigma_p^{-1}\frac{1}{\rho}\right)s_p-\beta\frac{1}{\tau}\sigma_F^2\kappa \\ m_F-\frac{1}{\tau}\sigma_F^2\kappa \end{pmatrix}$$

with $\mathbb{V}_i(\theta) = \left(\left(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\varepsilon^2}\right)I+\Sigma_p^{-1}\right)^{-1}$, and $\Sigma_p = \sigma_p^2 I$ where $\sigma_p^2 = \frac{1-\rho^2}{\rho^2}\sigma_\theta^2+\frac{1}{\rho^2}\sigma_z^2\left(\frac{\sigma_\varepsilon^2}{\tau}\right)^2$. Plugging the expression for Σ_p back into the expression for $\vec{P}_U$, plugging the expression for $\mathbb{V}_i(\theta)$ back into the expression for Γ, and collecting terms yields the expressions in the Proposition. □

C.3 Levered economy

Now, consider the setting in which firms have leverage. Our goal in this section is to establish the following result.

Proposition 7. *There exists an equilibrium in the financial market. The vector of equilibrium asset prices has the representation*

$$P = g'\left(\begin{pmatrix} \mathbf{1}_2 e_1' \\ \vdots \\ \mathbf{1}_2 e_N' \\ e_F' \end{pmatrix}\Gamma^{-1}\left(\int \vec{\mu}_i di + \frac{1}{\tau}\Gamma\vec{z} - \frac{1}{\tau}\Gamma\begin{pmatrix} \mathbf{0}_N \\ \kappa \end{pmatrix}\right)\right) \tag{55}$$

where $e_n \in \mathbb{R}^{N+1}$ and $e_F \in \mathbb{R}^{N+1}$ are vectors with ones in the n^{th} and $(N+1)^{st}$ elements, respectively, and zeros in all other elements, and the function $g' : \mathbb{R}^{2N+1} \to \mathbb{R}^{2N+1}$ is the gradient of a function given in closed-form in eq. (61) in the proof.

The equilibrium price vector can be written in terms of the underlying random variables and parameters as

$$\vec{P} = g'\left(\begin{pmatrix} \mathbf{1}_2 e_1' \\ \vdots \\ \mathbf{1}_2 e_N' \\ e_F' \end{pmatrix}\Gamma^{-1}\vec{P}_U\right) \tag{56}$$

where $\vec{P}_U$ is the vector of unlevered prices from Lemma 7 in which the equilibrium price signal coefficient matrix is diagonal $B = \frac{\sigma_\varepsilon^2}{\tau}I$.

Furthermore, this vector of prices can be understood as a vector of risk-neutral expected payoffs under the joint cash flow distribution

$$\vec{\mathcal{V}} \sim^{\mathbb{Q}} N\left(\vec{P}_U, \Gamma\right) \tag{57}$$

Under the generalized linear equilibrium conjecture, the $N+1$ dimensional vector of cash flows $\vec{\mathcal{V}}$ is jointly normally distributed with mean vector $\vec{\mu}_i$ and variance matrix Γ characterized in Section C.1 above. We can use this fact to derive the conditional distribution of the overall $2N+1$ dimensional vector of security payoffs $\vec{V} = ((V_{1D}, V_{1E}), \dots (V_{ND}, V_{NE}), V_F)$, which is the key step in the equilibrium derivation.

Let $\vec{u} = ((u_{1D}, u_{1E}), \dots, (u_{ND}, u_{NE}), u_F)' \in \mathbb{R}^{2N+1}$ be an arbitrary vector and let $u_{E,F} = (u_{1E}, \dots, u_{NE}, u_F)' \in \mathbb{R}^{N+1}$ be the subvector of elements associated with the equity security "slots" in $\vec{u}$, including the factor security. Explicitly integrating against the joint density of the cash flows $\vec{\mathcal{V}} \in \mathbb{R}^{N+1}$, the conditional MGF of $\vec{V} \in \mathbb{R}^{2N+1}$ can be written

