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Choosing to Disagree: Endogenous Dismissiveness and Overconfidence in Financial Markets

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ABSTRACT

The psychology literature documents that individuals derive current utility from their beliefs about future events. We show that, as a result, investors in financial markets choose to disagree about both private information and price information. When objective price informativeness is low, each investor dismisses the private signals of others and ignores price information. In contrast, when prices are sufficiently informative, heterogeneous interpretations arise endogenously: most investors ignore prices, while the rest condition on it. Our analysis demonstrates how observed deviations from rational expectations (e.g., dismissiveness, overconfidence) arise endogenously, interact with each other, and vary with economic conditions.

THE STANDARD APPROACH IN ECONOMICS assumes that market participants have rational expectations and learn efficiently from the information in prices. Yet there is ample evidence that people do not behave this way: returns exhibit excess predictability and volatility, investors are overconfident and trade too often, and individuals appear to underreact to prices in some settings but overreact in others.[1] To explain this evidence, existing literature explores how

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[1] For instance, Shiller (1981) documents that stock returns exhibit excess volatility relative to fundamentals, and Jegadeesh and Titman (1993) document that stocks exhibit momentum. Odean (1999) documents that individual investors exhibit overconfidence as evidenced by excessive trading, while Greenwood and Shleifer (2014) document that investor expectations of future returns exhibit extrapolation.

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informational frictions, endowed behavioral biases, and cognitive limits affect investors' interpretation of information. However, these models are usually unable to explain the variation in interpretations across investors and over economic conditions without assuming exogenous heterogeneity across investors.[2]

To study how the interpretation of information evolves endogenously, we require a model of subjective belief choice. We build on the large literature in psychology and behavioral economics that documents that individuals experience anticipatory utility from their beliefs about future events.[3] For instance, the anticipation of a positive *future* experience generates a positive *contemporaneous* utility flow (e.g., excitement about an upcoming vacation). In contrast, the prospect of negative future outcomes may lower current utility (e.g., anxiety about an annual medical checkup). In such cases, individuals often distort their beliefs by engaging in "wishful thinking": they choose subjective beliefs to make themselves happier about the future, even when such distortions come at a cost.[4]

In an economy with symmetrically informed, homogeneous investors, we show that wishful thinking leads to endogenous disagreement that varies predictably with economic conditions. We consider a setting in which a continuum of symmetrically informed investors trade a risky asset against noise traders (as in Hellwig (1980)). We allow each investor to entertain subjective beliefs about the informativeness of her own private signal as well as the private information of others. The equilibrium price aggregates investors' private information and provides an endogenous (noisy) signal about asset payoffs. The cost of belief distortion is given by the loss in the average ex post experienced utility due to trading on subjective beliefs.[5]

In the rational-expectations benchmark, all investors are constrained to agree on the interpretation of signals, and so efficiently condition on both their private signal and the price when submitting their demand for the risky asset. Under wishful thinking, however, we show that investors always choose to disagree about the interpretation of these signals. Moreover, the nature of this disagreement depends endogenously on the information environment. When

[2] The empirical evidence in Banerjee (2011) suggests that the extent to which investors condition on prices varies substantially across stocks. More recent evidence suggests that both disagreement (e.g., Andrade et al. (2016), Fischer, Kim, and Zhou (2020)) and overconfidence (e.g., Merkle (2017)) vary over time.

[3] We follow Bénabou and Tirole (2016) and use the term "anticipatory utility" to refer to the contemporaneous utility that an individual derives from the anticipation of future outcomes. As such, anticipatory utility is distinct from the notion of *anticipated* utility, which refers to settings in which agents treat parameters that they learn about as constant when formulating decisions (e.g., Kreps (1998), Cogley and Sargent (2008)).

[4] As we discuss further in Section I, our approach parallels the literature on robust expectations (Hansen and Sargent (2008)), where investors choose subjective beliefs to optimize against "worst-case" scenarios. In contrast, investors in our setting optimize behavior anticipating "best-case" scenarios.

[5] We refer to this as the *experienced utility penalty*. As we discuss in Appendix B.3, many of our results are qualitatively robust to other cost specifications (e.g., the Kullback-Leibler (K-L) distance).

prices are not very informative, there exists a unique, symmetric equilibrium in which each investor believes her own signal is informative but dismisses the information of others. As a result, investors choose to ignore the information in prices. However, when prices are sufficiently informative, the model yields a novel source of endogenous heterogeneity: while the majority of investors continue to treat the price as uninformative, the remaining investors use the information in prices to update their beliefs.

To highlight the key intuition for our results, we begin with a benchmark in which each investor is constrained to correctly interpret her own signal. Believing that others are less informed has two opposing effects on an investor's anticipatory utility. On the one hand, this implies that the price is less informative about payoffs, which increases the investor's perceived uncertainty and reduces her anticipatory utility. We refer to this as the *information effect*. On the other hand, when others are less informed, the perceived trading gains from speculating against them is higher, which increases anticipatory utility. We term this the *speculative effect*. Importantly, the speculative effect generates a type of strategic substitutability across investors' chosen beliefs. When others condition on prices more heavily, the price is more sensitive to their private information. This increases the relative benefit from perceiving others to be less informed, which leads the investor to underreact to price information more. We first show that, when all other investors are constrained to hold objective beliefs (i.e., exhibit rational expectations), any individual investor strictly prefers to dismiss the information of others and completely ignores the price. In this case, the speculative effect dominates the information effect, and the utility cost of ignoring price information is not very high.[6]

Next, we characterize the equilibrium when all investors choose their subjective beliefs. When the objective price informativeness is sufficiently low (e.g., aggregate risk tolerance is low, or noise trading volatility is high), there exists a unique, symmetric equilibrium in which *all* investors choose to dismiss the information of others. We refer to this as the *dismissive* equilibrium. The speculative effect dominates even though all investors ignore price information because prices are objectively not very informative, and so the information effect is relatively small.

When prices are sufficiently informative, a dismissive equilibrium cannot be sustained. This is a consequence of the strategic substitutability in belief choice: when all others ignore the information in prices, the speculative effect is relatively small. Furthermore, since the price information is very precise, the information effect dominates. This leads an individual investor to deviate and condition on price information. In such settings, we show that there exists a unique mixed-strategy equilibrium in which the majority of investors ignore price information while a minority condition efficiently on it. Moreover,

[6] Since all other investors exhibit rational expectations, the price is informationally efficient and sufficiently "close" to fundamentals. As we discuss in Section III.B, this implies the experienced utility loss from taking a suboptimal position in the risky asset is smaller than the corresponding gain in anticipatory utility.

we show that the fraction of investors who ignore price information initially decreases but then increases in price informativeness. In fact, we show that as prices become arbitrarily informative (e.g., noise trading volatility goes to zero), the fraction of investors who ignore price information approaches one because the cost of doing so is very small.[7]

We extend our benchmark analysis to allow for subjective beliefs about the volatility of supply shocks. Since subjective beliefs about others' signals and noise trading affect anticipatory utility only through their effect on the perceived precision of the price signal, our results remain qualitatively unchanged. However, the mixed-strategy equilibrium that arises features underreaction to price information by some investors and *overreaction* by others. Existing models generate such differences in interpretation and investment strategies by assuming that investors are ex ante heterogeneous (e.g., in the quality of their private information, or their ability to process such information). In contrast, our model endogenously generates these opposing interpretations with ex ante homogeneous investors and therefore helps us better understand how such disagreement varies with economic conditions.

We then explore the implications of subjective belief choice about private information. Increasing the perceived precision about one's own signal unambiguously increases anticipatory utility by reducing uncertainty, that is, only the information effect is in force. As a result, investors generically exhibit overconfidence with respect to their private information. When investors choose subjective beliefs only about their own private signal, the equilibrium degree of overconfidence decreases with prior uncertainty about payoffs, volatility of noise trading, and risk aversion, and is U-shaped in the objective precision of private signals.[8] When investors choose subjective beliefs about their own signals as well as those of others, then symmetric equilibria are characterized by overconfidence in private information and dismissiveness of others' signals. Moreover, we show that investors trade off belief distortion along these two dimensions: when an investor's subjective belief about price information is closer to the objective distribution, the degree of overconfidence increases. This negative relation between dismissiveness and overconfidence arises endogenously as a result of wishful thinking, and distinguishes our model from those that consider these biases separately.

We next characterize our model's distinguishing predictions for market observables. A key takeaway from our analysis is that investors' choice of subjective beliefs and the "behaviorial bias" they generate depend crucially on economic conditions. Our model predicts that periods with high price informativeness are associated with more diversity in investment strategies, lower volatility, and a positive relation between return predictability and volatility.

[7] As we discuss further in Section III.C, this is in sharp contrast to standard, noisy rational-expectations models (e.g., Grossman and Stiglitz (1980)) in which the fraction of investors who condition on prices usually *increases* with price informativeness.

[8] These comparative statics distinguish our model's implications from a setting in which investors acquire private information endogenously (e.g., Verrecchia (1982)). See Section IV.

In contrast, periods of high volatility and low price informativeness are associated with higher correlation in investment styles, low (or even negative) serial correlation in returns, and a negative relation between predictability and volatility.[9] Moreover, relative to the rational-expectations benchmark, the subjective beliefs equilibria generate higher expected returns, trading volume, and serial correlation in returns.

We also characterize the model's predictions for individual and consensus forecasts of investors. In particular, in equilibria where investors overreact to private information but underreact to the information in prices, consensus forecast revisions exhibit underreaction while individual forecast revisions exhibit overreaction.[10] To assess our model's implications more directly, we propose a novel empirical test: regress individual forecast errors on lagged returns. Our analysis predicts that the distribution of the regression coefficient across investors should be state-dependent. When price informativeness is low, the regression coefficient for all investors should be positive since investors underreact to price information. When price informativeness is high, the regression coefficient is positive for some investors but negative for others.

The rest of the paper is as follows. Section I briefly discusses the phenomenon of anticipatory utility and reviews the related literature. Section II introduces the benchmark model, discusses our assumptions, and characterizes the financial market equilibrium given investor beliefs. Section III characterizes the trade-offs associated with subjective belief choice and characterizes the equilibrium in the benchmark model. Section IV generalizes the analysis to accommodate subjective beliefs about noise trading volatility and private signals, and explores how our results change with general cost functions. Section V characterizes the empirical implications of our analysis. Section VI concludes. Proofs and extensions can be found in Appendices A and B, respectively.

I. Background and Related Literature

A. Anticipatory Utility and Subjective Belief Choice

Bénabou and Tirole (2016) survey the now extensive literature on motivated beliefs (e.g., Akerlof and Dickens (1982), Loewenstein (1987), Caplin and Leahy (2001), Eliaz and Spiegler (2006)). The concept of anticipatory utility, or current subjective expected utility, dates to at least Jevons (1905), who considers agents who derive contemporaneous utility not simply from current actions but also from the anticipation of future utility flows. As a result, an agent's subjective beliefs about future events will affect not just an agent's actions but also

[9] As discussed further in Section V, a number of our predictions are broadly consistent with existing empirical evidence on time variation in momentum returns and crashes (e.g., Cooper, Gutierrez Jr, and Hameed (2004), Moskowitz, Ooi, and Pedersen (2012), Daniel and Moskowitz (2016)) and price informativeness (e.g., Bai, Philippon, and Savov (2016), and Dávila and Parlatore (2019)), while others offer novel implications for future empirical work.

[10] This result is broadly consistent with the recent empirical evidence on analyst forecast revisions (e.g., Bordalo et al. (2020b)).

her current utility. This creates a tension between holding beliefs that are "accurate" (and therefore lead to optimal actions) and beliefs that are "desirable" (and therefore increase current utility).

The most closely related papers are Brunnermeier and Parker (2005) and Caplin and Leahy (2019). Brunnermeier and Parker (2005) show how subjective belief choice ("optimal expectations") is useful in understanding risk-taking, preference for skewness, optimism/pessimism, portfolio underdiversification, and consumption/savings patterns. Similarly, Caplin and Leahy (2019) show that wishful thinking can help explain a number of behavioral biases, including optimism, procrastination, confirmation bias, and polarization.

We view our work as complementary, both building on these insights and offering a new perspective focused on understanding how wishful thinking affects the interpretation of endogenous information in a market setting. As such, our model derives novel predictions not only about how overconfidence and dismissiveness can arise endogenously among investors, but also about how these subjective belief choices interact and the way in which they depend on market conditions. These aspects are missing from the earlier literature and have important consequences for our understanding of financial markets.

Note that subjective belief choice is not just of theoretical interest—substantial direct empirical evidence suggests that individuals experience anticipatory utility, and as a result distort their subjective beliefs in systematic ways. Individuals engage in information avoidance, for instance, by choosing not to learn about the risk of deadly disease even if the test is approximately costless (Oster, Shoulson, and Dorsey (2013)). At the same time, individuals may actively seek (and pay) to learn about potential good news, such as the outcome of a lottery-like event (Ganguly and Tasoff (2017)) or the performance of their portfolios on days when the market has done well (Karlsson, Loewenstein, and Seppi (2009)). Individuals also update asymmetrically when information is revealed, placing more weight on good news (e.g., a positive signal about one's IQ in Mobius et al. (2014)) than bad news (e.g., a negative signal about one's attractiveness in Eil and Rao (2011)). Finally, many individuals interpret information in ways that are favorable to their current well-being, updating in ways consistent with their political beliefs (Kahan (2013)) or interpreting uninformative signals of ability as positive indicators (Exley and Kessler (2019)). This literature suggests that such wishful thinking is not generated by individuals' inability to understand their environment. Indeed, both Kahan (2013) and Kahan et al. (2017) show that cognitive ability can even exacerbate the effect because more sophisticated individuals can better "rationalize" their beliefs and interpretations. Given this evidence, we expect our analysis to apply, to varying degrees, to both retail and institutional investors, and believe it is important to understand the impact of such behavior on market outcomes.[11]

[11] For instance, it is likely that sophisticated traders are particularly adept at justifying their favored bets, attributing their successes to skill and ability, while blaming losses to bad luck.

As discussed by Caplin and Leahy (2019), an alternative, parallel approach to modeling subjective belief choice is robust control (e.g., Hansen and Sargent (2001), Hansen and Sargent (2008)). Agents who exhibit robust control are unsure about their model of the world, but choose actions optimally under the "worst-case" subjective beliefs.[12] Like wishful thinking, robust control is motivated by a large literature in psychology and economics (which documents evidence of ambiguity aversion) and is useful in understanding a number of stylized facts about aggregate financial markets (e.g., limited participation, the equity premium puzzle). We view these approaches as complementary. While beyond the scope of the current paper, it would be interesting to explore the implications of investors who endogenously choose to exhibit wishful thinking in some domains and robust control preferences in others.

B. Distorted Beliefs in Financial Markets

Our paper contributes to three strands of the literature studying the impact of deviations from rational expectations, which help explain stylized facts about financial markets that are difficult to reconcile in the standard framework (e.g., excess trading volume and return predictability). The first strand focuses on differences of opinion, whereby investors "agree to disagree" about the joint distribution of payoffs and signals and therefore incorrectly condition on the information in prices (e.g., Harrison and Kreps (1978), Kandel and Pearson (1995), Banerjee, Kaniel, and Kremer (2009), and Banerjee (2011)). The second strand focuses on the impact of overconfidence, specifically, settings in which agents believe their private information is more precise than it objectively is (e.g., Daniel, Hirshleifer, and Subrahmanyam (1998), Odean (1998), Daniel, Hirshleifer, and Subrahmanyam (2001), Gervais and Odean (2001), Scheinkman and Xiong (2003)). Our equilibria also feature investors who are dismissive of price information, disagree about the interpretation of a public signal, and overestimate the precision of their private signal, but our focus is in understanding how such diversity in beliefs can arise from a single, psychologically motivated feature of individual decision making (i.e., wishful thinking). This foundation also helps us understand how such behaviors interact with each other (e.g., the negative relation between overconfidence and dismissiveness in our model) and how they vary with economic conditions.

[12] More concretely, a robust control agent chooses action a and subjective beliefs μ to solve

$$\min_{\mu}\max_{a}\mathbb{E}_{\mu}[u(a)]+C(\mu), \tag{1}$$

where $\mathbb{E}_{\mu}[u(a)]$ reflects the subjective expected utility from action a under "worst-case" beliefs μ and $C(\mu)$ reflects the penalty of choosing subjective beliefs μ that differ from the reference distribution. Analogously, a wishful thinking agent chooses action a and subjective beliefs μ to solve

$$\max_{\mu}\max_{a}\mathbb{E}_{\mu}[u(a)]-C(\mu), \tag{2}$$

where μ reflects the wishful thinking that the agent engages in to maximize anticipatory utility $\mathbb{E}_{\mu}[u(a)]$.

The final strand includes some of the alternative settings in which investors do not fully condition on the information in prices, including models that feature rational inattention (e.g., Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016)), cursedness (e.g., Eyster, Rabin, and Vayanos (2018)), and costly learning from prices (e.g., Mondria, Vives, and Yang (2021)).[13] Notably, in the models of Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) and Mondria, Vives, and Yang (2021), updating one's beliefs using prices is costly, in either an attention or pecuniary sense, and as such investors choose to discount this information. We view our analysis of endogenous belief choice as complementary to this earlier work. In particular, even when there is no explicit cost to learning (efficiently) from prices, we show that investors may choose to dismiss price information when they experience anticipatory utility.[14]

Our analysis also highlights that the benefits and costs of using the information in prices depends on how other investors use this information. This is reminiscent of, but distinct from, the channel discussed by Mondria, Vives, and Yang (2021), who show that there can exist complementarity in learning from prices: an investor may choose to learn more from prices when others become more sophisticated about learning from prices because prices become more informative. In contrast, we show that endogenous beliefs can give rise to substitutability: when others are learning from prices, one's incentive to do so decreases.

II. Benchmark Model

This section introduces the model setup, provides some preliminary analysis, and discusses the key assumptions of our setting.

A. Model Setup

Asset Payoffs. There are two securities. The gross return on the risk-free security is normalized to one. The terminal payoff (fundamental value) of the risky security is F, which is normally distributed with mean m and prior precision τ, that is,

$$F \sim \mathcal{N}\left(m, \frac{1}{\tau}\right). \tag{3}$$

[13] While Eyster, Rabin, and Vayanos (2018) show that cursedness can generate distinct predictions from a model of differences of opinions (which they term dismissiveness) when there is imperfect competition and no noise trading, our setting features perfectly competitive markets and noise in prices, and so cursedness and differences of opinions are effectively isomorphic.

[14] In our setting, the "opportunity cost" of choosing subjective beliefs is the forgone experienced utility. An alternative framing of investors' objective function would be to adopt the subjective beliefs that maximize investors' anticipatory utility as the reference distribution: in that setting, any deviations (toward rational expectations, for instance) would be costly in terms of the forgone anticipatory utility, but our results would be qualitatively unchanged.

We denote the market-determined price of the risky security by P, and the aggregate supply of the risky asset by $Z + z$, where

$$z \sim \mathcal{N}\left(0, \frac{1}{\tau_z}\right), \tag{4}$$

and we normalize the mean aggregate supply to $Z = 0$.[15] We interpret shocks to the aggregate supply (i.e., z) as resulting from trades by liquidity or noise traders who trade the risky asset for noninformative reasons.

Information. There is a continuum of investors, indexed by $i \in [0, 1]$. Before trading, each investor is endowed with a private signal s_i, where

$$s_i = F + \varepsilon_i \qquad \varepsilon_i \sim N\left(0, \frac{1}{\tau_e}\right) \tag{5}$$

and ε_i is independent and identically distributed across investors so that $\int \varepsilon_i \, di = 0$. Investors can also update their beliefs about F by conditioning on the price, P.

Beliefs and Preferences. Each investor i is endowed with initial wealth W_0 and zero shares of the risky security, and exhibits constant absolute risk aversion (CARA) utility with coefficient of absolute risk aversion γ over terminal wealth W_i,

$$W_i = W_0 + x_i(F - P), \tag{6}$$

where x_i denotes her demand for the risky security. In our benchmark model, we assume that each investor has correct beliefs about her own signal, but allow for subjective beliefs about the private signals of others. Specifically, we assume that investor i believes that other investors observe

$$s_j =_i F + \sqrt{1 - \rho_i^2}\eta_i + \rho_i \varepsilon_j, \quad \eta_i, \varepsilon_j \sim_i \mathcal{N}\left(0, \frac{1}{\pi_i \tau_e}\right), \tag{7}$$

where $=_i$ and $\sim_i$ denote investor i's subjective beliefs. Similarly, we denote the expectation and variance of random variable X under investor i's subjective beliefs by $\mathbb{E}_i[X]$ and $\text{var}_i[X]$.

Intuitively, equation (7) captures the idea that investor i can distort her beliefs about both the amount of noise in others' signals ($\pi_i \in [0, \infty]$) and the average correlation in this noise ($\rho_i \in [0, 1]$). In particular, investor i perceives the error in others' signals to consist of a common η_i shock and idiosyncratic ε_j shocks, where η_i and ε_j are independent for all i and j.[16] When $\rho_i = \pi_i = 1$, investor i's beliefs satisfy *rational expectations*: her beliefs coincide with the

[15] We make this assumption for analytic tractability, and in Section V we consider an extension in which $Z > 0$. We can show numerically that the resulting equilibria are qualitatively similar to our benchmark results, and that wishful thinking can lead to higher unconditional expected returns than rational expectations.

