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Signal or noise? Uncertainty and learning about whether other traders are informed ☆

Snehal Banerjee [a,*], Brett Green [b]

[a] Northwestern University, Kellogg School of Management, 2001 Sheridan Road, Evanston, IL 60208, United States
[b] University of California, Berkeley, Haas School of Business, 545 Student Services, #1900, Berkeley, CA 94720-1900, United States

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ABSTRACT

We develop a model where some investors are uncertain whether others are trading on informative signals or noise. Uncertainty about others leads to a nonlinear price that reacts asymmetrically to news. We incorporate this uncertainty into a dynamic setting where traders gradually learn about others and show that it generates empirically relevant return dynamics: expected returns are stochastic but predictable, and volatility exhibits clustering and the "leverage" effect. The model nests both the rational expectations (RE) and differences of opinions (DO) approaches and highlights a link between disagreement about fundamentals and uncertainty about other traders.

1. Introduction

As early as Keynes (1936), it has been recognized that investors face uncertainty not only about fundamentals, but also about the underlying characteristics and trading motives of other market participants. Asset pricing models have focused primarily on the former, taking the latter as common knowledge. For instance, in Grossman and Stiglitz (1980), uninformed investors know the number of informed investors in the market and the precision of their signals. Similarly, each investor in Hellwig (1980) is certain about both the number of other investors and the distribution of their signals. Arguably, this requires an unrealistic degree of knowledge about the economy — it seems unlikely that investors who are uncertain about fundamentals, know, *with certainty*, whether other investors are privately informed.

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* Corresponding author. Tel.: +1 847 491 8335; fax: +1 847 491 5719.
E-mail addresses: snehal-banerjee@kellogg.northwestern.edu (S. Banerjee), bgreen@haas.berkeley.edu (B. Green).

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We develop a framework in which rational uninformed traders are uncertain whether others trade on informative signals or noise. This uncertainty generates an equilibrium price that is nonlinear in the information about fundamentals, and reacts more strongly to bad news than to good news. When risk considerations are large enough, the price may even *decrease* with additional good news. We incorporate this uncertainty into a dynamic environment in which uninformed traders gradually learn whether others are informed by observing prices and dividends. Return dynamics are also asymmetric — future return moments are more sensitive to lagged returns for negative realizations. The combination of uncertainty and learning generates predictability in expected returns, and can lead to volatility clustering, in which large absolute return realizations are followed by higher expected returns and volatility.

In order to explain the intuition for these results, a brief overview of the model is useful. There is a risky asset in fixed supply that pays a stream of dividends, and there are two groups of investors in the market at any given time. The first group of traders (θ) may be one of two types. They are either informed investors ($\theta = I$), such as institutions, who trade on a signal that is informative about next period's dividend. Or, they are noise traders ($\theta = N$), such as retail investors, who trade on a spurious signal that they (incorrectly) believe to be informative.[1] The second group consists of uninformed rational traders (U), such as hedge funds or liquidity providers, who do not have private information and are uncertain about the type of other investors in the market. All agents have mean–variance preferences and trade competitively in a centralized market by submitting limit orders.

Our benchmark model is static: uninformed traders face uncertainty about whether θ traders are informed but they do not learn about them. In equilibrium, the price and residual demand reveals the signal of the θ investors to uninformed traders, but they are uncertain whether it is informative. Because of this, a surprise in the signal (in either direction) increases the uninformed traders' posterior variance about fundamentals. As a result, the equilibrium price is (i) nonlinear in the signal, and (ii) depends on the probability that uninformed traders assign to θ investors being informed.

An immediate implication is that the price reacts asymmetrically to information about fundamentals.[2] Because they are uncertain whether it is informative, a surprise in the signal increases the uninformed investors' posterior variance about fundamentals. A negative surprise also lowers their expectation about fundamentals and both effects lead to a decrease in the price. A positive surprise increases their conditional expectation, but is offset by the increase in uncertainty. As a result, the price is more sensitive to negative surprises than to positive surprises. When the overall risk concerns are sufficiently large, the effect of the additional uncertainty dominates and the price *decreases* following additional good news about fundamentals. This occurs despite the fact that with good news, θ investors demand strictly more of the asset at any price.

We extend the benchmark model to a dynamic setting. The asymmetry in price reactions to news about fundamentals leads to an asymmetry in return dynamics. Using simulated data, we find that the sensitivity of future return moments to lagged returns is larger for negative realizations of lagged returns. This is consistent with the so-called "leverage" effect, though the mechanism in our model underlying this prediction does not rely on leverage.[3] Rather, it is driven by uninformed traders' uncertainty about others, which causes return changes associated with negative realizations of the θ investor's signal to be larger than the changes corresponding to positive realizations of the same magnitude.

An additional feature of the dynamic setting is that, over time, uninformed traders update their beliefs about whether others are informed using realized prices and dividends. The endogenous evolution of their beliefs (combined with (i) and (ii) above) implies that expected returns and volatility are stochastic, but predictable, and vary with uninformed traders' beliefs about others. Learning about whether others are informed also naturally gives rise to volatility clustering.[4] Since uninformed traders form their conditional expectations of next period's dividends based on their inference about other traders' signal, a dividend realization that is far from their conditional expectation (i.e., a large dividend surprise) leads them to revise downward the probability of others being informed. In other words, large dividend surprises, which are accompanied by large absolute return realizations, reduce the likelihood that θ investors are informed. This can increase uninformed traders' overall uncertainty and, therefore, leads to higher volatility and higher expected returns in future periods.

Our framework bridges the gap between two common approaches to modeling belief heterogeneity and disagreement across investors: *rational expectations* (RE) and *difference of opinions* (DO). In DO models, disagreement arises due to heterogeneous priors, while in RE models, it is due to differences in information. Both are nested within our framework — when the probability that other investors are informed is one (zero), our model is a standard RE

[1] DeLong, Shleifer, Summers, and Waldmann (1990), Hirshleifer, Subrahmanyam, and Titman (2006), and Mendel and Shleifer (2012) use similar specifications for noise (or *sentiment*) traders.

[2] Asymmetric price reactions have been well documented in the empirical literature. For instance, Campbell and Hentschel (1992) document asymmetric price reactions to dividend shocks at the aggregate stock market level. At the firm level, using a sample of voluntary disclosures, Skinner (1994) documents that the price reaction to bad news is, on average, twice as large as that for good news. Skinner and Sloan (2002) document that the price response to negative earnings surprises is larger, especially for growth stocks.

[3] The leverage effect, which refers to the negative correlation between lagged returns and future volatility, has also been widely documented empirically (e.g., Black, 1976; Christie, 1982; Schwert, 1989; Glosten, Jagannathan, and Runkle, 1993; Andersen, Bollerslev, Diebold, and Ebens, 2001; Bollerslev, Litvinova, and Tauchen, 2006).

[4] Since Mandelbrot (1963), a large number of papers have documented the phenomenon of volatility clustering for various asset classes, and at different frequencies. See Bollerslev, Chou, and Kroner (1992) for an early survey.

(DO) model. In general, the nature of the uncertainty about whether others are informed, which is inherently missing from RE and DO models, affects the degree of disagreement across investors. When there is a high probability of other investors being informed, all investors agree that the signal is informative and disagreement is low. When there is a low probability of others being informed, disagreement is high since θ investors think the signal is informative while uninformed traders believe it is noise.

Furthermore, the model predicts that the relation between uninformed investors' beliefs about others traders and returns is nonmonotonic and varies over time as uninformed traders' beliefs evolve. This dependence of disagreement and returns on the likelihood of informed trading has been overlooked in the existing literature. As a result, the standard empirical approach, which assumes a stable, monotonic relation between disagreement and returns, may be misspecified.[5] Importantly, our analysis suggests that in order to empirically uncover the underlying relationship, one must condition on the likelihood that investors assign to others being informed.

Our noise trader specification — as investors who trade on noise as if it were information — is different from the typical noisy supply approach (e.g., Grossman and Stiglitz, 1980). In Section 6, we explore the robustness of our results by considering a setting in which N investors know they are uninformed, but the aggregate supply of the asset is noisy. We show that this alternative specification generates qualitatively similar results to our benchmark model. From this analysis, we conclude that the positive implications of our theory are not driven by a particular specification of noise trading. Rather, the key feature is that some investors face uncertainty about whether other investors are informed.

The rest of the paper is organized as follows. We discuss the related literature in the next section. In Section 3, we solve the benchmark model, which allows us to highlight the intuition for many of our results transparently in a static setting. Section 4 extends the analysis to a dynamic setting, which allows us to focus on the effects of learning, and Section 5 discusses the implications of the model. In Section 6, we study robustness under an alternative specification in which the supply of the asset is noisy. Section 7 concludes. Proofs and supplementary analysis are located in the Appendices.

2. Related literature

A small number of recent asset-pricing models consider the effects of uncertainty about other traders. Easley, O'Hara, and Yang (2014) study a single-period economy in which ambiguity-averse investors face uncertainty about the effective risk tolerance of other traders and show that reducing ambiguity decreases expected returns. Gao, Song, and Wang (2013) also explore a static environment where risk-averse, uninformed traders are uncertain whether the proportion of informed traders is either low or high.[6] They show that in addition to the fully revealing equilibrium, a continuum of partially revealing rational expectations equilibria can exist. One advantage of our benchmark model relative to theirs is that we obtain a unique equilibrium, which facilitates a sharper set of predictions. More generally, by analyzing a dynamic setting, we contribute to this literature by exploring the effects of *learning about others* on return dynamics.

Our paper is also related to a subset of the market microstructure literature that studies environments where investors face multiple dimensions of uncertainty. Gervais (1997) considers a static Glosten and Milgrom (1985) model in which the market maker is uncertain about the precision of informed trader's signal. Romer (1993) and Avery and Zemsky (1998) consider models in which the proportion of informed traders is uncertain (but is not learned over time). Li (2012) and Back, Crotty, and Li, (2013) consider generalizations of the continuous-time, Kyle-model of Back (1992) that allow for uncertainty about whether the strategic trader is informed or not. While these papers focus on the market microstructure implications in settings with risk-neutral agents (e.g., market depth, insider's profit), we analyze a setting with risk-averse investors and focus on the implications for risk-premia and volatility (e.g., predictability, clustering).[7]

While the majority of the rational expectations literature has focused on linear equilibria in an exponential-normal setting, a number of papers, including most recently Breon-Drish (2012) and Albagli, Hellwig, and Tsyvinski (2011), have explored the effects of relaxing the assumption that fundamental shocks and signals are normally distributed in static environments.[8] Our paper contributes to this literature in two ways. First, we develop a model in which a nonlinear price function arises because of the composition of traders in the market and the information structure rather than the distribution of payoffs.[9] Second, we explore the implications of the nonlinearity on return dynamics.

[5] The empirical evidence on how returns vary with disagreement and information quality is inconclusive. For instance, Diether, Malloy, and Scherbina (2002) and Johnson (2004) document a negative relation between disagreement (as proxied by analyst forecast dispersion) and expected returns, but Qu, Starks, and Yan (2004) and Banerjee (2011) document a positive relation.

[6] In a less closely related environment, Stein (2009) explores market efficiency in a setting where arbitrageurs are uncertain about the total arbitrage capacity in the market.

[7] In a series of papers, Easley, O'Hara and co-authors analyze the probability of informed trading (PIN) in a sequential trade model similar to Glosten and Milgrom (1985) (e.g., Easley, Kiefer, and O'Hara, 1997a,b; Easley, Hvidkjaer, and O'Hara, 2002). In these papers, the risk-neutral market maker updates her valuation of the asset based on whether a specific trade is informed or not, but does not face uncertainty about the presence of informed traders in the market. In contrast, the uninformed investors in our model must update their beliefs, not only about the value of the asset, but also about the probability of other investors being informed, which leads to nonlinearity in prices.

[8] Earlier papers in this literature include Ausubel (1990), Foster and Viswanathan (1993), Rochet and Vila (1994), DeMarzo and Skiadas (1998), Barlevy and Veronesi (2000), and Spiegel and Subrahmanyam (2000).

[9] Specifically, even though shocks to fundamentals and signals are normally distributed in our model, since the uninformed investor is uncertain whether other investors are informed, her beliefs about the price signal are given by a mixture of normals distribution.

A related nonlinearity arises in the incomplete information, regime-switching models of David (1997), Veronesi (1999), David and Veronesi (2013, 2014), and others, in which a representative investor updates her beliefs about the current macroeconomic regime using signals about fundamental shocks (e.g., dividends). In these models, the nonlinearity in the representative investor's filtering problem leads to time-variation in uncertainty and, therefore, variation in expected returns and stochastic volatility. Stochastic volatility also arises in noisy rational expectations models, like Collin-Dufresne and Fos (2012), in which noise trader volatility is stochastic and persistent. These features arise *endogenously* in our model even though shocks to both fundamentals and news are independent and identically distributed (i.i.d.)

Cao, Coval, and Hirshleifer (2002) show that limited participation can also generate stochastic volatility, as well as large price movements in response to little, or no, apparent information.[10] Because of participation costs, sidelined investors update the interpretation of their private signals based on what they learn from prices, and only enter the market once they are sufficiently confident. In our model, the friction is purely informational — uninformed investors trade less aggressively because they are uncertain about the trading motives of other investors, and consequently, the informativeness of the price.

Finally, our model contributes to the differences of opinion (DO) literature, which has been important in generating empirically observed features of price and volume dynamics (e.g., Harrison and Kreps, 1978; Harris and Raviv, 1993; Kandel and Pearson, 1995; Scheinkman and Xiong, 2003; Banerjee and Kremer, 2010). With the exception of Banerjee, Kaniel, and Kremer (2009), the DO models in the literature have largely ignored the role of learning from prices, since investors agree to disagree about fundamentals, and therefore find the information in the price irrelevant.[11] In our model, investors may exhibit differences of opinion (since all θ investors believe their signals are payoff relevant), but uninformed investors still condition on prices to update their beliefs about fundamentals.

3. The benchmark model

This section presents the analysis for the (two-date) benchmark model. This simple setting will allow us to isolate the effects of uncertainty about other traders from the effects of learning about them, which obtain in the dynamic setting of Section 4. The static model also allows us to solve for equilibrium prices in closed form and develop the underlying intuition more transparently.

Agents: There are three different groups of traders in the model. Traders within each group are identical and behave competitively.

- *Informed traders* (I): I traders are rational agents who receive a private and informative signal about the dividend (e.g., institutional investors).
- *Noise/sentiment traders (N)*. N traders are irrational agents who observe and trade on a signal that they believe is informative, but is purely noise (e.g., retail investors).
- *Uninformed traders* (U). U traders are rational agents who receive no private signal about fundamentals but update their beliefs by observing prices and quantities (e.g., hedge funds or liquidity providers).

The key feature we want to capture in the benchmark model is that uninformed traders are uncertain about who they are trading against. To this end, we assume that either I or N traders are present in the market but not both, and further, U traders do not know which type of other traders they are facing. We let $\theta \in \{I, N\}$ denote the random variable that represents the type of other traders that are present in the market.[12]

Securities: There are two assets: a risk-free asset and a risky asset. The gross risk-free rate is normalized to $R \equiv 1+r > 1$. At date 1, the risky asset pays a dividend $D = \mu + d$, where $\mu > 0$ and $d \sim \mathcal{N}(0, \sigma^2)$. The aggregate supply of the risky asset is constant and equal to Z. At date 0, the risky asset is traded in a competitive market. Let P denote the market clearing price and $Q \equiv D - RP$ denote the excess (dollar) return per share of the risky asset.

Information and beliefs: The θ traders are either informed traders (i.e., $\theta = I$) or noise traders (i.e., $\theta = N$), where the prior probability of being informed is $\pi_0 \equiv \Pr(\theta = I)$. Prior to submitting their order, θ investors receive a signal S_θ of the form:

$$S_\theta = \begin{cases} d + \varepsilon & \text{if } \theta = I \\ u + \varepsilon & \text{if } \theta = N, \end{cases}$$

where $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, u is distributed identically to d, and (ε, u, d) are mutually independent.[13] Conditional on $\theta = I$, the informativeness of the signal is captured by the signal-to-noise ratio:

$$\lambda \equiv \frac{\mathrm{cov}[S_I, d]}{\mathrm{var}[S_I]} = \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2}.$$

[10] Other papers that study the informational effects of limited participation include Romer (1993), Lee (1998), Hong and Stein (2003), and Alti, Kaniel, and Yoeli (2012).

[11] In the DO model of Banerjee, Kaniel, and Kremer (2009), investors agree to disagree but use the price to update their beliefs about higher order expectations, which is useful for them to speculate against each other.

[12] One could also consider a setting in which all three types of traders are present in the market, and uninformed traders are uncertain about the proportion of informed traders vs. noise traders they face. We discuss such a setting in Section 7.

[13] As we shall see, the assumption that u and d are identically distributed implies that the optimal demand for θ investors is also identically distributed across types, and consequently, uninformed traders do not learn about θ from the price and residual demand. If we relax the assumption, uninformed traders would learn about θ even in a static setting. We maintain the assumption in order to separately characterize the effects of uncertainty about others in the benchmark model and the effects of learning about others that arise in the dynamic setting of Section 4.

Noise traders ($\theta = N$) behave as if their signal has the same information content as that of I investors (i.e., that the signal-to-noise ratio of S_N is λ), when in fact it is uncorrelated with dividends.

Preferences: Traders have mean-variance preferences over terminal wealth, and trade competitively (i.e., are price takers). In particular, each trader belonging to group $i \in \{I, N, U\}$ submits a limit order, x_i, such that

$$x_i = \arg\max_x \mathbb{E}_i[W_i R + xQ] - \frac{\alpha}{2}\text{var}_i[W_i R + xQ]. \quad (1)$$

We use $\mathbb{E}_i[\cdot]$ and $\text{var}_i[\cdot]$ to denote the conditional expectation and variance given the trader's information set, W_i denotes investor i's wealth, and α represents the degree of risk aversion. Given these preferences, investor i's optimal demand for the risky asset is given by

$$x_i = \frac{\mathbb{E}_i[Q]}{\alpha\,\text{var}_i[Q]} = \frac{\mathbb{E}_i[D] - RP}{\alpha\,\text{var}_i[D]}. \quad (2)$$

The (reduced-form) specification for investor preferences in Eq. (1) facilitates tractability, since it leads to optimal demand of the conventional form given by Eq. (2).