$$
\begin{aligned}
&\mathbb{E}_i\left[\exp\left\{\vec{u}'\vec{V}\right\}\right] \\
&= \int \exp\left\{\sum_{n=1}^{N} u_{nD}\psi_n(v_n) + \sum_{n=1}^{N} u_{nE}(v_n - \psi_n(v_n)) + u_F v_F\right\} \\
&\qquad \times \frac{1}{(2\pi)^{-(N+1)/2}|\Gamma|^{1/2}} \exp\left\{-\frac{1}{2}(\vec{v}-\vec{\mu}_i)'\Gamma^{-1}(\vec{v}-\vec{\mu}_i)\right\} d\vec{v} &(58)\\
&= \int \exp\left\{\left(\sum_{n=1}^{N}(u_{nD}-u_{nE})\psi_n(v_n)\right) + u'_{E,F}\vec{v}\right\} \\
&\qquad \times \frac{1}{(2\pi)^{-(N+1)/2}|\Gamma|^{1/2}} \exp\left\{-\frac{1}{2}(\vec{v}-\vec{\mu}_i)'\Gamma^{-1}(\vec{v}-\vec{\mu}_i)\right\} d\vec{v} &(59)\\
&= \exp\left\{\frac{1}{2}\left(\Gamma^{-1}\vec{\mu}_i + u_{E,F}\right)'\Gamma\left(\Gamma^{-1}\vec{\mu}_i + u_{E,F}\right) - \frac{1}{2}\left(\Gamma^{-1}\vec{\mu}_i\right)'\Gamma\left(\Gamma^{-1}\vec{\mu}_i\right)\right\} \\
&\quad \times \int \frac{1}{(2\pi)^{-(N+1)/2}|\Gamma|^{1/2}} \exp\left\{\left(\sum_{n=1}^{N}(u_{nD}-u_{nE})\psi_n(v_n)\right) - \frac{1}{2}\left(\vec{v}-\Gamma\left(\Gamma^{-1}\vec{\mu}_i+u_{E,F}\right)\right)'\Gamma^{-1}\left(\vec{v}-\Gamma\left(\Gamma^{-1}\vec{\mu}_i+u_{E,F}\right)\right)\right\} d\vec{v} \\
& &(60)
\end{aligned}
$$

where the first equality writes the expectation explicitly as an integral and writes the debt and equity payoffs as functions of the underlying cash flows, the second equality groups terms in the exponential, and the final equality completes the square in the exponential.

Let $\vec{y} = ((y_{1D}, y_{1E}), \dots, (y_{ND}, y_{NE}), y_F)' \in \mathbb{R}^{2N+1}$ be an arbitrary vector and let $y_{E,F} = (y_{1E}, \dots, y_{NE}, y_F) \in \mathbb{R}^{N+1}$ denote the subvector associated with the equity security "slots". Define the function $g : \mathbb{R}^{2N+1} \to \mathbb{R}$ by

$$
\begin{aligned}
g(\vec{y}) = \log\Bigg(&\exp\left\{\frac{1}{2}y'_{E,F}\Gamma y_{E,F}\right\} \\
&\times \int \frac{1}{(2\pi)^{-(N+1)/2}|\Gamma|^{1/2}} \exp\left\{\sum_{n=1}^{N}(y_{nD}-y_{nE})\psi_n(v_n) - \frac{1}{2}\left(\vec{v}-\Gamma y_{E,F}\right)'\Gamma^{-1}\left(\vec{v}-\Gamma y_{E,F}\right)\right\} d\vec{v}\Bigg) \qquad (61)
\end{aligned}
$$

and note that when $\vec{y} = ((y_1, y_1), \dots, (y_N, y_N), y_F)'$ has the same argument within each pair of firm-level slots, we have

$$
\begin{aligned}
g\left(\left(y_1, y_1\right), \ldots, \left(y_N, y_N\right), y_F\right) &= \log\left(\exp\left\{\frac{1}{2} y_{E,F}' \Gamma y_{E,F}\right\}\right. \\
&\qquad \left. \int \tfrac{1}{(2\pi)^{-(N+1)/2}|\Gamma|^{1/2}} \exp\left\{-\tfrac{1}{2}\left(\vec{v}-\Gamma y_{E,F}\right)' \Gamma^{-1}\left(\vec{v}-\Gamma y_{E,F}\right)\right\} d\vec{v}\right) \\
&= \frac{1}{2} y_{E,F}' \Gamma y_{E,F}
\end{aligned}
$$

