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Discussion of "disclosure processing costs, investors' information choice, and equity market outcomes: A review"☆

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A R T I C L E I N F O

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A B S T R A C T

In their review of investor processing costs, Blankespoor, deHaan, and Marinovic (2020) encourage researchers to consider "rational inattention" as a way to explain stock market underreaction to public information. We formalize their argument and propose alternatives using a set of formal models. To distinguish between the various drivers of underreaction, we encourage future researchers to consider the opportunity cost of being inattentive, the historical evolution of underreaction, and the difference between information-based and return-based underreaction.

Blankespoor et al. (2020), hereafter BDM, review over 500 papers concerning disclosure and processing costs. Their organization of processing costs into awareness, acquisition and integration types is intuitive and useful. Moreover, their review is thorough in many areas, such as the empirical evidence linking processing costs to equity market outcomes. However, it is less explicit in others, such as a precise theoretical definition of "rational inattention" or a specific model which is consistent with the empirical phenomenon they summarize.

The purpose of this discussion is to fill in some of these gaps. We begin with BDM's argument about the scope of processing costs within a rational framework to explain stock market anomalies. BDM write, "One key insight we draw from the analytical literature review is that disclosure processing costs can provide rational explanations for many market phenomena that may otherwise appear anomalous. For example, post-earnings announcement drift (PEAD), accruals mispricing, and portfolio under-diversification can all be generated by rational models with disclosure processing costs." While this is true theoretically within a rational "processing cost" model a la Grossman and Stiglitz (1980), there are other predictions of this model that do not seem consistent with the empirics. Perhaps this is why even Fama (1998) struggles to explain PEAD within a rational framework and calls it "the granddaddy of all underreaction events."[1]

The empirical phenomenon BDM wish to explain is *ex-post underreaction* – that is, situations where individuals appear to "underreact" to available information, from the perspective of an ex-post observer. In archival data, academics often identify publicly available information (e.g., earnings surprises) that would have been useful for predicting returns in real time. If investors are perfectly rational and attentive, and the relation between the information source and returns is ex-

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[1] It is worth clarifying what we mean by rationality. In the classical asset pricing literature, a "rational" explanation for return predictability would generally consist of identifying relevant underlying risk factors which generate predictability. Our notion of rationality is that of BDM (see, e.g., Section 2.2.1 of their article), which is based upon the behavior of economic agents and requires that investors have common priors, use Bayes' rule, and maximize expected utility.

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ante knowable, such “excess predictability” should be eliminated in equilibrium, as long as markets are sufficiently frictionless. Consequently, “underreaction” to public information (from an ex-post, econometrician's perspective) is typically interpreted as inattentiveness. In this discussion, we present three possible models for why such underreaction might occur.

To illustrate the differences between the three models, consider a similar exercise in a different domain: suppose an alien arrived on earth and was tasked with forecasting a human's height at future time T with only information available at birth (time 0). The three different models we consider are like three different assumptions about the alien's behavior in this forecasting exercise.

(Costly processing) The first possibility is that the relationship between the relevant variables and time-T is known, but the time 0 values of the variables themselves are knowable only at a cost. For instance, the alien econometrician at time 0 knows that paternal and maternal height are useful predictors and that maternal diet and age are not (Bienertová-Vasku et al., 2017), but he does not know the values of the relevant variables for the baby. Perhaps the alien can interview the baby's parents, collect their measurements and health history, as well as the baby's birthweight and forecast the time T height based on this data. But this would take considerable effort and the alien would have to balance this effort against the benefits from the forecasting exercise.

(Behavioral) The second possibility is that the alien simply makes mistakes in the forecasting exercise. Perhaps the alien brings with him some preconceived notion of what should matter based on what he observed in his home planet and this leads him to ignore relevant data and put weight on irrelevant data.

(Unknowable relevance) The third possibility is that the alien has some guesses about what time 0 information might be useful to predict height - such as maternal height, paternal height, maternal diet and maternal age at birth − but he will only learn the correct relationships over time as he sees humans grow. *Ex-post*, the alien econometrician at time T will know that paternal and maternal height are useful predictors and that maternal diet and age are not (Bienertová-Vasku et al., 2017), but this was not knowable to the alien at time 0, even if the values of the variables themselves were. The true data-generating process only becomes known to the alien as more data becomes available.