3.1. Remarks on the model

The two standard approaches to modeling belief heterogeneity are nested in our framework. On the one hand, when $\pi_0 = 1$, investors have common priors over the joint distribution of payoffs and signals, as in a rational expectations equilibrium. On the other hand, when $\pi_0 = 0$, investors "agree to disagree" about the distribution of S_θ and d, and so exhibit a difference of opinion.[14]

In our benchmark model, noise trading arises from investors who trade on a spurious signal and we consider the extreme case in which their signal is not informative. This specification is in line with models of utility-maximizing traders that are subject to sentiment shocks (e.g., DeLong, Shleifer, Summers, and Waldmann, 1990; Hirshleifer, Subrahmanyam, and Titman, 2006; Mendel and Shleifer, 2012).[15] However, it differs from the typical assumptions made in the noisy RE literature (e.g., aggregate supply shocks). In Section 6, we consider a setting in which the aggregate supply of the asset is noisy, and derive qualitatively similar results to our benchmark model: the price remains nonlinear in the I investors information, which leads to asymmetric price reactions. This analysis suggests that our results are robust to the particular specification of noise trading that is assumed and the key feature for our conclusions pertains to the uncertainty that U investors face about whether θ investors are informed.

Our noise trader specification offers several advantages over the noisy supply approach. First, it allows us to encapsulate both the RE and DO models as extreme cases, and facilitates the interpretation of disagreement that would not obtain in a reduced-form specification. Second, it isolates the effects of *uncertainty* about other traders from the effects of *learning* about them. While both forces are important, we find it pedagogically easier to first explain the effects of uncertainty and then incorporate learning. Finally, this specification also allows us to derive closed-form solutions for the nonlinear price in the static model, which facilitates analytic comparative statics and clearly illustrates the role of uncertainty about others.[16]

One can interpret the behavior of noise traders in our model as a form of overconfidence (e.g., Daniel, Hirshleifer, and Subrahmanyam, 1998; Odean, 1998), which has been shown to have important implications for trading behavior in financial markets (e.g., Odean, 1999; Barber and Odean, 2000; Grinblatt and Keloharju, 2000). Our results remain qualitatively the same if, instead, noise traders receive an informative signal about the asset, but overestimate its informativeness. In particular, suppose the $\theta = N$ investor receives a signal

$$S_N = \psi d + \sqrt{(1-\psi^2)}\; u + \varepsilon, \quad (3)$$

for some $\psi \in (0,1)$, but believes she observes a signal $S_N = d + \varepsilon$. In this case, the true informativeness of her signal is given by

$$\frac{\text{cov}[S_N, d]}{\text{var}[S_N]} = \frac{\psi\sigma^2}{\sigma^2 + \sigma_e^2} = \psi\lambda, \quad (4)$$

while she believes the informativeness of the signal is λ. As mentioned earlier, we focus on the extreme case in which $\psi = 0$ for ease of exposition.

As is common in the literature on asymmetric information in financial markets, we consider a model with a single risky asset (e.g., Grossman and Stiglitz, 1980; Kyle, 1985; Wang, 1993). This assumption is made primarily for expositional purposes. Given that there is empirical evidence for many of our predictions for both portfolio and individual asset returns (e.g., asymmetric price reactions, time-varying expected returns, volatility clustering), the mechanism that we highlight may be applicable at both the aggregate-level and the firm-level (due to limits to arbitrage or other frictions). One could interpret the risky asset in the model as an industry-level portfolio, which bears aggregate risk, and about which investors may have private information.

As we will show below, in the unique equilibrium of the benchmark model, the θ investors signal is perfectly revealed to U investors through prices and quantities. Thus, the implications of the benchmark model remain unchanged if we instead assume that S_θ is a *public* signal, and U traders face uncertainty about whether it is informative about fundamentals. This equivalence facilitates an alternative interpretation of S_θ as public news. In

[14] See Morris (1995) for a discussion of the implications of relaxing the common prior assumption.

[15] One difference from DeLong, Shleifer, Summers, and Waldmann (1990), Hirshleifer, Subrahmanyam, and Titman (2006), and Mendel and Shleifer (2012) is that, in our model, noise traders are not present in all states (e.g., when $\theta = I$). That is, we have introduced the minimal amount of noise trading necessary to avoid trivial outcomes. Of course, one could introduce additional noise trading across all states if desired.

[16] Our specification is also arguably closer to Black's (1986) notion of noise traders than aggregate supply shocks: "*Noise trading is trading on noise as if it were information… Perhaps they think the noise they are trading on is information.*" It also appears to have empirical relevance (see, e.g., Shleifer and Summers, 1990; Hirshleifer, 2001).

alternative specifications (e.g., in a model with supply shocks as in Section 6), equilibrium prices and quantities need not fully reveal the signal. The advantage of our benchmark setting relative to this alternative specification is that it allows us to isolate the effects of uncertainty about the quality of the information (Sections 3.2– 3.4) from learning about that quality (Section 4). Both of these forces are present in the static model with supply shocks (Section 6).

3.2. Equilibrium characterization

An equilibrium consists of a price function for the risky asset, P, and investor demands, x_i, such that: (i) investor i's demand are optimal, given their beliefs and information, (i.e., satisfy Eq. (2)) and (ii) the market for the risky asset clears i.e.,

$$x_U + x_\theta = Z. \tag{5}$$

Since there are no additional sources of noise, one expects that in equilibrium, uninformed traders will be able to infer S_θ from the price and the aggregate residual supply and use this to update their beliefs about the dividend. As we will see, because the equilibrium price is nonmonotonic in S_θ, observing the price alone does not necessarily reveal the signal S_θ. We follow Kreps (1977) and allow the U traders to condition their order on both price and quantity.[17]

Definition 1. An equilibrium is *signal-revealing*, if the equilibrium price and allocations reveal S_θ, but not θ, to U traders. Formally, that S_θ is measurable with respect to U's information set but θ is not.

Because U is uncertain about θ, a signal-revealing equilibrium differs from a fully revealing equilibrium in which *both* S_θ and θ are revealed. Below we show that the unique equilibrium is signal-revealing.

Regardless of type, a θ investor believes that her signal is informative about dividends. This implies that the conditional beliefs of θ investors are symmetric across $\theta \in \{I, N\}$ and are given by

$$\mathbb{E}_\theta[d] = \lambda S_\theta \quad \text{and} \quad \mathrm{var}_\theta[d] = \sigma^2(1-\lambda). \tag{6}$$

In a signal-revealing equilibrium, U traders face uncertainty about θ and their beliefs will generically differ from those of a θ investor. Conditional on inferring the signal, S_θ, U's beliefs about d are given by

$$\begin{aligned}\mathbb{E}_U[d] &= \pi_0 \mathbb{E}_U[d|\theta = I] + (1-\pi_0)\mathbb{E}_U[d|\theta = N] \\ &= \pi_0 \lambda S_\theta \end{aligned} \tag{7}$$

and

$$\mathrm{var}_U[d] = \underbrace{\pi_0\sigma^2(1-\lambda) + (1-\pi_0)\sigma^2}_{\text{expected conditional variance}} + \underbrace{\pi_0(1-\pi_0)(\lambda S_\theta)^2}_{\text{variance of conditional expectation}}. \tag{8}$$

Eq. (8) highlights the key novel force of the benchmark model: U traders' conditional variance depends on the realization of the signal S_θ. If U traders are certain that θ is informed (i.e., $\pi_0 = 1$), then their conditional expectation of d depends on S_θ. If U traders are certain that θ is *not* informed (i.e., $\pi_0 = 0$), then their conditional expectation is identical to their prior and is unaffected by S_θ. In either case, since they are *certain about* θ, their conditional variance is independent of S_θ. When U traders are *uncertain about* θ, the variance of their conditional expectation is (generically) not zero, and this leads to additional uncertainty about dividends. Furthermore, this additional uncertainty is increasing in the magnitude of the signal: larger realizations of the signal are further from the signal's unconditional mean (recall $\mathbb{E}[S_\theta] = 0$), and therefore will increase the disparity between the expected dividend conditional on $\theta = I$ and the expected dividend conditional on $\theta = N$.[18] As we shall see, this dependence of U traders' conditional variance on the realization of the signal plays an important role in our results.

The following result characterizes the equilibrium of the benchmark model.

Proposition 2. In the benchmark model, there exists a unique equilibrium. This equilibrium is signal-revealing and the price is given by

$$P = \frac{1}{R}\left(\mu + (\kappa + (1-\kappa)\pi_0)\lambda S_\theta - \kappa\alpha\sigma^2(1-\lambda)Z\right), \tag{9}$$

where the weight κ is given by

$$\begin{aligned}\kappa &\equiv \frac{\mathrm{var}_U[d]}{\mathrm{var}_U[d] + \mathrm{var}_\theta[d]} \\ &= \frac{\sigma^2(1-\pi_0\lambda) + \pi_0(1-\pi_0)(\lambda S_\theta)^2}{\sigma^2(1-\lambda) + \sigma^2(1-\pi_0\lambda) + \pi_0(1-\pi_0)(\lambda S_\theta)^2} \in \left[\frac{1}{2}, 1\right]. \end{aligned} \tag{10}$$

The equilibrium price can be decomposed into a market expectations component and a risk-premium component, since

$$P = \frac{1}{R}\left(\underbrace{\mu + \kappa\mathbb{E}_\theta[d] + (1-\kappa)\mathbb{E}_U[d]}_{\text{expectations}} - \underbrace{\kappa\alpha\sigma^2(1-\lambda)Z}_{\text{risk premium}}\right). \tag{11}$$

The market expectations component is a weighted average of investors' conditional expectations of future dividends, where the weight on θ investors is given by the relative precision of their beliefs, as measured by κ. Intuitively,

[17] As in one of the examples in Kreps (1977), an equilibrium fails to exist if we do not allow U traders to condition on both price and quantity. To see this, suppose the price is signal-revealing; then, as we show in the paper, it must be nonmonotonic in S_θ, which is a contradiction. Suppose, instead, the price is not signal-revealing. Then the uninformed investor demands the same amount at the same price, but the θ investor demands different amounts, which means the price must be different. Again, this is a contradiction.

[18] That some realizations increase uncertainty has a similar flavor to Nimark (2014), in which the presence of an exogenous public signal indicates that extreme events are more likely. In our model, uninformed investors always observe the price, but more extreme prices increase their uncertainty about future dividends.

when U investors are relatively more uncertain (κ closer to 1), the expectations component reflects the conditional expectation of θ investors more. The risk-premium component is increasing in κ — higher uncertainty for U investors implies a larger price discount.

Clearly, κ plays an important role in determining the equilibrium price. Fig. 1 provides an illustration of how κ depends on the signal S_θ and the prior π_0. Since the (perceived) uncertainty of θ investors is weakly less than that of U investors, $\kappa \geq \frac{1}{2}$. When $\pi_0 \in \{0,1\}$, κ is independent of the signal since U investors are certain about whether the signal is informative. When the realization of S_θ is zero, κ decreases linearly in π_0 — the more likely θ investors are to be informed, the lower the conditional variance of U investors, and consequently, the lower is κ. When S_θ is nonzero, recall that the conditional variance of U investors is increasing in S_θ^2 and hump-shaped in π_0, and consequently, so is κ. Importantly, κ, and hence the equilibrium price, depends nonlinearly on S_θ. This distinguishes our model from standard RE and DO models as illustrated by Corollary 3.

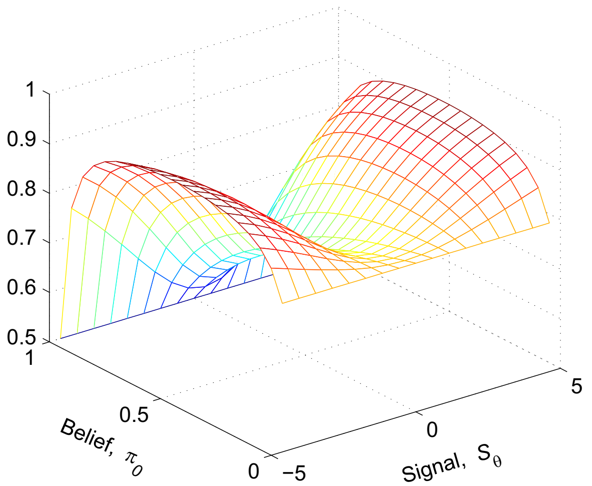

Fig. 1. This figure plots how the relative precision of investors' beliefs, κ, depends on the signal S_θ and the prior π_0.

Corollary 3. When U investors face no uncertainty about other traders, the price is a linear function of S_θ. More specifically,

(i) *When $\pi_0 = 1$, the price corresponds to a fully revealing, rational expectations equilibrium and is given by*
$$P = \frac{1}{R}\left(\mu + \lambda S_\theta - \frac{1}{2}\alpha\sigma^2(1-\lambda)Z\right). \tag{12}$$

(ii) *When $\pi_0 = 0$, the price corresponds to a difference of opinions model and is given by*
$$P = \frac{1}{R}(\mu + \kappa_0 \lambda S_\theta - \kappa_0 \alpha\sigma^2(1-\lambda)Z), \tag{13}$$
where $\kappa_0 = \sigma^2/(\sigma^2 + \sigma^2(1-\lambda))$ is a constant.

Returning to expression (11), we see that the risk aversion coefficient, α, and the aggregate supply of the asset, Z, scale the risk-premium component, but not the expectations component. Thus, the product, αZ, determines the relative role of each component in the price. When risk aversion is low or the aggregate supply of the asset is small, the price is primarily driven by the expectations component. On the other hand, when risk aversion is high, or the aggregate supply of the asset is large, the risk-premium component drives the price. As such, it is useful to characterize separately how each component of the price behaves. The prior belief, π_0, is the key parameter of interest; comparative statics with respect to π_0 help develop the intuition for the dynamic model, in which π_t evolves over time.

Proposition 4.

(i) *The expectations component of the price is increasing in S_θ, increasing in π_0 for $S_\theta > 0$, and decreasing in π_0 for $S_\theta < 0$.*

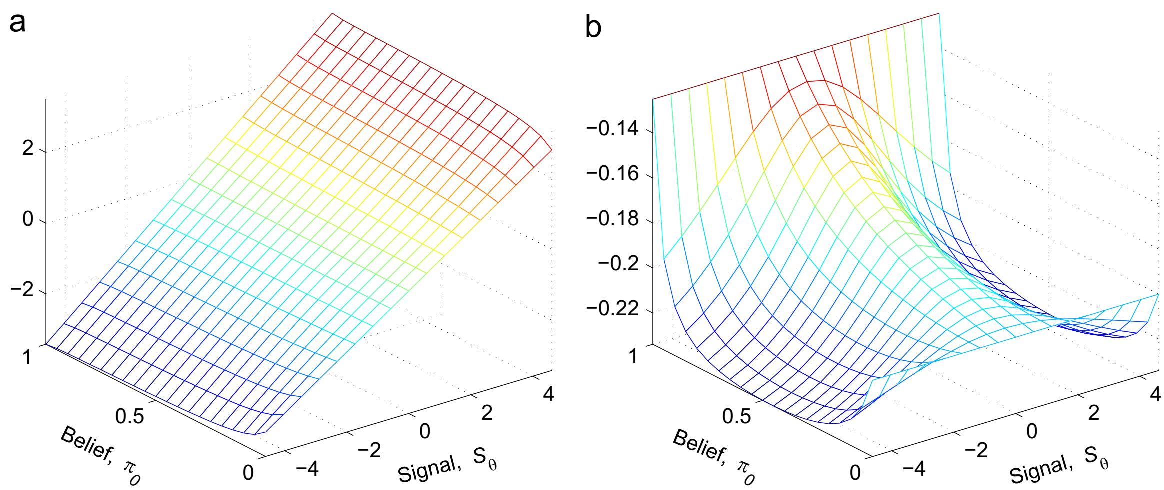

Fig. 2. The two components of the equilibrium price function as they depend on the prior π_0 and the realization of information S_θ. (a) Expectations component of P. (b) Risk premium component of P.

(ii) *The risk-premium component of the price is hump-shaped in S_θ around zero, and U-shaped in π_0 around* $\frac{1}{2}\left(1-\frac{\sigma^2}{\lambda S_\theta^2}\right)$.

Fig. 2 presents an illustration of these results. Intuitively, the comparative statics for the expectations component follow because it is a weighted average of investors' conditional expectations, which are increasing in S_θ. The risk-premium component of prices depends (negatively) on the uncertainty faced by the θ investor (i.e., $\sigma^2(1-\lambda)$) scaled by the weight κ. As a result, comparative statics for the risk-premium term with respect to π_0 and S_θ are the same as those for κ — it decreases in S_θ^2, for $\pi_0 \in (0,1)$, and is U-shaped in π_0.

3.3. Asymmetric price reaction to news

The overall effect of S_θ on the price distinguishes our model from linear price functions in RE and DO models that are standard in the literature. While the expectations component of price is monotonic in S_θ, the risk-premium component is hump-shaped in S_θ around zero. This implies that the two components of the price reinforce each other when there is negative information ($S_\theta < 0$), but offset each other when there is positive information ($S_\theta > 0$).

Corollary 5. The equilibrium price reacts asymmetrically to information about fundamentals: it decreases more with bad news than it increases with good news. For any $s>0$,

$$\frac{d}{dS_\theta}P(s) < \frac{d}{dS_\theta}P(-s).$$

Since the risk-premium component is bounded, the expectations component dominates when $|S_\theta|$ is large enough. However, for S_θ small enough, the risk-premium component dominates. This means that the price can actually *decrease* with additional good news, as documented by the following proposition.

Proposition 6. For any two signal realizations s_1, s_2 such that $0 < s_1 < s_2$, there exists a $\gamma > 0$ such that if $\alpha Z > \gamma$, the equilibrium price is strictly greater when s_1 is realized than it is when s_2 is realized.

Intuitively, if the overall risk concerns in the market (as measured by αZ) are large enough, more positive news about fundamentals can have a bigger impact on prices through the uncertainty it generates for uninformed investors than through its effect on the market's expectations about future dividends.