[16] Note that neither these shocks nor the realizations of the signals s_j are observed by investor i.

objective distribution of the underlying shocks. When $\rho_i < 1$, investor i overestimates the correlation in others' signals, and when $\pi_i < 1$ ($\pi_i > 1$) she overestimates (underestimates) the noise in their signals. As such, the parameters π_i and ρ_i reflect the degree to which investor i distorts her subjective beliefs. We assume that such deviations from the objective distribution impose a utility cost, denoted by $C(\pi_i, \rho_i)$.[17]

Given her choice of subjective beliefs, each investor optimally chooses her position in the risky security. Thus, optimally chosen subjective beliefs maximize her anticipatory utility, net of cost $C(\cdot)$. Formally, denote investor i's optimal demand, given her beliefs, by

$$x_i^*(\pi_i, \rho_i) = \arg\max_{x_i} \mathbb{E}_i\big[-\gamma \exp\{-\gamma x_i(F-P) - \gamma W_0\}\big|s_i, P\big], \tag{8}$$

and denote investor i's *anticipatory utility* by

$$AU_i(\pi_i, \rho_i) \equiv \mathbb{E}_i\big[\mathbb{E}_i\big[-\gamma \exp\big\{-\gamma x_i^*(F-P) - \gamma W_0\big\}\big|s_i, P\big]\big]. \tag{9}$$

Then investor i optimally chooses subjective beliefs ρ_i and π_i to maximize

$$\max_{\pi_i, \rho_i} AU_i(\pi_i, \rho_i) - C(\pi_i, \rho_i). \tag{10}$$

In our benchmark analysis, the cost each investor incurs by distorting her subjective beliefs is the reduction in expected utility (under the *objective* distribution) when her position in the risky asset, $x_i^*(\pi_i, \rho_i)$, is determined under her chosen *subjective* distribution. As is well established, any deviation from the rational-expectations benchmark ($\pi_i = \rho_i = 1$) is objectively inefficient: the investor is over- or underweighting the information she receives. We refer to this cost specification as the "experienced utility" penalty.

DEFINITION 1: *Investor i incurs the experienced utility penalty if the cost of choosing π_i, ρ_i is given by*

$$C_{obj}(\pi_i, \rho_i) \equiv \begin{array}{c} \mathbb{E}\big[-\gamma \exp\big\{-\gamma x_i^*(1,1) \times (F-P) - \gamma W_0\big\}\big] \\ -\mathbb{E}\big[-\gamma \exp\big\{-\gamma x_i^*(\pi_i, \rho_i) \times (F-P) - \gamma W_0\big\}\big]. \end{array} \tag{11}$$

When investors incur the experienced utility penalty, we can show that their subjective belief choice problem can be represented as

$$\max_{\pi_i, \rho_i} AU_i(\pi_i, \rho_i) + \mathbb{E}\big[-\gamma \exp\big\{-\gamma x_i^*(\pi_i, \rho_i) \times (F-P) - \gamma W_0\big\}\big]. \tag{12}$$

This is closely related to the objective function in the "optimal expectations" approach of Brunnermeier and Parker (2005).[18] We use this cost function as

[17] As the discussion in Section II.C highlights, alternative specifications of (7) would leave the analysis unchanged: the key feature is that investor i's subjective beliefs allow for common noise in investors' aggregated information. The specification above is chosen for expositional clarity.

[18] Under their approach, an investor optimally chooses actions under subjective beliefs $\mathbb{E}_i[\cdot]$, and the optimal choice of beliefs maximizes the investor's well-being under the objective distribution, that is,

our benchmark because of its clear interpretation, intuitive appeal, and direct quantitative implications. As we discuss below, we explore how our analysis changes for more general cost functions in Appendix B.3.

B. Discussion of Assumptions

Our benchmark analysis focuses on subjective beliefs about others' signals. In Section III.D, we allow for subjective beliefs about supply shocks, and in Section IV we allow investors to choose their beliefs regarding the precision of their own private signal. In this richer setting, we show that investors tend to exhibit overconfidence about their private information, and we characterize how this overconfidence is endogenously related to the degree to which investors dismiss the information of others.

Throughout our analysis, we restrict investors to choose subjective beliefs about the precision of these signals, and we assume that they make these choices *before* observing the realizations of their signals. These assumptions allow us to tractably model how investors interpret different types of information: we can explicitly characterize the financial market equilibrium since the resulting equilibrium price is a linear signal about fundamentals. Moreover, this specification is naturally interpreted as the stage game of a dynamic repeated setting in which investors experiment with (and update about) different models of the world.[19] While allowing for more flexibility in subjective belief choice may lead to additional implications (e.g., a preference for skewness as in Brunnermeier and Parker (2005)), we expect our main results to be qualitatively similar in these settings. A formal analysis is beyond the scope of this paper, however, and left for future work.

The penalty function in (11) does not necessarily imply that the investor knows the objective distribution; instead, it should be interpreted as a tractable specification for the utility cost of subjective beliefs, from the perspective of the modeler (or observer). As in other models of subjective belief choice (including models of robust control), the cost of choosing subjective beliefs depends on how they deviate from a reference distribution. While a literal interpretation of this specification is that each investor has "multiple selves,"

$$\max_{\pi_i,\rho_i} \mathbb{E}\begin{bmatrix} \mathbb{E}_i\left[-\gamma \exp\left\{-\gamma x_i^*(\pi_i,\rho_i)(F-P)-\gamma W_0\right\}\middle|s_i,P\right] \\ -\gamma \exp\left\{-\gamma x_i^*(\pi_i,\rho_i)\times(F-P)-\gamma W_0\right\}\end{bmatrix} \tag{13}$$

$$= \max_{\pi_i,\rho_i} \mathbb{E}\left[AU_i(\pi_i,\rho_i)\right] + \mathbb{E}\left[-\gamma \exp\left\{-\gamma x_i^*(\pi_i,\rho_i)\times(F-P)-\gamma W_0\right\}\right]. \tag{14}$$

In our setting, $AU_i(\pi_i, \rho_i) = \mathbb{E}[AU_i(\pi_i, \rho_i)]$ and so the two objectives coincide.

[19] In Appendix B.1, we explore how our results are affected when investors choose their interpretations *after* observing the signals. Unfortunately, solving for the general equilibrium in which all investors choose their beliefs is not feasible in this setting. The perceived precision depends on the realizations of signals, and the linearity of the market-clearing price is not preserved, so that "closing the model" is intractable. However, the partial equilibrium analysis of a single investor's interpretation suggests that similar biases can arise even under this alternative timing.

we view it as one in which investors evaluate their actions and outcomes under a single, subjective model of the world, which they reach through motivated reasoning.[20] In particular, there is ample evidence (see, e.g., Epley and Gilovich (2016)) that individuals seek and evaluate information in such a way that their resultant model of the world helps them balance their competing objectives. In our setting, this model may result from a more complicated process of experimentation, learning, and experience, that trades off "desirable" models (that increase anticipatory utility) and "accurate" models (that increase experienced utility). The specification in (11) provides a tractable characterization of this process from the perspective of economic modeling.

Note that the experienced utility penalty depends on the equilibrium subjective beliefs of other investors via investor i's beliefs about the distribution of the equilibrium price. This is in contrast to models of subjective belief choice (e.g., Caplin and Leahy (2019)) that use a statistical, distance-based cost function (e.g., the K-L divergence). Our exploration of alternative cost functions in Appendix B.3, however, suggests that this dependence does not play a critical role for our main results. Even in settings in which the beliefs of others do not directly affect the cost of belief distortion, they affect the subjective belief choice of investor i through her anticipatory utility.

C. Preliminary Analysis

We begin by solving for the financial market equilibrium, taking investors' chosen subjective beliefs (i.e., π_i and ρ_i for all $i \in [0, 1]$) as given. We proceed in two steps. First, we conjecture and verify that for each investor i, the price P is an affine function of the average signal $\bar{s} = \int_j s_j dj$ and the aggregate supply shock z. Second, given this observation, we derive equilibrium beliefs and the market-clearing price.

Step 1: Conjecture that investor i believes that the price is an affine signal of $\bar{s}$ and z, that is, $P =_i a_i \bar{s} + b_i z + c_i$. Since her beliefs about others' signals are given by (7), her belief about the average signal is

$$\bar{s} =_i \int_j s_j dj = F + \sqrt{1 - \rho_i^2}\,\eta_i, \tag{15}$$

and so conditioning on the price is equivalent to conditioning on a signal of the form

$$s_{p,i}(P) =_i \bar{s} + \beta_i z \quad \Leftrightarrow \quad s_{p,i}|F \sim_i \mathcal{N}\left(F, \frac{1}{\tau_{p,i}}\right), \tag{16}$$

[20] Furthermore, as discussed by Caplin and Leahy (2019), one could replace the objective distribution in the cost function by an alternate reference distribution (e.g., the consensus distribution) without qualitatively changing the economic trade-offs—the objective distribution imposes additional discipline on the modeler.

where β_i and $\tau_{p,i}$ are specified below and where $s_{p,i}$ is independent of her private signal s_i, conditional on F. Then her conditional beliefs about F are given by

$$F|s_i, P \sim_i \mathcal{N}\left(\mu_i, \frac{1}{\omega_i}\right), \text{ where} \tag{17}$$

$$\mu_i \equiv \mathbb{E}_i[F|s_i, P] = m + A_i(s_i - m) + B_i\big(s_{p,i}(P) - m\big), \tag{18}$$

$$\omega_i \equiv \frac{1}{\text{var}_i[F|s_i, P]} = \frac{\tau}{1 - A_i - B_i}, \text{ and} \tag{19}$$

$$A_i \equiv \frac{\tau_e}{\tau + \tau_e + \tau_{p,i}}, \text{ and } B_i \equiv \frac{\tau_{p,i}}{\tau + \tau_e + \tau_{p,i}}. \tag{20}$$

Given her beliefs, the optimal demand for investor i is given by

$$x_i^*(s_i, P) = \frac{\mathbb{E}_i[F|s_i, P] - P}{\gamma \text{var}_i[F|s_i, P]} = \frac{\omega_i}{\gamma}(\mu_i - P), \tag{21}$$

and the market-clearing condition is given by $\int_j x_j^*(s_j, P)dj = z$, which implies

$$P = \frac{\int_j \omega_j\big(m + A_j(s_j - m) + B_j\big(s_{p,j}(P) - m\big)\big)dj}{\int_j \omega_j di} - \frac{\gamma z}{\int_j \omega_j di} \tag{22}$$

$$= \frac{\int_j \omega_j\big(m + B_j\big(s_{p,j}(P) - m\big)\big)dj}{\int_j \omega_j di} + \frac{\int_j \omega_j A_j dj}{\int_j \omega_j dj}(\bar{s} - m) - \frac{\gamma z}{\int_j \omega_j di}, \tag{23}$$

where the second equality follows from the fact that each investor takes others' beliefs and strategies as given.[21] Furthermore, this expression implies that conditioning on the price is equivalent to conditioning on the signal

$$s_p(P) =_i \bar{s} + \beta z, \qquad \text{where } \beta = -\frac{\gamma}{\int_j \omega_j A_j dj} \tag{24}$$

for all i. Importantly, this establishes that the coefficient $\beta_i = \beta = -\frac{\gamma}{\tau_e}$ for all investors, and so the only source of disagreement about the information in prices is driven by variation in subjective beliefs about $\bar{s}$.

Step 2: Substituting (5) into the definition of $\bar{s}$ implies that the objective price signal is $s_p = F + \beta z$ and the objective conditional distribution of s_p is given by

$$s_p|F \sim \mathcal{N}\left(F, \frac{1}{\tau_p}\right), \quad \text{where} \quad \tau_p = \frac{\tau_z}{\beta^2} = \frac{\tau_z \tau_e^2}{\gamma^2}. \tag{25}$$

[21] In particular, investor i takes $\{\beta_j, A_j, B_j, \omega_j\}_j$ and the mapping $\{s_{p,j}(P)\}_j$ as given.

However, given her subjective beliefs, as in (15), investor i believes that

$$s_p =_i F + \sqrt{1-\rho_i^2}\eta_i + \beta z, \tag{26}$$

which implies

$$s_p|F \sim_i \mathcal{N}\left(F, \frac{1-\rho_i^2}{\pi_i \tau_e} + \frac{\beta^2}{\tau_z}\right) \equiv \mathcal{N}\left(F, \frac{1}{\tau_{p,i}}\right). \tag{27}$$

It is convenient to parameterize investor i's subjective beliefs about the informativeness of the price signal by the ratio

$$\delta_{p,i} \equiv \frac{\tau_{p,i}}{\tau_p} = \frac{\frac{\beta^2}{\tau_z}}{\frac{\beta^2}{\tau_z} + \frac{1-\rho_i^2}{\pi_i \tau_e}}. \tag{28}$$

Since $\rho_i \in [0, 1]$ and $\pi_i \in [0, \infty)$, it is easy to see that $\delta_{p,i} \in [0, 1]$.

The representations (27) and (28) highlight that investor i's subjective beliefs about others' information (i.e., π_i and ρ_i) are relevant only to the extent that they distort her perception of the precision of the price signal, s_p. When $\delta_{p,i} < 1$, investor i underreacts to the information in prices, either because she believes others' signals contain more noise (i.e., $\pi_i > 1$) or because they are more correlated (i.e., $\rho_i < 1$).[22] Under rational expectations, $\delta_{p,i} = 1$; moreover, we note that $\delta_{p,i} = 1$ any time $\rho_i = 1$, since in the absence of any perceived correlation, the noise in others' signals would be aggregated away under market clearing.

Plugging $\tau_{p,i}$ into (17) to (20), and noting that $\bar{s} = F$ under the *objective* distribution, allows us to solve for the market-clearing price using (23). This gives us the following characterization of the financial market equilibrium.

LEMMA 1: *Given investor i's subjective beliefs $\delta_{p,i}$, there always exists a unique, linear financial market equilibrium in which*

$$P = m + \Lambda(s_p - m), \ \textit{where}\ \Lambda = \frac{\tau_e + \bar{\delta}_p \tau_p}{\tau + \tau_e + \bar{\delta}_p \tau_p}, \tag{29}$$

$s_p = \bar{s} + \beta z$, $\tau_p = \tau_z/\beta^2$, $\beta = -\frac{\gamma}{\tau_e}$, *and* $\bar{\delta}_p = \int_i \delta_{p,i} di$ *reflects the average subjective beliefs across investors.*

The financial market equilibrium is standard—the price is linear in the signal $s_p = \bar{s} + \beta z$, where $\beta = -\frac{\gamma}{\tau_e}$. Investors disagree about how informative $\bar{s}$ is about F and so disagree about the informativeness of s_p. However, since $\bar{s} = F$

[22] Given the objective distribution of signals, the above expression implies that investors cannot overreact to the information in prices (i.e., $\delta_{p,i} \leq 1$). To allow for this possibility, in Section III.D we allow investors to have subjective beliefs about the volatility of noise trading, or equivalently, the precision of supply shocks (i.e., τ_z)—in this case, $\delta_{p,i} > 1$ corresponds to subjective beliefs that overestimate the precision of aggregate supply shocks.

under the objective distribution, this implies that the objective informativeness of prices is unaffected by subjective beliefs, that is, τ_p does not depend on $\delta_{p,i}$.[23] In contrast, subjective beliefs do affect the objective price sensitivity, Λ, to the signal s_p. For example, when investors deviate from rational expectations and perceive s_p to be less informative (i.e., $\delta_{p,i} < 1$), they put less weight and trade less aggressively on this information. As a result, the price is less sensitive to s_p.

In what follows, we characterize the subjective beliefs of investor i using $\delta_{p,i}$ instead of $\{\rho_i, \pi_i\}$. The definition of the experienced utility penalty $C_{obj}(\delta_{p,i})$ is modified to correspond to the analogous function when we replace $x_i^*(\pi_i, \rho_i)$ by $x_i^*(\delta_{p,i})$.

III. Subjective Belief Choice

In this section, we characterize the equilibrium subjective belief choice of investors, given the financial market equilibrium characterized above. We do so in three steps. First, in Section III.A, we characterize the effects of subjective belief choice on anticipatory utility. Second, in Section III.B, we characterize an individual investor's choice of subjective beliefs when all other investors exhibit rational expectations, highlighting the partial equilibrium implications of subjective belief choice. Finally, in Section III.C, we allow all investors to choose their subjective beliefs, taking as given the behavior of others. Importantly, each investor also takes as given the subjective beliefs of other investors, that is, she does not assume they hold rational expectations. Comparing these results to the partial equilibrium analysis allows us to highlight how general equilibrium considerations can give rise to endogenously different responses to price information.

A. Anticipatory Utility

Given the optimal demand for the risky asset, anticipatory utility is given by

$$AU_i\left(\delta_{p,i}\right) = \mathbb{E}_i\left[-\exp\left\{-\frac{(\mathbb{E}_i[F|s_i,P]-P)^2}{2\text{var}_i[F|s_i,P]}\right\}\right]. \tag{30}$$

Moreover, given the characterization of the equilibrium price in Lemma 1, investor i's beliefs about the conditional return are given by

$$\mathbb{E}_i[\mathbb{E}_i[F|s_i,P]-P] = m - m = 0, \text{ and} \tag{31}$$

$$\text{var}_i[\mathbb{E}_i[F|s_i,P]-P] = \text{var}_i[F-P] - \text{var}_i[F|s_i,P], \tag{32}$$

[23] In Section IV, we show how subjective beliefs about *private* signals can affect τ_p through β.

where the first equality follows from the law of iterated expectations and the second equality follows from the law of total variance.[24] From this, we derive the following result.

LEMMA 2: *Anticipatory utility for investor i is given by*

$$AU_i(\delta_{p,i}) = -\sqrt{\frac{var_i[F|s_i,P]}{var_i[F-P]}}, \tag{33}$$

where $AU_i(\delta_{p,i})$ is nonmonotonic in $\delta_{p,i}$: there exists some $\bar{\delta} > 0$ such that for all $\delta_{p,i} < \bar{\delta}$, anticipatory utility is decreasing in $\delta_{p,i}$, while for all $\delta_{p,i} > \bar{\delta}$, it is increasing.

Increasing the perceived precision of the price signal (i.e., increasing $\delta_{p,i}$) has two competing effects, as highlighted by the following expression:

$$\frac{\partial AU_i}{\partial \delta_{p,i}} \propto \left(\underbrace{\frac{1}{\text{var}_i[F-P]} \frac{\partial \text{var}_i[F-P]}{\partial \delta_{p,i}}}_{\text{speculative effect}} - \underbrace{\frac{1}{\text{var}_i[F|s_i,s_p]} \frac{\partial \text{var}_i[F|s_i,P]}{\partial \delta_{p,i}}}_{\text{information effect}} \right). \tag{34}$$

First, the *information effect* of learning from prices reduces the conditional variance $\text{var}_i[F|s_i,P]$: the investor has better information about the asset's value, which increases anticipatory utility. This information effect reduces the volatility of the perceived return on the risky security, a benefit in and of itself, but it also allows the investor to scale up her trading position. Second, the *speculative effect* of believing that prices are more informative decreases the perceived variance of the return (i.e., $\text{var}_i[F-P]$), which lowers anticipatory utility. Intuitively, when the price is more informative, it tracks fundamentals more closely, and as a result the trading opportunity is less profitable.

The overall effect on anticipatory utility of changing $\delta_{p,i}$ depends on the relative magnitude of these two effects. When $\delta_{p,i}$ is low, the speculative effect dominates, while the information effect dominates when $\delta_{p,i}$ is high. As a result, anticipatory utility first decreases and then increases in $\delta_{p,i}$. Moreover, note that the information effect can be expressed as

$$-\frac{1}{\text{var}_i[F|s_i,s_p]} \frac{\partial \text{var}_i[F|s_i,P]}{\partial \delta_{p,i}} = \frac{\tau_p}{\tau + \tau_e + \delta_{p,i}\tau_p} > 0, \tag{35}$$

[24] The law of total variance implies

$$\text{var}_i[F-P] = \mathbb{E}_i[\text{var}_i[F-P|s_i,P]] + \text{var}_i\big[\mathbb{E}_i[F-P|s_i,P]\big],$$

which implies the above expression in turn.

which does not depend on the behavior of others. In contrast, the speculative effect is

$$\frac{1}{\text{var}_i[F-P]}\frac{\partial \text{var}_i[F-P]}{\partial \delta_{p,i}} = -\frac{\Lambda^2\tau}{\delta_{p,i}\left(\Lambda^2\tau + (1-\Lambda)^2\delta_{p,i}\tau_p\right)} < 0. \tag{36}$$

Recall that Λ captures the sensitivity of the price to fundamentals, which is increasing in $\bar{\delta}_p = \int_i \delta_{p,i} di$. Thus, the above expression implies that the speculative effect is relatively more important when other investors interpret prices to be more informative (i.e., $\bar{\delta}_p$ is high).[25] Intuitively, when others condition on prices more heavily, the perceived loss of speculative opportunities is larger for investor i. This source of *relative strategic substitutability* plays an important role in determining the nature of equilibrium, as we demonstrate below.[26]

It is worth noting that these competing effects may generalize beyond our specific setting. It can be shown that anticipatory utility is a monotonic transformation of

$$\frac{\text{var}_i[F-P]}{\text{var}_i[F|s_i,P]} = 1 + \text{var}_i\left(\frac{\mathbb{E}_i[F-P|s_i,P]}{\sqrt{\text{var}_i[F-P|s_i,P]}}\right) \equiv 1 + \text{var}_i\big(SR_i\big), \tag{37}$$

where

$$SR_i \equiv \frac{\mathbb{E}_i[F-P|s_i,P]}{\sqrt{\text{var}_i[F-P|s_i,P]}} \tag{38}$$

is investor i's conditional Sharpe ratio, given her beliefs. As shown by the partial equilibrium analysis of Van Nieuwerburgh and Veldkamp (2010), investor utility is increasing in the squared Sharpe ratio under more general preference and payoff assumptions. As such, we expect that the key effects of belief distortion on anticipatory utility (the information and speculative effects) should be qualitatively robust. Our focus is the CARA-normal setting, however, because this allows us to characterize in closed form the (general) equilibrium effects of subjective belief choice.