where the second equality uses the fact that the integral in the first equality is simply the integral of an $N+1$ dimensional normal density with mean vector $\Gamma y_{E,F}$ and variance matrix Γ over all of $\mathbb{R}^{N+1}$ and hence has a value of one.

Let $e_n \in \mathbb{R}^{N+1}$ and $e_F \in \mathbb{R}^{N+1}$ be the vectors with ones in the n^{th} and $(N+1)^{\text{st}}$ elements, respectively, and zeros in all other elements. With the above definition of g, we can therefore write the MGF in eq. (60) as

$$
\mathbb{E}_i\left[\exp\left\{\vec{u}'\vec{V}\right\}\right] = \exp\left\{ g\begin{pmatrix} \binom{u_{1D}}{u_{1E}} + \mathbf{1}_2 e_1'\Gamma^{-1}\vec{\mu}_i \\ \vdots \\ \binom{u_{ND}}{u_{NE}} + \mathbf{1}_2 e_N'\Gamma^{-1}\vec{\mu}_i \\ u_F + e_F'\Gamma^{-1}\vec{\mu}_i \end{pmatrix} - g\begin{pmatrix} \mathbf{1}_2 e_1'\Gamma^{-1}\vec{\mu}_i \\ \vdots \\ \mathbf{1}_2 e_N'\Gamma^{-1}\vec{\mu}_i \\ e_F'\Gamma^{-1}\vec{\mu}_i \end{pmatrix} \right\}.
$$

We record for completeness that, under this representation, g is precisely the cumulant generating function (CGF) of $\vec{V}$.

Now, using this expression for the conditional MGF, we can write the problem for an arbitrary investor i as

$$
\begin{aligned}
&\max_{\vec{x}_i} \mathbb{E}_i\left[-\exp\left\{-\frac{1}{\tau}\left(\vec{x}_i'\left(\vec{V}-\vec{P}\right) + \kappa P_F\right)\right\}\right] \\
&= \max_{\vec{x}_i} \; -\exp\left\{ g\left(\begin{pmatrix} \mathbf{1}_2 e_1'\Gamma^{-1}\vec{\mu}_i \\ \vdots \\ \mathbf{1}_2 e_N'\Gamma^{-1}\vec{\mu}_i \\ e_F'\Gamma^{-1}\vec{\mu}_i \end{pmatrix} - \frac{1}{\tau}\vec{x}_i\right) - g\begin{pmatrix} \mathbf{1}_2 e_1'\Gamma^{-1}\vec{\mu}_i \\ \vdots \\ \mathbf{1}_2 e_N'\Gamma^{-1}\vec{\mu}_i \\ e_F'\Gamma^{-1}\vec{\mu}_i \end{pmatrix} + \frac{1}{\tau}\vec{x}_i'\vec{P} - \frac{1}{\tau}\kappa P_F \right\}.
\end{aligned}
$$

Given that the CGF $g(\cdot)$ is strictly convex, twice continuously differentiable, and finite on all of $\mathbb{R}^{2N+1}$, this objective function is twice continuously differentiable, strictly concave, and defined on all of $\mathbb{R}^{2N+1}$. Hence the first-order condition is necessary and sufficient for an optimum, and the optimum is unique whenever a solution to the FOC exists. Rearranging the FOC yields the optimal demand

$$
\vec{x}_i = \tau\left(\begin{pmatrix} \mathbf{1} e_1'\Gamma^{-1}\vec{\mu}_i \\ \vdots \\ \mathbf{1} e_N'\Gamma^{-1}\vec{\mu}_i \\ e_F'\Gamma^{-1}\vec{\mu}_i \end{pmatrix} - \left(g'\right)^{-1}\left(\vec{P}\right)\right)
$$

where we have used the fact that g is strictly convex to conclude that the gradient g' is injective and therefore invertible on its range.