The mechanism through which the asymmetry in prices arises in our model differs from those in the regime-switching models of Veronesi (1999) and others. Specifically, in Veronesi (1999), the asymmetry in price reaction is driven by uncertainty about whether the underlying state of the economy is good or bad. The representative investor "over-reacts" to bad news only if he believes with sufficiently high probability that the current state is good, and "under-reacts" to good news only if he believes that the current state is bad, because these are the instances in which the realization of the news increases uncertainty about the underlying state. Thus, these regime-switching models predict the nature of the asymmetry is related to the business cycle. In contrast, the asymmetry in our model is not state-dependent: the price is more sensitive to bad news even in the absence of any learning about θ.[19] This is because the asymmetry is driven by uncertainty about the informativeness of the price signal, not the underlying fundamentals.[20]

3.4. Expected returns and volatility

We now turn to investigating the moments of returns. The decomposition in expression (11) implies that excess returns can be expressed as

$$Q = d - (\kappa \mathbb{E}_\theta[d] + (1-\kappa)\mathbb{E}_U[d]) + \kappa\alpha\sigma^2(1-\lambda)Z. \tag{14}$$

Return moments are computed based on the information set of the U investor, since she has rational expectations. This also corresponds to the information set of an econometrician who observes the price and quantity of executed trades as well as dividends. We refer to *conditional* expected returns as the expected returns conditional on all information prior to the realization of the dividend (i.e., conditional on the price and residual demand, and consequently, S_θ). *Unconditional* returns are computed from an ex ante perspective.

Proposition 7. The conditional expected return and volatility are given by

$$\mathbb{E}[Q|P, x_\theta] = -(1-\pi_0)\lambda\kappa S_\theta + \kappa\alpha\sigma^2(1-\lambda)Z \quad \textit{and} \tag{15}$$

$$\mathrm{var}[Q|P, x_\theta] = \sigma^2(1-\pi_0\lambda) + \pi_0(1-\pi_0)(\lambda S_\theta)^2. \tag{16}$$

The unconditional expected return and volatility are given by

$$\mathbb{E}[Q] = \mathbb{E}[\kappa]\alpha\sigma^2(1-\lambda)Z \quad \textit{and} \tag{17}$$

$$\mathrm{var}[Q] = \sigma^2(1-\pi_0^2\lambda) + (1-\pi_0)^2\lambda^2\mathrm{var}[\kappa S_\theta] + (\sigma^2(1-\lambda)\alpha Z)^2\mathrm{var}[\kappa]. \tag{18}$$

[19] Though, of course, the magnitude of the asymmetry depends on π_0 and disappears in the extremes (i.e., $\pi_0 \in \{0,1\}$). Note that in the dynamic version of our model, the uninformed investor updates her beliefs based on realizations of fundamentals, but this is not what drives the asymmetric reaction of prices to signals.

[20] As discussed in Section 3.1, given the unique equilibrium is signal-revealing, our model has an alternate interpretation in which S_θ is a *public* signal. Under this interpretation, a natural benchmark to consider is a setting in which there is a representative U investor who is uncertain about the quality of the signal. In the representative agent benchmark, the price will still be a nonlinear function of S_θ and several of our results in the benchmark model will continue to hold (e.g., asymmetric price reaction). However, alternative specifications will yield different implications. One difference, highlighted in Section 6, is that uninformed traders can learn about the signal quality even without observing dividends by conditioning on prices and residual order flow. In the benchmark with a representative investor, the price will not convey any incremental information about the quality of the signal. Moreover, a representative investor model does not lend itself to predictions about disagreement across investors, and how these relate to expected returns.

To gain some intuition for the expressions in Proposition 7, we note that the expectation of Q (in Eq. (14)) with respect to an arbitrary information set $\mathcal{I}$ can be decomposed into the following two components:

$$\mathbb{E}[Q|\mathcal{I}] = \underbrace{\mathbb{E}\left[\kappa(\mathbb{E}_U[d]-\mathbb{E}_\theta[d])|\mathcal{I}\right]}_{\text{expectations}} + \underbrace{\mathbb{E}[\kappa\alpha\sigma^2(1-\lambda)Z|\mathcal{I}]}_{\text{risk-premium}}. \quad (19)$$

As noted earlier, because the U investor is uncertain about the interpretation of S_θ, her conditional variance about d depends on both π_0 and S_θ. This means that κ and, as a result, the risk-premium component of expected returns also depend on both π_0 and S_θ.

The expression for the unconditional volatility of returns given in Eq. (18) can be decomposed into three terms, each of which captures a different source of risk,

$$\begin{aligned}\text{var}[Q] = &\underbrace{\sigma^2(1-\pi_0^2\lambda)}_{\text{fundamental}} + \underbrace{(1-\pi_0)^2\lambda^2\text{var}[\kappa S_\theta]}_{\text{expectations}} \\ &+ \underbrace{(\sigma^2(1-\lambda)\alpha Z)^2\text{var}[\kappa]}_{\text{risk premium}}. \end{aligned} \quad (20)$$

The first term is the expectation of the conditional variance in returns and so captures the volatility in returns due to uncertainty about next period's fundamental dividend shock d. The second term in Eq. (20) reflects the volatility in returns due to variation in the expectations component of conditional expected returns. Finally, the third term is volatility due to variation in the risk-premium component of conditional expected returns. As in the case of expected returns, each of these components depends on π_0. In the next section, we will see that this dependence on π_0 gives rise to expected returns and volatility that are stochastic but predicted by the beliefs of uninformed traders. For the interested reader, we explore these components in greater detail and discuss comparative statics on return moments in the benchmark model in Appendix B.

4. The dynamic model

In this section, we extend our analysis to a dynamic, overlapping generations (OLG) model in which the key state variable is the belief about θ investors. Two key considerations distinguish the dynamic setting from the benchmark model. First, the price is affected not only by investors' beliefs about fundamentals and other traders, but also their beliefs about future prices. Second, uninformed investors' beliefs about other traders change over time as they learn from realized prices and dividends.

We retain the main features of the benchmark model as described in Section 3 with the following natural extensions.

Agents: As before, there are three groups of traders: uninformed traders (U), informed traders (I), and noise/sentiment traders (N). In each generation, U traders are uncertain about which type of other traders they face, and $\theta_t \in \{I, N\}$ denotes the random variable that represents the type of these other traders at date t, where $\pi_0 = \Pr(\theta_0 = I)$. We allow θ to vary over time, and consider two specifications: (i) θ_t is i.i.d. over time with $\Pr(\theta_t = I) = \pi_0$ for all t, and (ii) θ_t exhibits serial correlation according to a Markov switching process. These cases together allow us to model the composition of traders in the market quite generally.

Securities: In date t, the risky asset pays a dividend D_t, which evolves according to an $AR(1)$ process:

$$D_{t+1} = (1-\rho)\mu + \rho D_t + d_{t+1}, \quad (21)$$

where $d_{t+1} \sim \mathcal{N}(0, \sigma^2)$, and $\rho < 1$. The excess dollar return at time t on a share of the risky asset is given by $Q_t \equiv P_t + D_t - RP_{t-1}$.

Preferences: Each generation of investors lives for two dates, and has mean–variance preferences over terminal wealth. An investor i, who is born in date t and consumes in date $t+1$, has optimal demand for the risky asset given by

$$x_{i,t} = \frac{\mathbb{E}_{i,t}\left[P_{t+1}+D_{t+1}\right] - RP_t}{\alpha \text{var}_{i,t}\left[P_{t+1}+D_{t+1}\right]}. \quad (22)$$

Information and beliefs: In addition to the information structure described in Section 3, each generation of investors can observe the history of dividend realizations, prices, and trades. The noise terms in the signals (i.e., ε_t) are independent of other random variables and identically distributed over time.

4.1. General characterization

We begin with a characterization of the price in any signal-revealing equilibrium that extends the results from the previous section.

Proposition 8. In any signal-revealing equilibrium, investor beliefs are given by

$$\mathbb{E}_{U,t}[D_{t+1}] = (1-\rho)\mu + \rho D_t + \pi_t \lambda S_{\theta,t}, \quad (23)$$

$$\mathbb{E}_{\theta,t}[D_{t+1}] = (1-\rho)\mu + \rho D_t + \lambda S_{\theta,t}, \quad (24)$$

$$\text{var}_{U,t}[D_{t+1}] = \sigma^2(1-\pi_t\lambda) + \pi_t(1-\pi_t)(\lambda S_{\theta,t})^2, \quad (25)$$

$$\text{var}_{\theta,t}[D_{t+1}] = \sigma^2(1-\lambda), \quad (26)$$

and the price of the risky asset is given by

$$P_t = \frac{1}{R}\left(\underbrace{\overline{\mathbb{E}}_t[P_{t+1}+D_{t+1}]}_{\text{expectations}} - \underbrace{\alpha\kappa_t \text{var}_{\theta,t}[P_{t+1}+D_{t+1}]Z}_{\text{risk premium}}\right), \quad (27)$$

where $\overline{\mathbb{E}}_t[\cdot] \equiv \kappa_t \mathbb{E}_{\theta,t}[\cdot] + (1-\kappa_t)\mathbb{E}_{U,t}[\cdot]$, *and* κ_t *is given by*

$$\kappa_t = \frac{\text{var}_{U,t}[P_{t+1}+D_{t+1}]}{\text{var}_{U,t}[P_{t+1}+D_{t+1}] + \text{var}_{\theta,t}[P_{t+1}+D_{t+1}]}. \quad (28)$$

The decomposition of the price is familiar from the benchmark model (see Eq. (11)). It is a weighted average of investors' conditional expectations about future payoffs, adjusted for a risk-premium. In contrast to linear equilibria in standard models, both components of the price are nonlinear functions of the signal $S_{\theta,t}$ and beliefs π_t.

As in the benchmark model, we proceed by first presenting the limit cases in which uninformed traders are *not* uncertain about whether θ investors are informed. This will help to illustrate our main results are driven by U's uncertainty about θ_t and subsequent learning (when θ_t

is serially correlated), neither of which are present in these limit cases. We then reintroduce the uncertainty by considering the two different specifications for investor composition dynamics: Section 4.3 considers the case in which the distribution of θ_t is i.i.d. and Section 4.4 allows for serial correlation in θ_t.

4.2. When there is no uncertainty about other investors

In this subsection, we briefly discuss the two limit cases of the dynamic model in which there is no uncertainty about other traders. When θ_t investors are informed and U traders are certain about this (i.e., $\pi_t=1$), the model is analogous to a standard RE environment. When θ_t investors are uninformed and U traders are certain about this (i.e., $\pi_t=0$), the model is analogous to a typical DO model.

In both cases, without uncertainty about other traders, the model's predictions are standard — the equilibrium price is linear, return volatility is constant, and expected returns are either constant or i.i.d. The following proposition summarizes these findings.

Proposition 9. If $\pi_0 \in \{0,1\}$, then there exists a unique, linear stationary equilibrium. The equilibrium price is signal-revealing, expected returns are affine in the signal, and return volatility is constant.

When investors do not face uncertainty about other traders, we obtain a unique equilibrium. This is in contrast to standard (linear) OLG models (e.g., Spiegel, 1998) which generally exhibit two equilibria. In these models, the price is exposed to two types of shocks: fundamental shocks and noise shocks. The multiplicity in equilibria arise due to multiplicity in self-fulfilling beliefs about the price sensitivity to noise shocks. Generically, there is a low volatility equilibrium (when investors believe prices are not very sensitive to noise) and a high volatility equilibrium (when investors believe prices are sensitive to noise). In the two limit cases of our model, the price is a function of a single shock ($S_{\theta,t}$) and the price sensitivity of this shock is pinned down by the traders' beliefs of its informativeness.

4.3. When the distribution of types is i.i.d.

When the distribution of other traders' type (i.e., the distribution of θ_t) is independent and identical across generations, there exists a stationary equilibrium of the dynamic model which closely resembles the equilibrium of the benchmark model. Moreover, this stationary equilibrium is the limit of the unique equilibrium of a finite horizon (T-period) version of the model in which the price is normalized to zero at date T.

Proposition 10. Suppose $\alpha Z<(R-\rho)/(2\sigma)$ and the distribution of θ_t is i.i.d. Then, there exists a stationary equilibrium, which is signal-revealing and the price is given by $P_t=A\mu+BD_t+p(S_{\theta,t})$, where $A\equiv R(1-\rho)/((R-1)(R-\rho))$, $B\equiv\rho/(R-\rho)$,

$$p(S_{\theta,t})\equiv\frac{1}{R}\begin{pmatrix}(1+B)(\kappa_t+(1-\kappa_t)\pi_0)\lambda S_{\theta,t}+m\\ -\alpha\kappa_t((1+B)^2\sigma^2(1-\lambda)+v)Z\end{pmatrix}, \tag{29}$$

$$\kappa_t\equiv\frac{(1+B)^2(\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)(\lambda S_{\theta,t})^2)+v}{(1+B)^2(\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)(\lambda S_{\theta,t})^2)+(1+B)^2\sigma^2(1-\lambda)+2v}, \tag{30}$$

and where (m,v) are characterized implicitly by the solution to

$$m=\mathbb{E}[p(S_{\theta,t})] \quad and \quad v=\mathrm{var}[p(S_{\theta,t})]. \tag{31}$$

Moreover, there exists a solution to the system of equations given in (31) such that the price above corresponds to the limit of the unique equilibrium price of the T-period model, as $T\to\infty$.

As in other OLG models (e.g., Spiegel, 1998), the sufficient condition for existence $\alpha Z<(R-\rho)/(2\sigma)$ ensures that the aggregate risk in holding the risky asset is not too large. Intuitively, when risk considerations (αZ) or fundamental volatility (σ) increase, the risk-premium component of the current price grows and becomes more sensitive to shocks (in κ_t), which in turn increases the risk-premium in the previous period. As a result, if $\alpha Z\cdot\sigma$ is too large, the risk-premium terms may explode and a stationary equilibrium may not exist.

Note that the equilibrium price in Proposition 10 depends not only on beliefs about current dividends and whether others are informed, but also on investors' beliefs about the price next period. However, because the distribution of the type of other traders is i.i.d., uninformed traders do not learn about θ over time. The effect of learning about θ on the equilibrium price is the focus of the next subsection.

4.4. When the distribution of types is serially correlated

Next, suppose that θ_t follows a symmetric Markov switching process with transition probability $1-q$, i.e., $\Pr(\theta_{t+1}=I|\theta_t=I)=\Pr(\theta_{t+1}=N|\theta_t=N)=q$. In this case, prices and dividends at date t are informative about the composition of traders in the market at date $t+1$.

Uninformed traders' beliefs about other investors change over time. We denote their beliefs about whether others are informed at date t by $\pi_t\equiv\Pr_{U,t}(\theta_t=I)$. Following the realization of D_{t+1}, the posterior probability that uninformed traders assign to $\theta_t=I$, is given by

$$\Pr(\theta_t=I|S_{\theta,t},d_{t+1})=\frac{\pi_t\Pr(S_{\theta,t}|\theta_t=I,d_{t+1})}{\pi_t\Pr(S_{\theta,t}|\theta_t=I,d_{t+1})+(1-\pi_t)\Pr(S_{\theta,t}|\theta_t=N,d_{t+1})} \tag{32}$$

$$=\frac{\frac{\pi_t}{\sigma_\varepsilon}\phi\left(\frac{S_{\theta,t}-d_{t+1}}{\sigma_\varepsilon}\right)}{\frac{\pi_t}{\sigma_\varepsilon}\phi\left(\frac{S_{\theta,t}-d_{t+1}}{\sigma_\varepsilon}\right)+\frac{1-\pi_t}{\sqrt{\sigma^2+\sigma_\varepsilon^2}}\phi\left(\frac{S_{\theta,t}-0}{\sqrt{\sigma^2+\sigma_\varepsilon^2}}\right)}, \tag{33}$$

where $\phi(\cdot)$ is the probability distribution function for a standard normal random variable.

Accounting for the likelihood of a transition, the probability that uninformed traders assign to $\theta_{t+1}=I$ at date

$t+1$ is given by

$$\pi_{t+1} = q\Pr(\theta_t = I|S_{\theta,t}, d_{t+1}) + (1-q)(1-\Pr(\theta_t = I|S_{\theta,t}, d_{t+1})) \tag{34}$$

$$\pi_{t+1} = (1-q) + \frac{(2q-1)\frac{\pi_t}{\sigma_\varepsilon}\phi\left(\frac{S_{\theta,t}-d_{t+1}}{\sigma_\varepsilon}\right)}{\frac{\pi_t}{\sigma_\varepsilon}\phi\left(\frac{S_{\theta,t}-d_{t+1}}{\sigma_\varepsilon}\right) + \frac{1-\pi_t}{\sqrt{\sigma^2+\sigma_\varepsilon^2}}\phi\left(\frac{S_{\theta,t}-0}{\sqrt{\sigma^2+\sigma_\varepsilon^2}}\right)}. \tag{35}$$

Uninformed traders' belief about other traders serves as a state variable that evolves stochastically and leads to predictability and time-variation in return moments. Note that in the extreme case that $q=1$, so that $\theta_t = \theta_{t+1} = \theta$ for all t, uninformed traders will eventually learn whether θ investors are informed. On the other hand, for $q \in (0,1)$, π_t is bounded between $(1-q, q)$ and does not converge to a degenerate belief even as $t \to \infty$.

Our solution technique is motivated by the equilibrium of a finite horizon version of the model, which is characterized in the following result.

Proposition 11. In the T-period model, there exists at most one signal-revealing equilibrium. The corresponding equilibrium price is of the form $P_t = A_t\mu + B_tD_t + p_t(S_{\theta,t}, \pi_t)$, *where* $p_T \equiv 0$, $A_T \equiv 0$, $B_T \equiv 0$, $A_t \equiv (1/R)(A_{t+1} + (1-\rho)(1+B_{t+1}))$, $B_t \equiv (\rho/R)(1+B_t)$,

$$p_t(S_{\theta,t}, \pi_t) \equiv \frac{1}{R}\left\{\begin{array}{c} \overline{\mathbb{E}}_t\left[(1+B_{t+1})d_{t+1} + p_{t+1}(S_{\theta,t+1}, \pi_{t+1})\right] \\ -\alpha\kappa_t \mathrm{var}_{\theta,t}\left[(1+B_{t+1})d_{t+1} + p_{t+1}(S_{\theta,t+1}, \pi_{t+1})\right]Z \end{array}\right\}, \tag{36}$$

$$\kappa_t \equiv \frac{\mathrm{var}_{U,t}\left[(1+B_{t+1})d_{t+1} + p_{t+1}(S_{\theta,t+1}, \pi_{t+1})\right]}{\mathrm{var}_{U,t}\left[(1+B_{t+1})d_{t+1} + p_{t+1}(S_{\theta,t+1}, \pi_{t+1})\right] + \mathrm{var}_{\theta,t}\left[(1+B_{t+1})d_{t+1} + p_{t+1}(S_{\theta,t+1}, \pi_{t+1})\right]}, \tag{37}$$

and $\overline{\mathbb{E}}_{i,t}[\cdot] \equiv \kappa_t \mathbb{E}_{\theta,t}[\cdot] + (1-\kappa_t)\mathbb{E}_{U,t}[\cdot]$.