In the analysis that follows, we characterize equilibria using conditions on the price informativeness measure τ_p, which is a "derived" parameter (i.e., it depends on model primitives τ_e, τ_z, and γ). Objective price informativeness is the model-relevant measure that determines equilibrium beliefs, and we present our results in this manner to highlight the economic intuition. However, we establish these conditions while keeping other parameters fixed, and

[25] The term capturing the proportionality in (34) also depends on Λ, but does not affect the relative magnitude of the two effects.

[26] Note that subjective belief choice is not always a strategic substitute in the standard sense, since $\frac{\partial^2 AU_i}{\partial\delta_{p,i}\partial\delta_p}$ is not negative everywhere. However, the above expressions suggest that the relative effect characterized in (34) is decreasing in Λ and, more formally, we can show that $\frac{\partial}{\partial\Lambda}(\frac{1}{AU_i}\frac{\partial AU_i}{\partial\delta_{p,i}}) < 0$.

so one should interpret changes in τ_p as corresponding to changes in either τ_z or γ, while keeping τ_e and τ constant.

B. Belief Choice When Others Exhibit Rational Expectations

Before characterizing the equilibrium subjective belief choice across all investors, we begin by characterizing the optimal belief choice of investor i when all other investors exhibit rational expectations. The price faced by investor i is the same as in a rational-expectations equilibrium: taking others' beliefs as given, she sets $\bar{\delta}_p = 1$ and so $P = m + \Lambda(s_p - m)$, where $\Lambda = \frac{\tau_e+\tau_p}{\tau+\tau_e+\tau_p}$. We show that because Λ is relatively high, so is the speculative effect. As a result, strategic substitutability in subjective belief choice implies that investor i chooses to underweight the information in prices.

The following proposition characterizes the optimal beliefs of investor i in this setting.

PROPOSITION 1: *Suppose all other investors exhibit rational expectations. If investor i is subject to the experienced utility penalty, she chooses to completely ignore price information, that is, $\delta_{p,i} = 0$, when price informativeness, τ_p, is sufficiently high or sufficiently low.*

While we are able to analytically prove the result for τ_p sufficiently high and sufficiently low, numerical simulations suggest that the result holds generally for all τ_p. Intuitively, investor i's anticipatory utility is high when she believes that others are uninformed, that is, when $\delta_{p,i} = 0$. While this implies that investor i faces more uncertainty about fundamentals and so trades less aggressively (the information effect), she expects her trades to be more profitable because she thinks others are effectively noise traders: the private and price signals on which they condition are essentially noise (the speculative effect). Moreover, the cost of belief distortion under the experienced utility penalty is relatively small when others exhibit rational expectations. Because the price is (objectively) informationally efficient, choosing an objectively inefficient position in the risky asset ($x_i^*(\delta_{p,i})$ instead of $x_i^*(1)$) is less costly. Together, this implies that the investor chooses to completely dismiss the information in prices (set $\delta_{p,i} = 0$) when others exhibit rational expectations.

Standard intuition suggests that behavioral investors are worse off in an environment with rational investors. With wishful thinking, however, it is the presence of the rational investors that allows the investor to deviate from rational expectations. The key insight is that it is not as costly to dismiss the information in prices as long as others trade on it efficiently because the loss (per dollar of trade) is minimized in this case (i.e., $|F - P|$ is minimized). This suggests that the increased presence of rational investors need not drive out investors who engage in wishful thinking and may actually encourage such investors to distort their beliefs further. This result provides a distinct prediction of our model relative to settings in which investor biases are exogenously specified.

Ignoring price information when others condition on it is a result of strategic substitutability. In the next section, we use this same channel to show that when others dismiss the information in prices, investor i may choose to condition on this information efficiently. Moreover, we show how this desire to deviate from the beliefs of others can lead to endogenous heterogeneity in the interpretation of price information.

C. General Equilibrium

We now turn to the general setting in which all investors optimally choose their beliefs about the quality of the information contained in prices. The following result characterizes the equilibrium when all investors choose their subjective beliefs about price information.

PROPOSITION 2: *Suppose all investors incur the experienced utility penalty. Then, there exists $\bar{\tau}_p \geq \underline{\tau}_p > 0$, such that:*

1. *For all $\tau_p \leq \underline{\tau}_p$, there exists a unique equilibrium in which all investors ignore the information in prices (i.e., $\delta_{p,i} = \delta_p = 0$ for all i).*
2. *For all $\tau_p \geq \bar{\tau}_p$, there does not exist a pure symmetric equilibrium in which all investors choose the same subjective beliefs $\delta_{p,i} = \delta_p$. The unique equilibrium is one in which investors mix between two sets of beliefs: a fraction λ optimally chooses $\delta_{p,i} = 0$, while the remaining fraction $1 - \lambda$ optimally chooses $\delta_{p,i} = 1$, where λ is given by*

$$\lambda = 1 + \frac{\tau_e}{\tau_p} - \sqrt{\frac{\tau}{3\tau_p}\left(\frac{\tau_p - 3\tau - 3\tau_e}{\tau + \tau_e + \tau_p}\right)}. \tag{39}$$

The above result establishes sufficient conditions for the existence and uniqueness of two types of equilibria. When price informativeness is sufficiently low, the unique equilibrium is one in which all investors dismiss the information in prices *completely*. We refer to this as the *dismissive* equilibrium. On the other hand, when price informativeness is sufficiently high, we show that there cannot exist an equilibrium in which all investors choose the same subjective beliefs (i.e., $\delta_{p,i} = \delta_p$ for all i).[27] We refer to this as the *mixed* equilibrium.

The intuition for this result builds on our earlier observations. When price informativeness is low, the information effect is small since dismissing the information in prices has little impact on investors' perceived uncertainty (and their resulting position in the risky asset). However, there is still a meaningful speculative effect regardless of the information others infer from the price. This is because each investor believes that others are trading on uninformative *private* signals, which introduces more opportunities for profitable trade.

[27] Technically, the first part establishes sufficient conditions for the existence of a pure strategy, symmetric equilibrium in pure strategies, while the second part establishes the nonexistence of such equilibria and the existence of mixed-strategy, symmetric equilibria.

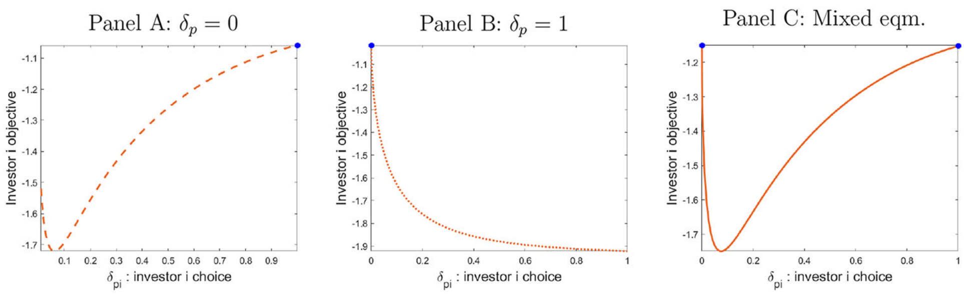

Figure 1. Anticipatory utility net of costs versus $\delta_{p,i}$. The figure plots the anticipatory utility net of costs for investor i as a function of her choice $\delta_{p,i}$. Other parameters are: $\tau = \tau_e = \tau_z = 1$, $\gamma = 0.3$. The solid blue dots in each panel indicate the maxima of the objective function. (Color figure can be viewed at wileyonlinelibrary.com)

In fact, in the limit, as $\tau_p \to 0$, we have

$$\underbrace{-\frac{1}{\text{var}_i[F|s_i, s_p]}\frac{\partial \text{var}_i[F|s_i, P]}{\partial \delta_{p,i}}}_{\text{information effect}} \to 0, \quad \underbrace{\frac{1}{\text{var}_i[F-P]}\frac{\partial \text{var}_i[F-P]}{\partial \delta_{p,i}}}_{\text{speculative effect}} \to \frac{1}{\delta_{p,i}}. \tag{40}$$

Moreover, the experienced utility cost of dismissing the information in prices is not too large because the price is not very informative to begin with. Taken together, these effects lead to an equilibrium in which everyone dismisses the information in the price.

On the other hand, when price informativeness is objectively high, whether or not the speculative effect dominates the information effect depends on the equilibrium behavior of others. For instance, suppose all other investors choose not to efficiently condition on the price so that $\int_j \delta_{p,j} di = \delta_p < 1$ for all i. Then as $\tau_p \to \infty$, we have

$$-\frac{1}{\text{var}_i[F|s_i, s_p]}\frac{\partial \text{var}_i[F|s_i, P]}{\partial \delta_{p,i}} \to \frac{1}{\delta_{p,i}}, \quad \frac{1}{\text{var}_i[F-P]}\frac{\partial \text{var}_i[F-P]}{\partial \delta_{p,i}} \to 0. \tag{41}$$

This implies that in the limit because others underreact to the information in prices, the information effect dominates and investor i has an incentive to choose a *higher* $\delta_{p,i}$. Moreover, since the cost of holding rational expectations (i.e., setting $\delta_{p,i} = 1$) is zero, investor i would choose to deviate to this point. However, Proposition 1 implies that there cannot exist an equilibrium in which all investors choose $\delta_{p,i} = 1$. As we show in the proof of Proposition 2, similar reasoning implies that for any sufficiently high τ_p, there cannot exist an equilibrium in which all investors choose the same subjective beliefs $\delta_{p,i} = \delta_p$.

Panels A and B of Figure 1 provide a numerical illustration of this nonexistence argument. The panels show investor i's anticipatory utility, net of costs, as a function of $\delta_{p,i}$, given the behavior of others. In Panel A, all other investors choose $\delta_p = 0$. In this case, investor i has an incentive to deviate by setting

$\delta_{p,i} = 1$, since the information effect dominates the speculative effect. In Panel B, all other investors choose $\delta_p = 1$. Now, the speculative effect dominates and investor i strictly prefers to ignore the information in prices.[28] In both cases, a symmetric equilibrium is ruled out because an individual investor has an incentive to deviate from the equilibrium behavior of others.

Panel C of Figure 1 illustrates an instance of the mixed equilibrium. In this case, each investor is indifferent between two beliefs. In equilibrium, fraction $\lambda = 0.98$ of investors ignore the information in prices completely (i.e., choose $\delta_{p,i} = 0$) while the remaining fraction ($1 - \lambda = 0.02$) interpret the information in prices correctly (i.e., set $\delta_{p,i} = 1$). Both choices are maxima, given the beliefs of others, and so the mixed equilibrium arises.

Proposition 2 also provides a characterization of how the composition of equilibrium beliefs (i.e., λ) depends on the underlying parameters of the model, which we illustrate numerically in Figure 2. The shaded region of each panel corresponds to the dismissive equilibrium (i.e., $\lambda = 1$), while the unshaded region corresponds to the mixed equilibrium (i.e., $\lambda < 1$). Recall that price informativeness τ_p is increasing in τ_e and τ_z, but decreasing in γ, since $\tau_p = \frac{\tau_e^2 \tau_z}{\gamma^2}$. This naturally implies that the dismissive equilibrium obtains when γ is sufficiently high, or when τ_e or τ_z are sufficiently low.

Within the mixed equilibrium parameter space, however, λ is U-shaped in τ_e, τ_z, and γ. To understand this nonmonotonicity, consider the case of τ_z (the other parameters are analogous). When τ_z is sufficiently large (i.e., τ_p is sufficiently large), the dismissive equilibrium cannot be sustained. For initial increases in τ_z in this region, the mass of investors who condition on the price increases (i.e., λ falls). This is because the information effect dominates the speculative effect and it is relatively costly to dismiss price information. As τ_z increases, however, and more investors condition on the price, Λ increases, making the speculative channel more relevant. When τ_z is sufficiently high, this swamps the information effect and thus λ increases again. This is because when prices are sufficiently informative and a sufficient measure of other investors are incorporating this information into the price, it is no longer as costly to deviate from rational expectations.

The relation between λ and the precision of investors' prior beliefs, τ, is qualitatively and economically distinct from that found with the parameters that drive τ_p. Specifically, the dismissive equilibrium obtains when prior precision is sufficiently high or sufficiently low, while intermediate values give rise to the mixed equilibrium. When prior precision is extremely high, the information effect is relatively small and the cost of distorting beliefs is low. Intuitively, when investor i faces little uncertainty about fundamentals, dismissing the information in prices is not very costly in terms of uncertainty reduction or inefficient investment. In contrast, when the prior precision is low, it is costly to dismiss the information in prices; however, the speculative effect is also large, and it dominates the information effect. To see this, it is useful to return to (35) and

[28] The speculative effect is sufficiently strong as $\delta_{p,i}$ approaches zero that investor i's objective function is either downward-sloping (as in Panel B or U-shaped (as in Panels A and C).

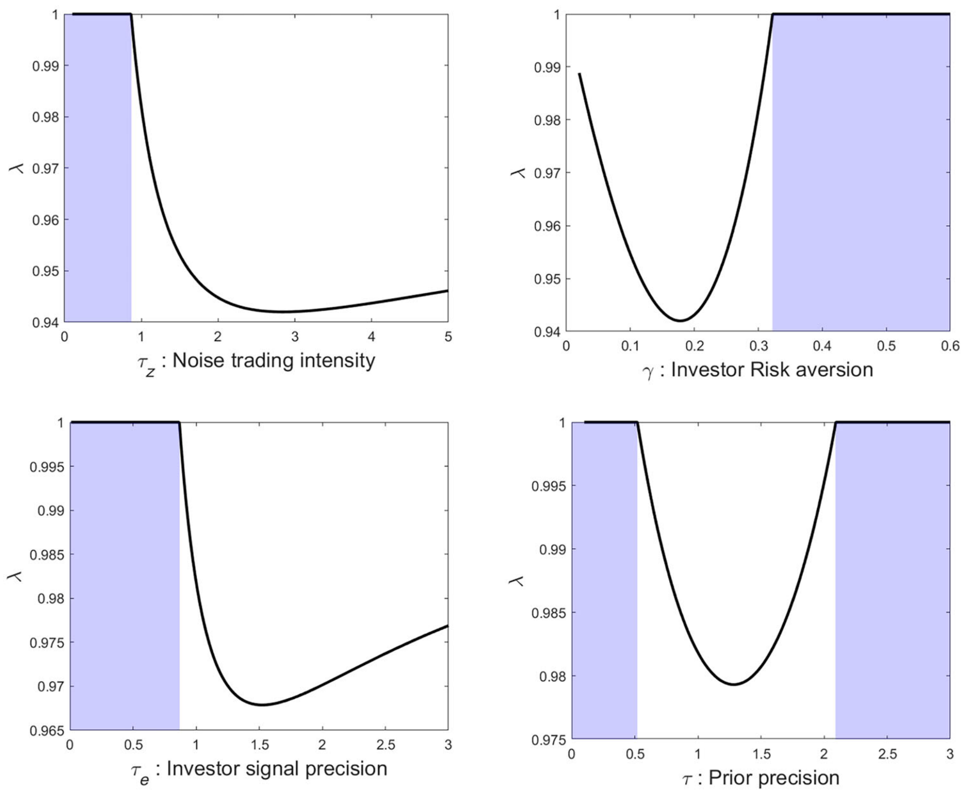

Figure 2. Composition of equilibrium beliefs: λ **versus underlying parameters**. The figure plots the equilibrium fraction of investors choosing $\delta_{p,i} = 0$, λ, as a function of the model primitives. Other parameters are: $\tau = \tau_e = \tau_z = 1$, $\gamma = 0.3$. The blue regions indicate the parameter combinations in which we have the dismissive equilibrium, that is, all agents ignore price information (i.e., $\delta_{p,i} = 0$). (Color figure can be viewed at wileyonlinelibrary.com)

(36), the expressions that characterize the marginal impact of the information and speculative effects. As prior precision decreases (i.e., $\tau \to 0$), these expressions show that the information effect is bounded above, but the speculative effect continues to decrease with $\delta_{p,i}$, that is,

$$-\frac{1}{\mathrm{var}_i[F|s_i, s_p]}\frac{\partial \mathrm{var}_i[F|s_i, P]}{\partial \delta_{p,i}} \to \frac{\tau_p}{\tau_e + \delta_{p,i}\tau_p}, \quad \frac{1}{\mathrm{var}_i[F-P]}\frac{\partial \mathrm{var}_i[F-P]}{\partial \delta_{p,i}} \to \frac{1}{\delta_{p,i}}. \tag{42}$$

Thus, for sufficiently low $\delta_{p,i}$, the speculative effect dominates irrespective of what other investors choose, and the resulting equilibrium features dismissiveness (i.e., $\delta_{p,i} = 0$ for all i). For intermediate levels of prior precision, the relative magnitude of the speculative effect depends on the behavior of other investors, and so the mixed equilibrium obtains.

The following is a notable corollary of Proposition 2.

COROLLARY 1: *As* $\tau_z \to \infty$, $\lambda \to 1$.

This result implies that even as the noise in prices becomes arbitrarily small, the fraction of investors who use price information must be zero. Intuitively, this follows from the fact that as prices become arbitrarily informative, the cost of distorting beliefs (i.e., ignoring the price) becomes smaller.[29]

This is in sharp contrast to the implications of a rational-expectations equilibrium in standard settings. For instance, in the noisy rational-expectations model of Grossman and Stiglitz (1980), the fraction of uninformed investors who condition on prices *increases* as the informativeness of prices increases. In the limit, as $\tau_z \to \infty$, rational-expectations investors ignore their private signals and only condition on the price (i.e., $A_i \to 0$, $B_i \to 1$). In contrast, under wishful thinking, as $\tau_z \to \infty$, more investors ignore the price and condition only on their private information (i.e., $A_i > 0$, $B_i = 0$). This is true even though the cost of choosing rational expectations is zero in our setting. This qualitative difference suggests that allowing for subjective belief choice about how investors interpret information is likely to play an important role in understanding how markets aggregate and reflect information.

Finally, it is worth noting that there are two closely related but distinct types of disagreement that arise in our model. In the dismissive equilibrium, each investor believes that their own signal is informative but all other investors disagree. As a result, all investors *agree* that the price is uninformative and choose to ignore it. In the mixed equilibrium, positive measures of investors disagree about the quality of others' signals and so *also* disagree about the quality of price information. In models with difference of opinions or cursedness, investors also underweight or dismiss the information in prices, but the extent to which they do so is determined as an exogenous parameter (e.g., Banerjee (2011) or Eyster, Rabin, and Vayanos (2018)).[30] In contrast, our model generates the fraction of dismissive investors λ as an *endogenous* outcome that depends on other primitive parameters of the model.

D. Beliefs about Noise Traders

Assuming that investors hold correct beliefs about the volatility of noise trading (i.e., τ_z) constrains $\delta_{p,i} \leq 1$ in the benchmark model, that is, investors cannot overestimate the quality of price information. In what follows, we

[29] Note that the limit of the equilibria is not an equilibrium of the limit. If there is no aggregate noise in price (i.e., $\tau_z = \infty$), then we revert to a version of the Grossman (1977) paradox, since investors ignore their private information when forming demands.

[30] In other words, disagreement about private signals is driven by exogenous parameters. In Mondria, Vives, and Yang (2021), underreaction to price information does not stem from disagreement about private information.

analyze a setting in which investor i's subjective beliefs about z are given by

$$z \sim_i \mathcal{N}\left(0, \frac{1}{\psi_i \tau_z}\right), \tag{43}$$

where $\psi_i \in [0, \infty)$ is chosen along with $\{\pi_i, \rho_i\}$ to maximize (12). It is then straightforward to show that investors' beliefs about price informativeness depend jointly on their subjective beliefs about the informativeness of others' signals (i.e., ρ_i and π_i) and beliefs about the supply shocks (i.e., ψ_i), that is, we can redefine $\delta_{p,i}$ as

$$\delta_{p,i} \equiv \frac{\frac{\beta^2}{\tau_z}}{\frac{\beta^2}{\psi_i \tau_z} + \frac{1-\rho_i^2}{\pi_i \tau_e}} \in [0, \infty]. \tag{44}$$

Then, for sufficiently large ψ_i, investor i chooses to overestimate the precision of the price signal (i.e., $\delta_{p,i} > 1$).[31]

PROPOSITION 3: *Suppose all investors incur the experienced utility penalty and choose $\delta_{p,i}$ as in (44). Then there exists some $\bar{\bar{\tau}}_p \geq \underline{\underline{\tau}}_p > 0$, such that:*

1. *For all $\tau_p \leq \underline{\underline{\tau}}_p$, there exists a unique equilibrium in which all investors ignore the information in prices (i.e., $\delta_{p,i} = \delta_p = 0$ for all i).*
2. *For all $\tau_p \geq \bar{\bar{\tau}}_p$, there does not exist a pure symmetric equilibrium in which all investors choose the same subjective beliefs $\delta_{p,i} = \delta_p$. If there exists a solution (λ, δ_p^*) to equations (A20) and (A21) in Appendix A, then there exists an equilibrium in which investors mix between two sets of beliefs: a fraction λ optimally chooses $\delta_{p,i} = 0$, while the remaining fraction $1 - \lambda$ optimally chooses $\delta_{p,i} = \delta_p^* > 1$.*

The intuition parallels that of Proposition 2, but since $\delta_{p,i}$ is unconstrained, the mixed equilibrium features a mass $1 - \lambda$ of investors who *overweight* the information in prices due to the relative importance of the information effect when $\tau_p > \bar{\bar{\tau}}_p$. Thus, our model predicts that conflicting biases and investment styles naturally arise in otherwise ex ante identical traders. This suggests that the observation of such differences does not require that investors are endowed with differential ability, preferences, or information.[32] Moreover, our model

[31] In practice, investors are likely to have subjective beliefs about both aspects of the market environment, and distinguishing the primary source of variation in $\delta_{p,i}$ remains an open empirical question. An earlier version of this paper restricted attention to the case of subjective beliefs about noise trading volatility exclusively, while assuming that subjective beliefs about others were correct. In this case, $\delta_{p,i} = \psi_i$.