We claim that the range of g' is the set of no-arbitrage prices. Let

$$S = \left\{ (v_{1D}, v_{1E}, \ldots, v_{ND}, v_{NE}, v_F) \in \mathbb{R}^{2N+1} : \begin{array}{c} (v_{nD} < K_n, v_{nE} = 0) \text{ or } (v_{nD} = K_n, v_{nE} > 0) \quad \forall n \\ v_F \in \mathbb{R} \end{array} \right\}$$

denote the support of the overall payoff vector $\vec{V}$. As in the baseline setting, the range of g' is the closed convex hull of S. This again follows from results about exponential family distributions in Barndorff-Nielsen (2014). Because the function g is defined on all of $\mathbb{R}^{2N+1}$ and furthermore because this set is open, the exponential family described by g is "regular". Hence, it follows from Theorem 8.2 in Barndorff-Nielsen (2014) that the exponential family is "steep" and therefore from Theorem 9.2 in Barndorff-Nielsen (2014) that the gradient g' maps $\mathbb{R}^{2N+1}$ onto the interior of the closed convex hull of S, $\operatorname{int}\operatorname{conv}(S)$. This set coincides with the set of candidate prices in which each firm's debt price is strictly less than its face value K_n, each firm's equity price is strictly greater than zero, and the price of the factor asset can take any value, which is precisely the set of prices that do not admit arbitrage.

We can now enforce the market clearing condition, where we use the fact that liquidity trade is identical within each firm's securities, $(z_{nD}, z_{nE})' = \mathbf{1}_2 e_n' \vec{z}$:

$$\begin{aligned} \begin{pmatrix} \mathbf{0}_2 \\ \vdots \\ \mathbf{0}_2 \\ \kappa \end{pmatrix} &= \int \vec{x}_i di + \begin{pmatrix} \mathbf{1}_2 e_1' \vec{z} \\ \vdots \\ \mathbf{1}_2 e_1' \vec{z} \\ e_F' \vec{z} \end{pmatrix} \\ \Rightarrow \vec{P} &= g' \left(\begin{pmatrix} \mathbf{1}_2 e_1' \Gamma^{-1} \int \vec{\mu}_i di \\ \vdots \\ \mathbf{1}_2 e_N' \Gamma^{-1} \int \vec{\mu}_i di \\ e_F' \Gamma^{-1} \int \vec{\mu}_i di \end{pmatrix} - \frac{1}{\tau} \left(\begin{pmatrix} \mathbf{0}_2 \\ \vdots \\ \mathbf{0}_2 \\ \kappa \end{pmatrix} - \begin{pmatrix} \mathbf{1}_2 e_1' \\ \vdots \\ \mathbf{1}_2 e_N' \\ e_F' \end{pmatrix} \vec{z} \right) \right) \end{aligned} \tag{62}$$

Because $\begin{pmatrix} \mathbf{0}_2 \\ \vdots \\ \mathbf{0}_2 \\ \kappa \end{pmatrix} = \begin{pmatrix} \mathbf{1}_2 e_1' \left(\begin{smallmatrix} \mathbf{0}_N \\ \kappa \end{smallmatrix} \right) \\ \vdots \\ \mathbf{1}_2 e_N' \left(\begin{smallmatrix} \mathbf{0}_N \\ \kappa \end{smallmatrix} \right) \\ e_F' \left(\begin{smallmatrix} \mathbf{0}_N \\ \kappa \end{smallmatrix} \right) \end{pmatrix} = \begin{pmatrix} \mathbf{1}_2 e_1' \\ \vdots \\ \mathbf{1}_2 e_N' \\ e_F' \end{pmatrix} \left(\begin{smallmatrix} \mathbf{0}_N \\ \kappa \end{smallmatrix} \right)$ we can further write the argument of g' above as