We solve for the (stationary) equilibrium price function numerically, by using a recursive procedure to compute the limit of the finite horizon model. We initialize $k=1$ and $P_T = 0$ and compute $P_{T-k}(S_{\theta,t}, \pi)$ over a grid of signals and beliefs using Eq. (27). We iterate this procedure over k until the mean squared difference in the price functions across iterations is negligible. Under the sufficient parametric restriction given in Proposition 10 (i.e., $\alpha Z < (R-\rho)/(2\sigma)$), we find that the price function converges for a wide range of parameters.[21]

As Fig. 3 illustrates, the components of the equilibrium price are similar to those in the benchmark model: the expectations component increases in $S_{\theta,t}$, while the risk-premium component is hump-shaped. For negative $S_{\theta,t}$ these two effects reinforce each other, while for positive $S_{\theta,t}$, the effects offset each other. As before, this leads to the asymmetric reaction of the price to $S_{\theta,t}$.

5. Implications of the dynamic model

To explore the implications of the model, we focus on the dynamic model with serial correlation in investor types since it provides a richer environment than the i.i.d. case. Since the model does not permit closed-form solutions, we explore its implications numerically. While the analysis in the earlier sections has focused on characterizing properties of dollar returns per share, $Q_{t+1} = P_{t+1} + D_{t+1} - RP_t$, we will now characterize properties of the excess rate of return, $R_{e,t+1} \equiv Q_{t+1}/P_t$, in order to highlight the robustness of the results and to facilitate comparisons to the broader literature. Unless otherwise specified, we use the parameters in Table 1. For these parameter values, the expected excess return on the risky asset is 7.5% and the volatility is 22%, when evaluated at $\pi_t = 1$ and $\lambda = 0.75$.[22]

5.1. Predictability in expected returns and volatility

When uninformed traders are uncertain about θ_t, the effect of π_t on the price generates novel empirical predictions that distinguish our model from linear, dynamic rational expectations models. In particular, the belief π_t is an endogenous state variable of the model, which evolves stochastically and is persistent. As a result, in addition to generating *stochastic* expected returns and volatility, these moments are *persistent* and vary *predictably* with π_t, despite the fact that shocks to fundamentals and signals are i.i.d.

As Fig. 4 shows, excess returns are first increasing in π_t but decreasing for larger π_t. Both moments are decreasing in λ for high π_t, but are *increasing* in λ for low π_t. The plots also suggest that the magnitude of the comparative static results are economically meaningful. For instance, at the baseline parameters (where $\pi_t = 1$), an increase in λ from 0.25 to 0.75 implies a decrease in expected returns from 9.2% to 7.5% and a decrease in volatility from 25% to 22%; a decrease in π_t from 1 to 0.5 implies an increase in expected returns from 7.5% to 10% and an increase in volatility from 22% to 30% (for $\lambda = 0.75$).

[21] We do not have a proof of existence for the infinite horizon case with persistent types. However, we have verified existence numerically for a wide range of parameters. We have also established existence in the dynamic model with i.i.d. types, as well as existence and uniqueness in the limiting cases where $\pi_t \in \{0, 1\}$.

[22] Because we are using normally distributed random variables, the population moments of $R_{e,t+1}$ are not well defined, since prices can be arbitrarily close to zero, or even negative — see Campbell, Grossman, and Wang (1993) and Llorente, Michaely, Saar, and Wang (2002) for a discussion. We adopt the conventional approach and choose D_t large enough relative to the volatility of dividend shocks (setting $D_t=1$ and $\sigma = 0.06$) such that the numerical estimation of these moments is well behaved.

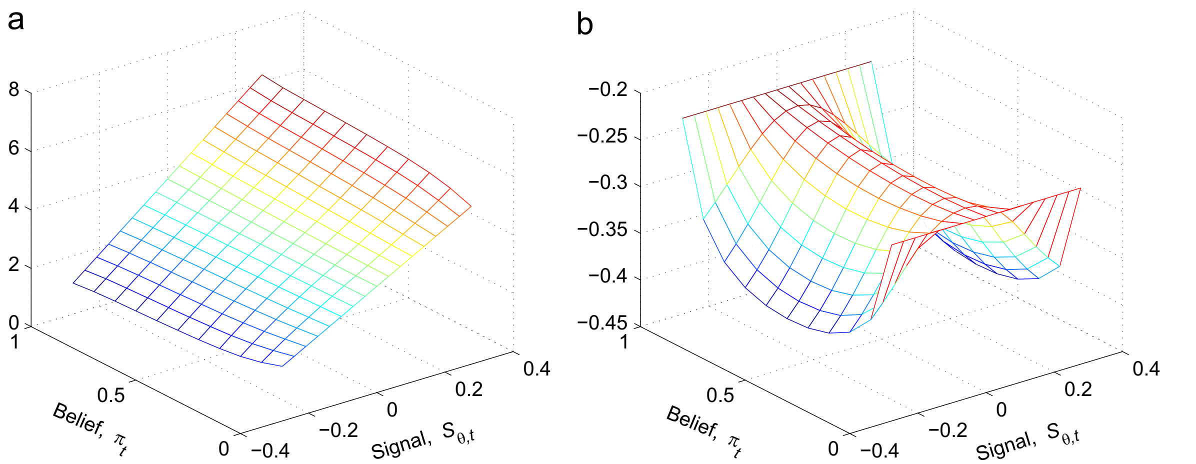

Fig. 3. The two components of the equilibrium price function as they depend on the underlying state variable and the realization of information. As in the benchmark model, the sum of these two components reacts asymmetrically to good versus bad news. (a) Expectations component of P_{t+1}. (b) Risk premium component of P_{t+1}.

Table 1
The table reports the baseline parameters used in numerical results and simulations.

Parameter	Value
Risk-free rate (r)	3%
Dividend growth rate (μ)	4%
Dividend volatility (σ)	6%
Dividend persistence (ρ)	0.95
Aggregate supply (Z)	1
Risk aversion coefficient (α)	1
Persistence in trader type (q)	0.75

5.2. *Disagreement, information quality, and returns*

In our model, the *information quality* of S_θ and the *disagreement* across investors are linked through their dependence on the likelihood of informed traders, π_t, and the Kalman gain of the informed investors' signal, λ. Specifically, the information quality of the signal, as measured by its (unconditional) correlation with the dividend, is given by

$$\mathrm{cor}(S_{\theta,t}, D_{t+1}) = \pi_t \sqrt{\lambda}. \tag{38}$$

Similarly, to see the effect of π_t and λ on disagreement, note that the absolute difference in dividend forecasts across investors, conditional on the realization of $S_{\theta,t}$, is given by $|\mathbb{E}_{U,t}[D_t] - \mathbb{E}_{\theta,t}[D_t]| = (1-\pi_t)\lambda|S_{\theta,t}|$. Since $S_{\theta,t}$ has a half-normal distribution, integrating over signal realizations gives us a measures of the (expected) disagreement across investors

$$(1-\pi_t)\lambda\mathbb{E}[|S_{\theta,t}|] = (1-\pi_t)\lambda\sqrt{\frac{2\mathrm{var}[S_{\theta,t}]}{\boldsymbol{\pi}}}$$
$$= (1-\pi_t)\sigma\sqrt{\frac{2\lambda}{\boldsymbol{\pi}}}, \tag{39}$$

where $\boldsymbol{\pi}$ denotes the mathematical constant (not to be confused with uninformed traders' belief).

Therefore, to understand the relation between information quality, disagreement, and returns, we return to comparative statics with respect to π_t and λ, which are illustrated in Fig. 4. When uninformed traders assign a low probability to others being informed (i.e., π_t is low), an increase in π_t increases their uncertainty about others, which results in higher expected returns and volatility. Similarly, an increase in λ leads θ investors to trade more aggressively on their signal, but this simply generates additional noise in prices from the perspective of uninformed traders and, therefore, also results in higher expected returns and volatility. Next, suppose uninformed traders believe other investors are very likely to be informed (i.e., π_t is high). In this case, an increase in π_t decreases uncertainty about others and an increase in λ implies a more informative signal — both lead to a decrease in expected returns and volatility.

When changes in π_t drive disagreement, the intuition is consistent with the predictions of Banerjee (2011): expected returns are negatively related to disagreement in DO models, but positively related to disagreement in RE models. In our model, when π_t is low, investors behave as if they "agree to disagree," since U traders do not believe $S_{\theta,t}$ is informative, but θ traders do. In this case, higher disagreement (lower π_t) is associated with lower uncertainty and lower expected returns. On the other hand, when π_t is high, both groups of investors agree on the informativeness of $S_{\theta,t}$, as they would in a rational expectations model. Now, higher disagreement (lower π_t) increases uncertainty, and consequently, expected returns.

These results imply that the relation between disagreement, information quality, and returns is nonmonotonic and varies over time as π_t evolves. How returns vary with either information quality or disagreement depends on: (i) whether the variation is primarily driven by λ or π_t, and (ii) whether π_t is high or low. The model suggests that standard empirical specifications in the literature, which

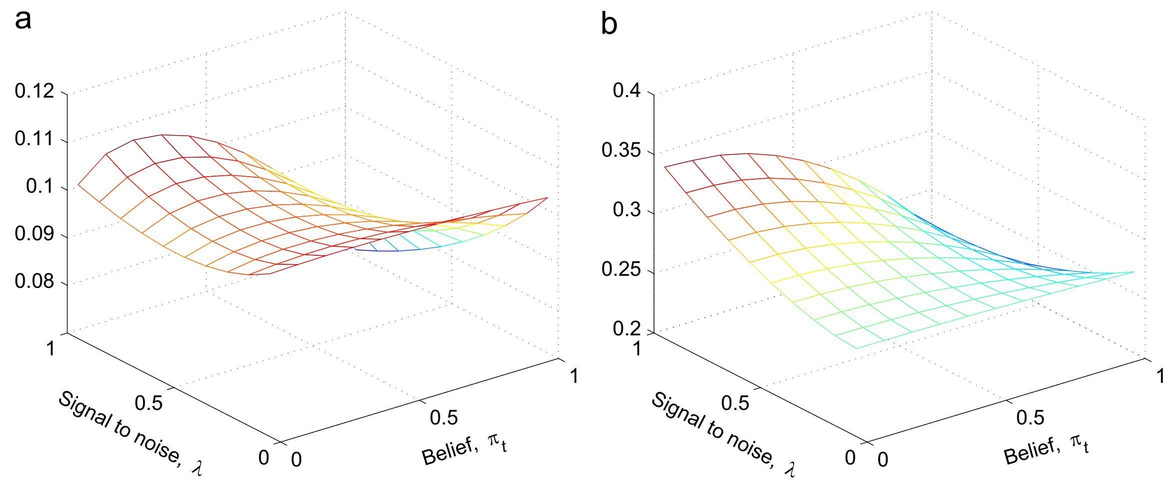

Fig. 4. This figure plots the expected excess rate of return and volatility as a function of π_t and λ. The parameters are set to the following baseline values unless otherwise specified: $r=3\%$, $\mu=4\%$, $\rho=0.95$, $\sigma=6\%$, $Z=1$, $\alpha=1$, and $q=0.75$. (a) Expected excess rate of return. (b) Volatility of rate of return.

posit a monotonic relation between returns and these variables, are misspecified.[23] Moreover, when trying to uncover the underlying empirical relation between these variables, it is crucial to control for the likelihood that investors are informed. While developing an empirical proxy that captures π_t is a challenge, existing measures of informed trading (e.g., *PIN* and institutional ownership) may be a useful starting point.

Empirical studies have associated the likelihood of informed trading (and proxies thereof) as a measure of the degree of asymmetric information in the market (e.g., Easley, Hvidkjaer, and O'Hara, 2002). However, our model suggests this relationship need not be monotonic. This is because uncertainty about *whether informed traders are present* can serve as the *source* of the asymmetric information. When uninformed traders place a very high (low) likelihood on informed traders being present, they know that the price is informative (uninformative) about fundamentals and the asymmetric information problem is mitigated. In our model, the asymmetric information problem is *most severe* when uninformed traders are *most uncertain* about whether informed traders our present. As such, data that do not exhibit a clear relation between expected returns and proxies of the likelihood of informed trading need not imply that asymmetric information does not affect expected returns.

5.3. Volatility clustering

The model can generate *volatility clustering* — return surprises in either direction are followed by an increase in both volatility and expected returns. This obtains when θ_t is serially correlated and π_t is close to one. The intuition for these predictions follows from how U updates her beliefs about whether θ_t is informed. An unanticipated realization of D_{t+1} leads the U investor to revise her beliefs about θ being informed downwards (i.e., $\pi_{t+1}<\pi_t$).[24] This revision in beliefs generates additional uncertainty for U traders, and as a result, leads to higher future volatility and higher expected returns going forward. Fig. 5 illustrates this clustering effect. Specifically, the figure plots expected returns and volatility in period $t+1$ as a function of the current realization of D_{t+1} (scaled by its standard error) starting from π_t close to one. Starting from zero on the x-axis, increasing the dividend surprise in either direction implies π_{t+1} is closer to $\frac{1}{2}$ and therefore U is more uncertain about θ, which leads to higher expected returns and volatility. Note that for sufficiently large (and unlikely) surprises, the posterior, π_{t+1}, tends to zero and expected returns and volatility may decrease.

For the baseline parameters, Fig. 5 provides magnitudes for the volatility clustering effect. For $\lambda=0.75$, a one-standard deviation surprise in dividend realizations predicts an increase in future expected excess returns of roughly 60 basis points (from 9% to 9.6%) and an increase in future volatility of 2% (from 25% to 27%), while a two-deviation surprise generates increases of 1% in expected returns and about 7% in volatility. The effect is increasing in λ — for $\lambda=0.9$, a one-standard deviation surprise corresponds to a 2.5% increase in expected returns and an 8% increase in volatility. Finally, note that in the limiting cases without uncertainty about others (i.e., where $\pi_t\in\{0,1\}$), these plots are perfectly flat. Thus, even for small deviations from the standard model ($\pi_t=0.95$ instead of $\pi_t=1$), the clustering effect can be quite economically significant.

For π_t close to zero, the opposite relationship can obtain: returns in line with expectations cause the U investor to

[23] See footnote 5 for a discussion of the mixed evidence on the disagreement-return relation. Similarly, while some papers document a negative relation between information quality and expected returns (e.g., Easley, Hvidkjaer, and O'Hara, 2002; Francis, LaFond, Olsson, and Schipper, 2005; Francis, Nanda, and Olsson, 2008), others find either limited or no evidence of a relation (e.g., Core, Guay, and Verdi, 2008; Duarte and Young, 2009).

[24] This follows from the evolution of π_t in (35), q being close to one, and π_t being large initially.

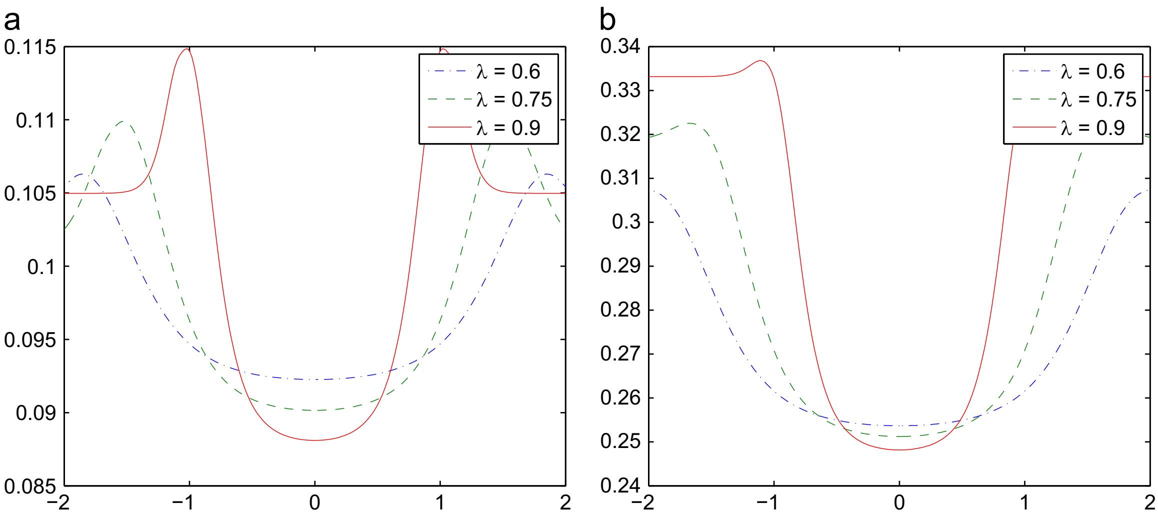

Fig. 5. The figure plots expected returns and volatility in period $t+1$ as they depend on dividend surprises in period t and the quality of the information (in both figures $\pi_t = 0.95$). The parameters are set to the following baseline values unless otherwise specified: $r=3\%$, $\mu=4\%$, $\rho=0.95$, $\sigma=6\%$, $Z=1$, $\alpha=1$, $\pi_t=0.95$ and $q=0.95$. (a) Future excess returns. (b) Future volatility.