[32] Similar heterogeneity in investment strategies can arise in models in which investors are heterogeneously informed by assumption. For example, in noisy rational-expectations models with heterogeneously informed investors (e.g., Wang (1993)), better-informed investors behave as fundamental traders or contrarians, while less well-informed traders condition on price information and behave like momentum traders (see also Brown and Jennings (1989)).

provides predictions about when we are more likely to see such heterogeneity arise endogenously as a result of market conditions, a feature we explore in Section V.

IV. Beliefs about Private Signals

In the benchmark model, investors are assumed to have correct beliefs about the precision of their own, private signal. We now relax this assumption and explore how the predictions that arise distinguish our model from existing analyses.

In what follows, we now suppose that investor i's subjective belief about her private signal is given by

$$s_i|F \sim_i \mathcal{N}\left(F, \frac{1}{\delta_{e,i}\tau_e}\right), \tag{45}$$

where $\delta_{e,i} \in [0, \infty)$ denotes the extent to which investor i under- or overestimates the precision of her private signal. As before, $\delta_{e,i} = 1$ corresponds to rational expectations and deviations from this generate a cost due to the experienced utility penalty (appropriately modified).

The key difference from our benchmark analysis is that subjective beliefs about the private signal affect anticipatory utility only via the information effect. The perceived precision of one's own signal affects each investor's posterior uncertainty about fundamentals (i.e., $\mathrm{var}_i[F|s_i, s_p]$), but does not impact their perception of the unconditional uncertainty about returns (i.e., $\mathrm{var}_i[F - P]$). Specifically,

$$\frac{\partial AU_i}{\partial \delta_{e,i}} \propto -\frac{1}{\mathrm{var}_i[F|s_i, s_p]} \frac{\partial \mathrm{var}_i[F|s_i, s_p]}{\partial \delta_{e,i}} = \frac{\tau_e}{\tau + \delta_{e,i}\tau_e + \delta_{p,i}\tau_p} \geq 0. \tag{46}$$

The absence of any speculative effect implies that investors never choose to underweight their private information in equilibrium. To illustrate this effect, we first consider a special case in which investors can only choose their beliefs about private signals.

PROPOSITION 4: *Suppose all investors incur the experienced utility penalty and exhibit objective beliefs about the price signal, that is, $\delta_{p,i} = 1$ for all i. Then there exists a unique equilibrium in which the optimal choice of $\delta_{e,i} = \delta_e$ satisfies*

$$\frac{\left(\tau + \tau_p + \tau_e\delta_e(2 - \delta_e)\right)^{\frac{3}{2}}}{\left(\tau + \tau_p + \tau_e\delta_e\right)^{\frac{3}{2}}} = 2(\delta_e - 1). \tag{47}$$

All investors exhibit overconfidence (i.e., $\delta_e > 1$) and the equilibrium degree of overconfidence, δ_e, (i) increases with τ and τ_z, (ii) decreases with risk aversion γ, and (iii) is U-shaped in τ_e.

Proposition 4 demonstrates how overconfidence is shaped by the economic environment. As prior uncertainty falls (τ increases) and as the quality of the information in prices rises (τ_z increases or γ decreases), the marginal benefit of overconfidence falls, providing a smaller increase in the investor's perceived information advantage. Interestingly, however, the cost of overconfidence falls even faster when investors have access to better outside information, and so equilibrium overconfidence is higher. Similar logic applies with respect to the quality of the investor's private signal when τ_e is high (as it too increases the quality of price information). However, when τ_e is low, the cost of increasing overconfidence in a relatively noisy private signal outweighs the benefit and hence δ_e is nonmonotonic in τ_e.

Overconfidence leads investors to trade more aggressively, which at first glance is analogous to a setting in which investors have objectively more precise information. However, when we compare our results to a setting in which investors acquire information endogenously, distinct predictions arise. For instance, Verrecchia (1982) shows that investors choose to acquire more precise information when (i) prior uncertainty is higher (i.e., τ is lower) and (ii) noise in prices is higher (i.e., τ_z is lower). Intuitively, the benefit of acquiring private information is larger when investors' given information sources are of lower quality. In contrast, the opposite comparative statics arise for endogenous beliefs: it is less costly for investors to exhibit overconfidence when (i) prior uncertainty is lower or (ii) prices are more informative. As such, one may be able to distinguish our model's predictions from specific models of costly information acquisition by comparing how dispersion in expectations or trading positions vary with prior uncertainty or price informativeness.[33]

Next, we analyze the implications when each investor chooses her beliefs about both her private signal and the price signal, as in (44). In this case, the partial derivative of equation (46) with respect to $\delta_{p,i}$ yields

$$\frac{\partial^2 AU}{\partial \delta_{e,i} \partial \delta_{p,i}} \propto \left(\frac{\kappa \left(\delta_{e,i} \tau_e + \tau \right) - 2\delta_{p,i} \tau_p \kappa - 3\delta_{p,i}^2 \tau_p}{2\delta_{p,i} \left(\kappa + \delta_{p,i} \right)} \right), \text{where} \tag{48}$$

$$\kappa \equiv \left(\frac{\Lambda}{1 - \Lambda} \right)^2 \frac{\tau}{\tau_p} \quad \text{and} \quad \Lambda = \frac{\bar{\delta}_{e,i} \tau_e + \bar{\delta}_p \tau_p}{\tau + \bar{\delta}_{e,i} \tau_e + \bar{\delta}_p \tau_p}. \tag{49}$$

In general, this implies that the marginal benefit of overconfidence depends nonmonotonically on the investor's subjective interpretation of the price signal $\delta_{p,i}$. For instance, when $\delta_{p,i}$ is sufficiently small, $\frac{\partial^2 AU}{\partial \delta_{e,i} \delta_{p,i}} > 0$: a decrease in $\delta_{p,i}$ lowers the marginal utility of increasing $\delta_{e,i}$. Intuitively, when $\delta_{p,i}$ is low (close to zero), choosing to be increasingly dismissive of the price (i.e., moving further

[33] As suggested by the Associate Editor, we acknowledge that the implications of noisy rational-expectations models are sensitive to the underlying assumptions about information acquisition technology (e.g., costly precision vs. entropy-based cost functions). While it is difficult to ensure that wishful thinking always generates different predictions from all rational-expectations models, our discussion highlights a key difference in the underlying economic mechanism between the two settings.

away from rational expectations) lowers the marginal value of overconfidence about one's private signal. On the other hand, when $\delta_{p,i}$ is sufficiently large, the effect is reversed. For instance, when $\delta_{p,i}$ is greater than one, a decrease in $\delta_{p,i}$ implies *more* objective beliefs about price information, which increases the marginal benefit of distorting beliefs about private information.

Taken together, the above suggests that an investor gains less from distorting her subjective beliefs about private signals when her beliefs about the price information are more distorted. Moreover, our earlier analysis implied that, in a symmetric equilibrium, investors optimally choose to set $\delta_{p,i}$ as low as possible because of the relative importance of the speculative effect. In Proposition 5, we characterize how these two channels interact when we extend our benchmark analysis to allow for subjective beliefs about private signals.

PROPOSITION 5: *Suppose all investors incur the experienced utility penalty. There exists a $\underline{\tau}_p > 0$ such that for all $\tau_p \leq \underline{\tau}_p$, there exists a symmetric equilibrium in which $\delta_{p,i} = \bar{\delta}_p \equiv \delta_p$ and $\delta_{e,i} = \bar{\delta}_{e,i} \equiv \delta_e$ for all i.*

(i) If subjective belief choice is unconstrained (i.e., $\delta_{p,i}, \delta_{e,i} \in [0, \infty)$), then the equilibrium choices are $\delta_e = 1$ and $\delta_p = 0$.

(ii) If subjective belief choices about the price signal are bounded below (i.e., $\delta_{p,i} \geq \underline{\delta} > 0$), then the symmetric equilibrium choices are $\delta_p = \underline{\delta}$ and δ_e satisfies

$$\frac{1}{2\left(1+\frac{\kappa}{\underline{\delta}}\right)^{\frac{1}{2}}\left(\tau_e\delta_e+\tau_p\underline{\delta}+\tau\right)^{\frac{3}{2}}} = \frac{(1+\kappa)(\delta_e-1)}{\left[(1+\kappa)\left(\tau+\tau_e+\tau_p-\tau_e(\delta_e-1)^2\right)-\tau_p\left(\underline{\delta}-1\right)^2\right]^{\frac{3}{2}}}, \tag{50}$$

where $\kappa = \frac{(\delta_e\tau_e+\underline{\delta}\tau_p)^2}{\tau\tau_p}$.

As in Proposition 2, we show in the proof of Proposition 5 that there exists a unique symmetric equilibrium in which all investors choose the same subjective beliefs (i.e., $\delta_{p,i} = \delta_p$ and $\delta_{e,i} = \delta_e$ for all i) when the price is not too informative.[34] When beliefs about price informativeness are unconstrained, investors choose to completely dismiss price information, that is, set $\delta_{p,i} = 0$. However, this also implies that investors condition on private information efficiently, that is, $\delta_{e,i} = 1$. This is because when $\delta_{p,i} = 0$, investor i does not benefit from distorting her beliefs about the private signal (i.e., $\frac{\partial AU}{\partial \delta_{e,i}}|_{\delta_{p,i}=0} = 0$). When beliefs about price informativeness are constrained below at $\underline{\delta}$, investors still choose to maximally distort their beliefs about price informativeness (i.e., $\delta_{p,i} = \underline{\delta}$) but now also exhibit overconfidence in their private signal (i.e., $\delta_{e,i} = \delta_e > 1$). Figure 3 illustrates this result: as the lower bound $\underline{\delta}$ increases, the equilibrium choice of δ_e also increases. This is consistent with the intuition above: each investor gains more from distorting her beliefs about private information when her beliefs about prices are closer to the objective distribution.

[34] While numerical analysis shows that analogous mixed equilibria exist when τ_p is sufficiently high, and we can characterize the equations that pin down such equilibria, we are unable to analytically establish their existence or uniqueness.

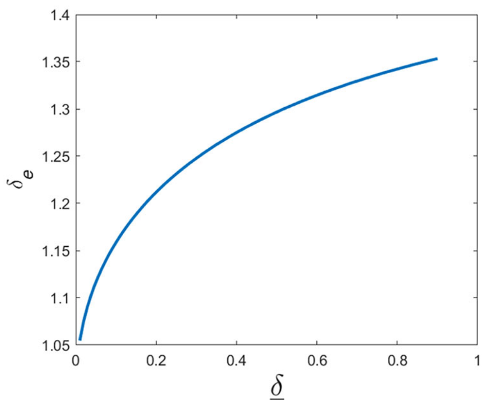

Figure 3. Optimal δ_e versus the lower bound $\underline{\delta}$. The figure plots the equilibrium choice of δ_e in a symmetric equilibrium as a function of the lower bound on $\delta_{p,i}$, given by $\underline{\delta}$. Other parameters are: $\tau = \tau_e = \tau_z = 1$ and $\gamma = 3$. (Color figure can be viewed at wileyonlinelibrary.com)

Much of the existing literature separately considers overconfidence (e.g., Odean (1998)) or dismissiveness of price information (e.g., Eyster, Rabin, and Vayanos (2018), Mondria, Vives, and Yang (2021)) in which superficially similar behavior can arise (e.g., investors place less weight on price information). In contrast, in our model both types of biases arise as distinct phenomena as a consequence of a single assumption: individuals' experience of anticipatory utility. Moreover, the endogenous interaction of these biases provides a distinct prediction relative to these existing frameworks. For instance, our analysis predicts a negative relation between price dismissiveness and overconfidence in private information, which may be testable empirically. We discuss this further in Section V.

V. Implications

When investors experience anticipatory utility, our model predicts that they systematically and predictably deviate from rational expectations. In this section, we explore how these deviations lead to distinct predictions for observables. Section V.A provides predictions for return moments.[35] Section V.B provides implications for investor forecasts and disagreement.

Our primary focus is to distinguish our model's predictions from the rational-expectations benchmark. To be clear, a number of our predictions appear similar to those that arise in existing behavioral models. For instance, as in models of difference of opinions or costly price information, returns can exhibit positive predictability in our setting. However, the key distinction of our model is that this prediction is state-dependent, since investors' bias is endogenous and its relation with other observables depends on economic conditions.

[35] In Appendix B.2, we also analyze the model's implications for trading volume.

A. Return Moments

Since the risk-free security is the numeraire, the (net) return on it is zero. As is standard in the literature, we focus on the dollar return on the risky asset, denoted by $R = F - P$.[36] We begin by characterizing implications for return volatility and predictability, since these help distinguish our model from related models. We then characterize predictions for expected returns in an extension of our benchmark analysis.

A.1. Return Volatility and Return Predictability

Return Volatility. The unconditional variance in returns, which we also refer to as return volatility, is given by

$$\sigma_R^2 = \text{var}(R) = \frac{(1-\Lambda)^2}{\tau} + \frac{\Lambda^2}{\tau_p}. \tag{51}$$

Return Predictability. Return predictability is measured by the coefficient θ, which denotes the degree to which the current price P predicts return R, that is,

$$\mathbb{E}[R|P] = m + \theta(P - m), \quad \text{where} \qquad \theta \equiv \frac{\text{cov}(R,P)}{\text{var}(P)} = \frac{1}{\Lambda}\left(\frac{\tau_p}{\tau + \tau_p} - \Lambda\right). \tag{52}$$

When $\theta < 0$, higher current prices predict lower future returns on average. We refer to this phenomenon as *reversals*. In contrast, when $\theta > 0$, higher current prices predict higher future returns on average, which we refer as *continuation*.[37]

To gain some intuition for the above characterizations, note that when investors choose how to interpret both their own private signal and others' private signals,

$$\Lambda \equiv \frac{\bar{\delta}_{e,i}\tau_e + \bar{\delta}_p\tau_p}{\tau + \bar{\delta}_{e,i}\tau_e + \bar{\delta}_p\tau_p}. \tag{53}$$

Believing that others are less well-informed in aggregate (i.e., $\delta_{p,i} < 1$) decreases Λ relative to rational expectations, while overconfidence in private information (i.e., $\delta_{e,i} > 1$) increases Λ. This leads to the following observations.

[36] While dollar returns are distinct from rates of return commonly used in the empirical literature, earlier work in similar settings shows that predictions on dollar returns remain qualitatively unchanged when converted to rates of return (e.g., Banerjee (2011)).

[37] Ideally, we would prefer to characterize return predictability in a dynamic model of wishful thinking. Unfortunately, this is not analytically tractable in our setting. However, the return predictability coefficient θ in our static framework is closely related to time-series reversals and continuation (drift) in dynamic settings (e.g., see Banerjee, Kaniel, and Kremer (2009)). As such, we characterize the implications in terms of "reversals" and "continuation" in returns.

First, returns exhibit reversals if and only if $\Lambda > \frac{\tau_p}{\tau+\tau_p}$. Thus, equation (53) implies that (i) returns always exhibit reversals under rational expectations (since $\delta_{e,i} = \delta_{p,i} = 1$) and (ii) returns cannot exhibit continuation unless investors underreact to price information (i.e., $\delta_{p,i} < 1$). Second, return volatility increases with Λ if and only if $\Lambda > \frac{\tau_p}{\tau+\tau_p}$. When Λ is higher, prices reflect fundamentals more closely and this reduces volatility in returns (via the $\frac{(1-\Lambda)^2}{\tau}$ term in (51)). Prices, however, are also more sensitive to the noise in prices, which increases volatility (via the $\frac{\Lambda^2}{\tau_p}$ term). The first effect dominates when Λ is low, but the second dominates otherwise.

These observations lead to the following predictions, which describe how return volatility and predictability change endogenously in our model as a function of price informativeness, and which distinguish our model from the rational-expectations benchmark.

PROPOSITION 6:

(1) In the rational-expectations equilibrium, return volatility is decreasing in price informativeness while return predictability is increasing in price informativeness (i.e., $\frac{\partial}{\partial\tau_p}\sigma_R^2 < 0$ *and* $\frac{\partial}{\partial\tau_p}\theta > 0$*).*

(2) Suppose all investors incur the experienced utility penalty and choose subjective beliefs about the price signal only.

(i) When τ_p *is sufficiently low, return volatility is decreasing in price informativeness while return predictability is increasing in price informativeness (i.e.,* $\frac{\partial}{\partial\tau_p}\sigma_R^2 < 0$ *and* $\frac{\partial}{\partial\tau_p}\theta > 0$*).*

(ii) When τ_p *is sufficiently high, both return volatility and predictability decrease with price informativeness (i.e.,* $\frac{\partial}{\partial\tau_p}\sigma_R^2 < 0$ *and* $\frac{\partial}{\partial\tau_p}\theta < 0$*).*

Note that volatility is negatively related to price informativeness for both types of models, consistent with the empirical evidence documented by Dávila and Parlatore (2019). However, while predictability always increases with price informativeness for rational expectations, it *decreases* with price informativeness in the mixed equilibrium. This provides a distinctive prediction: while a positive relation between volatility and predictability can arise due to wishful thinking, it is inconsistent with rational expectations in our setting.[38]

Figure 4 provides a numerical illustration of these results. Specifically, the figure plots volatility and predictability for the rational expectations (dashed) and subjective beliefs equilibria (solid) as a function of risk aversion, γ. Recall that an increase in risk aversion makes prices objectively less informative (i.e., τ_p is decreasing in γ). The kink in the solid lines corresponds to the value of γ at which the subjective beliefs equilibrium switches from the mixed equilibrium (low γ, high τ_p) to the symmetric equilibrium (high γ, low τ_p).

[38] While a negative relation arises in rational expectations, it can also follow from the symmetric equilibrium and so does not necessarily rule out wishful thinking equilibria. In Appendix B.2, we formally establish that predictability is always lower under rational expectations.

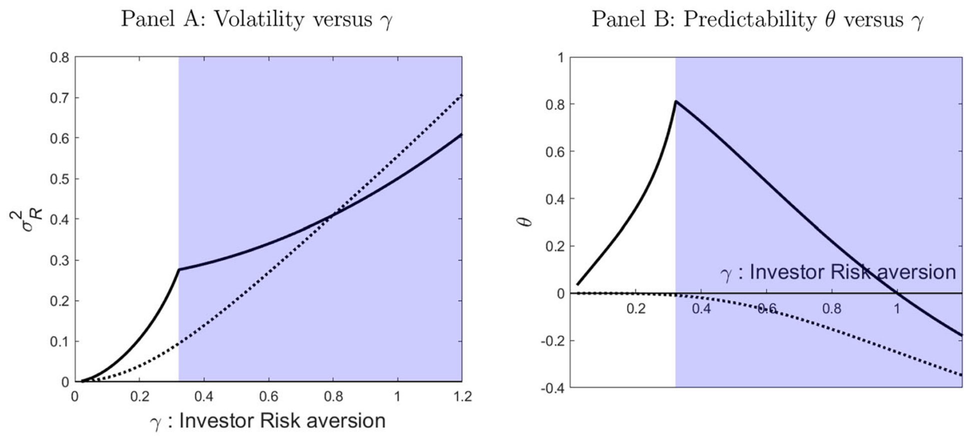

Figure 4. Return volatility and predictability. The figure plots return volatility (variance) and return predictability as a function of risk aversion for subjective beliefs (solid line) and rational expectations (dotted line). Other parameters are set to $\tau = \tau_e = \tau_z = 1$ and $Z = 0$. The blue shaded region corresponds to the symmetric equilibrium. (Color figure can be viewed at wileyonlinelibrary.com)

As in Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), one can interpret variation in investor risk aversion as a source of business-cycle variation: high risk aversion is associated with higher (aggregate) volatility and recessions, while low risk aversion is associated with low volatility and economic expansions. Under this interpretation, our model predicts that expansions are more likely to feature (i) more informative prices and (ii) a positive relation between return autocorrelation and volatility.[39] In contrast, recessions are periods of high market stress, characterized by (i) low price informativeness and (ii) a negative relation between predictability and volatility. Moreover, as Figure 4 illustrates, such periods are also associated with lower, or even negative, return predictability (i.e., reversals). These predictions are broadly consistent with the evidence on time-series variation in momentum returns and crashes (e.g., Cooper, Gutierrez Jr, and Hameed (2004), Moskowitz, Ooi, and Pedersen (2012), Daniel and Moskowitz (2016)).[40]

[39] Our prediction of a positive relation between economic conditions and price informativeness is consistent with other channels (e.g., Veldkamp (2006)). However, other models predict the opposite relation. For example, in Hong and Stein (2003), negative information is hidden during booms due to short-sales constraints, but revealed during market declines, and so prices may be more informative during recessions.

[40] It is important to note that return predictability in our model corresponds to what is described as time-series momentum in the literature, and not cross-sectional momentum. However, as Banerjee, Kaniel, and Kremer (2009) show in a similar setting with multiple assets, the two notions of momentum are closely related.