We nest these different models for information processing in a unified framework. The first model, which we refer to as “costly processing,” captures situations in which the relationship between public information and firm value is known, but one can only condition on public information at a cost. This type of inattentive behavior seems most similar to what BDM describe.[2] The second is a canonical behavioral model of disagreement in which investors “agree to disagree” about the interpretation of public information. The third model, which we refer to as “unknowable relevance,” reflects a setting in which the precise nature of the relationship between the data source (e.g., earnings announcements) and firm value is ex-ante unknowable but can potentially be learned by observing subsequent data. As we argue, all are plausible interpretations of behavior that appears “inattentive” ex post, but they produce distinct predictions that distinguish them in the data.

In the next section, we present the theoretical framework and consider each model's ability to explain underreaction to earnings surprises (PEAD).[3] We choose PEAD because it is the most well-known and cited capital market anomaly in the accounting literature.[4] However, there is nothing special about earnings surprises, and the mechanisms we propose could easily be applied to other settings in order to explain inattentiveness to other information. In Section 2, we discuss some takeaways from the theoretical exercise and propose ways in which empiricists can distinguish between the models.

1. Theoretical framework

We lay out a stylized framework which allows us to distinguish the key channels through which PEAD may arise. There are three dates $t \in \{0, 1, 2\}$. The risk-free asset is the numeraire and consequently we normalize the risk-free rate to zero. The risky security is in zero net supply and pays a terminal, liquidating dividend D at date $t = 2$.[5] There is a continuum of ex-ante identical investors, indexed by $i \in [0, 1]$, with initial wealth normalized to zero $W_0 = 0$, and exponential utility over terminal wealth with constant absolute risk aversion γ. Denote the price of the risky security at date t by P_t.

At date $t = 1$, a signal s is publicly revealed to investors. The joint distribution of payoffs and signals is characterized by:

$$D = \theta s + \varepsilon, \tag{1}$$

[2] While BDM do not offer a particular model of costly processing, our reading of the paper, particularly of Section 2.3.2, suggests a re-interpretation of Grossman and Stiglitz (1980) as a simple benchmark for the type of frictions that they propose.

[3] While the majority of the accounting and finance literature seems to accept PEAD as an underreaction phenomenon, some papers have explored alternative explanations. For example, Hou et al. (2015) argue that some of PEAD's profitability can be explained by a q-factor risk model.

[4] As of June 2020, Ball and Brown (1968) has more citations than any article ever published in the *Journal of Accounting and Economics*, the *Journal of Accounting Research* and the *Accounting Review* according to Google Scholar.

[5] We assume that the net supply of the risky security is zero to simplify the model and abstract away from “risk-premium” effects. This allows us to clearly highlight how predictability can arise through purely informational channels.

where θ, s, and ε are independent. Further, we assume $s \sim N(0, \sigma_S^2)$, $\theta \in \{0, 1\}$ with probability $p = \Pr(\theta = 1)$, and $\varepsilon \sim N(0, \sigma^2)$. We interpret the signal s as the earnings surprise (SUE), the indicator θ as the relevance of the earnings signal to firm value, and ε as an unlearnable shock to firm value.

The above specification nests two natural benchmarks:

1. When $\theta = 1$ and this is commonly known, the above specification reduces to a variant of the standard "truth plus noise" specification i.e., conditional on observing s, the conditional distribution of D is given by:

$$D \big| s \sim N\left(s, \sigma^2\right). \tag{2}$$

2. When θ is unknowable to investors, then the above specification reflects the notion that investors face uncertainty about the payoff relevance of the signal s and cannot learn it by simply paying a cost. Specifically, this implies that conditional on observing s, beliefs about D are given by:

$$E[D|s] = ps, \;\; var(D|s) = p(1-p)s^2 + \sigma^2. \tag{3}$$

Since investors have no information at date 0 and the asset is in zero net supply, the date zero price is $P_0 = 0$. Moreover, since the payoff is revealed at date 2, we have $P_2 = D$. We say that prices exhibit ***SUE PEAD*** if for some $\Lambda > 0$, we have:

$$E[P_2 - P_1 | s, \theta = 1] = \Lambda s \tag{4}$$

and exhibit ***CAR PEAD*** if for some $\Gamma > 0$, we have:

$$E[P_2 - P_1 | P_1 - P_0, \theta = 1] = \Gamma (P_1 - P_0). \tag{5}$$

Implicitly, we assume that the econometrician (who is measuring PEAD ex-post) conditions on the realizations of the earnings surprise (i.e., s) or the announcement return (i.e., $P_1 - P_0$), and the relevance of the earnings news (i.e., $\theta = 1$) when evaluating predictability.[6]

In the following subsections, we characterize three possible mechanisms through which PEAD can arise. In Section 1.1, we consider a setting in which investors must pay a cost c to process the information in s, and update their beliefs accordingly – this is a stylized version of the rational inattention model suggested by BDM. In Section 1.2, we present a behavioral model in which investors "agree to disagree" about the interpretation of s (as in Kandel and Pearson (1990) or Banerjee and Kremer (2010)), that shares features of the bounded-rationality "news-watchers" model of Hong and Stein (1999). Finally, in Section 1.3, we propose an alternative in which investors are unable to learn the relevance of the signal ex-ante. While all three mechanisms are likely to be at play in the real world, we outline some predictions that would help distinguish them.

The relevance variable θ plays a key role in our analysis, particularly in contrasting the "costly processing" and "unknowable relevance" models. In the costly processing model, we suppose that θ is common knowledge, and investors must pay a processing cost in order to condition on s. This corresponds to a setting in which the relation between earnings surprises and returns is *known* by investors ex-ante, but they must pay a cost (explicit or implicit) to correctly condition on the earnings information. In contrast, in the unknowable relevance model, θ is not known by investors but the signal s is a traditional public signal freely available and interpretable to all investors. This corresponds to a setting in which investors do not, ex-ante, know the precise relation between earnings surprises and firm value and cannot resolve their uncertainty simply by paying a cost. Instead, the true relation can only be learned by observing data on earnings and returns.[7] In all cases, we assume that the econometrician, who observes earnings surprises and returns *ex-post* knows the relation between the two (i.e., knows the true value of θ). Indeed, with a sufficiently large sample, θ can be estimated with arbitrary accuracy by simply running a regression of realized returns on realized earnings surprises.

[6] Brandt et al. (2007) document a version of CAR PEAD, which they refer to as earnings announcement return (EAR), by measuring the drift in returns for portfolios formed on the stock price reaction around earnings announcements. This phenomenon is also discussed in the review by Richardson et al. (2010), and in the taxonomy of anomalies by Novy-Marx and Velikov (2016) and Green et al. (2017), who refer to it as CAR PEAD and EAR, respectively.

[7] Note that the sense of "unknowable relevance" in this model is fundamentally different from "awareness" and "integration" costs as discussed by BDM, which refer to costs associated with becoming aware of the existence of disclosures and with understanding the implication of such disclosures for firm value, respectively. In BDM's framework, investors can pay a cost to become aware of disclosures and to learn the correct relationship between those disclosures and firm value. In contrast, when relevance is unknowable, they do not know the relation between payoffs and disclosures and cannot learn the true relation simply by paying a cost. In a sense, this is related to models in which investors are uncertain about the precision of signals available to them (e.g., Subramanyam (1996)) but in which precision is unknowable until more data is generated.

1.1. Costly information processing approach

This mechanism builds on the insights of Grossman and Stiglitz (1980).[8] We assume that it is commonly known that $\theta = 1$ (i.e., it is commonly known that the public signal is payoff relevant). Investors must pay a cost $c > 0$ to process the payoff relevant information in contained in s. If they do not do so, they condition only on the asset price P_1. Let $\lambda \in [0, 1]$ denote the (endogenous) proportion of traders who choose to pay the processing cost. As is standard in the literature, we assume that aggregate supply of the risky security is subject to a random shock to ensure that prices are not fully revealing.[9]

$$z \sim N\left(0, \sigma_z^2\right) \tag{6}$$

This setting is essentially a reinterpretation of Grossman and Stiglitz (1980), where the cost of processing the public signal takes the place of the cost of private information, and a "processed" public signal takes the place of a classic private signal. Consequently, solving the model is analogous to characterizing the overall equilibrium in Grossman and Stiglitz (1980). Because this solution is so well-known, we suppress the details and highlight only the key points.