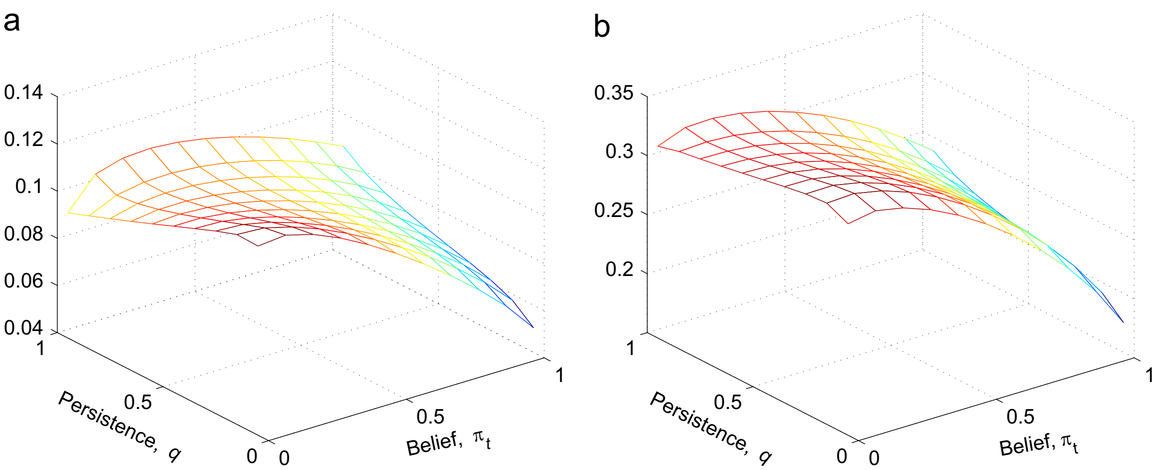

Fig. 6. This figure plots the expected excess rate of return and volatility as a function of π_t and q. The parameters are set to the following baseline values unless otherwise specified: $r=3\%$, $\mu=4\%$, $\rho=0.95$, $\sigma=6\%$, $Z=1$, $\alpha=1$, and $\lambda=0.75$. (a) Expected excess rate of return. (b) Volatility of rate of return.

revise her belief upwards, which again increases the uncertainty about other traders and hence volatility and expected returns. In this sense, no *news* (i.e., little to no surprise in returns) can either be good news (when π_t is close to one) or bad news (when π_t is close to zero). More generally, our model highlights a channel through which cash-flow news (i.e., dividend surprises) can affect discount rates (i.e., expected returns) in the future through its effect on uncertainty about other investors.[25]

5.4. Switching probability

Comparative statics on return moments with respect to the switching probability also depend on π_t. As an instance, Fig. 6 plots the expected excess rate of return and volatility as a function of π_t and q for the baseline parameters. The plots suggest that except near the boundaries of $q=0$ and $q=1$, expected returns and volatility are increasing in q for large π_t, but decreasing in q for low π_t. Intuitively, changing q does not change beliefs about next period's dividends, but it does affect beliefs about future prices. When the likelihood of other traders being informed is low in the current period (i.e., π_t is low), an increase in persistence of θ implies that the likelihood of other traders being informed is lower in future periods. In contrast, an increase in q when π_t is high implies that the likelihood of other traders being informed is higher in future periods. Since future prices are more sensitive to signals, and therefore riskier, when the likelihood of $\theta = I$ is higher, expected returns and volatility is increasing in q for high π_t but decreasing in q for low π_t.

These results suggest a novel prediction of the model, which has not been tested in the literature (to the best of

[25] We thank Karl Diether for this observation.

our knowledge). Namely, higher variation in ownership composition (lower q) should be associated with higher expected returns and volatility when the likelihood of informed trading is high, and lower expected returns and volatility when the likelihood of informed trading is low. In order to test this prediction, measures of informed trading such as *PIN* or fraction of institutional ownership can be used to proxy for π_t.

5.5. Simulations

To explore additional empirical implications of our model, we generate and analyze simulated data in this section. We maintain the parameter assumptions as in Table 1 and set $\lambda = 0.75$. For each initial value of $\pi_0 \in \{0, 0.05, 0.1, \dots, 1\}$, we generate a sample of 25,000 paths by generating realizations of θ_t, d_t, and $S_{\theta,t}$. We then compute excess returns ($R_{e,t}$), updated beliefs (π_t), and disagreement based on these realizations and on the numerical solution of the price function.[26] The sample is trimmed at the 1st and 99th percentiles of observations for excess returns to ensure that the empirical specifications are not driven by outliers.

Table 2 provides summary statistics of our main variables of interest. As expected, the mean (10.5%) and median (8.9%) excess return is higher than the excess return of 7.5% in the benchmark rational expectations equilibrium, when there is no uncertainty about others (i.e., when $\pi_t = 1$). Although the autocorrelation in disagreement is positive (0.825), the autocorrelation in excess returns is negative (-0.218). This implies that, for our benchmark parameters, even though excess returns are predictable (as we shall see), they are not positively serially correlated.

Table 3 reports the results from predictive regressions of excess returns ($R_{e,t+1}$) and volatility, as proxied by squared excess returns ($R^2_{e,t+1}$), on lagged excess returns. The regression coefficient of excess returns on lagged excess returns is negative, which implies excess returns exhibit reversals, even though π_t and D_t are persistent. Consistent with our prediction of volatility clustering, both excess returns and volatility are positively related to squared lagged returns (i.e., $R^2_{e,t}$). Moreover, the economic impact is significant: the first and third rows of Table 3 suggest that a one-standard deviation increase in $R^2_{e,t}$ corresponds to a $0.217 \times 0.215 = 4.7$ percentage point increase in average excess returns, and a $0.217 \times 0.326 = 0.07$ (or, equivalently, a 0.326 standard-deviation increase) increase in squared excess returns.

The estimates in Table 3 are also consistent with an asymmetric response to lagged returns. Specifically, the sensitivity of future volatility and excess returns to lagged returns is significantly larger for negative lagged returns (i.e., the coefficient on $R_{e,t}1_{\{R_{e,t}<0\}}$ is negative and significant). This asymmetric reaction partially explains the estimated volatility clustering effect, and is economically meaningful in and of itself. The second and fourth rows of Table 3 imply that a one-standard deviation change in lagged return increases expected excess returns by $0.285 \times 0.346 = 9.9$ percentage points, more if it is negative; the same change in returns increases future squared returns by $0.285 \times 0.770 = 0.2194$ (or, equivalently, one-standard deviation) more when it is negative. The conclusions from the regression analysis are confirmed by the plots in Fig. 7: excess returns and squared excess returns decrease sharply across lower deciles of lagged excess returns, when these are negative, but more gradually across the higher deciles.

These results are consistent with the so-called "leverage" effect — see Black (1976) and the subsequent literature. Of course, the mechanism in our model that generates this asymmetric relation between return moments and lagged returns does not rely on leverage. Instead, it is driven by the asymmetric price reaction to signals that we discussed in Section 3.3. The intuition is as follows. Since the signals are conditionally i.i.d. over time, they are a source of reversals — a large negative realization at date t corresponds to a lower excess return between dates $t-1$ and t, but a higher excess return (and higher volatility) between dates t and $t+1$, on average. However, due to the asymmetric price reaction in our model, these reversals are amplified for negative realizations of $S_{\theta,t}$ — all else equal, the change in returns associated with a negative realization of $S_{\theta,t}$ is larger than for the change in returns associated with a positive realization of the same magnitude, and this generates an asymmetry in the return dynamics.

We next turn to the relation between disagreement and returns generated by our model. The plot in Fig. 8 highlights the nonmonotonicity: disagreement and return moments are positively related when disagreement is low, but negatively related when disagreement is high. The results in Table 4 suggest how the mixed evidence documented in the literature may arise due to misspecification. Since our model generates a nonmonotone relation between disagreement and returns, the linear specifications in Table 4 generate inconsistent estimates based on the sample chosen. For the full sample, the estimated relation is positive, while for the high-disagreement sample, it is negative. A nonlinear specification on the full sample (as reported in the second row) generates a more accurate depiction of the underlying relation between the two variables.

6. Robustness: noisy aggregate supply

In this section, we consider an alternative specification to our benchmark model. We focus on the two-date version (and normalize $R=1$, $\mu=0$) using the same setup as in Section 3, with the two exceptions: N investors are fully rational (i.e., they know their signal is uninformative) and the aggregate supply of the risky asset is stochastic. This specification is useful in highlighting the robustness

[26] As discussed earlier, to ensure nonnegative prices, we set the initial dividend level at large value relative to its volatility (i.e., $D_0 = 1$ and $\sigma = 0.06$). However, since dividends mean-revert, and the growth rate of dividends is relatively low, we restrict our sample to a large cross-section of short price paths (three periods long). This ensures that prices stay positive, and consequently, rates of returns are well-behaved. The results are robust to increasing the length of each price path, if parameters are modified to ensure positive prices.

Table 2
This table reports summary statistics of the simulated data generated by the model. The variables reported are the excess return (i.e., $R_{e,t+1}=Q_{t+1}/P_t$), squared excess returns ($R^2_{e,t+1}$), and the disagreement across investors. The summary statistics reported are the mean, standard deviation, median, 5th and 95th percentiles, and the autocorrelation (AC) in the variable.

Variable	Mean	Std. dev	Median	5th Pctl	95th Pctl	AC
Exc. ret	0.105	0.285	0.089	−0.325	0.585	−0.218
Sq. exc. ret	0.092	0.217	0.032	0.000	0.353	−0.018
Disagreement	0.021	0.013	0.021	0.002	0.039	0.825

Table 3
Predictability in returns. The table reports the regression coefficients from the following specification:

$$y_{t+1}=a+bR_{e,t}+cR^2_{e,t}+dR_{e,t}1_{\{R_{e,t}<0\}}+\varepsilon_{t+1},$$

where y_{t+1} is either the excess return next period (i.e., $R_{e,t+1}$) or the squared excess return next period (i.e., $R^2_{e,t+1}$). Since the returns are from a cross-section of simulations with independent shocks, we report ordinary least squares (OLS) standard errors. The adjusted R^2s are also reported in percentage points.

Specification	Constant	$R_{e,t}$	$R^2_{e,t}$	$R_{e,t}1_{\{R_{e,t}<0\}}$	Adj. R^2
$y_{t+1}=R_{e,t+1}$	0.149	−0.378	0.215		5.801
s.e.	0.001	0.003	0.005		
$y_{t+1}=R_{e,t+1}$	0.123	−0.225	0.093	−0.346	6.037
s.e.	0.001	0.004	0.004	0.011	
$y_{t+1}=R^2_{e,t+1}$	0.170	−0.453	0.326		11.965
s.e.	0.001	0.004	0.008		
$y_{t+1}=R^2_{e,t+1}$	0.112	−0.111	0.056	−0.770	13.701
s.e.	0.001	0.003	0.003	0.010	

of the positive implications of uncertainty about others in a more familiar setting — the model reduces to the noisy rational expectations model of Grossman and Stiglitz (1980) when $\pi_0=1$.

Unlike in the previous specification, N investors know that S_N is not informative about dividends, or equivalently, N investors do not observe a signal prior to submitting orders. The optimal demand for a θ investor is given by

$$x_\theta=\begin{cases}\dfrac{\lambda S_\theta-P}{\alpha\sigma^2(1-\lambda)} & \text{if } \theta=I\\[2mm] \dfrac{0-P}{\alpha\sigma^2} & \text{if } \theta=N.\end{cases} \tag{40}$$

Note that without an additional source of noise, observing price and quantities perfectly reveals both S_θ and θ. Thus, we follow the noisy rational expectations literature and introduce aggregate supply shocks. In particular, the aggregate supply of the risky asset is $Z+z$, where $z\sim \mathrm{N}(0,\sigma_z^2)$.[27] The market clearing condition is given by

$$x_\theta+x_U=Z+z. \tag{41}$$

[27] Alternatively, one could assume that in addition to being potentially informed, θ investors anticipate an endowment to their wealth of zd in the next period, where z is known to θ investors but not to uninformed traders, and $z\sim \mathrm{N}(0,\sigma_z^2)$.

Finally, as in the benchmark model, we assume uninformed traders can condition on the equilibrium price and residual supply when determining their optimal demand. As a result, they can construct a signal

$$y\equiv\alpha\sigma^2(1-\lambda)(x_\theta-z)+P,$$

which is informative about both the type of θ investors and the fundamental dividend shock. Given the optimal demand of the θ investors, y takes the form

$$y=\begin{cases}\lambda S_\theta-\alpha\sigma^2(1-\lambda)z & \text{if } \theta=I\\ \lambda P-\alpha\sigma^2(1-\lambda)z & \text{if } \theta=N,\end{cases} \tag{42}$$

and so U traders can condition on y to update the likelihood of $\theta=I$. Further, conditional on $\theta=I$, U's belief about d is given by

$$\mathbb{E}_U\left[d|y,\theta=I\right]=\lambda_y y,\quad \mathrm{var}_U\left[d|y,\theta=I\right]=\sigma^2(1-\lambda_y),$$

where

$$\lambda_y\equiv\frac{\mathrm{cov}(y,\mathrm{d}|\theta=I)}{\mathrm{var}(y|\theta=I)}=\frac{\lambda\sigma^2}{\lambda\sigma^2+\lambda^2\alpha^2\sigma_\epsilon^4\sigma_z^2}.$$

In this setup, there exists a rational expectations equilibrium which is characterized by the following proposition.

Proposition 12. There exists a rational expectations equilibrium in which the price is given by the solution to

$$P=\underbrace{(\kappa^*+(1-\kappa^*)\pi^*\lambda_y)y}_{\text{expectations}}-\underbrace{\kappa^*\alpha\sigma^2(1-\lambda)Z}_{\text{risk premium}},$$

where $y=\alpha\sigma^2(1-\lambda)(x_\theta-z)+P$, $\pi^*=\Pr(\theta=I|y,P)$, *and*

$$\kappa^*=\frac{\sigma^2(1-\pi^*\lambda_y)+\pi^*(1-\pi^*)(\lambda_y y)^2}{\sigma^2(1-\lambda)+\sigma^2(1-\pi^*\lambda_y)+\pi^*(1-\pi^*)(\lambda_y y)^2}.$$

Analogous to the decomposition in Eq. (11), the price can be decomposed into the expectations and risk-premium components. Fig. 9 illustrates these two components and suggests that uncertainty about whether others are informed has qualitatively similar implications in this setup. As in the static model of Section 3, the expectations component is monotonic in the price signal. Moreover, U traders are unsure about the informativeness of y, which implies that their posterior variance, and therefore, the risk-premium component of price depends on the realization of y. Therefore, as in the benchmark model, the price reacts asymmetrically to good news versus bad news.

In contrast to the benchmark model, uninformed traders learn directly about θ from the signal y; large realizations of y lead to large updates in π^* (either towards zero or one). That is, both uncertainty and learning are present even in the static environment. As a result, the risk-premium component is dampened for large realizations of y, since for these realizations π^* is near zero or one.

7. Final remarks

Asset pricing models primarily focus on uncertainty about the underlying fundamentals and assume that the

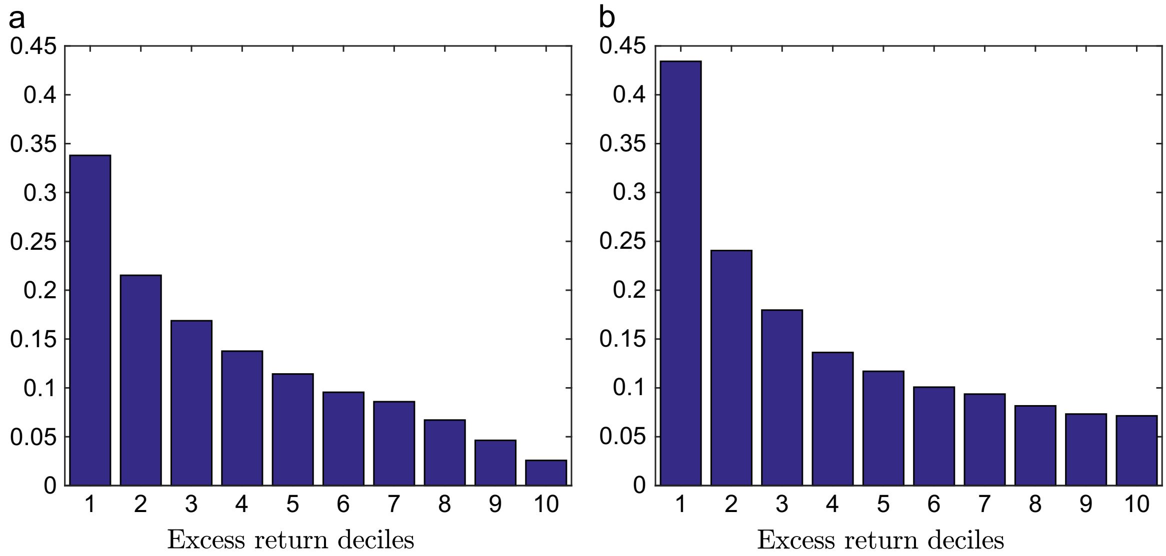

Fig. 7. Average excess returns ($R_{e,t+1}$) and squared excess returns ($R^2_{e,t+1}$) by lagged excess return deciles. (a) Excess return ($R_{e,t+1}$). (a) Squared excess return ($R^2_{e,t+1}$).

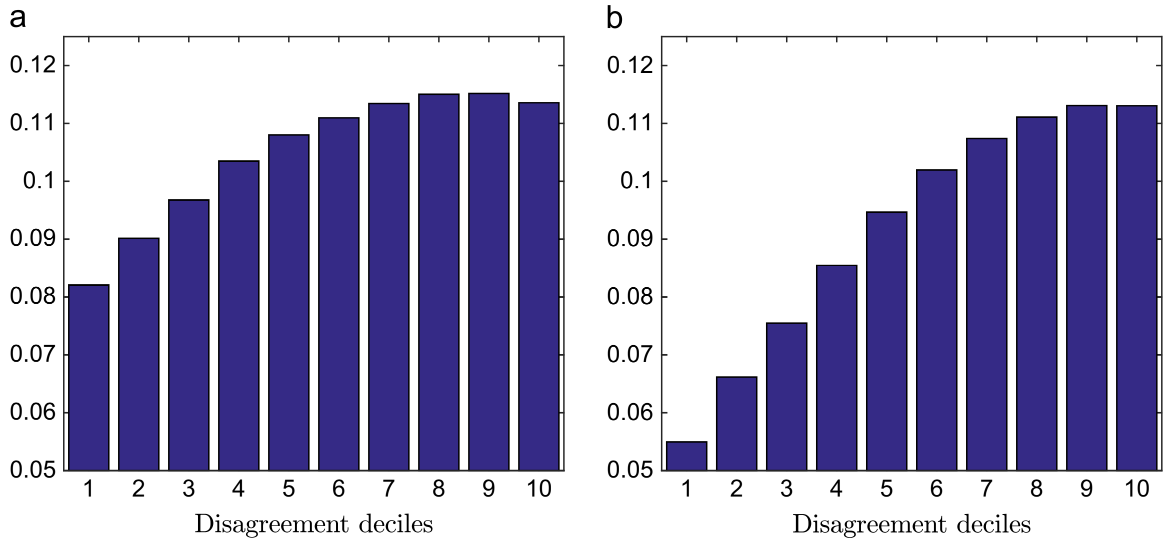

Fig. 8. Average excess returns ($R_{e,t+1}$) and squared excess returns ($R^2_{e,t+1}$) by disagreement deciles. (a) Excess return ($R_{e,t+1}$). (b) Squared excess return ($R^2_{e,t+1}$).

characteristics of other traders in the market are common knowledge. We consider a framework in which investors are uncertain whether others are informed and gradually learn about them by observing prices and dividends. We show that these channels have important implications for return dynamics. Specifically, the model generates nonlinear prices that are more sensitive to bad news than good news; stochastic, predictable expected returns and volatility, that vary with beliefs about other investors; volatility clustering and the "leverage" effect; time-variation in the relation between disagreement, information quality, and returns. From a theoretical perspective, the model connects two widely adopted approaches to modeling belief heterogeneity (RE and DO), and seems, to us, a useful framework for future research.