$$\begin{aligned} & \begin{pmatrix} \mathbf{1}_2 e_1' \Gamma^{-1} \int \vec{\mu}_i di \\ \vdots \\ \mathbf{1}_2 e_N' \Gamma^{-1} \int \vec{\mu}_i di \\ e_F' \Gamma^{-1} \int \vec{\mu}_i di \end{pmatrix} - \frac{1}{\tau} \left(\begin{pmatrix} \mathbf{0}_2 \\ \vdots \\ \mathbf{0}_2 \\ \kappa \end{pmatrix} - \begin{pmatrix} \mathbf{1}_2 e_1' \\ \vdots \\ \mathbf{1}_2 e_N' \\ e_F' \end{pmatrix} \vec{z} \right) \\ &= \begin{pmatrix} \mathbf{1}_2 e_1' \\ \vdots \\ \mathbf{1}_2 e_N' \\ e_F' \end{pmatrix} \left(\Gamma^{-1} \int \vec{\mu}_i di + \frac{1}{\tau} \vec{z} - \frac{1}{\tau} \left(\begin{smallmatrix} \mathbf{0}_N \\ \kappa \end{smallmatrix} \right) \right) \end{aligned}$$

$$= \begin{pmatrix} \mathbf{1}_2 e_1' \\ \vdots \\ \mathbf{1}_2 e_N' \\ e_F' \end{pmatrix} \Gamma^{-1} \left(\int \vec{\mu}_i di + \frac{1}{\tau} \Gamma \vec{z} - \frac{1}{\tau} \Gamma \left(\begin{smallmatrix} \mathbf{0}_N \\ \kappa \end{smallmatrix} \right) \right).$$

As in the unlevered case, we obtain that the equilibrium price signal coefficient matrix is $B = \frac{\sigma_\varepsilon^2}{\tau} I$ by plugging in for $\vec{\mu}_i$ from the expression in Section C.1 above, grouping terms, and enforcing the initial linear conjecture on s_p.

With this value of B pinned down, we recognize that $\int \vec{\mu}_i di + \frac{1}{\tau}\Gamma\vec{z} - \frac{1}{\tau}\Gamma \left(\begin{smallmatrix} \mathbf{0}_N \\ \kappa \end{smallmatrix} \right)$ is precisely the vector of prices from the unlevered economy, $\vec{P}_U$. Therefore, the equilibrium price vector in eq. (62) can be written

$$\vec{P} = g' \left(\begin{pmatrix} \mathbf{1}_2 e_1' \\ \vdots \\ \mathbf{1}_2 e_N' \\ e_F' \end{pmatrix} \Gamma^{-1} \vec{P}_U \right).$$

Differentiating the expression for $g(\cdot)$ defined above yields, upon inspection, that this expression is equivalent to representing each security's price as the expectation of the security's payoff under a risk-neutral joint distribution for cash flows given by $\vec{\mathcal{V}} \sim^{\mathbb{Q}} N\left(\vec{P}_U, \Gamma\right)$.

D Limited Liability

Our baseline model can capture the limited liability feature of levered equity (when $K \geq 0$). However, it does not capture the limited liability feature of debt: debt payoffs in the model are unbounded below. Furthermore, in the case that the firm is unlevered, equity payoffs are unbounded below. In this section, we alter our baseline model by specifying that firm cash flows follow a distribution that is bounded below by zero. This ensures that the payoffs on all securities satisfy limited liability.

In particular, suppose that the firm's cash flows $\mathcal{V}$ follow a truncated $N(m, \sigma_\theta^2)$ distribution, truncated below at zero. The unconditional density of $\mathcal{V}$ is

$$f_{\mathcal{V}}(v) = \frac{\mathbf{1}_{\{v \geq 0\}} \frac{1}{\sqrt{2\pi\sigma_\theta^2}} \exp\left\{ -\frac{1}{2} \frac{(v-m)^2}{\sigma_\theta^2} \right\}}{1 - \Phi\left(\frac{-m}{\sigma_\theta}\right)}. \tag{63}$$

We continue to assume that the firm has both debt and equity outstanding. In order to keep the equilibrium non-trivial, we assume that the face value of debt satisfies $K > 0$.