The optimal demand for an investor who processes the information in s takes the standard mean-variance form:

$$x_I(s, P_1) = \frac{E[D - P_1 | s, P_1]}{\gamma \ var(D|s, P_1)} = \frac{s - P_1}{\gamma\sigma^2}.$$

An investor who do not process the information themselves and conditions only on the price has optimal demand given by:

$$x_U(P_1) = \frac{E[D - P_1 | P_1]}{\gamma \ var(D|P_1)}.$$

It is easy to show that the "price signal" $s_p \equiv s - \frac{\gamma\sigma^2}{\lambda} z$, is informationally equivalent to the asset price and therefore that:

$$E[D|P_1] = \alpha s_p \quad and \quad var(D|P_1) = \sigma_s^2(1 - \alpha) + \sigma^2 \tag{7}$$

for $= \frac{\sigma_s^2}{\sigma_s^2 + \left(\frac{\gamma\sigma^2}{\lambda}\right)^2 \sigma_z^2}$. Combined with the above demand functions, it follows that the equilibrium price is:

$$P_1 = A s_p, \ \ where \ \ A \equiv \frac{\left(\frac{\lambda}{\sigma^2} + \frac{(1-\lambda)\alpha}{\sigma_s^2(1-\alpha) + \sigma^2}\right)}{\frac{\lambda}{\sigma^2} + \frac{(1-\lambda)}{\sigma_s^2(1-\alpha) + \sigma^2}} \le 1. \tag{8}$$

The equilibrium proportion of investors who process the signal s is characterized by comparing the expected utility from paying the cost to condition on the earnings signal i.e.,

$$U_I(\lambda) = E\left[E\left[-e^{-\gamma(x(s,P_1)(D-P_1)-c)}\middle| s, P_1\right]\right]$$

to the expected utility from avoiding the cost and conditioning on the price i.e.,

$$U_U(\lambda) = E\left[E\left[-e^{-\gamma(x(P_1)(D-P_1)-c)}\middle| P_1\right]\right].$$

As Grossman and Stiglitz (1980) show, there exist cost thresholds $0 < \underline{c} < \bar{c}$ such that:

(i) when costs are sufficiently low (i.e., $c \le \underline{c}$), all investors process the signal,
(ii) when costs are sufficiently high (i.e., $c \ge \bar{c}$), no investor processes the signal, and
(iii) when costs are in between (i.e., $\underline{c} < c < \bar{c}$), the equilibrium proportion is such that the marginal investor is exactly indifferent to processing the signal, $U_I(\lambda) = U_U(\lambda)$, and is given by

[8] Andrei et al. (2019) build on a similar setting to explore how higher economic uncertainty can lead investors to rationally allocate more attention to firm-specific information.

[9] This is equivalent to specifying exogenous noise traders who demand $-z$ shares.

$$\lambda(c) = \gamma\sigma\sigma_z\sqrt{\frac{1}{e^{2\gamma c}-1} - \frac{\sigma^2}{\sigma_s^2}}. \tag{9}$$

In this setting, prices exhibit SUE PEAD as long as costs are not so low that all traders process the signal. In particular,

$$E\left[D - P_1 | s, \theta = 1\right] = s - AE\left[s_p | s\right] = \Lambda s, \tag{10}$$

where $\Lambda \equiv 1 - A \geq 0$, with equality if and only if $\lambda = 1$. In aggregate, investors underweight the earnings signal because only a fraction of them are able to condition on it. However, because all investors correctly weight the information in prices, prices do not exhibit CAR PEAD. Specifically, since $P_0 = 0$ and $s_p = P_1/A$, we have:

$$E[D - P_1 | P_1 - P_0, \theta = 1] = (\alpha - A)s_p = \Gamma(P_1 - P_0), \tag{11}$$

where $\Gamma = \frac{(\alpha - A)}{A} < 0$.[10] This is consistent with the observation of Banerjee et al. (2009), who show that when investors exhibit rational expectations and condition on prices correctly, it is difficult to generate predictability based on lagged returns.