To illustrate the dynamic implications of uncertainty and learning, we use an OLG setting with mean–variance investors. This is primarily for tractability and to highlight the key forces in a parsimonious way. With long-lived investors, hedging demands would complicate the equilibrium characterization and analysis, though we believe the main forces are robust.

We focus on a setting in which investors are uncertain about whether other traders are informed. However, one could also consider alternative settings in which uninformed investors are uncertain about other characteristics of other traders such as their risk aversion or hedging demands (Section 2 discusses some recent advances along similar lines). The predictions of such models will depend on the exact source of uncertainty, yet a number of similarities should arise: the multi-dimensional uncertainty will generally lead to a nonlinearity in prices, and learning about others should generate rich return dynamics.

In the specifications considered, we assume that either I or N traders are present in the market, but not both. One could instead consider a setting in which both informed investors and noise traders are present simultaneously, but uninformed traders are uncertain about the *proportion* of each type of investor. By conditioning on the information in the residual demand and the price, uninformed traders will be able to update their beliefs about the proportion of informed investors even in a static setting. Conditional on these beliefs, the residual demand provides a noisy signal about the dividend next period, which uninformed traders can use to update their beliefs about fundamentals. A complete analysis of such a model is left for future work.

Table 4

Returns and disagreement. The table reports the regression coefficients from the following specification:

$$R_{e,t+1} = a + bDis_t + cDis_t 1_{\{Dis_t > 80^{th} Pct\}} + \varepsilon_{t+1},$$

where $R_{e,t+1}$ is the excess return at date $t+1$, Dis_t is the disagreement at date t, and $1_{\{Dis_t > 80^{th} Pct\}}$ is an indicator variable for disagreement in the top 80th percentile. The first and second specifications use the entire sample, while the third specification uses only the sample of observations where disagreement is in the top 80th percentile (high-disagreement sample). Since the returns are from a cross-section of simulations with independent shocks, we report ordinary least squares (OLS) standard errors. The adjusted R^2s are also reported in percentage points.

	Full sample			
Specification	Constant	Dis_t	$Dis_t 1_{\{Dis_t > 80^{th} Pct\}}$	Adj. R^2
Coeff.	0.088	0.818		0.129
s.e.	0.001	0.031		
Coeff.	0.084	1.118	−0.315	0.143
s.e.	0.001	0.045	0.040	

	High disagreement sample		
	Constant	Dis_t	Adj. R^2
Coeff.	0.126	−0.306	0.001
s.e.	0.008	0.231	

Appendix A. Proofs

Proof of Proposition 2. First, note that in the static model the optimal demand given in (2) reduces to

$$x_i = \frac{\mu + \mathbb{E}_i[d] - RP}{\alpha \mathrm{var}_i[d]}. \tag{A.1}$$

For θ investors, this can be expressed as

$$x_\theta = \frac{\mu + \lambda S_\theta - RP}{\alpha(1-\lambda)\sigma^2}. \tag{A.2}$$

We argue that any equilibrium must be signal-revealing. If the equilibrium is not signal-revealing, then there must exist two signal realizations, $s_1 > s_2$, for which P is the same. But in this case, from (A.2), the θ investor would demand strictly more after observing s_1, which implies that the U investor can distinguish between s_1 and s_2 using residual demand (i.e., $Z - x_\theta$). Next, since I and N have symmetric optimal strategies, prices and quantities cannot reveal information about θ. Hence, the equilibrium cannot be fully revealing and therefore, U's beliefs about the dividend must be given by (7) and (8). Existence and uniqueness follow by plugging the formulas for the optimal demand of U and θ investors given by (A.1) into the market clearing condition and solving for P as given by (9).□

Proof of Proposition 4. To demonstrate the results, it will be useful to establish the following properties of κ:

$$\begin{aligned}\frac{\partial}{\partial \lambda}\kappa &= \frac{\sigma^2(1-\pi_0)(\pi_0 S_\theta^2(2-\lambda)\lambda + \sigma^2)}{(\sigma^2(1-\lambda)+\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)(\lambda S_\theta)^2)^2} \\ &= \frac{(1-\pi_0)(\pi_0 S_\theta^2(2-\lambda)\lambda+\sigma^2)}{\sigma^2(1-\lambda)^2}(1-\kappa)^2 \geq 0\end{aligned} \tag{A.3}$$

$$\begin{aligned}\frac{\partial}{\partial \pi_0}\kappa &= -\frac{(\sigma^2-(1-2\pi_0)\lambda S_\theta^2)\lambda(1-\lambda)\sigma^2}{(\sigma^2(1-\lambda)+\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)(\lambda S_\theta)^2)^2} \\ &= -\frac{(\sigma^2-(1-2\pi_0)\lambda S_\theta^2)\lambda}{\sigma^2(1-\lambda)}(1-\kappa)^2\end{aligned} \tag{A.4}$$

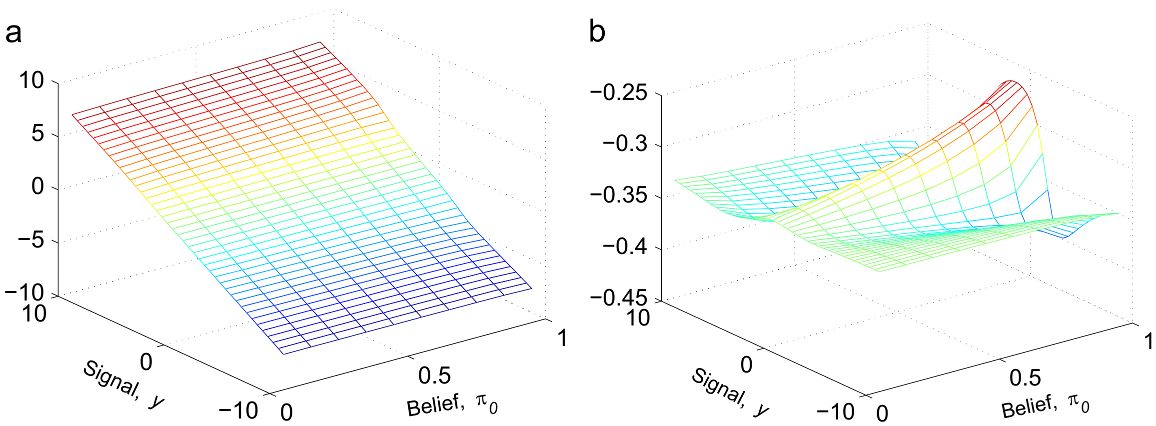

Fig. 9. Using the alternative specification given in Section 6, this figure illustrates the two components of the equilibrium price function as they depend on the price signal y and the prior beliefs π_0. (a) Expectations component of price. (b) Risk premium component of price.

$$\frac{\partial}{\partial S_\theta}\kappa = \frac{2\pi_0(1-\pi_0)(1-\lambda)\lambda^2\sigma^2 S_\theta}{(\sigma^2(1-\lambda)+\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)(\lambda S_\theta)^2)^2} = \frac{2\pi_0(1-\pi_0)\lambda^2 S_\theta}{\sigma^2(1-\lambda)}(1-\kappa)^2 \tag{A.5}$$

which imply κ is (i) increasing in λ, (ii) hump-shaped in π_0 around $\frac{1}{2}\left(1-\sigma^2/(\lambda S_\theta^2)\right)$, and (iii) U-shaped in S_θ around zero.

Effect of λ: The derivative of the expectations component of P with respect to λ is given by

$$\frac{\partial}{\partial\lambda}((\kappa+(1-\kappa)\pi_0))\lambda S_\theta) = \left(\pi_0+(1-\pi_0)\left(\kappa+\lambda\frac{\partial}{\partial\lambda}\kappa\right)\right)S_\theta. \tag{A.6}$$

From (i) above, this component increases with λ for $S_\theta>0$ and decreases in λ otherwise. The derivative of the risk-premium component is given by

$$\frac{\partial}{\partial\lambda}(-\alpha\sigma^2(1-\lambda)\kappa Z) = \frac{\alpha Z\sigma^2\left((-1+\pi_0)^2\pi_0^2 S_\theta^4\lambda^4+2\pi_0\sigma^2 S_\theta^2\lambda\left(-1+\pi_0+3\lambda-3\pi_0\lambda-\lambda^2+\pi_0^2\lambda^2\right)+\sigma^4\left(1+\pi_0^2\lambda^2+\pi_0\left(1-4\lambda+\lambda^2\right)\right)\right)}{\left((-1+\pi_0)\pi_0 S_\theta^2\lambda^2+\sigma^2(-2+\lambda+\pi_0\lambda)\right)^2}. \tag{A.7}$$

The expression can be positive or negative depending on $|S_\theta|$. For $S_\theta=0$ and as $|S_\theta|\to\infty$, the derivative is strictly positive and so the risk-premium component of price increases in λ at these extremes. However, for intermediate values of $|S_\theta|$, the derivative is negative.

Effect of π_0: The derivative of the expectations component of P with respect to π_0 is given by

$$\frac{\partial}{\partial\pi_0}((\kappa+(1-\kappa)\pi_0))\lambda S_\theta) = \left((1-\kappa)+(1-\pi_0)\frac{\partial}{\partial\pi_0}\kappa\right)\lambda S_\theta. \tag{A.8}$$

Inserting the expression from (A.4) for $(\partial/\partial\pi_0)\kappa$ gives

$$\left((1-\kappa)+(1-\pi_0)\frac{\partial}{\partial\pi_0}\kappa\right)\lambda S_\theta = \frac{\sigma^2(1-\lambda)(((1-\pi_0)S_\theta\lambda)^2+2(1-\lambda)\sigma^2)}{(1-\lambda)\sigma^2+(1-\pi_0\lambda)\sigma^2+(\lambda S_\theta)^2(1-\pi_0)}\lambda S_\theta. \tag{A.9}$$

Therefore, the derivative of the expectations component of prices with respect to π_0 has the same sign as S_θ. The risk-premium component of the price is U-shaped in π_0 around $\frac{1}{2}\left(1-\sigma^2/(\lambda S_\theta^2)\right)$. This can be seen by using (ii) above and

$$\frac{\partial}{\partial\pi_0}(-\alpha\sigma^2(1-\lambda)\kappa Z) = -\alpha\sigma^2(1-\lambda)Z\frac{\partial}{\partial\pi_0}\kappa.$$

Effect of S_θ: The expectations component of P is increasing in S_θ. This can be seen by using (iii) above and

$$\frac{\partial}{\partial S_\theta}((\kappa+(1-\kappa)\pi_0))\lambda S_\theta) = (\kappa+(1-\kappa)\pi_0))\lambda+(1-\pi_0)\lambda S_\theta\frac{\partial}{\partial S_\theta}\kappa>0. \tag{A.10}$$

The risk-premium component of the price is hump-shaped in S_θ around zero. This can also be seen by using (iii) above and

$$\frac{\partial}{\partial S_\theta}(-\alpha\sigma^2(1-\lambda)\kappa Z) = -\alpha\sigma^2(1-\lambda)Z\frac{\partial}{\partial S_\theta}\kappa.$$

This completes the proof of the comparative static results.□

Proof of Corollary 5. Follows immediately from Proposition 4 since (i) the derivative of the expectations component (with respect to) S_θ is symmetric around zero and (ii) the derivative of the risk-premium component is increasing for positive signal levels and decreasing for negative ones.□

Proof of Proposition 6. Let $P(s)$ denote the equilibrium price (as given by (11)) for an arbitrary signal realization s, and similarly for $\kappa(s)$ (which is given by (10)). Note that for $0<s_1<s_2$, the difference in the price is given by

$$P(s_2)-P(s_1) = \frac{1}{R}\begin{pmatrix}(\kappa(s_2)+(1-\kappa(s_2))\pi_0)\lambda s_2\\ -(\kappa(s_1)+(1-\kappa(s_1))\pi_0)\lambda s_1\\ -\alpha\sigma^2(1-\lambda)Z(\kappa(s_2)-\kappa(s_1))\end{pmatrix}. \tag{A.11}$$

Since $\kappa(s_2)>\kappa(s_1)$ (see proof of Proposition 4), setting

$$\gamma = \frac{(\kappa(s_2)+(1-\kappa(s_2))\pi_0)\lambda s_2-(\kappa(s_1)+(1-\kappa(s_1))\pi_0)\lambda s_1}{\sigma^2(1-\lambda)(\kappa(s_2)-\kappa(s_1))} \tag{A.12}$$

gives the result.□

Proof of Proposition 7. The expressions for the conditional expected return and volatility follow from the observation that the only source of randomness in returns, conditional on P and x_θ, is the realization of the dividend d. In particular, this implies that $\mathrm{var}[Q|P,x_\theta]=\mathrm{var}_U[d|P,x_\theta]$. To derive the expression for unconditional expected return, take the expectation of the right-hand side (RHS) of (14) and using that $\mathbb{E}_U[d]=\pi_0\lambda S_\theta$, we have

$$\mathbb{E}[Q]=\mathbb{E}[\mathbb{E}[Q|P,x_\theta]] = \alpha\sigma^2(1-\lambda)Z\mathbb{E}[\kappa]-(1-\pi_0)\lambda\mathbb{E}[\kappa S_\theta], \tag{A.13}$$

Thus, it suffices to show that $\mathbb{E}[\kappa S_\theta]=0$. For this, note that $\kappa\cdot S_\theta$ is an odd-function (of S_θ) and the distribution of S_θ is symmetric around zero. Thus, $\mathbb{E}[\kappa S_\theta|S_\theta>0] = -\mathbb{E}[\kappa S_\theta|S_\theta<0]$, which implies $\mathbb{E}[\kappa S_\theta]=0$.

For unconditional volatility of returns, we have that

$$\mathrm{var}[Q]=\mathbb{E}[\mathrm{var}[Q|P,x_\theta]]+\mathrm{var}[\mathbb{E}[Q|P,x_\theta]] \tag{A.14}$$

$$\mathrm{var}[Q]=\begin{matrix}\mathbb{E}[\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)(\lambda S_\theta)^2]\\ +\mathrm{var}\left[\alpha\sigma^2(1-\lambda)\kappa Z-(1-\pi_0)\kappa\lambda S_\theta\right]\end{matrix} \tag{A.15}$$

$$\mathrm{var}[Q]=\begin{array}{c}\sigma^2(1-\pi_0^2\lambda)+(\alpha\sigma^2(1-\lambda)Z)^2\mathrm{var}[\kappa]\\ +(1-\pi_0)^2\lambda^2\mathrm{var}[\kappa S_\theta]\\ -2\alpha\sigma^2(1-\lambda)\lambda Z(1-\pi_0)\mathrm{cov}(\kappa,\kappa S_\theta).\end{array}\tag{A.16}$$

Stein's Lemma implies that for $Y\sim\in(0,\sigma_Y^2)$, and g(Y) such that $\mathbb{E}[g(Y)Y]<\infty$ and $\sigma_Y^2\mathbb{E}[g'(Y)]<\infty$, we have $\mathrm{cov}(g(Y),X)=\mathbb{E}[g'(Y)]\mathrm{cov}(Y,X)$. Therefore,

$$\mathrm{cov}(\kappa,S_\theta)=\mathbb{E}\left[\frac{\partial}{\partial S_\theta}\kappa\right]\mathrm{var}(S_\theta)\tag{A.17}$$

$$\mathrm{var}[\kappa S_\theta]=\mathbb{E}[\kappa^2S_\theta^2]-(\mathbb{E}[\kappa S_\theta])^2\tag{A.18}$$

$$\mathrm{var}[\kappa S_\theta]=\mathrm{cov}(\kappa^2S_\theta,S_\theta)-\mathrm{cov}(\kappa,S_\theta)\tag{A.19}$$

$$\mathrm{var}[\kappa S_\theta]=\left(\mathbb{E}\left[\kappa^2+2\kappa S_\theta\frac{\partial}{\partial S_\theta}\kappa\right]-\mathbb{E}\left[\frac{\partial}{\partial S_\theta}\kappa\right]\right)\mathrm{var}(S_\theta)\tag{A.20}$$

$$\mathrm{cov}(\kappa,\kappa S_\theta)=\mathbb{E}[\kappa^2S_\theta]-\mathbb{E}[\kappa]\mathbb{E}[\kappa S_\theta]\tag{A.21}$$

$$\mathrm{cov}(\kappa,\kappa S_\theta)=\mathrm{cov}(\kappa^2,S_\theta)-\mathbb{E}[\kappa]\mathrm{cov}(\kappa,S_\theta)\tag{A.22}$$

$$\mathrm{cov}(\kappa,\kappa S_\theta)=\left(\mathbb{E}\left[2\kappa\tfrac{\partial}{\partial S_\theta}\kappa\right]-\mathbb{E}[\kappa]\mathbb{E}\left[\frac{\partial}{\partial S_\theta}\kappa\right]\right)\mathrm{var}(S_\theta).\tag{A.23}$$

Since $(\partial/\partial S_\theta)\kappa(S_\theta)=-(\partial/\partial S_\theta)\kappa(-S_\theta)$, we have that $\mathbb{E}[(\partial/\partial S_\theta)\kappa]=0$, and $\mathbb{E}[\kappa(\partial/\partial S_\theta)\kappa]=0$. This implies that volatility can be expressed as

$$\mathrm{var}[Q]=\begin{array}{c}\sigma^2(1-\pi_0^2\lambda)+(1-\pi_0)^2\lambda^2\mathrm{var}[\kappa S_\theta]\\ +(\alpha\sigma^2(1-\lambda)Z)^2\mathrm{var}[\kappa]\end{array}\tag{A.24}$$

since $\lambda=\sigma^2/\mathrm{var}(S_\theta)$.□

Proof of Proposition 8. Optimality of $x_{i,t}$ follows from (2), the expressions for beliefs are given by (6)– (7), and the expression for the price follows from the market clearing condition.□

Proof of Proposition 9. One can conjecture and verify the specified price function in each case. In particular, suppose $P_{t+1}=A\mu+BD_{t+1}+CS_{\theta,t+1}+F$. Since $S_{\theta,t+1}$ is uncorrelated with d_{t+1}, we have that optimal demand for investor i is given by

$$x_{i,t}=\frac{\mathbb{E}_{i,t}[P_{t+1}+D_{t+1}]-RP_t}{\mathrm{var}_{i,t}[P_{t+1}+D_{t+1}]}\tag{A.25}$$

$$x_{i,t}=\frac{A\mu+(B+1)((1-\rho)\mu+\rho D_t+\mathbb{E}_{i,t}[d_{t+1}])+F-RP_t}{(B+1)^2\ \mathrm{var}_{i,t}[d_{t+1}]+C^2(\sigma^2+\sigma_\varepsilon^2)}.\tag{A.26}$$