We continue to assume that investors observe private signals of the form

$$s_i = (\mathcal{V} - m) + \varepsilon_i$$

where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ and perceive others signal as

$$s_j = \rho(\mathcal{V} - m) + \sqrt{1-\rho^2}\xi_i + \varepsilon_j \tag{64}$$

where $\xi_i \sim N(0, \sigma_\xi^2)$ is independent of all other random variables. As in the baseline model, we also continue to assume that liquidity traders submit identical demands $z \sim N(0, \sigma_z^2)$ in both the debt and equity of the firm. Finally, we also consider to search for equilibria in the generalized linear class, in which security prices depend on the underlying random variable through a linear statistic

$$s_p = \overline{s} + bz \tag{65}$$

with b an endogenous constant to be determined.

The next proposition characterizes the equilibrium.

Proposition 8. *Suppose that the firm cash flow $\mathcal{V}$ is unconditionally distributed as a truncated normal, truncated below by zero, with parameters m and σ_θ^2. Then there exists a generalized linear equilibrium, unique within this class. In this equilibrium:*

1. *Each investor's conditional distribution of firm cash flow $\mathcal{V}$ is truncated normal with parameters μ_i and σ_s^2 that are identical to those in the baseline model.*
2. *The equilibrium prices of debt and equity can be represented as the securities' expected payoffs under a risk-neutral cash flow distribution. The risk-neutral cash flow distribution is truncated normal with parameters P_U and σ_s^2 that are identical to those in the baseline model.*[27]

Proof. The proof takes a similar form to the derivation in the baseline model, and so we omit some of the algebraic details to focus on the key differences.

Under the generalized linear conjecture, Bayes' rule implies that the conditional density for an arbitrary investor i satisfies

$$f_{\mathcal{V}|s_i,s_p}(v|s_i,s_p) \propto f_{\mathcal{V}}(v)\, f_{s_i,s_p|\mathcal{V}}(s_i,s_p|v)$$
$$\propto \mathbf{1}_{\{v\geq 0\}} \exp\left\{-\frac{1}{2}\frac{(v-m)^2}{\sigma_\theta^2}\right\} \exp\left\{-\frac{1}{2}\frac{(s_i-(v-m))^2}{\sigma_\varepsilon^2} - \frac{1}{2}\frac{\left(\frac{s_p}{\rho}-(v-m)\right)^2}{\sigma_p^2}\right\}$$

[27] Note that for easy comparability of the expressions for equilibrium price and returns, we abuse notation by continuing to let $P_U = m + \sigma_s^2\left(\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)(\overline{s} + bz) - \frac{\kappa}{\tau}\right)$. The actual equity price in the unlevered version of the truncated normal economy is itself nonlinear and is not equal to P_U.

$$\propto \mathbf{1}_{\{v\geq 0\}} \exp\left\{-\frac{1}{2}\left(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v^2+\left(\frac{m}{\sigma_\theta^2}+\frac{m+s_i}{\sigma_\varepsilon^2}+\frac{m+\frac{s_p}{\rho}}{\sigma_p^2}\right)v\right\}$$
$$\propto \mathbf{1}_{\{v\geq 0\}} \exp\left\{-\frac{1}{2}\frac{(v-\mu_i)^2}{\sigma_s^2}\right\}$$

where $\sigma_p^2 = \frac{1-\rho^2}{\rho^2}\sigma_\xi^2 + \frac{1}{\rho^2}\sigma_z^2 b^2$ and where

$$\mu_i = m + \sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2}s_i + \frac{1}{\sigma_p^2}\frac{1}{\rho}s_p\right) \tag{66}$$
$$\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)^{-1}. \tag{67}$$