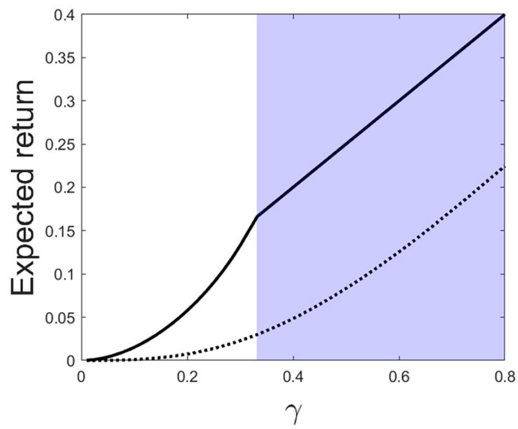

Figure 5. Expected returns. The figure plots the unconditional expected return as a function of risk aversion for subjective beliefs (solid line) and rational expectations (dotted line). Other parameters are set to $\tau = \tau_e = \tau_z = 1$ and $Z = 1$. The blue shaded region corresponds to the symmetric equilibrium. (Color figure can be viewed at wileyonlinelibrary.com)

A.2. Expected Returns

In this section, we extend the analysis to consider a positive aggregate supply of the risky asset, that is, $Z > 0$. While this extension is not as analytically tractable, we solve it numerically and find that the resulting equilibria are qualitatively similar to our benchmark. The unconditional expected return is given by

$$\mathbb{E}[R] = \frac{\gamma}{\int_i \omega_i di} Z, \tag{54}$$

where ω_i is investor i's posterior precision about F (i.e., $\omega_i = (\text{var}_i[F|s_i, s_p])^{-1}$), γ is the coefficient of risk aversion, and Z is the aggregate supply of the risky asset. Our benchmark analysis restricts the mean aggregate supply of the risky asset to $Z = 0$, which implies that the unconditional expected return is always zero. In general, however, subjective belief choice has two, potentially offsetting, effects on the posterior precision (ω_i) relative to the rational-expectations equilibrium: overconfidence in private information increases ω_i, while dismissing price information decreases it.

In the dismissive equilibrium with unconstrained subjective beliefs, all investors put zero weight on prices and correctly interpret their private information (see Proposition 5, part (i)). As a result, only the second effect is relevant, and expected returns are higher than under rational expectations. In the mixed equilibrium, we cannot characterize the net effect analytically but, as Figure 5 illustrates, we numerically find that the second effect dominates the first. As a result, expected returns are higher in the subjective beliefs equilibrium. Moreover, the difference in the risk premium relative to rational expectations falls more quickly within the mixed equilibrium (as price informativeness increases) since a positive measure of investors choose to condition on the price.

B. Investor Forecasts and Disagreement

In this section, we explore the implications of our model for investor forecasts about fundamentals and propose a new, simple test that distinguishes our model from both the rational-expectations benchmark and the standard difference-of-opinions models. We then discuss the model's implications for disagreement across investors.

B.1. Predictability of Forecast Errors

Since investors start with the common prior that $\mathbb{E}_i[R] = \mathbb{E}_i[F - P] = 0$, investor i's forecast revision (FR_i) and forecast error (FE_i) about return R are given by

$$FR_i \equiv \mathbb{E}_i[R|s_i, P], \quad \text{and} \quad FE_i \equiv R - \mathbb{E}_i[R|s_i, P]. \tag{55}$$

Taking averages across i gives us analogous expressions for the consensus forecast revision, $\bar{FR}$, and consensus forecast error, $\bar{FE}$.

Recent literature on information processing and belief updating (e.g., Coibion and Gorodnichenko (2012, 2015)) focuses on regressing consensus forecast errors on consensus forecast revisions and documents underreaction, that is, $\text{cov}(\bar{FE}, \bar{FR}) > 0$. This test does not help distinguish the equilibria we derive, since

$$\text{cov}(\bar{FE}, \bar{FR}) = \text{cov}(F - \bar{\mu}, \bar{\mu}) = \frac{\bar{\delta}_{e,i}\tau_e + \bar{\delta}_p(1 - \bar{\delta}_p)\tau_p}{(\tau + \bar{\delta}_{e,i}\tau_e + \bar{\delta}_p\tau_p)^2}. \tag{56}$$

Notably, the above is positive in any model in which $\bar{\delta}_{e,i} > 0$ and $\bar{\delta}_p \leq 1$, which includes rational-expectations equilibria (where $\bar{\delta}_{e,i} = \bar{\delta}_p = 1$), symmetric difference-of-opinions or cursedness equilibria, and our symmetric wishful thinking equilibria.

Recently, Bordalo et al. (2020a) find that individual forecasts exhibit overreaction, that is, $\text{cov}(FE_i, FR_i) < 0$ even while consensus forecasts exhibit underreaction. In our setting,

$$\text{cov}(FE_i, FR_i) = \text{cov}(F - \mu_i, \mu_i) = \frac{\delta_{e,i}(1 - \delta_{e,i})\tau_e + \delta_{p,i}(1 - \delta_{p,i})\tau_p}{(\tau + \delta_{e,i}\tau_e + \delta_{p,i}\tau_p)^2}. \tag{57}$$

This expression is zero both in rational-expectations models (since $\delta_{e,i} = \delta_{p,i} = 1$) and pure difference-of-opinions models (since $\delta_{e,i} = 1$ while $\delta_{p,i} = 0$). However, the coefficient above can be negative in our model when investors choose to overreact to private signals (i.e., set $\delta_{e,i} > 1$) while discounting price information (i.e., set $\delta_{p,i} < 1$) (as in Section IV). Notwithstanding, this does not distinguish our model from a setting in which investors exogenously exhibit overconfidence in their private signals.

To test our model's predictions more directly, we propose a regression of *individual forecast errors on lagged returns*, that is,

$$FE_i = \alpha_i + \beta_i R_{-1} + u, \tag{58}$$

where β_i measures the sensitivity of investor i's forecast error to lagged returns.[41] The regression coefficient β_i is proportional to

$$\beta_i \propto \text{cov}(FE_i, P) = \text{cov}(F - \mu_i, P) \propto \frac{1 - \delta_{p,i}}{\tau + \delta_{e,i}\tau_e + \delta_{p,i}\tau_p}. \tag{59}$$

Under rational expectations, $\beta_i = 0$ for all investors—investors efficiently use their information and so ex post forecast errors are unpredictable given date-t information. In models with difference of opinions, dismissiveness, or cursedness, $\beta_i > 0$ for all investors since each investor underreacts to the information in prices (i.e., $\delta_{p,i} < 1$).

However, in our model, the distribution of β_i across investors is state-dependent. Specifically, when price informativeness is low, all investors symmetrically underreact to the information in prices (i.e., $\delta_{p,i} = \delta_p < 1$), and so our model predicts a common, positive regression coefficient, that is, $\beta_i = \bar{\beta} > 0$. However, when price informativeness is high, the model predicts that investors exhibit heterogeneous reactions to price information. Specifically, we should observe that while some investors underreact to prices, so that $\beta_i > 0$, the rest (weakly) overreact to prices and so $\beta_i \leq 0$. As such, running regressions of the form (58) may help identify the presence of wishful thinking in financial markets. While an empirical analysis of this type is beyond the scope of the current paper, we hope future work explores such implications in more detail.

B.2. Investor Heterogeneity and Disagreement

Our model provides a mechanism through which investors, who are ex ante identical and symmetrically informed, endogenously choose to exhibit different interpretations of public and private information in response to changing economic conditions.

For instance, disagreement about price information is nonmonotonic in the objective price informativeness (see Figure 2). When prices are extremely noisy (e.g., τ_z is very low) or very precise (e.g., τ_z is very high), disagreement about price information is low because most investors choose to dismiss the information in prices. However, for intermediate levels of price informativeness, many

[41] In our model, "lagged" returns are given by $R_{-1} = P - 0$. The advantage of running the regression at the investor level is that it controls for persistent investor-level differences in forecaster optimism (e.g., due to different priors).

investors dismiss prices but others condition on it and so disagreement is highest. Furthermore, the extent of disagreement depends on the behavior of others. When all others exhibit rational expectations, an individual chooses to ignore prices completely (see Section III.B). In contrast, when most investors dismiss price information but prices are sufficiently informative, some investors choose to overreact to this information (see Section III.D).

More generally, our analysis provides sharp predictions about how heterogeneity in investment strategies varies with economic conditions. For instance, the dismissive investors in our model resemble "fundamentals-based" or value investors who identify mispriced securities using their private information. In contrast, the investors who choose to overweight the price information engage in behavior that (arguably) resembles technical or momentum trading, where investors overextrapolate from past price changes. To the extent that market booms are associated with higher price informativeness, our model suggests that such periods exhibit greater heterogeneity in investment strategies, and in particular in popularity of price/return-based strategies. Periods of market stress, however, are associated with less diversity in investment strategies and greater incidence of value investing.

VI. Concluding Remarks

We develop a model in which investors who experience anticipatory utility choose how to interpret the information available to them before trading in financial markets. We show that wishful thinking endogenously gives rise to a rich set of behavior that is consistent with existing empirical evidence, while providing new insight into how such behavior varies with economic conditions and context. We view this as a promising approach to understanding observed behavior and briefly discuss potential extensions and areas for future work.

Generalized Cost Functions. In our benchmark analysis, the cost function (the experienced utility penalty) is endogenous and depends on equilibrium choices of others through Λ. In other models of subjective belief choice (e.g., Caplin and Leahy (2019)), the cost function is often specified in terms of a statistical distance measure and does not depend on equilibrium choices. While a complete analysis with general cost functions is beyond the scope of this paper, we provide a partial characterization of our results in Appendix B.3. First, for a general class of cost functions, we show that investors choose to dismiss price information in any symmetric equilibrium. Next, we numerically show that our benchmark results from Section III remain qualitatively unchanged using a cost function based on the K-L divergence between subjective and objective beliefs.

Public Information. Our analysis focuses on the subjective interpretation of price and private information in financial markets. However, public signals (e.g., regulatory disclosures) are an important part of the economic information environment. In future work, we hope to study how the interpretation of "exogenous" public information (e.g., disclosures) and "endogeneous" public information (e.g., prices) interact as a result of wishful thinking.[42]

Policy Implications and Welfare. The notion of welfare in settings with heterogeneous beliefs and subjective interpretations is nuanced (see Brunnermeier, Simsek, and Xiong (2014)). In Appendix B.4, we present some preliminary analysis using a measure of welfare that is conservative in that it ignores the gain in anticipatory utility that investors experience by distorting their beliefs. First, we show that in mixed-strategy equilibria, investors who condition on prices have higher objective expected utility (and so lower anticipatory utility). Second, we show that noise traders, who are responsible for aggregate supply shocks in the risky asset, can be better off in the presence of wishful thinking investors. Moreover, overall (objective) experienced utility may be higher with wishful thinking investors than with rational expectations.[43] These results suggest that understanding the role of wishful thinking has potentially important implications for regulatory policy (e.g., disclosure regulations).

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Appendix A: Proofs

A. *Proof of Lemma 2*

We make use of the standard result that for a normally distributed random variable, $X \sim N(\mu, \sigma^2)$, we have

$$\mathbb{E}\left[\exp\left\{-aX - \frac{1}{2}AX^2\right\}\right] = \frac{e^{\frac{a^2\sigma^2 - \mu(2a + A\mu)}{2A\sigma^2 + 2}}}{\sqrt{A\sigma^2 + 1}}. \tag{A1}$$

Given the definition of AU_i in (30) and setting $a = 0$, $A = \frac{1}{\mathrm{var}_i[F|s_i, P]}$, and $X = \mathbb{E}_i[F|s_i, P] \sim N(0, \mathrm{var}_i[F - P] - \mathrm{var}_i[F|s_i, P])$ in the above expression yields (33).

Lemma 1 implies that the price is of the form $P = m + \Lambda(s_p - m)$. Substituting this into the anticipatory utility expression (in equation (33)), we get

$$AU\left(\delta_{p,i}\right) = -\sqrt{\frac{\frac{1}{\tau + \tau_e + \delta_{p,i}\tau_p}}{(1 - \Lambda)^2\frac{1}{\tau} + \Lambda^2\frac{1}{\delta_{p,i}\tau_p}}}. \tag{A2}$$

[42] In a companion paper, Banerjee, Davis, and Gondhi (2023) study how individuals who exhibit wishful thinking interpret private and exogenous public information in a generalized coordination game with externalities (e.g., Angeletos and Pavan (2007)). They show that the interpretation of public information depends on how nonfundamental aggregate volatility affects an individual's payoffs.

[43] Because wishful thinking leads investors to dismiss price information on average, it has two potentially offsetting effects: (i) it makes the price less informative (relative to rational expectations) and (ii) it decreases the price impact of trades. We show that for noise traders, the second effect dominates the first.

Note that given other investors' choices, investor i's marginal anticipatory utility is

$$\frac{\partial}{\partial \delta_{p,i}} AU = \frac{(1-\Lambda)^2 \delta_{p,i}^2 \tau_p^2 - \Lambda^2 \tau(\tau_e + \tau)}{2\delta_{p,i}\left(\Lambda^2 \tau + (1-\Lambda)^2 \delta_{p,i} \tau_p\right)\left(\tau_e + \delta_{p,i}\tau_p + \tau\right)} \times \sqrt{\frac{\frac{1}{\tau+\tau_e+\delta_{p,i}\tau_p}}{(1-\Lambda)^2 \frac{1}{\tau} + \Lambda^2 \frac{1}{\delta_{p,i}\tau_p}}}. \tag{A3}$$

This implies anticipatory utility is increasing in $\delta_{p,i}$ when

$$\frac{\delta_{p,i}^2}{\tau_e + \tau} > \frac{\Lambda^2}{(1-\Lambda)^2} \frac{\tau}{\tau_p^2}, \tag{A4}$$

that is, it is initially decreasing and then increasing in $\delta_{p,i}$. Moreover, note that

$$\lim_{\delta_{p,i} \to 0} \frac{\partial}{\partial \delta_{p,i}} AU = -\infty, \tag{A5}$$

and $\frac{\partial}{\partial \delta_{p,i}} AU$ equals zero at

$$\bar{\delta} = \frac{1}{\tau_p}\left(\frac{\Lambda}{1-\Lambda}\right)\sqrt{\tau(\tau_e + \tau)}. \tag{A6}$$

B. Lemma A1 and Its Proof

LEMMA A1: *With experienced utility penalty, the cost function is the disutility that the investor incurs under the objective distribution and is given by*

$$C(\delta_{p,i}) = \frac{1}{\sqrt{\Lambda^2 (\delta_{p,i} - 1)^2 + var(F-P)\left(\tau + \tau_e + \tau_p \delta_{p,i}(2 - \delta_{p,i})\right)}}. \tag{A7}$$

PROOF: Based on Definition 1 and ignoring the first term (which is constant) in it, the cost function is

$$\begin{aligned} C(\delta_{p,i}) &= -\mathbb{E}\left[-\gamma \exp\left\{-\gamma x_i^*(\delta_{p,i}) \times (F-P)\right\}\right] \\ &= \mathbb{E}[\gamma \exp\{-\omega_i(\mu_i - P) \times (F-P)\}]. \end{aligned}$$

Suppose we have

$$\begin{pmatrix} \mu_i - P \\ F - P \end{pmatrix} \sim N\left(\begin{pmatrix} m \\ m \end{pmatrix}, \begin{pmatrix} \sigma_{ERi}^2 & \sigma_{ERi,ER} \\ \sigma_{ERi,ER} & \sigma_{ER}^2 \end{pmatrix}\right). \tag{A8}$$

In this case, the cost function simplifies to

$$C(\delta_{p,i}) = \sqrt{\frac{\omega_i^{-2}}{(\omega_i^{-1} + \sigma_{ERi,ER})^2 - \sigma_{ER}^2 \sigma_{ER,i}^2}}. \tag{A9}$$

Note that

$$\sigma^2_{ER,i} = \text{var}(\mu_i - P) = \text{var}\big(A_i(s_i - m) + B_i(s_p - m) - \Lambda(s_p - m)\big)$$
$$= \frac{\big(A_i + B_i - \Lambda\big)^2}{\tau} + \frac{A_i^2}{\tau_e} + \frac{(B_i - \Lambda)^2}{\tau_p},$$
$$\sigma^2_{ER} = \text{var}(F - P) = \text{var}\big(F - m - \Lambda(s_p - m)\big) = \tfrac{(1-\Lambda)^2}{\tau} + \tfrac{\Lambda^2}{\tau_p},$$
$$\sigma_{ERi,ER} = \text{cov}(\mu_i - P, F - P) = \text{cov}\big(A_i s_i + B_i s_p - \Lambda s_p, F - \Lambda s_p\big)$$
$$= \frac{\big(A_i + B_i - \Lambda\big)(1 - \Lambda)}{\tau} - \frac{(B_i - \Lambda)\Lambda}{\tau_p}.$$

Substituting these coefficients into the cost function given in equation (A9) and simplifying, we get

$$C(\delta_{p,i}) = \frac{1}{\sqrt{\Lambda^2(\delta_{p,i} - 1)^2 + \text{var}(F - P)\big(\tau + \tau_e + \tau_p \delta_{p,i}(2 - \delta_{p,i})\big)}}.$$

□

C. Proof of Proposition 1

The objective of investor i is given by

$$\max_{\delta_{p,i}} AU(\delta_{p,i}) - C(\delta_{p,i}),$$

which translates into

$$\max_{\delta_{p,i}} - \sqrt{\frac{1}{\left(1 + \frac{\kappa}{\delta_{p,i}}\right)(\tau + \tau_e + \delta_{p,i}\tau_p)}} - \sqrt{\frac{1}{(1 - \delta_{p,i})^2 \kappa \tau_p + (1 + \kappa)\big(\tau + \tau_e + \tau_p \delta_{p,i}(2 - \delta_{p,i})\big)}},$$

where $\kappa = (\frac{\Lambda}{1-\Lambda})^2 \frac{\tau}{\tau_p}$. Since all other investors are rational, $\Lambda = \frac{\tau_e + \tau_p}{\tau + \tau_e + \tau_p}$ and κ reduces to $\kappa = \frac{\gamma^2(\tau_e + \tau_p)^2}{\tau \tau_e^2 \tau_z}$. Investor i chooses $\delta_{p,i} = 0$ if and only if

$$AU(0) - C(0) > AU(\delta_{p,i}) - C(\delta_{p,i})$$

for all $\delta_{p,i} \in (0, 1]$, or equivalently

$$1 + \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})} - \frac{C(0)}{C(\delta_{p,i})} > 0. \tag{A10}$$

Let $R \equiv \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})}$ and $L \equiv \frac{C(0)}{C(\delta_{p,i})}$. First, we examine the case in which $\tau_p \to 0$. Note that

$$\lim_{\tau_p \to 0} R = \sqrt{\delta_{p,i}}, \quad \lim_{\tau_p \to 0} L = 1.$$

This implies that as $\tau_p \to 0$, condition (A10) reduces to

$$\lim_{\tau_p \to 0} 1 + \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})} - \frac{C(0)}{C(\delta_{p,i})} = 1 + \sqrt{\delta_{p,i}} - 1 > 0,$$

which implies that for τ_p sufficiently low, investor i chooses $\delta_{p,i} = 0$.

Next, we examine the case in which $\tau_p \to \infty$. Note that $\lim_{\tau_p \to \infty} R = \lim_{\tau_p \to \infty} L = 1$, so

$$\lim_{\tau_p \to \infty} 1 + \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})} - \frac{C(0)}{C(\delta_{p,i})} > 0,$$

which implies that for τ_p high enough, investor i chooses $\delta_{p,i} = 0$.

While we prove this partial equilibrium result analytically for low and high τ_p, numerical simulations show that the result holds for all τ_p.

D. *Proof of Proposition 2*

Proof of part 1: Note that $\delta_{p,i} = 0 \;\; \forall\, i$ is a symmetric equilibrium if and only if

$$AU(0) - C(0) > AU(\delta_{p,i}) - C(\delta_{p,i}) \tag{A11}$$

for all $\delta_{p,i}$, or equivalently,

$$1 + \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})} - \frac{C(0)}{C(\delta_{p,i})} > 0.$$

Let $R \equiv \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})}$ and $L \equiv \frac{C(0)}{C(\delta_{p,i})}$. Note that

$$\lim_{\tau_p \to 0} R = \sqrt{\delta_{p,i}}, \quad \lim_{\tau_p \to 0} L = 1, \tag{A12}$$

which implies that as $\tau_p \to 0$, condition (A11) reduces to

$$\lim_{\tau_p \to 0} 1 + \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})} - \frac{C(0)}{C(\delta_{p,i})} = \sqrt{\delta_{p,i}} > 0,$$

which implies in turn that for τ_p low enough, the only equilibrium is $\delta_{p,i} = 0 \;\forall\, i$.

Proof of part 2:

Step 1: For τ_p high, there is no pure-strategy symmetric equilibrium.

Suppose all other investors choose $\bar{\delta}_p \neq 0$. Note that

$$\lim_{\tau_p \to \infty} R = \lim_{\tau_p \to \infty} L = 1, \tag{A13}$$

so

$$\lim_{\tau_p \to \infty} \left(1 + \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})} - \frac{C(0)}{C(\delta_{p,i})} \right) > 0, \tag{A14}$$

which implies that for sufficiently high τ_p, an investor prefers to choose $\delta_{p,i} = 0$ for any $\bar{\delta}_p \neq 0$. Next, suppose all other investors choose $\bar{\delta}_p = 0$. In this case

$$\lim_{\tau_p \to \infty} R = \lim_{\tau_p \to \infty} \sqrt{2 - \bar{\delta_p}}, \tag{A15}$$

$$\lim_{\tau_p \to \infty} L = \lim_{\tau_p \to \infty} \tfrac{1}{\gamma} \sqrt{\left(2 - \bar{\delta_p}\right)} = \infty, \tag{A16}$$

which suggests that

$$\lim_{\tau_p \to \infty} \left(1 + \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})} - \frac{C(0)}{C(\delta_{p,i})} \right) < 0, \tag{A17}$$

and hence investor i will not choose $\delta_{p,i} = 0$. Taken together, this analysis implies that there is no pure-strategy symmetric equilibrium for sufficiently high τ_p.

Step 2: For τ_p high enough, the objective function of investor i is either U-shaped or downward-sloping in $\delta_{p,i}$ for $\delta_{p,i} \in [0, 1]$.