The above suggests that while "rational" inattention may play a role in generating SUE PEAD, it does not naturally give rise to CAR PEAD, unless investors also systematically under-react to the information in prices.[11] Moreover, the model suggests that the magnitude of SUE PEAD should decrease as the fraction of investors who process the signal directly increases (i.e., as λ increases). Note that the fraction λ is decreasing in the processing cost (i.e., c), increasing in the information content of the signal (i.e., σ_s^2) and hump-shaped in risk aversion (i.e., γ) and the unlearnable component of dividends (i.e., σ^2). The key implications of this approach are given by the following.

Implications 1. Suppose investors exhibit rational inattention with respect to conditioning on the earnings surprise s. Then, returns generally exhibit SUE PEAD, unless costs are sufficiently low, but cannot exhibit CAR PEAD. Moreover, the magnitude of SUE PEAD, when it arises, increases with the level of information processing costs.

1.2. Behavioral approach

The next mechanism again assumes that the relevance of the earnings signal is common knowledge, $\theta = 1$, but relies on two key assumptions. First, suppose investor i's processing of signal s is imperfect (but costless) and introduces additional noise. Specifically, we assume that she extracts a noisy version s_i, where

$$s_i = s + e_i, \tag{12}$$

and $e_i \sim N(0, \sigma_e^2)$ are independent across investors. Second, suppose that investors "agree to disagree" about the information content of one another's signals and hence do not condition on prices.

A number of underlying assumptions about information processing can lead to such beliefs. For instance, in difference of opinions models (e.g., Harrison and Kreps (1978); Harris and Raviv (1993); Kandel and Pearson (1990); Scheinkman and Xiong (2003); Banerjee and Kremer (2010)), investors have heterogenous priors over the joint distribution of signals and payoffs and therefore "agree to disagree" on the interpretation of the same signals. On the other hand, in models of bounded rationality (e.g., Hong and Stein (1999)), each investor is only able to utilize a subset of the publicly available information.

The optimal demand for investor i is given by:

$$x_i(s_i) = \frac{E[D|s_i] - P_1}{\gamma\ var(D|s_i)} = \frac{\kappa s_i - P_1}{\gamma\left(\sigma^2 + \sigma_s^2(1-\kappa)\right)}, \tag{13}$$

where $\kappa = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_e^2} \in (0, 1)$. Since the risky security is in zero net supply, market clearing implies:

$$P_1 = \kappa s,\ where \quad \kappa = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_e^2} < 1. \tag{14}$$

In this setting, prices exhibit SUE PEAD, since

$$E[D - P_1 | s, \theta = 1] = (1-\kappa)s, \quad and \quad (1-\kappa) > 0, \tag{15}$$

and CAR PEAD since

[10] Note that from equation (9), we know that $A = w + (1-w)\alpha$ for some $w \in (0,1)$, so that $\Gamma = -\frac{w(1-\alpha)}{\alpha + w(1-\alpha)} < 0$.

[11] Indeed, this benchmark model generates *negative* predictability in returns.

$$E[D - P_1|P_1 - P_0, \theta = 1] = E[D - P_1|s] = \frac{1-\kappa}{\kappa}(P_1 - P_0). \quad (16)$$

Because investors condition on noisy versions of the public signal, in aggregate they under-react to it which leads to SUE PEAD. Moreover, because traders neglect the information in prices, they under-weight the price signal, which leads to CAR PEAD. While the model is stylized, note that the underlying mechanism is simply driven by "under-reaction" to public information (i.e., prices and earnings).[12] As such, we expect similar results to hold in alternative settings that generate similar under-reaction effects due to differences in preferences or beliefs, or due to bounds on information processing or rationality.

Implications 2. Suppose investors process the information in s imperfectly and agree to disagree about the interpretation of the signal (as specified above). Then prices always exhibit both SUE PEAD and CAR PEAD.