Hence, the conditional cash flow distribution remains truncated normal but with parameters μ_i and σ_s^2 in place of m and σ_θ^2. We can use this to derive the conditional MGF of the vector of security payoffs $(V_D, V_E) = (\min\{\mathcal{V}, K\}, \max\{\mathcal{V}-K, 0\})$:

$$\begin{aligned}
&\mathbb{E}_i\left[\exp\{u_D V_D + u_E V_E\}\right]\\
&= \int_0^K \exp\{u_D v\}\frac{\frac{1}{\sqrt{2\pi\sigma_s^2}}\exp\left\{-\frac{1}{2}\frac{(v-\mu_i)^2}{\sigma_s^2}\right\}}{1-\Phi\left(-\frac{\mu_i}{\sigma_s}\right)}dv\\
&\quad + \int_K^\infty \exp\{u_D K + u_E(v-K)\}\frac{\frac{1}{\sqrt{2\pi\sigma_s^2}}\exp\left\{-\frac{1}{2}\frac{(v-\mu_i)^2}{\sigma_s^2}\right\}}{1-\Phi\left(-\frac{\mu_i}{\sigma_s}\right)}dv\\
&= \exp\left\{\frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}+u_D\right)^2 - \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}\right)^2\right\}\frac{\Phi\left(\frac{K-\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}+u_D\right)}{\sigma_s}\right)-\Phi\left(\frac{-\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}+u_D\right)}{\sigma_s}\right)}{1-\Phi\left(-\frac{\mu_i}{\sigma_s}\right)}\\
&\quad + \exp\left\{(u_D-u_E)K + \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}+u_E\right)^2 - \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}\right)^2\right\}\frac{1-\Phi\left(\frac{K-\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}+u_E\right)}{\sigma_s}\right)}{1-\Phi\left(-\frac{\mu_i}{\sigma_s}\right)}.
\end{aligned}$$

Defining the function $g : \mathbb{R}^2 \to \mathbb{R}$ as

$$g\begin{pmatrix} y_D \\ y_E\end{pmatrix} \equiv \log\left(\exp\left\{\frac{1}{2}\sigma_s^2 y_D^2\right\}\left(\Phi\left(\frac{K-\sigma_s^2 y_D}{\sigma_s}\right)-\Phi\left(\frac{-\sigma_s^2 y_D}{\sigma_s}\right)\right)\right. \tag{68}$$
$$\left. + \exp\left\{(y_D-y_E)K + \frac{1}{2}\sigma_s^2 y_E^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_E}{\sigma_s}\right)\right)\right), \tag{69}$$

we can concisely write the MGF as

$$\mathbb{E}_i\left[\exp\left\{u'V\right\}\right] = \exp\left\{g\left(u + \mathbf{1}\frac{\mu_i}{\sigma_s^2}\right) - g\left(\mathbf{1}\frac{\mu_i}{\sigma_s^2}\right)\right\},$$

which is of the exponential family form. It follows that the demand function of investor i is

$$x_i = \tau\left(\mathbf{1}\frac{\mu_i}{\sigma_s^2} - \left(g'\right)^{-1}(P)\right). \tag{70}$$

Imposing the market-clearing condition and rearranging yields equilibrium price vector

$$P = g'\left(\mathbf{1}\frac{\int \mu_i di + \frac{1}{\tau}\sigma_s^2 z - \frac{1}{\tau}\sigma_s^2 \kappa}{\sigma_s^2}\right). \tag{71}$$

Plugging in the expression for μ_i from (66), grouping terms, and imposing the generalized linear conjecture, we find that there is a unique solution $b = \frac{\sigma_\varepsilon^2}{\tau}$ for the price signal coefficient. Finally, recognizing that with $b = \frac{\sigma_\varepsilon^2}{\tau}$ we have $\int \mu_i di + \frac{1}{\tau}\sigma_s^2 z - \frac{1}{\tau}\sigma_s^2\kappa = P_U$, where P_U is as defined in the baseline model, we can write the equilibrium price vector as

$$P = g'\left(\mathbf{1}\frac{P_U}{\sigma_s^2}\right). \tag{72}$$

Differentiating the expression for g above yields, upon inspection, that the debt and equity prices can be interpreted as expected payoffs under a risk-neutral distribution for the cash flow $\mathcal{V}$ that is truncated normal, with parameters P_U and σ_s^2, truncated below at zero. To see this, consider the equity (the expression for debt is analogous). We have