This implies that the equilibrium is signal-revealing, since the optimal demand for θ investors is linear in $S_{\theta,t}$. Moreover, note that for $\pi_t=1$ and $\theta=I$, we have

$$\mathbb{E}_{i,t}[d_{t+1}]=\lambda S_{\theta,t}\quad\text{and}\tag{A.27}$$

$$\mathrm{var}_{i,t}[d_{t+1}]=\sigma^2(1-\lambda),\tag{A.28}$$

for $i\in\{U,\theta\}$, while for $\pi_t=0$ and $\theta=N$, we have

$$\mathbb{E}_{\theta,t}[d_{t+1}]=\lambda S_{\theta,t}\quad\text{and}\tag{A.29}$$

$$\mathrm{var}_{\theta,t}[d_{t+1}]=\sigma^2(1-\lambda),\tag{A.30}$$

$$\mathbb{E}_{U,t}[d_{t+1}]=0\quad\text{and}\tag{A.31}$$

$$\mathrm{var}_{U,t}[d_{t+1}]=\sigma^2.\tag{A.32}$$

Plugging in these beliefs into the optimal demand for each type of investor, and imposing the market clearing condition (i.e., $x_{U,t}+x_{\theta,t}=Z$) verifies the conjectured linear form. Matching coefficients implies uniqueness of the equilibrium. In particular, for $\pi_t=1$ and $\theta=I$, we have

$$C=\frac{\lambda}{R-\rho},\tag{A.33}$$

while for $\pi_t=0$ and $\theta=N$, we can show that C is the solution to the cubic equation:

$$C=\frac{\lambda\left(R^2\lambda+C^2(R-\rho)^2\right)}{\left(R^2(2-\lambda)\lambda+2C^2(R-\rho)^2\right)(R-\rho)}.\tag{A.34}$$

Since the discriminant of the above equation is less than zero, there is one real solution, which pins down the unique linear equilibrium in this case. The expressions for expected returns and volatility in returns can be verified by plugging in the expression for price and computing the moments.□

Proof of Proposition 10. Consider first the (T-period) finite horizon model in which all dividends and trading terminates at some arbitrary date T with $P_T=0$. We first establish that, for any T, there is a unique equilibrium price for all $t<T$, which is given by

$$P_t=A_t\mu+B_tD_t+p_t(S_{\theta,t}),\tag{A.35}$$

in which

$$B_t=\frac{\rho}{R}(1+B_{t+1}),\tag{A.36}$$

$$A_t=\frac{1}{R}(A_{t+1}+(1-\rho)(1+B_{t+1}))\quad\text{and}\tag{A.37}$$

$$p_t(S_{\theta,t})=\frac{1}{R}\left\{\begin{array}{c}\overline{\mathbb{E}}_t[(1+B_{t+1})d_{t+1}+p_{t+1}(S_{\theta,t+1})]\\ -\alpha\kappa_t\mathrm{var}_{\theta,t}[(1+B_{t+1})d_{t+1}+p_{t+1}(S_{\theta,t+1})]Z\end{array}\right\},\tag{A.38}$$

$$\kappa_t=\frac{\mathrm{var}_{U,t}[(1+B_{t+1})d_{t+1}+p_{t+1}(S_{\theta,t+1})]}{\mathrm{var}_{U,t}[(1+B_{t+1})d_{t+1}+p_{t+1}(S_{\theta,t+1})]+\mathrm{var}_{\theta,t}[(1+B_{t+1})d_{t+1}+p_{t+1}(S_{\theta,t+1})]}\tag{A.39}$$

whereand $\overline{\mathbb{E}}_{i,t}[\cdot]=\kappa_t\mathbb{E}_{\theta,t}[\cdot]+(1-\kappa_t)\mathbb{E}_{U,t}[\cdot]$. We shall establish the claim by induction.

Base step. The terminal date is T at which point $P_T=0$ and so P_{T-1} is given by

$$P_{T-1}=\frac{1}{R}\left(\begin{array}{c}(1-\rho)\mu+\rho D_{T-1}+\overline{E}_{i,T-1}[d_T]\\ -\alpha\kappa_{T-1}\mathrm{var}_{\theta,t}[d_T]Z\end{array}\right)\tag{A.40}$$

$$P_{T-1} = A_{T-1}\mu + B_{T-1}D_{T-1} + p_{T-1}(S_{\theta,T-1}). \tag{A.41}$$

Hence A_{T-1}, B_{T-1} and p_{T-1} satisfy the conjectured recursion. Also, note that $\mathbb{E}_{i,T-1}[p_T(S_{\theta,T})] = \text{var}_{i,T-1}[p_T(S_{\theta,T})] = 0$ for $i \in \{U, \theta\}$.

Recursive step. Suppose the price in the next period satisfies $P_{t+1} = A_{t+1}\mu + B_{t+1}D_{t+1} + p_{t+1}(S_{\theta,t+1})$, and $|\mathbb{E}_{i,t}[p_{t+1}(S_{\theta,t+1})]| \le m_t$ and $\text{var}_{i,t}[p_{t+1}(S_{\theta,t+1})] < v_t$ for $i \in \{U, \theta\}$ for some $m_t, v_t < \infty$. Then,

$$x_{i,t} = \frac{\mathbb{E}_{i,t}[P_{t+1}+D_{t+1}] - RP_t}{\alpha \text{var}_{i,t}[P_{t+1}+D_{t+1}]} \tag{A.42}$$

$$x_{i,t} = \frac{\mathbb{E}_{i,t}[A_{t+1}\mu + (1+B_{t+1})D_{t+1} + p_{t+1}(S_{\theta,t+1})] - RP_t}{\alpha \text{var}_{i,t}[(1+B_{t+1})D_{t+1} + p_{t+1}(S_{\theta,t+1})]}. \tag{A.43}$$

Market clearing then requires that

$$P_t = \frac{1}{R}\begin{pmatrix} (A_{t+1} + (1+B_{t+1})(1-\rho))\mu + (1+B_{t+1})\rho D_t \\ +\overline{\mathbb{E}}_{i,t}[(1+B_{t+1})d_{t+1} + p_{t+1}(S_{\theta,t+1})] \\ -\alpha\kappa_t \text{var}_{\theta,t}[(1+B_{t+1})d_{t+1} + p_{t+1}(S_{\theta,t+1})]Z \end{pmatrix} \tag{A.44}$$

$$P_t \equiv A_t\mu + B_t D_t + p_t(S_{\theta,t}), \tag{A.45}$$

which verifies (A.35)–(A.38) hold in period t. Note that $S_{\theta,t+1}$ is independent of d_{t+1}. Then, $p_t(S_{\theta,t})$ is given by

$$p_t(S_{\theta,t}) = \frac{1}{R}\left\{ \begin{matrix} \overline{\mathbb{E}}_t[(1+B_{t+1})d_{t+1} + p_{t+1}(S_{\theta,t+1})] \\ -\alpha\kappa_t \text{var}_{\theta,t}[(1+B_{t+1})d_{t+1} + p_{t+1}(S_{\theta,t+1})]Z \end{matrix} \right\} \tag{A.46}$$

$$p_t(S_{\theta,t}) \le \frac{1}{R}\begin{pmatrix} (1+B_{t+1})(\kappa_t + (1-\kappa_t)\pi_0)S_{\theta,t} \\ +m_t + \alpha Z\left(\sigma^2(1+B_{t+1})^2 + v_t\right) \end{pmatrix}, \tag{A.47}$$

where the inequality follows from $\kappa_t \le 1$, $\text{var}_{\theta,t}[(1+B_{t+1})d_{t+1}] \le \sigma^2(1+B_{t+1})^2$, $|\mathbb{E}_{i,t}[p_{t+1}(S_{\theta,t+1})]| \le m_t$, $\text{var}_{i,t}[p_{t+1}(S_{\theta,t+1})] < v_t$, and the fact that p_{t+1} and d_{t+1} are independent. Taking the expectation gives

$$\left|\mathbb{E}_{i,t-1}[p_t(S_{\theta,t})]\right| \le \frac{1}{R}\left(m_t + \alpha Z\left(\sigma^2(1+B_{t+1})^2 + v_t\right)\right) \equiv m_{t-1}, \tag{A.48}$$

since $\mathbb{E}[(\kappa_t + (1-\kappa_t)\pi_0)S_{\theta,t}] = 0$ (recall κ_t is an odd-function of $S_{\theta,t}$ and $S_{\theta,t}$ is mean-zero). For the variance term, we have that

$$\text{var}_{i,t-1}[p_t(S_{\theta,t})] = (1+B_{t+1})^2 \text{var}_{i,t-1}[(\kappa_t + (1-\kappa_t)\pi_0)S_{\theta,t}] \tag{A.49}$$

$$\text{var}_{i,t-1}[p_t(S_{\theta,t})] \le (1+B_{t+1})^2(\sigma^2 + \sigma_\varepsilon^2) \equiv v_{t-1}, \tag{A.50}$$

where the inequality follows from $\mathbb{E}_{i,t-1}[(\kappa_t + (1-\kappa_t)\pi_0)^2 S_{\theta,t}^2] \le \mathbb{E}_{i,t-1}[S_{\theta,t}^2]$.

Stationary solution to the infinite horizon model with i.i.d. θ. Now we turn to the proof of existence in the infinite horizon. A stationary equilibrium requires that A_t and B_t be time-invariant, which from (A.37) implies that $A = R(1-\rho)/(R-1)(R-\rho)$, and $B = \rho/(R-\rho)$. Assuming they are well defined (which will be verified shortly), denote $m = \mathbb{E}_{i,t}[p_{t+1}]$ and $v = \text{var}_{i,t+1}[p_{t+1}]$ both of which can be made independent of t in a stationary equilibrium. Note that $\mathbb{E}[\kappa_t S_{\theta,t}] = 0$ and $\text{cov}[\kappa_t S_{\theta,t}, \kappa_t] = 0$, since κ_t is even in $S_{\theta,t}$, and also that $\text{var}[\kappa_t S_{\theta,t}] \le \sigma^2 + \sigma_\varepsilon^2$ and $\text{var}[\kappa_t] \le 1$, since $\kappa_t \le 1$. Finally, note that d_{t+1} and p_{t+1} are uncorrelated. Then,

$$m = \mathbb{E}_i[p_t] \tag{A.51}$$

$$= \mathbb{E}_i\left[\frac{1}{R}\left\{ \begin{matrix} \overline{\mathbb{E}}_t[(1+B)d_{t+1} + p_{t+1}] \\ -\alpha\kappa_t \text{var}_{\theta,t}[(1+B)d_{t+1} + p_{t+1}]Z \end{matrix} \right\}\right] \tag{A.52}$$

$$= \frac{1}{R}\left\{ \mathbb{E}\left[\begin{matrix} (1+B)(\kappa_t + (1-\kappa_t)\pi_0)\lambda S_{\theta,t} + m \\ -\alpha\kappa_t((1+B)^2\sigma^2(1-\lambda) + v)Z \end{matrix} \right] \right\} \tag{A.53}$$

which implies

$$m = -\frac{1}{R-1}\left((1+B)^2\sigma^2(1-\lambda) + v\right)\alpha Z \mathbb{E}[\kappa_t] \tag{A.54}$$

$$< \infty. \tag{A.55}$$

Let $\sigma_E^2 \equiv \text{var}[(\kappa_t + (1-\kappa_t)\pi_0)S_{\theta,t}]$ and $\sigma_\kappa^2 = \text{var}[\kappa]$. Then, we have

$$v = \text{var}_i\left[\frac{1}{R}\begin{pmatrix} (1+B)(\kappa_t + (1-\kappa_t)\pi_0)\lambda S_{\theta,t} + m \\ -\alpha\kappa_t((1+B)^2\sigma^2(1-\lambda) + v)Z \end{pmatrix}\right] \tag{A.56}$$

$$= \frac{1}{R^2}\begin{pmatrix} (1+B)^2\lambda^2\sigma_E^2 \\ +\alpha^2((1+B)^2\sigma^2(1-\lambda) + v)^2 Z^2 \sigma_\kappa^2 \end{pmatrix} \tag{A.57}$$

$$\Rightarrow v = J(v), \tag{A.58}$$

where

$$J(v) = \frac{1}{R^2}\begin{pmatrix} (1+B)^2\lambda^2\sigma_E^2 \\ +\alpha^2((1+B)^2\sigma^2(1-\lambda) + v)^2 Z^2 \sigma_\kappa^2 \end{pmatrix}. \tag{A.59}$$

Since, $\sigma_E^2 \le \sigma^2 + \sigma_\varepsilon^2$ and $\sigma_\kappa^2 \le 1$, we have that $J(v)$ is bounded above by the quadratic function

$$J(v) \le F + G(v+H)^2 \tag{A.60}$$

where $F = (1/R^2)(1+B)^2\lambda^2(\sigma^2 + \sigma_\varepsilon^2)$, $G = (1/R^2)\alpha^2 Z^2$, and $H = (1+B)^2\sigma^2(1-\lambda)$. The solution to the quadratic equation $v = F + G(v+H)^2$ is given by

$$v^* = \frac{1 - 2GH \pm \sqrt{1 - 4G(F+H)}}{2G} \tag{A.61}$$

and a sufficient condition for existence is

$$1 - 4G(F+H) \ge 1 - \frac{4\alpha^2 Z^2 \sigma^2}{(R-\rho)^2} > 0, \tag{A.62}$$

since $F + H = (1/R^2)(1+B)^2\sigma^2$. Under this sufficient condition, we know that $J(v^*) \le v^*$. Noting that $J(\cdot)$ is continuous on $\mathbb{R}_+$ and $J(0) > 0$, the intermediate value theorem implies there exists a solution to $J(v) = v$, and consequently, an equilibrium.

The equilibrium price is then given by

$$P_t = A\mu + BD_t + \frac{1}{R}\begin{pmatrix} (1+B)(\kappa_t + (1-\kappa_t)\pi_0)\lambda S_{\theta,t} \\ +m - \alpha\kappa_t((1+B)^2\sigma^2(1-\lambda) + v)Z \end{pmatrix}, \tag{A.63}$$

where A, B, m and v are characterized above. The equilibrium is signal-revealing in this case, since the optimal demand of the θ investor is linear in $S_{\theta,t}$.□

Proof of Proposition 11. Again, the claim will be established by backward induction on T.

Base step. The terminal date is T (i.e., $P_T=0$) and so P_{T-1} is given by

$$P_{T-1}=\frac{1}{R}\begin{pmatrix}(1-\rho)\mu+\rho D_{T-1}\\ +\overline{E}_{i,T-1}[d_T]-\alpha\kappa_{T-1}\text{var}_{\theta,t}[d_T]Z\end{pmatrix} \tag{A.64}$$

$$P_{T-1}=A_{T-1}\mu+B_{T-1}D_{T-1}+p_{T-1}(S_{\theta,T-1}), \tag{A.65}$$

since $A_T=B_T=p_T=0$. Also, note that $\mathbb{E}_{i,T-1}[p_T(S_{\theta,T})|\pi_T]=\text{var}_{i,T-1}[p_T(S_{\theta,T})]=0$ for $i\in\{U,\theta\}$.

Recursive step. Suppose the price in the next period satisfies $P_{t+1}=A_{t+1}\mu+B_{t+1}D_{t+1}+p_{t+1}(S_{\theta,t+1},\pi_{t+1})$, and $|\mathbb{E}_{i,t}[p_{t+1}(S_{\theta,t+1},\pi_{t+1})|d_{t+1}]|\le m_t$ and $\text{var}_{i,t}[p_{t+1}(S_{\theta,t+1},\pi_{t+1})]|\le v_t$ for some $m_t,v_t\le\infty$ and all d_{t+1}. Note that the first constraint implies that $|\mathbb{E}_{i,t}[p_{t+1}(S_{\theta,t+1},\pi_{t+1})]|\le m_t$. The optimal demand is given by

$$x_{i,t}=\frac{\mathbb{E}_{i,t}[P_{t+1}+D_{t+1}]-RP_t}{\alpha\text{var}_{i,t}[P_{t+1}+D_{t+1}]} \tag{A.66}$$

$$x_{i,t}=\frac{\mathbb{E}_{i,t}[A_{t+1}\mu+(1+B_{t+1})D_{t+1}+p_{t+1}(S_{\theta,t+1},\pi_{t+1})]-RP_t}{\alpha\text{var}_{i,t}[(1+B_{t+1})D_{t+1}+p_{t+1}(S_{\theta,t+1},\pi_{t+1}]}. \tag{A.67}$$

Market clearing implies $\sum_i x_{i,t}=Z$, or equivalently,

$$P_t=\frac{1}{R}\begin{pmatrix}(A_{t+1}+(1+B_{t+1})(1-\rho))\mu+(1+B_{t+1})\rho D_t\\ +\overline{\mathbb{E}}_{i,t}[(1+B_{t+1})d_{t+1}+p_{t+1}(S_{\theta,t+1},\pi_{t+1})]\\ -\alpha\kappa_t\text{var}_{\theta,t}[(1+B_{t+1})d_{t+1}+p_{t+1}(S_{\theta,t+1},\pi_{t+1})]Z\end{pmatrix} \tag{A.68}$$

$$P_t\equiv A_t\mu+B_tD_t+p_t(S_{\theta,t},\pi_t), \tag{A.69}$$

which verifies our conjectured form. To verify the price is well defined, we need to confirm that the conditional expectation and variance of $p_t(\cdot)$ is bounded. If the equilibrium is signal-revealing, it must be that:

$$p_t=\frac{1}{R}\begin{pmatrix}\overline{\mathbb{E}}_t[(1+B_{t+1})d_{t+1}+p_{t+1}]\\ -\alpha\kappa_t\text{var}_{\theta,t}[(1+B_{t+1})d_{t+1}+p_{t+1}]Z\end{pmatrix} \tag{A.70}$$

$$p_t=\frac{1}{R}\begin{pmatrix}(1+B_{t+1})(\kappa_t+(1-\kappa_t)\pi_t)\lambda S_{\theta,t}\\ +\overline{\mathbb{E}}_t[p_{t+1}]\\ -\alpha\kappa_t\text{var}_{\theta,t}[(1+B_{t+1})d_{t+1}+p_{t+1}]Z\end{pmatrix}. \tag{A.71}$$