1.3. Unknowable relevance

We now consider a setting in which all investors observe and condition on the same public signal s but in which the payoff-relevance of the public signal for payoffs is uncertain and ex-ante unknowable. Specifically, suppose investors do not know whether $\theta = 1$, but instead assign probability $p = \Pr(\theta = 1)$. We emphasize that no traders have private (or public) information about the relevance, θ, and consequently trade on the basis of their prior beliefs about it. Relevance is only learnable, in principle, from observing a time series of signals and prices.

In this case, the optimal demand for investor i is (approximately) given by:

$$x_i(s) \approx \frac{E[D - P_1|s, P_1]}{\gamma \ var(D|s)} = \frac{ps - P_1}{\gamma\left(\sigma^2 + p(1-p)s^2\right)}, \quad (17)$$

and the equilibrium price is given by[13]

$$P_1 = ps. \quad (18)$$

This immediately implies that prices exhibit SUE PEAD and CAR PEAD since $p < 1$:

$$E[D - P_1|s, \theta = 1] = (1-p)s, \quad (19)$$

and

$$E[D - P_1|P_1 - P_0, \theta = 1] = E[D - P_1|ps, \theta = 1] = \frac{(1-p)}{p}(P_1 - P_0). \quad (20)$$

Naturally, from the perspective of econometrician who knows that the earnings signal is relevant, traders appear to underweight the signal since they only place probability $p < 1$ on its relevance. Hence, prices exhibit SUE PEAD. Similarly, because prices convey no additional information beyond the earnings signal, ex-post traders appear to underweight the price signal, which generates CAR PEAD. However, from the perspective of an investor, there is no exploitable predictability based on the earnings surprise since

$$E[D - P_1|s] = p\, E[D - P_1|s, \theta = 1] + (1-p)\, E[D - P_1|s, \theta = 0] = p(1-p)s + (1-p)(-p)s = 0$$

Similarly, because prices convey the same information as earnings, there is no exploitable predictability on the basis of returns i.e., $E[D - P_1|P_1 - P_0] = E[D - P_1|s] = 0$.

Note that an analogous analysis also applies when $\theta = 0$, i.e., when the signal is not payoff relevant. However, in this case, prices exhibit over-reaction and reversals (i.e., "negative" PEAD) from the perspective of an econometrician since we have

$$E[D - P_1|s, \theta = 0] = -ps \quad and \quad E[D - P_1|P_1 - P_0, \theta = 0] = -(P_1 - P_0) \quad (21)$$

[12] Note that in order for CAR PEAD to arise, investors need not completely ignore the information in prices but simply under-react to it in aggregate. Specifically, we expect similar results to obtain when only a fraction of investors exhibit "agree to disagree" or "bounded rationality," as long as the rational investors are sufficiently risk averse or wealth constrained.

[13] In this case, traders' conditional beliefs about D are a mixture of normals − probability p of a $N(s, \sigma^2)$ random variable and probability $1-p$ of a $N(0, \sigma^2)$ random variable − so their problem *does not* reduce to mean variance optimization. Hence the given demand function is only an approximation, which we provide for comparability with the other models. Nevertheless, because agents have identical beliefs there exists a representative agent, from which it is easy to demonstrate that the expression for the asset price is exact. If instead we had specified directly that agents have mean-variance preferences, then the demand function would be exact, and the equilibrium price would be unchanged.

Finally, note that if investors learn about the relevance of s over time (e.g., by observing a longer and longer sample of $\{D, s\}$ pairs), their beliefs about θ to converge to the truth almost surely. i.e.,

$$\lim_{t \to \infty} p_t = \begin{cases} 1, & \theta = 1 \\ 0, & \theta = 0 \end{cases} \tag{22}$$

which implies the degree of PEAD (or the degree of over-reaction for irrelevant signals) will decline over time as traders learn about the structure of the economy.

Implications 3. Suppose investors do not know the relevance of a public signal ex-ante but learn about its relevance over time. Then prices always exhibit both SUE PEAD and CAR PEAD with respect to ex-post relevant signals, and over-reaction/reversals with respect to ex-post irrelevant signals. Moreover, the magnitude of under-reaction/over-reaction decreases over time as traders update their beliefs about the relevance of the signal.