$$\begin{aligned}
\frac{\partial}{\partial y_E} g\left(\begin{smallmatrix} y_D \\ y_E \end{smallmatrix}\right)\Big|_{y_E = y_D = \frac{P_U}{\sigma_s^2}} &= \frac{\left(\sigma_s^2 y_E - K + \sigma_s \frac{\phi\left(\frac{K-\sigma_s^2 y_E}{\sigma_s}\right)}{1-\Phi\left(\frac{K-\sigma_s^2 y_E}{\sigma_s}\right)}\right)\exp\left\{(y_D - y_E)K + \frac{1}{2}\sigma_s^2 y_E^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_E}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_D^2\right\}\left(\Phi\left(\frac{K-\sigma_s^2 y_D}{\sigma_s}\right) - \Phi\left(\frac{-\sigma_s^2 y_D}{\sigma_s}\right)\right) + \exp\left\{(y_D - y_E)K + \frac{1}{2}\sigma_s^2 y_E^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_E}{\sigma_s}\right)\right)}\Bigg|_{y_E = y_D = \frac{P_U}{\sigma_s^2}} \\
&= \frac{\left(P_U - K + \sigma_s \frac{\phi\left(\frac{K-P_U}{\sigma_s}\right)}{1-\Phi\left(\frac{K-P_U}{\sigma_s}\right)}\right)\left(1-\Phi\left(\frac{K-P_U}{\sigma_s}\right)\right)}{\left(\Phi\left(\frac{K-P_U}{\sigma_s}\right) - \Phi\left(\frac{-P_U}{\sigma_s}\right)\right) + \left(1-\Phi\left(\frac{K-P_U}{\sigma_s}\right)\right)} \\
&= \left(P_U - K + \sigma_s \frac{\phi\left(\frac{K-P_U}{\sigma_s}\right)}{1-\Phi\left(\frac{K-P_U}{\sigma_s}\right)}\right)\frac{1-\Phi\left(\frac{K-P_U}{\sigma_s}\right)}{1-\Phi\left(\frac{-P_U}{\sigma_s}\right)},
\end{aligned}$$

which is precisely the expectation of $\max\{0, \mathcal{V} - K\}$ for

$$\mathcal{V} \sim TruncatedNormal\left(P_U, \sigma_s^2 \middle| \mathcal{V} \geq 0\right).$$

□

This proposition demonstrates that, as in the baseline model, security prices can be expressed as their expected payoffs under a risk-neutral distribution with mean and variance parameters P_U and σ_s^2, respectively. The key difference is that this distribution is now the truncated normal, truncated below at zero. However, expected returns no longer follow simple analytical expressions as in the main text. Thus, we next numerically calculate expected returns to assess whether our main findings are robust.

Figure 7 illustrates the findings, comparing the results under limited liability to our baseline results. The figure reveals that the qualitative implications of our model continue to hold: private information quality, liquidity-trading volatility, and default risk impact expected debt and equity returns similarly to as in our baseline model. However, the lower right-hand plot reveals that there are two intuitive differences relative to the baseline model. First, as default risk converges to zero, expected equity returns converge to zero in our baseline model, but to a strictly negative value under limited liability. The reason is that, even when the firm is unlevered, limited liability causes the equity payoffs to remain positively skewed.

Second, for high levels of default risk, debt earns negative expected returns under limited liability. Intuitively, when the firm is extremely close to default, given limited liability, debt payoffs resemble those of an equity security, and thus become convex. In particular, debt has limited downside and large upside in the (unlikely) event that the firm produces high cash flows. We have confirmed across a range of parameters that this reversal only applies to firms with default risk that exceeds 50%, which represents an extremely small proportion of traded stocks, even among those with junk debt (e.g., Hilscher and Wilson (2017)).

Figure 7: Limited Liability Comparative Statics

This figure compares expected returns under our baseline model to those under the model considered in this section. The parameters are set to: $\sigma_\theta^2 = \sigma_\varepsilon^2 = m = \tau = K = \sigma_z^2 = 1; \kappa = 0; \rho = 0.5$.