Given that $\delta_{p,i} = 0 \;\; \forall\, i$ is not an equilibrium (from step 1), assume that the rest of the investors choose an average $\delta_{p,i}$ of $\bar{\delta}_p \neq 0$. The objective of investor i given by

$$\max_{\delta_{p,i}} - \sqrt{\frac{1}{\left(1 + \frac{\kappa}{\delta_{p,i}}\right)\left(\tau + \tau_e + \delta_{p,i}\tau_p\right)}} - \sqrt{\frac{1}{\left(1 - \delta_{p,i}\right)^2 \kappa \tau_p + (1 + \kappa)\left(\tau + \tau_e + \tau_p \delta_{p,i}\left(2 - \delta_{p,i}\right)\right)}},$$

where $\kappa = \frac{(\tau_e + \bar{\delta}_p \tau_p)^2}{\tau \tau_p}$. The first-order condition (FOC) for this objective is

$$\frac{1 - \frac{\kappa(\tau + \tau_e)}{\delta_{p,i}^2 \tau_p}}{\sqrt{\left(1 + \frac{\kappa}{\delta_{p,i}}\right)^3 \left(\tau + \tau_e + \delta_{p,i}\tau_p\right)^3}} - \frac{2\left(\delta_{p,i} - 1\right)}{\sqrt{\left((1 + \kappa)\left(\tau + \tau_e + \tau_p\right) - \left(1 - \delta_{p,i}\right)^2 \tau_p\right)^3}} = 0. \tag{A18}$$

Note that $\lim_{\tau_p \to \infty} \frac{\kappa}{\tau_p} = \frac{(\tau_e + \bar{\delta}_p \tau_p)^2}{\tau \tau_p^2} \to \frac{\bar{\delta}_p^2}{\tau}$. Dividing the FOC by τ_p and taking the limit as $\tau_p \to \infty$, the FOC reduces to

$$1 - \frac{\bar{\delta}_p^2(\tau + \tau_e)}{\tau \delta_{p,i}^2} = 2\big(\delta_{p,i} - 1\big),$$

which simplifies to

$$3\delta_{p,i}^2 - 2\delta_{p,i}^3 = \frac{\bar{\delta}_p^2(\tau + \tau_e)}{\tau}. \tag{A19}$$

In the range $\delta_{p,i} \in (0, 1)$, the left-hand side (LHS) of the equation above is increasing and the right-hand side (RHS) is constant. This implies that there is at most one solution to the FOC. Moreover, at $\delta_{p,i} = 0$, the LHS of the FOC (A18) is negative, which implies that the objective function is downward-sloping. These observations together imply that the objective function is either U-shaped or downward-sloping.

Step 3: In equilibrium, the objective function of investor i is U-shaped. This is because, if the objective function were downward-sloping for all investors, all investors would choose $\delta_{p,i} = 0$ and step 1 implies that this cannot be an equilibrium.

If the objective function is U-shaped and since there cannot be a pure-strategy symmetric equilibrium, the only other possible equilibrium is a mixed equilibrium in which a fraction λ of agents choose $\delta_{p,i} = 0$ and the remaining choose $\delta_{p,i} = 1$. For this to be the case, all investors have to be indifferent between $\delta_{p,i} = 0$ and $\delta_{p,i} = 1$ and this indifference equation pins down equilibrium λ. The indifference condition is

$$AU(0) - C(0) = AU(1) - C(1).$$

This simplifies to

$$\sqrt{\frac{1}{(1+\kappa)\big(\tau + \tau_e + \tau_p\big) - \tau_p}} = \frac{2}{\sqrt{(1+\kappa)\big(\tau + \tau_e + \tau_p\big)}},$$

which implies that

$$\lambda = 1 + \frac{\tau_e}{\tau_p} - \sqrt{\frac{\tau}{3\tau_p}\left(\frac{\tau_p - 3\tau - 3\tau_e}{\tau + \tau_e + \tau_p}\right)}.$$

E. Proof of Proposition 3

The proof of this proposition follows the same steps as the proof of Proposition 2 until equation (A19). In equation (A19), suppose we allow $\delta_{p,i} \in (0, \infty)$, in which case the LHS of this equation increases in $\delta_{p,i}$ up to $\delta_{p,i}=1$ and decreases thereafter. This implies that the FOC (i.e., equation (A19)) will have either no solutions or two solutions, the first less than one and the other greater than one. If the RHS is greater than one, that is, if $\frac{\bar{\delta}_p^2(\tau+\tau_e)}{\tau}>1$, then the equation has no solution. This implies that the objective is downward-sloping and all investors choose $\delta_{p,i}=0$, which from step 1 cannot be an equilibrium. This implies that, in equilibrium, the FOC will have two solutions. Moreover, at $\delta_{p,i}=0$, the LHS of the FOC (A18) is negative, which implies that the objective function is downward-sloping. This implies in turn that the solution of the FOC in $\delta_{p,i}\in(0,1)$ is a minima and the solution of the FOC in $\delta_{p,i}\in(1,\infty)$ is a maxima. The objective function therefore has a local maxima at $\delta_{p,i}=0$ and another local maxima at $\delta_{p,i}>1$.

Given the shape of the objective function and ruling out any pure-strategy symmetric equilibrium (as in step 2), the only possible equilibrium is a mixed equilibrium in which investors mix between two sets of beliefs: a fraction λ optimally chooses $\delta_{p,i}=0$, while the remaining fraction $1-\lambda$ optimally chooses $\delta_{p,i}=\delta_p^*>1$. Finally, δ_p^* and λ solve the FOC and the indifference condition given by

$$\frac{1-\frac{\kappa(\tau+\tau_e)}{\delta_p^{*2}\tau_p}}{\sqrt{\left(1+\frac{\kappa}{\delta_p^*}\right)^3\left(\tau+\tau_e+\delta_p^*\tau_p\right)^3}}-\frac{2\left(\delta_p^*-1\right)}{\sqrt{\left((1+\kappa)\left(\tau+\tau_e+\tau_p\right)-\left(1-\delta_p^*\right)^2\tau_p\right)^3}}=0. \tag{A20}$$

$$AU(0)-C(0)=AU(\delta_p^*)-C(\delta_p^*). \tag{A21}$$

F. Proof of Proposition 4

Following similar steps as in Lemma A1, the cost function with private signal choice is given by

$$C\left(\delta_{e,i}\right)=\frac{1}{\sqrt{\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)\left(\tau+\tau_p+\tau_e\delta_{e,i}\left(2-\delta_{e,i}\right)\right)}}.$$

The FOC is given by

$$\frac{\tau_e}{2\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)^{\frac{1}{2}}\left(\tau_e\delta_{e,i}+\tau_p+\tau\right)^{\frac{3}{2}}}=\frac{\tau_e\left(\delta_{e,i}-1\right)}{\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)^{\frac{1}{2}}\left[\left(\tau+\tau_p+\tau_e\delta_{e,i}\left(2-\delta_{e,i}\right)\right)\right]^{\frac{3}{2}}}, \tag{A22}$$

which simplifies to

$$\frac{\left(\tau+\tau_p+\tau_e\delta_{e,i}\left(2-\delta_{e,i}\right)\right)^{\frac{3}{2}}}{\left(\tau_p+\tau+\tau_e\delta_{e,i}\right)^{\frac{3}{2}}}=2\left(\delta_{e,i}-1\right) \tag{A23}$$

and thereby establishes the result. It is straightforward to see that the second-order conditions are satisfied and the comparative statics of δ_e follow directly from equation (A23).

G. *Proof of Proposition 5*

Proof of part 1: For an investor incurring the experienced utility penalty, choosing $(\delta_{e,i}, \delta_{p,i})$ yields anticipatory utility and costs

$$AU(\delta_{e,i}, \delta_{p,i}) = -\sqrt{\frac{\tau}{(1-\Lambda)^2}}\sqrt{\frac{1}{\left(1+\frac{\kappa}{\delta_{p,i}}\right)\left(\tau+\delta_{e,i}\tau_e+\delta_{p,i}\tau_p\right)}} \tag{A24}$$

$$C(\delta_{e,i}, \delta_{p,i}) = \sqrt{\frac{\tau}{(1-\Lambda)^2}}\sqrt{\frac{1}{(1-\delta_{p,i})^2\kappa\tau_p+(1+\kappa)\left(\tau+\tau_e\delta_{e,i}(2-\delta_{e,i})+\tau_p\delta_{p,i}(2-\delta_{p,i})\right)}}, \tag{A25}$$

where $\kappa \equiv (\frac{\Lambda}{1-\Lambda})^2\frac{\tau}{\tau_p}$. Suppose the average action across all other players is $\bar{\delta}_e, \bar{\delta}_p$. Then, $\Lambda = \frac{\tau_e\bar{\delta}_e+\tau_p\bar{\delta}_p}{\tau+\tau_e\bar{\delta}_e+\tau_p\bar{\delta}_p}$, and so

$$\kappa = \left(\frac{\Lambda}{1-\Lambda}\right)^2\frac{\tau}{\tau_p} = \frac{\gamma^2\left(\tau_e\bar{\delta}_e+\tau_p\bar{\delta}_p\right)^2}{\tau\tau_e^2\tau_z\bar{\delta}_e^2}. \tag{A26}$$

It follows that $(1, 0)$ is a symmetric equilibrium if and only if all investors prefer $(1, 0)$ over all other $(\delta_{e,i}, \delta_{p,i})$:

$$AU(1,0) - C(1,0) > AU(\delta_{e,i}, \delta_{p,i}) - C(\delta_{e,i}, \delta_{p,i}), \tag{A27}$$

or equivalently,

$$H \equiv 1 + R - L > 0,$$

where $R \equiv \frac{AU(\delta_{e,i},\delta_{p,i})}{-C(\delta_{e,i},\delta_{p,i})}$ and $L \equiv \frac{C(1,0)}{C(\delta_{e,i},\delta_{p,i})}$. Note that

$$\lim_{\tau_p\to 0} R = \sqrt{\frac{\left((2-\delta_{e,i})\delta_{e,i}\tau_e+\tau\right)\delta_{p,i}}{\delta_{e,i}\tau_e+\tau}}, \quad \lim_{\tau_p\to 0} L = \sqrt{\frac{(2-\delta_{e,i})\delta_{e,i}\tau_e+\tau}{\tau_e+\tau}}, \tag{A28}$$

which implies that

$$\lim_{\tau_p\to 0} H = 1 + \sqrt{\frac{\left((2-\delta_{e,i})\delta_{e,i}\tau_e+\tau\right)\delta_{p,i}}{\delta_{e,i}\tau_e+\tau}} - \sqrt{\frac{(2-\delta_{e,i})\delta_{e,i}\tau_e+\tau}{\tau_e+\tau}} \tag{A29}$$

$$\geq 1 + \sqrt{\frac{\left((2-\delta_{e,i})\delta_{e,i}\tau_e+\tau\right)\delta_{p,i}}{\delta_{e,i}\tau_e+\tau}} - \sqrt{\frac{\tau_e+\tau}{\tau_e+\tau}} \geq 0, \tag{A30}$$

and hence $(1, 0)$ is an equilibrium for τ_p sufficiently low.

Proof of part 2: The objective of investor i is given by

$$\max_{\delta_{e,i},\delta_{p,i}\in(\underline{\delta},\infty)} AU\left(\delta_{e,i},\delta_{p,i}\right) - C\left(\delta_{e,i},\delta_{p,i}\right).$$

This objective can be rewritten as

$$\max_{\delta_{p,i}\in(\underline{\delta},\infty)} \left[\max_{\delta_{e,i}\in(\underline{\delta},\infty)} AU\left(\delta_{e,i},\delta_{p,i}\right) - C\left(\delta_{e,i},\delta_{p,i}\right)\right].$$

Let us focus on the maximization inside the square bracket. The FOC is given by

$$\frac{\tau_e}{2\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\delta_{p,i}\tau_p}\right)^{\frac{1}{2}}\left(\tau_e\delta_{e,i}+\tau_p\delta_{p,i}+\tau\right)^{\frac{3}{2}}} = \frac{\mathrm{var}(F-P)\tau_e\left(\delta_{e,i}-1\right)}{\sqrt{\Lambda^2\left(\delta_{p,i}-1\right)^2+\mathrm{var}(F-P)\left(\tau+\tau_e\delta_{e,i}\left(2-\delta_{e,i}\right)+\tau_p\delta_{p,i}\left(2-\delta_{p,i}\right)\right)}}. \quad \text{(A31)}$$

The second-order condition is also satisfied. This implies that the solution to the above equation is the global maximum. This implies that the optimal $\delta_{e,i}$ chosen will always be interior and greater than one. From now on, we denote the solution to the FOC above by $\delta_{e,i}(\delta_{p,i})$.

Next, we prove that for optimization with respect to $\delta_{p,i}$, agent i will always choose $\delta_{p,i}=\underline{\delta}$. Note that $\delta_{p,i}=\underline{\delta}$ is an equilibrium if and only if

$$AU\left(\underline{\delta}\right) - C\left(\underline{\delta}\right) > AU\left(\delta_{p,i}\right) - C\left(\delta_{p,i}\right) \quad \text{(A32)}$$

for all $\delta_{p,i}$, or equivalently,

$$1+\frac{AU\left(\delta_{p,i}\right)}{-C\left(\delta_{p,i}\right)} - \frac{C\left(\underline{\delta}\right)}{C\left(\delta_{p,i}\right)} + \frac{AU\left(\underline{\delta}\right)}{C\left(\delta_{p,i}\right)} > 0. \quad \text{(A33)}$$

First, we examine what happens to $\delta_{e,i}(\delta_{p,i})$ as $\tau_p\to 0$. Taking the limit of equation (A31), we can show that $\lim_{\tau_p\to 0}\delta_{e,i}(\delta_{p,i})=1$. Let $R\equiv\frac{AU(\delta_{p,i})}{-C(\delta_{p,i})}$, $L\equiv\frac{C(\underline{\delta})}{C(\delta_{p,i})}$, and $G\equiv\frac{AU(\underline{\delta})}{C(\delta_{p,i})}$. Note that

$$\lim_{\tau_p\to 0} R = \lim_{\tau_p\to 0}\sqrt{\frac{\left(\left(2-\delta_{e,i}\right)\delta_{e,i}\tau_e+\tau\right)\delta_{p,i}}{\delta_{e,i}\tau_e+\tau}} = \sqrt{\delta_{p,i}},$$

$$\lim_{\gamma\to\infty} L = \lim_{\gamma\to\infty}\sqrt{\frac{\tau+\tau_e\delta_{ei}(2-\delta_{ei})}{\tau+\tau_e\delta_{ei}\left(\underline{\delta}\right)\left(2-\delta_{ei}\left(\underline{\delta}\right)\right)}} = 1$$

$$\lim_{\gamma\to\infty} G = -\sqrt{\underline{\delta}\frac{\tau+\tau_e\delta_{ei}(2-\delta_{ei})}{\tau+\delta_{e,i}\left(\underline{\delta}\right)\tau_e}} = -\sqrt{\underline{\delta}}.$$

Substituting these limits in inequality (A33), we get

$$\lim_{\tau_p\to\infty} 1+\frac{AU\left(\delta_{p,i}\right)}{-C\left(\delta_{p,i}\right)}-\frac{C\left(\underline{\delta}\right)}{C\left(\delta_{p,i}\right)}+\frac{AU\left(\underline{\delta}\right)}{C\left(\delta_{p,i}\right)}=1+\sqrt{\delta_{p,i}}-1-\sqrt{\underline{\delta}}>0,$$

which implies that $\delta_{p,i}=\underline{\delta}$ is an equilibrium for τ_p sufficiently low.

H. Proof of Proposition 6

Proof of part 1: Note that $\theta_{RE}=\frac{\tau_p(\tau+\tau_e+\tau_p)}{(\tau+\tau_p)(\tau_e+\tau_p)}-1$, and hence

$$\frac{\partial\theta_{RE}}{\partial\tau_p}=(\tau+\tau_e+2\tau_p)\frac{(\tau+\tau_p)(\tau_e+\tau_p)-\tau_p(\tau+\tau_e+\tau_p)}{(\tau+\tau_p)^2(\tau_e+\tau_p)^2}>0.$$

Note that $\sigma^2_{R,RE}=\frac{\tau\tau_p+(\tau_p+\tau_e)^2}{\tau_p(\tau_p+\tau_e+\tau)^2}$, and hence

$$\frac{\partial\sigma^2_{R,RE}}{\partial\tau_p}=-\frac{\tau_e^2\left(3\tau_p+\tau\right)+3\tau_e\tau_p^2+\tau_e^3+\tau_p^2\left(\tau_p+\tau\right)}{\tau_p^2\left(\tau_e+\tau_p+\tau\right)^3}<0.$$

Proof of part 2: When τ_p is sufficiently low, the equilibrium is symmetric and $\delta_{p,i}=0\ \ \forall\, i$. Note that $\theta_{SE}=\frac{\tau_p(\tau+\tau_e)}{(\tau+\tau_p)\tau_e}-1$, and hence $\frac{\partial\theta_{SE}}{\partial\tau_p}>0$. Note that $\sigma^2_{R,SE}=\frac{\tau\tau_p+(\tau_e)^2}{\tau_p(\tau_e+\tau)^2}$, and hence

$$\frac{\partial\sigma^2_{R,SE}}{\partial\tau_p}=-\frac{\tau_e^2}{(\tau_e+\tau)^2\tau_p^2}<0.$$

When τ_p is sufficiently high, the equilibrium is mixed and $\theta_{ME}=\frac{\tau_p(\tau+\tau_e+(1-\lambda)\tau_p)}{(\tau+\tau_p)(\tau_e+(1-\lambda)\tau_p)}-1$. In the mixed equilibrium,

$$(1-\lambda)\tau_p+\tau_e=\sqrt{\frac{\tau\tau_p}{3}\left(\frac{\tau_p-3\tau-3\tau_e}{\tau+\tau_e+\tau_p}\right)},$$

which implies that

$$\theta_{ME}=\frac{\tau_p(\tau+\sqrt{\frac{\tau\tau_p}{3}\left(\frac{\tau_p-3\tau-3\tau_e}{\tau+\tau_e+\tau_p}\right)})}{(\tau+\tau_p)(\sqrt{\frac{\tau\tau_p}{3}\left(\frac{\tau_p-3\tau-3\tau_e}{\tau+\tau_e+\tau_p}\right)})}-1$$

and $\lim_{\tau_p\to\infty}\frac{\partial\theta_{ME}}{\partial\tau_p}<0$. Note that

$$\sigma^2_{R,ME}=\frac{\tau\tau_p+\big((1-\lambda)\tau_p+\tau_e\big)^2}{\tau_p\big((1-\lambda)\tau_p+\tau_e+\tau\big)^2}=\frac{\tau\tau_p+\left(\sqrt{\frac{\tau\tau_p}{3}\left(\frac{\tau_p-3\tau-3\tau_e}{\tau+\tau_e+\tau_p}\right)}\right)^2}{\tau_p\left(\sqrt{\frac{\tau\tau_p}{3}\left(\frac{\tau_p-3\tau-3\tau_e}{\tau+\tau_e+\tau_p}\right)}+\tau\right)^2}. \tag{A34}$$

This implies that $\lim_{\tau_p\to\infty}\frac{\partial\sigma^2_{R,ME}}{\partial\tau_p}<0$.

Appendix B: Extensions

B.1. Ex Post Belief Choice

In the benchmark model, each investor chooses her subjective beliefs ($\delta_{p,i}$) before she observes the realization of her signals. In this section, we consider the implications when investor i chooses her beliefs ex post.

Anticipatory utility is now conditional on the realizations of P and s_i, that is, it can be written as

$$AU_i\big(\delta_{p,i};s_i,P\big)=-\exp\left\{-\frac{1}{2}\omega_i(\mu_i-P)^2\right\}.$$

This implies

$$\begin{aligned}\tfrac{\partial}{\partial\delta_{p,i}}AU_i&\propto(\mu_i-P)\times\big\{2\tau\big(s_p-m\big)+\omega_i(\mu_i-P)\big\}\\&\propto\left[s_i+\frac{B_i-\Lambda}{A_i}s_p\right]\big[(B_i\omega_i+2\tau)s_p+\omega_i\big(A_i-\Lambda\big)s_i\big].\end{aligned}$$

It can be shown (by rewriting the terms in brackets) that anticipatory utility is decreasing in $\delta_{p,i}$ when investor i's private signal is sufficiently distant from the price signal. As a result, and similar to our benchmark model, such an investor would dismiss others' information. In contrast, when investor i's private signal is sufficiently close to s_p, she has no incentive to deviate from rational expectations; under the extension allowing for more general beliefs about the price (found in Section III.D), such realizations would induce her to overweight the price, as in the mixed equilibrium we discuss in that section.

B.2. Return and Volume Moments

We begin by comparing return volatility and predictability across the rational expectations and wishful thinking equilibria.

PROPOSITION B1: *Suppose all investors incur the experienced utility penalty and choose subjective beliefs about price information and private signals. Then,*

(i) Return predictability is always higher than under rational expectations and can exhibit continuation.
(ii) Return volatility is higher than under rational expectations when prices are sufficiently informative.

PROOF: Denote the return characteristics in the rational expectations equilibrium, symmetric equilibrium, and mixed equilibrium by subscripts RE, SE, and ME, respectively.