Given our conjecture,

$$\text{var}_{\theta,t}[(1+B_{t+1})d_{t+1}+p_{t+1}] \tag{A.72}$$

$$=\begin{matrix}(1+B_{t+1})^2\text{var}_{\theta,t}[d_{t+1}]+\text{var}_{\theta,t}[p_{t+1}]\\ +2(1+B_{t+1})\text{cov}_{\theta,t}[\text{d}_{t+1},\text{p}_{t+1}]\end{matrix} \tag{A.73}$$

$$\le(1+B_{t+1})^2\sigma^2+v_t+2(1+B_{t+1})\sigma\sqrt{v_t} \tag{A.74}$$

$$\equiv V_t. \tag{A.75}$$

But this implies that

$$\begin{aligned}&\mathbb{E}_{i,t-1}[\kappa_t\text{var}_{\theta,t}[(1+B_{t+1})d_{t+1}+p_{t+1}]|d_t]\\ &\quad\le V_t+\sqrt{V_t},\end{aligned} \tag{A.76}$$

since $\kappa_t\in[0,1]$. Since $\mathbb{E}_{i,t-1}[(\kappa_t+(1-\kappa_t)\pi_t)\lambda S_{\theta,t}|d_t]=0$ (κ_t is odd and $S_{\theta,t}$ is mean-zero), we have

$$\left|\mathbb{E}_{i,t-1}[p_t|d_t]\right|\le\frac{1}{R}\left(m_t+\alpha Z(V_t+\sqrt{V_t})\right) \tag{A.77}$$

$$|\mathbb{E}_{i,t-1}[p_t|d_t]|\equiv m_{t-1}, \tag{A.78}$$

which verifies that the conditional mean of the price is bounded at date $t-1$. Next, note that

$$\text{var}_{i,t-1}[p_t|d_t]\le\frac{1}{R^2}\begin{pmatrix}(1+B_{t+1})^2\lambda^2(\sigma^2+\sigma_\varepsilon^2)\\ +m_t^2+\alpha^2Z^2V_t^2\\ +2(1+B_{t+1})\lambda\sqrt{\sigma^2+\sigma_\varepsilon^2}(m_t+\alpha ZV_t)\\ +2m_t\alpha ZV_t\end{pmatrix} \tag{A.79}$$

$$\text{var}_{i,t-1}[p_t|d_t]\equiv W_t. \tag{A.80}$$

But this implies that by applying the law of total variance, we have

$$\text{var}_{i,t-1}[p_t]=\mathbb{E}_{i,t-1}[\text{var}_{i,t-1}[p_t|d_t]]+\text{var}_{i,t-1}[\mathbb{E}_{i,t-1}[p_t|d_t]] \tag{A.81}$$

$$\text{var}_{i,t-1}[p_t]\le W_t+m_{t-1}^2\equiv v_{t-1}, \tag{A.82}$$

which verifies that conditional variance of the price is bounded in period $t-1$.□

Proof of Proposition 12. Given the distribution of y in Eq. (42), the U investor can use the "signal," y, to learn about θ. In particular, her updated belief conditional on (y,P) is given by

$$\pi^*(y,P)=\frac{\frac{\pi_0}{\sqrt{\sigma_{Y,I}^2}}\phi\left(\frac{y}{\sqrt{\sigma_{Y,I}^2}}\right)}{\frac{\pi_0}{\sqrt{\sigma_{Y,I}^2}}\phi\left(\frac{y}{\sqrt{\sigma_{Y,I}^2}}\right)+\frac{1-\pi_0}{\sqrt{\sigma_{Y,NI}^2}}\phi\left(\frac{y-\lambda P}{\sqrt{\sigma_{Y,NI}^2}}\right)}, \tag{A.83}$$

where

$$\sigma_{Y,I}^2=\lambda^2(\sigma^2+\sigma_\varepsilon^2)+\alpha^2\sigma^4(1-\lambda)^2\sigma_z^2 \tag{A.84}$$

$$\sigma_{Y,NI}^2=\alpha^2\sigma^4(1-\lambda)^2\sigma_z^2. \tag{A.85}$$

Conditional on $\theta=I$, U's belief about d is given by

$$\mathbb{E}_U[d|y,\theta=I]=\lambda_y y, \tag{A.86}$$

$$\text{var}_U[d|y,\theta=I]=\sigma^2(1-\lambda_y), \tag{A.87}$$

where

$$\lambda_y=\frac{\text{cov(y, d)}}{\text{var}(y)}=\frac{\lambda\sigma^2}{\sigma_{Y,I}^2}=\frac{\lambda\sigma^2}{\lambda\sigma^2+\lambda^2\alpha^2\sigma_\varepsilon^4\sigma_z^2}. \tag{A.88}$$

Optimal demand for the U investor is then given by

$$x_U(x_\theta-z,P)=\frac{1}{\alpha}\frac{\pi^*\lambda_y y-P}{\pi^*\sigma^2(1-\lambda_y)+(1-\pi^*)\sigma^2+\pi^*(1-\pi^*)(\lambda_y y)^2}. \tag{A.89}$$

The market clearing condition (41) implies that the equilibrium price, P, can be implicitly characterized as the solution to the following equation:

$$x_U(x_\theta-z,P)=Z-(x_\theta-z). \tag{A.90}$$

Note that since

$$\frac{\partial \pi^*}{\partial P} = \frac{\pi_0(1-\pi_0)}{\sigma^2_{Y,I}\sigma^2_{Y,NI}}\left(\begin{array}{c}\alpha\sigma^2(1-\lambda)(x_\theta - z)\left(\sigma^2_{Y,I}(1-\lambda)-\sigma^2_{Y,NI}\right)\\ +P\left(\sigma^2_{Y,I}(1-\lambda)^2-\sigma^2_{Y,NI}\right)\end{array}\right), \tag{A.91}$$

for any realization of $x_\theta - z$, we have

- If $\sigma^2_{Y,I}(1-\lambda)^2-\sigma^2_{Y,NI}>0$, the derivative is increasing in P and, for large enough values of P, it is positive, which implies $\lim_{|P|\to\infty}\pi^*=1$, and consequently, $\lim_{|P|\to\infty} x_U = \frac{\lambda_y y - P}{\alpha\sigma^2(1-\lambda_y)}$.
- If $\sigma^2_{Y,I}(1-\lambda)^2-\sigma^2_{Y,NI}<0$, the derivative is decreasing in P_t and, for large enough P_t, it is negative. This implies that $\lim_{|P|\to\infty}\pi^*=0$, and consequently, $\lim_{|P|\to\infty} x_U = \frac{0-P}{\alpha\sigma^2}$.

Since x_U is continuous in P and π^*, this implies that in either case, there exists a P that satisfies Eq. (A.90). Rearranging Eq. (A.90) gives the expression for the price in the proposition.□

Appendix B. Supplementary analysis

B.1. Comparative statics on return moments

To investigate comparative statics, we start by presenting the following result.

Proposition 13. In the static model,

(i) *The unconditional expected return is homogeneous of degree one* (HD1) *in σ^2 and αZ.*
(ii) *The unconditional volatility component due to fundamental shocks is* HD1 *in σ^2 and* HD0 *in αZ.*
(iii) *The unconditional volatility component due to the expectations component of returns is* HD1 *in σ^2 and* HD0 *in αZ.*
(iv) *The unconditional volatility component due to the risk premium component of returns is* HD2 *in σ^2 and αZ.*

Proof of Proposition 13. It suffices to show that $\mathbb{E}[\kappa]$ and var$[\kappa]$ are HD0 in σ^2, while var$[\kappa S_\theta]$ is HD1 in σ^2. Recall that

$$\kappa = \frac{\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda^2 S_\theta^2}{\sigma^2(1-\lambda)+\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda^2 S_\theta^2} \tag{B.1}$$

and by definition, $\lambda = \sigma^2/(\sigma^2+\sigma^2_\epsilon)$ and $S_\theta \sim \mathrm{N}(0,\sigma^2+\sigma^2_\epsilon) = \mathrm{N}(0,\sigma^2/\lambda)$, we have

$$\mathbb{E}[\kappa] = \frac{1}{\sqrt{2\pi\sigma^2/\lambda}} \int_{-\infty}^{\infty} \frac{\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda^2 s^2}{\sigma^2(1-\lambda)+\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda^2 s^2} e^{(-s^2/2\sigma^2/\lambda)}\, ds. \tag{B.2}$$

Using a change of variables, by letting $x \equiv (\sqrt{\lambda}/\sigma)s$, we get that

$$E[\kappa] = \frac{1}{\sqrt{2\pi\sigma^2/\lambda}}\frac{\sigma}{\sqrt{\lambda}} \int \frac{\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda^2\frac{\sigma^2}{\lambda}x}{\sigma^2(1-\lambda)+\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda^2\frac{\sigma^2}{\lambda}x} e^{(-x^2/2)}dx \tag{B.3}$$

$$E[\kappa] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda x}{(1-\lambda)+(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda x} e^{(-x^2/2)}dx. \tag{B.4}$$

And clearly, Eq. (B.4) is independent of σ. To see that var$[\kappa]$ is also independent of σ, note that the same proof as above applies to $\mathbb{E}[\kappa^2]$.

For var$[\kappa S_\theta]$, again using the same change of variables, we have that

$$\mathbb{E}[\kappa S_\theta] = \frac{1}{\sqrt{2\pi\sigma^2/\lambda}} \int_{-\infty}^{\infty} \frac{\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda^2 s^2}{\sigma^2(1-\lambda)+\sigma^2(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda^2 s^2} s e^{(-s^2/2\sigma^2/\lambda)}\, ds \tag{B.5}$$

$$\mathbb{E}[\kappa S_\theta] = \frac{\sigma}{\sqrt{2\pi}} \int \frac{(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda x}{(1-\lambda)+(1-\pi_0\lambda)+\pi_0(1-\pi_0)\lambda x}\frac{x}{\sqrt{\lambda}} e^{(-x^2/2)}\, dx, \tag{B.6}$$

which clearly scales with σ and hence, $(\mathbb{E}[\kappa S_\theta])^2$ scales with σ^2. The same change of variables can be used to show that the same is true of $\mathbb{E}[(\kappa S_\theta)^2]$, which completes the proof.□

As expected, (i) implies that unconditional expected returns are increasing in the fundamental volatility and the overall risk concerns in the market as captured by αZ. Results (ii) through (iv) are also fairly intuitive, but they have important implications for which component drives overall volatility. In particular, when overall concerns about risk in the market are relatively high, the risk-premium component of expression (20) is the key driver of overall return volatility. When αZ and σ^2 are relatively small, the first and second components of expression (20) drive overall volatility.

Proposition 13 is also useful for exploring comparative-static results with respect to λ and π_0. For example, (i) implies that when exploring how expected returns change with λ and π_0, it is without loss to normalize σ^2 and αZ. By doing so, we are left with a two-dimensional parameter space (i.e., $(\pi_0,\lambda)\in[0,1]^2$), over which the expected return can be plotted to obtain comparative-static results that obtain *for any parameter specification* of the model. Fig. 10 (a) illustrates the result; both higher quality information and greater likelihood of an informed trader decrease the expected return. This is because both higher quality information and a higher likelihood of an informed trader imply that the price is more informative about the fundamentals in expectation, and the uncertainty faced by the uninformed investor is lower.

Using (ii) through (iv), we can conduct a similar exercise to characterize the comparative-static effects of each of the individual components of volatility. Fig. 11(a) shows the volatility in returns due to fundamental dividend shocks is decreasing in π_0 and λ, since an increase in either parameter reduces the uncertainty that investors face about next period's dividend. Fig. 11(b) shows that the variance in the expectations component of conditional expected returns is decreasing in π_0 but increasing in λ. Recall that the expectations component of the conditional expected returns is nonzero because investors exhibit differences of opinion, and in particular, because

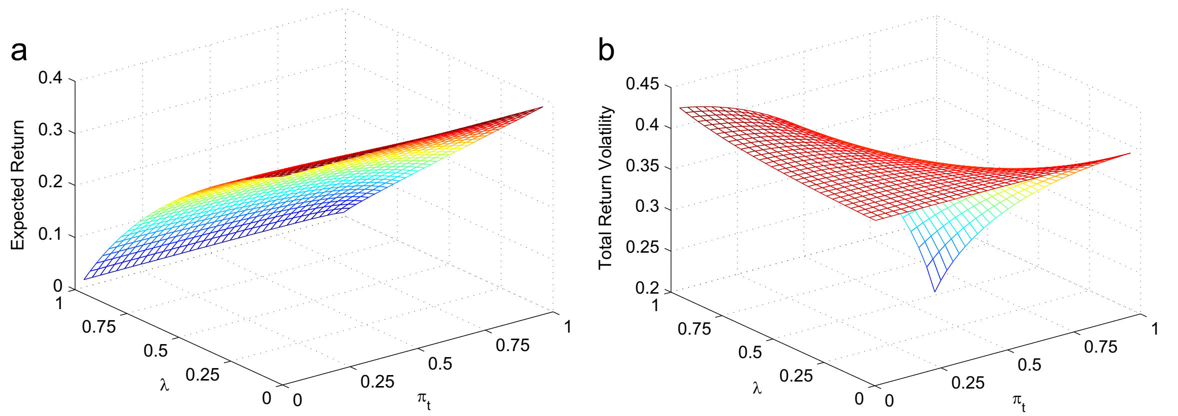

Fig. 10. Illustration of expected returns (a) and total volatility (b) as they depend on the quality of information λ and the probability of a θ being informed, i.e., π_0. The other parameters are $\sigma = 40\%$, and $\alpha Z = 3$. (a) Expected return. (b) Volatility.

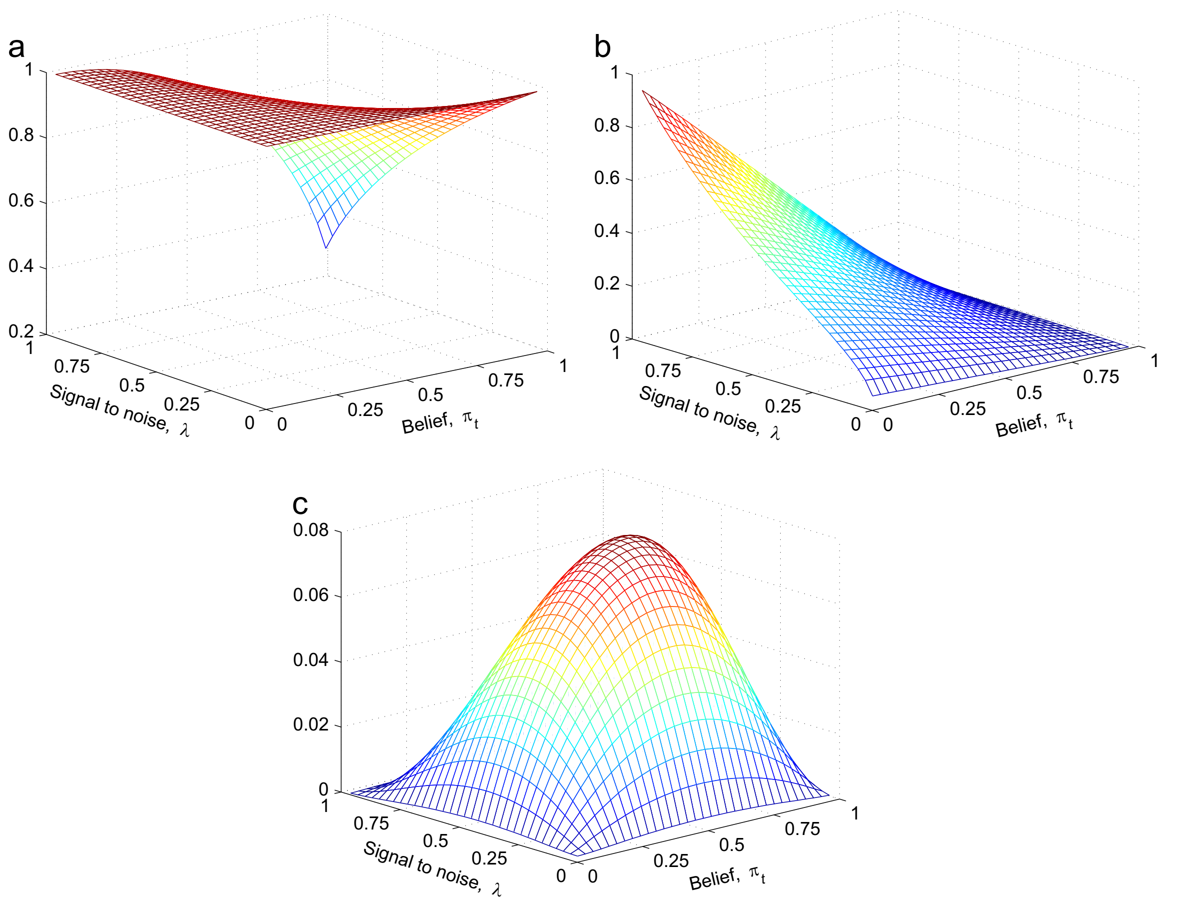

Fig. 11. The three components of volatility as they depend on the quality of information, λ, and the probability of a θ being informed, π_0. Panel (a) plots the fundamental component of volatility (i.e., $\sigma^2(1-\pi_0^2\lambda)$), Panel (b) plots the expectations component (i.e., $(1-\pi_0)^2\lambda^2\text{var}[\kappa S_\theta]$), and Panel (c) plots the risk-premium component (i.e., $(\sigma^2(1-\lambda)\alpha Z)^2\text{var}[\kappa]$). The other parameters are set as in Fig. 10.

uninformed θ investors believe they are informed. This effect is larger when π_0 is smaller (since θ investors are less likely to actually be informed) and when λ is larger (since uninformed θ investors put more weight on their signals), which leads to the effect on volatility. Fig. 11(c) shows the risk-premium component of volatility is non-monotonic in both π_0 and λ. This is because the risk-premium component of returns is stochastic only when both λ and π_0 are strictly between zero and one.[28]

Of course, comparative statics on the total return volatility depend on the relative magnitudes of σ^2 and αZ, which determine the relative weight on each component. For instance, Fig. 10(b) presents the effect of π_0 and λ on overall volatility for a given set of parameters, for which the fundamental and expectations components dominate the risk-premium component.

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[28] If $\pi_0 \in \{0, 1\}$, the conditional variance of U traders does not depend on S_θ. Consequently, κ and the risk-premium component of expected returns are constant. Similarly, when $\lambda = 1$, the risk-premium is zero, while when $\lambda = 0$, all θ investors are effectively uninformed, and so the risk-premium is, again, constant.

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