Proof of part (i): Note that $\theta_{RE} = \frac{\tau_p(\tau+\tau_e+\tau_p)}{(\tau+\tau_p)(\tau_e+\tau_p)} - 1$ and $\theta_{SE} = \frac{\tau_p(\tau+\tau_e)}{(\tau+\tau_p)(\tau_e)+1} - 1$, which implies that

$$\theta_{RE} - \theta_{SE} = -\frac{\tau\tau_e^2\tau_z^2}{\left(\gamma^2+\tau_e\tau_z\right)\left(\gamma^2\tau+\tau_e^2\tau_z\right)} < 0.$$

Let $\bar{\delta}_p = (1-\lambda)$ denote the average beliefs about the precision of price signal. Note that $\Lambda_{ME} = \frac{\tau_e+(1-\lambda)\tau_p}{\tau+\tau_e+(1-\lambda)\tau_p}$ and

$$\theta_{ME} - \theta_{RE} = \frac{\tau_z}{\Lambda_{ME}\left(\beta^2\tau+\tau_z\right)} - \frac{\tau_z}{\Lambda_{RE}\left(\beta^2\tau+\tau_z\right)}. \tag{B1}$$

Given that $\Lambda_{ME} < \Lambda_{RE}$, it is clear that $\theta_{ME} > \theta_{RE}$.

Proof of part (ii): Note that

$$\sigma_{R,RE}^2 - \sigma_{R,SE}^2 = \frac{\tau\left(\tau_p^2+\tau\tau_p-2\tau_e(\tau_e+\tau)\right)}{(\tau_e+\tau)^2\left(\tau_e+\tau_p+\tau\right)^2}, \tag{B2}$$

which is positive if and only if $\tau_p > \frac{1}{2}\sqrt{8\tau\tau_e+8\tau_e^2+\tau^2} - \frac{\tau}{2}$. Moreover,

$$\sigma_{R,ME}^2 = \frac{\tau\tau_p + \left((1-\lambda)\tau_p+\tau_e\right)^2}{\tau_p\left((1-\lambda)\tau_p+\tau_e+\tau\right)^2}.$$

In the mixed equilibrium,

$$(1-\lambda)\tau_p + \tau_e = \sqrt{\frac{\tau\tau_p}{3}\left(\frac{\tau_p-3\tau-3\tau_e}{\tau+\tau_e+\tau_p}\right)}.$$

Substituting this into the expression for $\sigma_{R,ME}^2$, we get

$$\lim_{\tau_p\to\infty}\sigma_{R,ME}^2 > \sigma_{R,SE}^2.$$

This implies that our results hold for τ_p high enough, which is the required condition for the mixed equilibrium. □

Since investors start without an endowment of the risky security, realized trading volume in our economy can be characterized as

$$\mathcal{V} \equiv \int_i |x_i^*|\, di. \tag{B3}$$

This implies that expected volume is given by

$$\mathbb{E}[\mathcal{V}] = \int_i \frac{\omega_i}{\gamma}|\mu_i - P|di = \int_i \frac{\tau}{\gamma(1-A_i-B_i)}\sqrt{\frac{2}{\pi}\left(\frac{A_i^2}{\tau_e} + \frac{(B_i-\Lambda)^2}{\tau_p} + \frac{\left(A_i+B_i-\Lambda\right)^2}{\tau}\right)}di, \tag{B4}$$

where A_i and B_i are appropriately redefined to reflect any subjective belief distortions about investors' own signals (as in Section IV). Note that volume reflects the cross-sectional variation across investor valuations (i.e., μ_i), scaled by their posterior variance (i.e., ω_i^{-1}). Such variation is driven by three channels: (i) the weight each investor places on her private signal (i.e., the $\frac{1}{\tau_e}$ term), (ii) the weight she places on price information, relative to others (i.e., the $\frac{1}{\tau_p}$ term), and (iii) the relative weight placed on her prior belief (i.e., the $\frac{1}{\tau}$ term). These observations give rise to the following result.

PROPOSITION B2:

(i) Fixing parameters, expected volume is higher in a symmetric equilibrium than in the corresponding rational expectations equilibrium if $\delta_{e,i} > 1$.
(i) Fixing parameters, expected volume is higher in a mixed equilibrium than in the corresponding rational expectations equilibrium.

If investors exhibit overconfidence (i.e., $\delta_{e,i} > 1$), then they (i) place more weight on their private signal, that is, A_i increases, and (ii) place relatively less weight on the price. Together, the first two terms imply that volume is higher under the symmetric equilibrium with overconfidence than under rational expectations. However, note that the last term is absent in symmetric equilibria, since $A_i + B_i = \Lambda$ in this case. In mixed equilibria, this final term reflects the variation in valuations due to the relative difference in weights each "type" of investor places on private signals and the information in prices. This difference across types generates increased trade among investors, even in the absence of overconfidence.

PROOF OF PROPOSITION B2: *Proof of part (i)*: In a symmetric equilibrium, $\delta_p = \bar{\delta}$ and δ_e solves equation (50). In this equilibrium, volume under the symmetric equilibrium ($\mathcal{V}_{SE}$) is given by

$$\mathbb{E}[\mathcal{V}_{SE}] = \frac{\omega}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{A^2}{\tau_e} + \frac{(B-\Lambda)^2}{\tau_p}\right)} \tag{B5}$$

$$= \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\delta_e^2\tau_e + \frac{\delta_e^2\tau_e^2}{\tau_p}\right)} \tag{B6}$$

$$> \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\tau_e + \frac{\tau_e^2}{\tau_p}\right)} = \mathbb{E}[\mathcal{V}_{RE}], \tag{B7}$$

where $\mathcal{V}_{RE}$ denotes volume under rational expectations.

Proof of part (ii): Let $\mathcal{V}_{ME}$ denote volume under the mixed equilibrium of Proposition 2. Then,

$$\mathbb{E}[\mathcal{V}_{ME}] = \lambda V_1 + (1-\lambda)V_2, \text{ where} \tag{B8}$$

$$V_1 \equiv \frac{(\tau_e+\tau)}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{1}{\tau}\left(\frac{\tau_e}{\tau+\tau_e} - \Lambda_{ME}\right)^2 + \frac{1}{\tau_e}\left(\frac{\tau_e}{\tau+\tau_e}\right)^2 + \frac{1}{\tau_p}\Lambda_{ME}^2\right)}, \tag{B9}$$

$$V_2 \equiv \frac{\tau+\tau_e+\tau_p}{\gamma}\sqrt{\frac{2}{\pi}\left(\begin{array}{c}\frac{1}{\tau}\left(\frac{\tau_e+\tau_p}{\tau+\tau_e+\tau_p} - \Lambda_{ME}\right)^2 + \frac{1}{\tau_e}\left(\frac{\tau_e}{\tau+\tau_e+\tau_p}\right)^2 \\ +\frac{1}{\tau_p}\left(\frac{\tau_p}{\tau+\tau_e+\tau_p} - \Lambda_{ME}\right)^2\end{array}\right)}. \tag{B10}$$

Let

$$A(x) = \frac{x\tau_e + (1-x)\tau_e}{x(\tau+\tau_e) + (1-x)\left(\tau+\tau_e+\tau_p\right)}, \tag{B11}$$

$$B(x) = \frac{(1-x)\left(\tau_p\right)}{x(\tau+\tau_e) + (1-x)\left(\tau+\tau_e+\tau_p\right)}, \tag{B12}$$

$$\omega(x) = x(\tau_e+\tau) + (1-x)\left(\tau+\tau_e+\tau_p\right), \tag{B13}$$

$$V(x) = \frac{\omega(x)}{\gamma}\sqrt{\frac{2}{\pi}\left(\tfrac{1}{\tau}\left(A(x)+B(x)-\Lambda\right)^2 + \tfrac{1}{\tau_e}A(x)^2 + \tfrac{1}{\tau_p}\left(B(x)-\Lambda\right)^2\right)}, \tag{B14}$$

and so we can rewrite the expected mixed-equilibrium volume as

$$\mathbb{E}[\mathcal{V}_{ME}] = \lambda V(1) + (1-\lambda)V(0). \tag{B15}$$

Note that

$$V(\lambda) = \frac{\omega(\lambda)}{\gamma}\sqrt{\frac{2}{\pi}\left(\tfrac{1}{\tau_e}A(\lambda)^2 + \tfrac{1}{\tau_p}\left(B(\lambda)-\Lambda\right)^2\right)} \tag{B16}$$

$$= \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{(\lambda\tau_e+(1-\lambda)\tau_e)^2}{\tau_e} + \frac{\left((1-\lambda)\left(\tau_p\right)-\lambda(\tau_e)-(1-\lambda)\left(\tau_e+\tau_p\right)\right)^2}{\tau_p}\right)} \tag{B17}$$

$$= \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{(\lambda\tau_e+(1-\lambda)\tau_e)^2}{\tau_e} + \frac{(\lambda\tau_e+(1-\lambda)\tau_e)^2}{\tau_p}\right)} \tag{B18}$$

$$= \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\tau_e + \frac{\tau_e^2}{\tau_p}\right)} = \mathbb{E}[\mathcal{V}_{RE}]. \tag{B19}$$

It remains to be shown that

$$\lambda V(1) + (1-\lambda)V(0) \geq V(\lambda). \tag{B20}$$

Note that

$$V(x) = \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\tfrac{1}{\tau}(\alpha(x)+\beta(x)-\Lambda\omega(x))^2 + \tfrac{1}{\tau_e}\alpha(x)^2 + \tfrac{1}{\tau_p}(\beta(x)-\Lambda\omega(x))^2\right)}, \tag{B21}$$

where

$$\alpha(x) = x\tau_e + (1-x)\tau_e \equiv a_0 + a_1 x, \tag{B22}$$

$$\beta(x) = (1-x)\tau_p \equiv b_0 + b_1 x, \tag{B23}$$

$$\omega(x) = x(\tau_e + \tau) + (1-x)\big(\tau + \tau_e + \tau_p\big) \equiv w_0 + w_1 x, \tag{B24}$$

$$\frac{V_{xx}}{V^3} = 4\tfrac{\tau+\tau_e+\tau_p}{\pi^2\gamma^4\tau\tau_e\tau_p}\big(-a_0 b_1 + a_0\Lambda w_1 + a_1 b_0 - a_1\Lambda w_0\big)^2 > 0, \tag{B25}$$

which implies $V(x)$ is convex, and hence

$$\mathbb{E}[\mathcal{V}_{ME}] = \lambda V(1) + (1-\lambda)V(0) \geq V(\lambda) \geq \mathbb{E}[\mathcal{V}_{RE}]. \tag{B26}$$

This completes the proof. □

B.3. Generalized Cost Functions

Our benchmark analysis shows that investors are generally dismissive of price information when the investor's cost of distorting beliefs is measured using the experienced utility penalty. In this section, we show that underreacting to price information is a robust consequence of wishful thinking that arises more generally for a broad class of cost functions. Specifically, suppose that the cost function $C(\delta_{p,i})$ is well-behaved as defined below.

DEFINITION B1: A cost function $C(\delta_{p,i})$ is well-behaved if $C(1) = \frac{\partial C}{\partial \delta}(1) = 0$, and $C(\cdot)$ is strictly convex (i.e., its global minimum is at $\delta_{p,i} = 1$).

The following result establishes that if a pure-strategy equilibrium exists, it must feature underreaction to price information. Notably, this result applies even in settings in which $\delta_{p,i}$ is unconstrained (i.e., $\delta_{p,i} \in [0, \infty]$) and when investors choose beliefs about their private signals as well as the price.

PROPOSITION B3: *Suppose the cost function is well-behaved. If there exists an equilibrium in which all investors choose the same subjective beliefs (i.e., $\delta_{p,i} = \delta_p$ for all i), then investors must discount the information in prices, that is, $\delta_{p,i} = \delta_p < 1$.*

PROOF: Lemma 1 implies that in any symmetric equilibrium (i.e., $\delta_{p,i} = \delta_p \;\forall i$), we have $\Lambda = \frac{\tau_e+\delta_p\tau_p}{\tau+\tau_e+\delta_p\tau_p}$. Moreover, note that $\frac{\partial}{\partial\delta_{p,i}}AU = 0$ at

$$\bar{\delta}_p = \frac{1}{\tau_p}\left(\frac{\Lambda}{1-\Lambda}\right)\sqrt{\tau(\tau_e+\tau)}, \tag{B27}$$

$$= \sqrt{1+\frac{\tau_e}{\tau}}\left(\delta_p + \tfrac{\tau_e}{\tau_p}\right) > \delta_p. \tag{B28}$$

But this implies $\frac{\partial}{\partial\delta_{p,i}}AU(\delta_{p,i} = \delta_p) < 0$ since $\frac{\partial AU}{\partial\delta_{p,i}} < (>)0$ for all $\delta_{p,i} < (>)\bar{\delta}_{p,i}$. Next, note that if $\delta_{p,i} = \delta_p \geq 1$, then $C'(\delta_{p,i}) > 0$. Taken together, this proves that at any proposed symmetric equilibrium in which $\delta_p > 1$, investor i has an incentive to deviate and choose $\delta_{p,i} < 1$. Thus, the only possible symmetric equilibrium is one in which each investor chooses $\delta_{p,i} < 1$. □

The equilibrium underreaction to price information is a consequence of the strategic substitutability in subjective belief choice we discussed in Section III.A. Consider the optimal choice for investor i in a symmetric equilibrium where all other investors choose δ_p. As Lemma 2 establishes, anticipatory utility is U-shaped in $\delta_{p,i}$. In the proof of Proposition B3, we take this result a step further, showing that investor i's anticipatory utility is always decreasing in $\delta_{p,i}$ at $\delta_{p,i} = \delta_p$. Intuitively, investor i can improve her ability to speculate against others by decreasing the perceived precision of price information, thereby decreasing the correlation between her conditional valuation (μ_i) and those of others ($\int_j \mu_j dj$).

Next, recall that for a well-behaved cost function, deviations away from rational expectations (i.e., $\delta_{p,i} = 1$) are penalized, that is, the cost function is decreasing below one and increasing above one. But this implies that the equilibrium choice of $\delta_{p,i}$ cannot be higher than one, since if it were, investor i could increase anticipatory utility and decrease costs by lowering $\delta_{p,i}$. As such, in any symmetric equilibrium, investors must set $\delta_{p,i} < 1$.

In general, whether a symmetric equilibrium exists will depend on the characteristics of the cost function. In our benchmark analysis, the experienced utility penalty depends on equilibrium choices Λ. In other models of subjective belief choice (e.g., robust control), the cost function is often specified in terms of a statistical distance measure (e.g., the K-L distance), and usually does not depend on equilibrium choices.

While an analytical characterization of the model with the K-L distance cost function is beyond the scope of the paper, we numerically explore the implications next. We show that when τ_p is sufficiently low, there exists a pure sym-

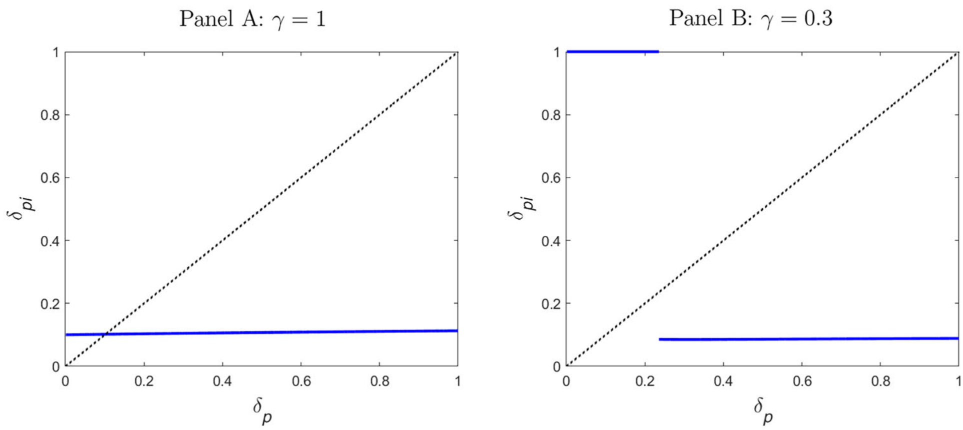

Figure B.1. Best response functions of investors. The figure plots the best response of investor i (in solid blue) as a function of average δ_p chosen by other investors. The dotted black line shows the 45° line. Panel A corresponds to high γ and Panel B corresponds to low γ. Other parameters are set to $\tau = \tau_e = \tau_z = 1$. (Color figure can be viewed at wileyonlinelibrary.com)

metric equilibrium in which all investors choose a $\delta_{p,i} = \delta_p \in (0, 1)$.[44] When τ_p is sufficiently high, there exists a mixed equilibrium in which a fraction λ optimally choose $\delta_{p,i} = \delta_p \in (0, 1)$, while the remaining $1 - \lambda$ investors choose to hold rational expectations.

Specifically, consider the setup of our benchmark analysis in Section II. Given our distributional assumptions, the cost of choosing $\delta_{p,i}$ is given by

$$C(\delta_{p,i}) = \frac{1}{2}\left(\log(\delta_{p,i}) + \frac{1}{\delta_{p,i}} - 1\right). \tag{B29}$$

Figure B.1 plots investor i's best response function. Note that at the intersection of the best response function and the 45° line, there is a symmetric equilibrium. Panel A shows that for relatively high risk aversion (i.e., low price informativeness), the equilibrium is symmetric: all investors choose $\delta_{p,i} \approx 0.1$, and so (partially) dismiss the information in prices. Investors do not completely dismiss the price as it is infinitely costly to choose $\delta_{p,i} = 0$.

However, Panel B shows that when risk aversion is low (i.e., high price informativeness), a symmetric equilibrium does not exist: there is no intersection between the best response function and the 45° line. Numerically, the only sustainable equilibrium is mixed: some investors discount the price, while others condition on it efficiently. The intuition mirrors the relative substitutability found in the baseline model: when other investors condition on (respectively, dismiss) the price, investor i chooses to dismiss (respectively, condition on) it.

[44] This is because the K-L distance-based cost function is more convex than the experienced utility penalty when $\delta_{p,i}$ is small. Hence, the optimal subjective belief choice is interior, and not $\delta_{p,i} = 0$ as with the experienced utility penalty.

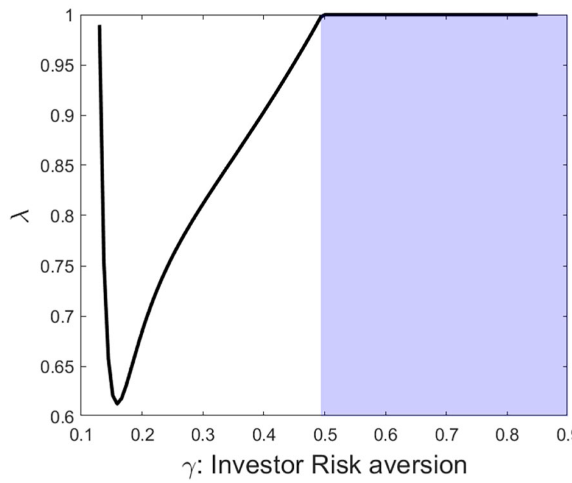

Figure B.2. Composition of equilibrium beliefs with K-L distance. The figure plots the equilibrium fraction of investors underweighting the information in prices, λ, as a function of γ. Other parameters are set to $\tau = \tau_e = \tau_z = 1$. The blue region indicates the parameter combinations that yield the symmetric equilibrium in which all agents underweight the information in prices. (Color figure can be viewed at wileyonlinelibrary.com)

Finally, we examine how the fraction of investors who underweight the information in prices, λ, changes with risk aversion. Figure B.2 plots the equilibrium fraction of investors who choose "low" $\delta_{p,i}$ as a function of risk aversion. Reassuringly, this plot is U-shaped, as in our benchmark (see Figure 2). This suggests that the comparative statics described in the main text are robust to alternative specifications of the cost function.

B.4. Welfare

In this section, we explore the welfare implications of motivated beliefs. Investors choose to deviate from rational expectations and so, under their chosen subjective beliefs, they are always better off. However, from the perspective of a social planner who holds objective beliefs and accounts only for expected utility, informed investors are strictly worse off when they deviate—their demand for the risky asset is suboptimal given their information sets.

In what follows, we use the objective distribution as the reference beliefs and define expected utility for an informed investor as

$$U_i \equiv \mathbb{E}\left[-\exp\left\{-\gamma x_i^*\left(\delta_{p,i}^*\right) \times (F - P) - \gamma W_0\right\}\right], \tag{B30}$$

where $x_i^*(\delta_{p,i}^*)$ is her optimal demand under her optimally chosen beliefs $\delta_{p,i}^*$. We emphasize that this is a conservative measure of expected utility as it only accounts for the costs of deviating from rational expectations and does not include any gains from anticipatory utility. Figure B.3 provides an illustration

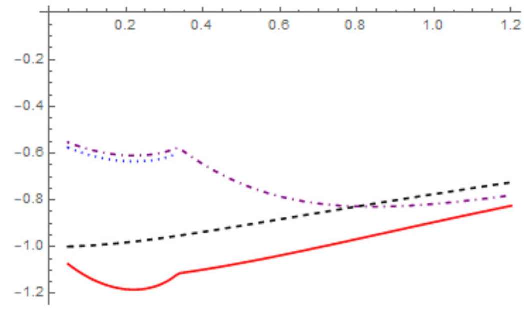

Figure B.3. Expected utility with subjective beliefs. The figure plots expected utility (y-axis) as a function of risk aversion, γ (x-axis). The dashed line plots U_i in the rational expectations equilibrium, the solid line plots U_i for investors who dismiss price information, the dotted line plots U_i for investors who overweight price information (when the mixed equilibrium exists), and the dot-dashed line plots U_i for a hypothetical, rational expectations investor in the subjective beliefs equilibrium. Other parameters are set to $\tau = \tau_e = \tau_z = 1$ and $\gamma_z = 0.75$. (Color figure can be viewed at wileyonlinelibrary.com)

of the relative levels of expected utility across both the rational expectations and subjective beliefs equilibria, focusing on the generalized model found in Section IV.

Unsurprisingly, a hypothetical rational expectations investor in the subjective beliefs equilibrium (dot-dashed) would experience a higher expected utility than those investors who hold subjective beliefs—she is optimally using all of the information available to her and exploiting the behavior of the other investors and the noise traders. However, it is interesting to note that (i) in the mixed equilibrium, investors who ignore the price are significantly worse off than those that overweight it, and (ii) investors in a rational expectations equilibrium (dashed) may be worse off than some investors in the mixed equilibrium. The first result follows from the fact overreacting to the price is more efficient than ignoring it, in equilibrium. The second result follows from the observation that expected speculative gains are lower in the rational expectations equilibrium since all investors use information efficiently. In contrast, the investors who overweight the price are able to exploit those that ignore it in the mixed equilibrium.

Next, we consider the effect of informed investors' deviations from rational expectations on the welfare of liquidity (or noise) traders. Recall that the aggregate supply, z, is noisy. Suppose that this reflects the sale of the risky asset by a liquidity trader, who has CARA utility with risk aversion γ_z and who is endowed with initial wealth W_0. Then her expected utility is given by

$$U_z \equiv \mathbb{E}[-\exp\{-\gamma_z(-z) \times (F - P) - \gamma_z W_0\}]. \quad \text{(B31)}$$

The following result characterizes the impact of motivated beliefs on both the expected utility of liquidity traders and total welfare ($\int U_i + U_z$).

PROPOSITION B4: *In equilibrium, the expected utility of a liquidity trader is given by*

$$U_z = -\sqrt{\frac{\tau_z}{\tau_z + 2\gamma_z\left(\beta\Lambda - \frac{1}{2\tau}\gamma_z(1-\Lambda)^2\right)}}\exp\{-\gamma_z W_0\}. \quad \text{(B32)}$$

Suppose $\gamma_z \leq \gamma$. Then,

(i) Liquidity traders have higher expected utility in the symmetric equilibrium than in the rational expectations equilibrium.
(ii) In any mixed equilibrium in which Λ is less than its rational-expectations counterpart, liquidity traders have higher expected utility in the mixed equilibrium.
(iii) There exists $\underline{\gamma} \geq 0$ such that for all $\gamma \geq \underline{\gamma}$, total welfare is higher under the subjective beliefs equilibrium than under the rational expectations equilibrium.

Expected utility for a liquidity trader depends on the equilibrium parameters through the term

$$\beta\Lambda - \tfrac{1}{2\tau}\gamma_z(1-\Lambda)^2. \quad \text{(B33)}$$

A liquidity trader's utility is driven by two components. The first component ($\beta\Lambda$) reflects her disutility from price impact—for instance, a larger sale (higher z) pushes prices downward, which reduces her proceeds. The second term ($-\frac{1}{2\tau}\gamma_z(1-\Lambda)^2$) reflects a standard risk-aversion channel—when prices are less informative about fundamentals, the liquidity trader faces more uncertainty about her payoff, which reduces utility.[45]

Price sensitivity, Λ, is generally higher when investors exhibit rational expectations. This has offsetting effects on the liquidity trader's utility. On the one hand, a lower Λ implies that the price is less sensitive to her trade and so utility increases through the price impact channel. On the other hand, a lower Λ implies that prices track fundamentals less closely, which increases the risk in the liquidity trader's payoff. As we show in the proof of Proposition B4, the price impact effect always dominates the risk-aversion effect if the risk aversion of investors is weakly higher than that of liquidity traders (i.e., $\gamma_z \leq \gamma$). In this case, liquidity traders are always better off when informed investors choose to deviate from rational expectations.

Note that $\gamma_z \leq \gamma$ is a sufficient condition, but it is not necessary for liquidity traders to be better off under the subjective beliefs equilibrium. Figure B.4

[45] It is important to note that expected utility is finite only when

$$\tau_z + 2\gamma_z\left(\beta\Lambda - \tfrac{1}{2\tau}\gamma_z(1-\Lambda)^2\right) > 0. \quad \text{(B34)}$$

Intuitively, if the combined disutility from the price impact and risk aversion terms are too large, the liquidity trader's expected utility from being forced to trade z units approaches negative infinity—she would rather exit the market and not trade if she could.

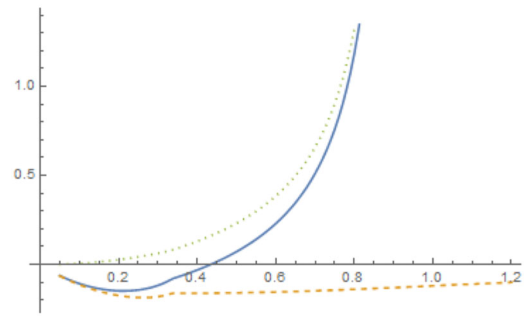

Figure B.4. Difference in U_i under rational and subjective expectations equilibria. The figure plots the difference in expected utility (y-axis) as a function of γ (x-axis). The dashed line plots the difference in expected utility of informed investors (under the objective distribution) (i.e., $\int U_{i,SE} - \int U_{i,RE}$), the dotted line plots the difference in expected utility for the noise traders (i.e., $U_{z,SE} - U_{z,RE}$), and the solid line plots the difference in utility across both groups (i.e., $\int U_{i,SE} + U_{z,SE} - (\int U_{i,RE} + U_{z,RE})$). Other parameters are set to $\tau = \tau_e = \tau_z = 1$ and $\gamma_z = 0.75$. (Color figure can be viewed at wileyonlinelibrary.com)

plots the difference in expected utility between the subjective beliefs equilibrium and the rational expectations equilibrium as a function of investor risk aversion γ for each group separately and for both groups as a whole. The plot illustrates that for this set of parameters, liquidity traders are always better off under subjective beliefs—the dotted line is always above zero—irrespective of whether informed investors are more or less risk-averse than them. In particular, note that noise trader risk aversion γ_z is fixed at 0.75, but informed investor risk aversion γ ranges from 0.1 to 1.2. Not surprisingly, under the objective distribution, informed investors are worse off under the subjective beliefs equilibrium—the dashed line is always below zero. The solid line in Figure B.4 illustrates the aggregate welfare ranking in Proposition B4, part (iii): aggregate welfare is higher in the rational expectations equilibrium when informed investor risk aversion is low, but higher under subjective beliefs otherwise. These results suggest that deviations from rational expectations may make liquidity traders better off.

B.5. Proof of Proposition B4

The utility of noise traders is

$$
\begin{aligned}
U_z &= -E(\gamma_z \exp\{+\gamma_z z(F - P)\}) \\
&= -E\left(\gamma_z \exp\{\gamma_z z F(1 - \Lambda) - \gamma \Lambda \beta z^2\}\right) \\
&= -E\left(\gamma_z \exp\left\{\left(\frac{\gamma_z^2(1 - \Lambda)^2}{2\tau} - \gamma_z \Lambda \beta\right) z^2\right\}\right)
\end{aligned}
$$

$$= -\gamma_z \frac{1}{\sqrt{1 - 2\frac{1}{\tau_z}\left(\left(\frac{\gamma_z^2(1-\Lambda)^2}{2\tau} - \gamma_z \Lambda \beta\right)\right)}}$$

$$= -\gamma_z \sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda)^2}{\tau} + 2\gamma_z \Lambda \beta}},$$

where we use the fact that $E(e^{a\varepsilon^2}) = \frac{1}{\sqrt{1-2a\sigma_\varepsilon^2}}$. This implies that the utility of noise traders is monotonically decreasing in $\frac{\gamma_z(1-\Lambda)^2}{2\tau} - \Lambda\beta$.

Proof of Part (i): In this case, all investors dismiss the price and so

$$\Lambda = \frac{\tau_e}{\tau + \tau_e}, \quad \Lambda_{RE} = \frac{\tau_e + \tau_p}{\tau + \tau_e + \tau_p}, \tag{B35}$$

so

$$U_{z,SE} - U_{z,RE} > 0 \tag{B36}$$

$$\Leftrightarrow \frac{\gamma \tau \tau_p}{\tau_e(\tau_e + \tau)(\tau_e + \tau_p + \tau)} > \frac{\tau \gamma_z}{2} \frac{\tau_p(2\tau_e + \tau_p + 2\tau)}{(\tau_e + \tau)^2(\tau_e + \tau_p + \tau)^2} \tag{B37}$$

$$\Leftrightarrow \frac{\gamma}{\gamma_z} > \frac{\tau_e}{\tau + \tau_e} \frac{2(\tau_e + \tau) + \tau_p}{2(\tau_e + \tau_p + \tau)}, \tag{B38}$$

which implies that if $\gamma \geq \gamma_z$, then $U_{z,SE} > U_{z,RE}$.

Proof of Part (ii): In the mixed equilibrium, some investors overweight the price. Let $\int \delta_{p,i} \equiv \delta_p$, and so we write

$$\Lambda = \frac{\delta_e + \delta_p \tau_p}{\tau + \tau_e + \delta_p \tau_p}. \tag{B39}$$

This implies that

$$U_{z,SE} - U_{RE} > 0 \tag{B40}$$

$$\Leftrightarrow \gamma\left(\frac{\Lambda_{RE}}{\tau_e} - \frac{\Lambda}{\tau_e}\right) > \gamma_z\left(\frac{(1-\Lambda)^2}{2\tau} - \frac{(1-\Lambda_{RE})^2}{2\tau}\right) \tag{B41}$$

$$\Leftrightarrow \frac{\gamma}{\tau_e}(\Lambda_{RE} - \Lambda) > \frac{\gamma_z}{2\tau}(\Lambda_{RE} - \Lambda)(2 - (\Lambda + \Lambda_{RE})). \tag{B42}$$

When $\Lambda_{RE} > \Lambda$, this is equivalent to

$$\frac{\gamma}{\gamma_z} > \frac{\tau_e}{2}\left(\frac{1}{\tau + \tau_e + \tau_p} + \frac{1}{\tau + \tau_e + \delta_p \tau_p}\right), \tag{B43}$$

which is always true if $\gamma \geq \gamma_z$.

Proof of Part (iii): Total welfare is given by

$$W(\delta_p) = -\frac{1}{\sqrt{\Lambda^2(\delta_p-1)^2+\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p\delta_p(2-\delta_p))}} - \sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda)^2}{\tau} + 2\gamma_z\Lambda\beta}}. \tag{B44}$$

Moreover, for the rational expectations equilibrium, we have $\delta_p = 1$. This implies that the difference in welfare is

$$W(\delta_p) - W_{RE} = \begin{array}{c} \frac{1}{\sqrt{\left(\frac{(1-\Lambda_{RE})^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p)}} + \sqrt{\frac{\tau_z}{\tau_z-\frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau}+2\gamma_z\Lambda_{RE}\beta}} \\ -\frac{1}{\sqrt{\Lambda^2(\delta_p-1)^2+\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p\delta_p(2-\delta_p))}} \\ -\sqrt{\frac{\tau_z}{\tau_z-\frac{\gamma_z^2(1-\Lambda)^2}{\tau}+2\gamma_z\Lambda\beta}}. \end{array} \tag{B45}$$

Above, we establish that when $\gamma_z \leq \gamma$ and $\Lambda < \Lambda_{RE}$, we have

$$U_{z,SE} = -\sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma^2(1-\Lambda)^2}{\tau} + 2\gamma\Lambda\beta}} > -\sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau} + 2\gamma_z\Lambda_{RE}\beta}} = U_{z,RE} \tag{B46}$$

$$\Leftrightarrow \tau_z - \frac{\gamma_z^2(1-\Lambda)^2}{\tau} + 2\gamma_z\Lambda\beta \geq \tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau} + 2\gamma_z\Lambda_{RE}\beta > 0. \tag{B47}$$

Let

$$\bar{\gamma} \equiv \frac{\tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau}}{\frac{2\gamma_z\Lambda_{RE}}{\tau_e}} = \frac{\tau_e\left(\tau_z(\tau_e+\tau_p+\tau)^2 - \tau\gamma_z^2\right)}{2\gamma_z(\tau_e+\tau_p)(\tau_e+\tau_p+\tau)}. \tag{B48}$$

Note that

$$\lim_{\gamma\uparrow\bar{\gamma}} \sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau} + 2\gamma_z\Lambda_{RE}\beta}} = \infty, \tag{B49}$$

but

$$\lim_{\gamma\uparrow\bar{\gamma}} \begin{array}{c} \frac{1}{\sqrt{\left(\frac{(1-\Lambda_{RE})^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p)}} - \sqrt{\frac{\tau_z}{\tau_z-\frac{\gamma_z^2(1-\Lambda)^2}{\tau}+2\gamma_z\Lambda\beta}} \\ -\frac{1}{\sqrt{\Lambda^2(\delta_p-1)^2+\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p\delta_p(2-\delta_p))}} \end{array} \geq -c \tag{B50}$$

for some $c \leq \infty$. This implies

$$\lim_{\gamma\to\bar{\gamma}} W(\delta_p) - W_{RE} > 0, \tag{B51}$$

or equivalently, $\exists \underline{\gamma} \leq \bar{\gamma}$, such that for all $\gamma > \underline{\gamma}$, $W(\delta_p) > W_{RE}$.

REFERENCES

Akerlof, George A., and William T. Dickens, 1982, The economic consequences of cognitive dissonance, *American Economic Review* 72, 307–319.

Andrade, Philippe, Richard K. Crump, Stefano Eusepi, and Emanuel Moench, 2016, Fundamental disagreement, *Journal of Monetary Economics* 83, 106–128.

Angeletos, George-Marios, and Alessandro Pavan, 2007, Efficient use of information and social value of information, *Econometrica* 75, 1103–1142.

Bai, Jennie, Thomas Philippon, and Alexi Savov, 2016, Have financial markets become more informative? *Journal of Financial Economics* 122, 625–654.

Banerjee, Snehal, 2011, Learning from prices and the dispersion in beliefs, *Review of Financial Studies* 24, 3025–3068.

Banerjee, Snehal, Jesse Davis, and Naveen Gondhi, 2023, Motivated beliefs in coordination games, Working paper, UC San Diego.

Banerjee, Snehal, Ron Kaniel, and Ilan Kremer, 2009, Price drift as an outcome of differences in higher-order beliefs, *Review of Financial Studies* 22, 3707–3734.

Bénabou, Roland, and Jean Tirole, 2016, Mindful economics: The production, consumption, and value of beliefs, *Journal of Economic Perspectives* 30, 141–164.

Bordalo, Pedro, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer, 2020a, Overreaction in macroeconomic expectations, *American Economic Review* 110, 2748–2782.

Bordalo, Pedro, Nicola Gennaioli, Rafael La Porta, and Andrei Shleifer, 2020b, Expectations of fundamentals and stock market puzzles, Technical report, National Bureau of Economic Research.

Brown, David P., and Robert H. Jennings, 1989, On technical analysis, *Review of Financial Studies* 2, 527–551.

Brunnermeier, Markus K., and Jonathan A. Parker, 2005, Optimal expectations, *American Economic Review* 95, 1092–1118.

Brunnermeier, Markus K., Alp Simsek, and Wei Xiong, 2014, A welfare criterion for models with distorted beliefs, *Quarterly Journal of Economics* 129, 1753–1797.

Caplin, Andrew, and John Leahy, 2001, Psychological expected utility theory and anticipatory feelings, *Quarterly Journal of Economics* 116, 55–79.

Caplin, Andrew, and John Leahy, 2019, Wishful thinking, Working paper, University of Michigan.

Cogley, Timothy, and Thomas J. Sargent, 2008, Anticipated utility and rational expectations as approximations of Bayesian decision making, *International Economic Review* 49, 185–221.

Coibion, Olivier, and Yuriy Gorodnichenko, 2012, What can survey forecasts tell us about information rigidities? *Journal of Political Economy* 120, 116–159.

Coibion, Olivier, and Yuriy Gorodnichenko, 2015, Information rigidity and the expectations formation process: A simple framework and new facts, *American Economic Review* 105, 2644–2678.

Cooper, Michael J., Roberto C. Gutierrez, Jr., and Allaudeen Hameed, 2004, Market states and momentum, *Journal of Finance* 59, 1345–1365.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under- and overreactions, *Journal of Finance* 53, 1839–1885.

Daniel, Kent, and Tobias J. Moskowitz, 2016, Momentum crashes, *Journal of Financial Economics* 122, 221–247.

Daniel, Kent D., David Hirshleifer, and Avanidhar Subrahmanyam, 2001, Overconfidence, arbitrage, and equilibrium asset pricing, *Journal of Finance* 56, 921–965.

Dávila, Eduardo, and Cecilia Parlatore, 2019, Volatility and informativeness, Technical report, National Bureau of Economic Research.

Eil, David, and Justin M. Rao, 2011, The good news-bad news effect: Asymmetric processing of objective information about yourself, *American Economic Journal: Microeconomics* 3, 114–138.

Eliaz, Kfir, and Ran Spiegler, 2006, Contracting with diversely naive agents, *Review of Economic Studies* 73, 689–714.

Epley, Nicholas, and Thomas Gilovich, 2016, The mechanics of motivated reasoning, *Journal of Economic Perspectives* 30, 133–140.

Exley, Christine L., and Judd B. Kessler, 2019, Motivated errors, Working paper, Harvard Business School.

Eyster, Erik, Matthew Rabin, and Dimitri Vayanos, 2018, Financial markets where traders neglect the informational content of prices, *Journal of Finance* 74, 371–399.

Fischer, Paul E., Chongho Kim, and Frank Zhou, 2020, Disagreement about fundamentals: Measurement and consequences, Working paper.

Ganguly, Ananda, and Joshua Tasoff, 2017, Fantasy and dread: The demand for information and the consumption utility of the future, *Management Science* 63, 4037–4060.

Gervais, Simon, and Terrance Odean, 2001, Learning to be overconfident, *Review of Financial Studies* 14, 1–27.

Greenwood, Robin, and Andrei Shleifer, 2014, Expectations of returns and expected returns, *Review of Financial Studies* 27, 714–746.

Grossman, Sanford, 1977, The existence of futures markets, noisy rational expectations and informational externalities, *Review of Economic Studies* 44, 431–449.

Grossman, Sanford J., and Joseph E. Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review* 70, 393–408.

Hansen, Lars Peter, and Thomas J. Sargent, 2001, Robust control and model uncertainty, *American Economic Review* 91, 60–66.

Hansen, Lars Peter, and Thomas J. Sargent, 2008, *Robustness* (Princeton University Press, Princeton, NJ).

Harrison, J. Michael, and David M. Kreps, 1978, Speculative investor behavior in a stock market with heterogeneous expectations, *Quarterly Journal of Economics* 92, 323–336.

Hellwig, Martin F., 1980, On the aggregation of information in competitive markets, *Journal of Economic Theory* 22, 477–498.

Hong, Harrison, and Jeremy C. Stein, 2003, Differences of opinion, short-sales constraints, and market crashes, *Review of Financial Studies* 16, 487–525.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Jevons, Herbert Stanley, 1905, *Essays on Economics* (Macmillan and Company, London).

Kacperczyk, Marcin, Stijn Van Nieuwerburgh, and Laura Veldkamp, 2016, A rational theory of mutual funds' attention allocation, *Econometrica* 84, 571–626.

Kahan, Dan M., 2013, Ideology, motivated reasoning, and cognitive reflection, *Judgment and Decision Making* 8, 407–424.

Kahan, Dan M., Ellen Peters, Erica Cantrell Dawson, and Paul Slovic, 2017, Motivated numeracy and enlightened self-government, *Behavioural Public Policy* 1, 54–86.

Kandel, Eugene, and Neil D. Pearson, 1995, Differential interpretation of public signals and trade in speculative markets, *Journal of Political Economy* 103, 831–872.

Karlsson, Niklas, George Loewenstein, and Duane Seppi, 2009, The ostrich effect: Selective attention to information, *Journal of Risk and Uncertainty* 38, 95–115.

Kreps, David M., 1998, Anticipated utility and dynamic choice, *Econometric Society Monographs* 29, 242–274.

Loewenstein, George, 1987, Anticipation and the valuation of delayed consumption, *Economic Journal* 97, 666–684.

Merkle, Christoph, 2017, Financial overconfidence over time: Foresight, hindsight, and insight of investors, *Journal of Banking and Finance* 84, 68–87.

Mobius, Markus M., Muriel Niederle, Paul Niehaus, and Tanya Rosenblat, 2014, Managing self-confidence, Working paper, Stanford University.

Mondria, Jordi, Xavier Vives, and Liyan Yang, 2021, Costly interpretation of asset prices, *Management Science* 68, 52–74.

Moskowitz, Tobias J., Yao Hua Ooi, and Lasse Heje Pedersen, 2012, Time series momentum, *Journal of Financial Economics* 104, 228–250.

Odean, Terrance, 1998, Volume, volatility, price, and profit when all traders are above average, *Journal of Finance* 53, 1887–1934.

Odean, Terrance, 1999, Do investors trade too much? *American Economic Review* 89, 1279–1298.

Oster, Emily, Ira Shoulson, and E. Ray Dorsey, 2013, Limited life expectancy, human capital, and health investments, *American Economic Review* 103, 1977–2002.

Scheinkman, José A., and Wei Xiong, 2003, Overconfidence and speculative bubbles, *Journal of Political Economy* 111, 1183–1220.

Shiller, Robert J., 1981, Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review* 71, 421–436.

Van Nieuwerburgh, Stijn, and Laura Veldkamp, 2010, Information acquisition and under-diversification, *Review of Economic Studies* 77, 779–805.

Veldkamp, Laura L., 2006, Information markets and the comovement of asset prices, *Review of Economic Studies* 73, 823–845.

Verrecchia, Robert E., 1982, Information acquisition in a noisy rational expectations economy, *Econometrica* 50, 1415–1430.

Wang, Jiang, 1993, A model of intertemporal asset prices under asymmetric information, *Review of Economic Studies* 60, 249–282.

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