2. Discussion

While all three models generate underreaction to information, how might a researcher distinguish among them empirically? BDM "encourage future researchers to more intentionally consider costly disclosure processing as an explanation for market phenomena, and to disentangle rational responses from behavioral biases where possible" (page 5). Based on the analysis above, we suggest some potential approaches to disentangle the underlying mechanisms.

2.1. Opportunity cost of inattentiveness

One distinguishing feature of the costly processing model is the ex-ante calculation made by investors: information is acquired *if and only if the expected benefit exceeds the acquisition costs*. Therefore, one way to infer whether costly processing is a reasonable explanation for observed underreaction is to compare the profits available from trading strategies that make use of public information to plausible estimates of processing costs. For example, Bernard (1993) estimates that the returns to a PEAD-based strategy are 4.5%, 8.9% and 9.9% for large, medium and small stocks respectively *after* transaction costs during the period 1974–1986. Since calculating a variable like SUE is relatively simple, even in the 1970's and 1980's, these foregone benefits seem too high to be justified by information processing costs. Such comparisons suggest that either the behavioral or "unknowable relevance" model is more appropriate. We encourage researchers to make similar calculations in other domains of predictability to assess the reasonableness of a costly processing explanation.

2.2. Information-based versus return-based predictability

A related implication is that while the behavioral and "unknowable relevance" models can generate CAR PEAD, the costly processing model does not. This is consistent with the observation that, even if collecting information on earnings surprises was sufficiently costly in the past, sorting stocks on price reactions to earnings announcements (as in CAR PEAD) should have been relatively trivial. As such, it is difficult to reconcile PEAD based on announcement returns with the costly processing model, without layering on additional assumptions.

In contrast, in our behavioral model, because investors "agree to disagree" about the interpretation of public information, they believe prices are not incrementally informative, and so effectively underreact to announcement returns. Similarly, when investors are uncertain about the relevance of public information, they collectively underreact to price information, and this leads to CAR PEAD. These differences suggest that whether predictability based on public information (e.g., SUE PEAD) is also accompanied by predictability based on returns (e.g., CAR PEAD) can also be used to empirically distinguish between models of underreaction.

A further, distinctive, prediction of the "unknowable relevance" model is that returns exhibit reversals around announcements when investors condition on information that turns out to be ex-post irrelevant – that is, when investors ex-post *overreact* to information. Without additional assumptions, such reversals do not arise in the behavioral model we discuss. While return reversals can arise in the costly processing model (due to noise trading), they occur simultaneously with investor *underreaction* to the public information.[14]

2.3. Disappearing underreaction over time

Finally, it is well known that many stock market anomalies are disappearing over time. For example, McLean and Pontiff (2016) examine 97 different anomalies published in the academic literature and estimate a decrease in long-short anomaly returns of 58% post-publication. Similarly, Calluzzo et al. (2019) find similar declines and increases in trading by institutions after the publication of 14 different anomalies.

[14] Indeed, predictability based on the public information and on the announcement return can be of opposite signs in the costly acquisition model precisely because noise-trading driven return reversals are nonetheless accompanied by underreaction to the public signal.

The weakening of underreaction over time is a direct prediction of the "unknowable relevance" model because investors learn about the correct relation between payoffs and public information as they observe more data. When investors are sufficiently certain about the true underlying relation, prices appropriately react to public information, and an econometrician examining these data years later should not be able to find any predictability after this point. However, if the relation between the information and payoffs changes over time (e.g., due to regulatory changes), the uncertainty about relevance may be reset and ex-post predictability may be higher.

The "costly processing" model also has a prediction for the time-series of underreaction: innovations that reduce the cost of acquiring information should result in immediate reductions in predictability.[15] For example, the introduction of the SEC's EDGAR platform presumably reduced the cost of information acquisition for investors. The model predicts a sharp reduction in return predictability from information contained in EDGAR filings. It should be noted, however, that the "costly processing" model, unlike the unknowable relevance model, is not unidirectional in its prediction. Where there are innovations that increase the cost of processing (such as the removal of a low-cost information acquisition platform) the model predicts an *increase* in underreaction. Nevertheless, since the overall cost of information acquisition has certainly shrunk over time, the "costly processing" model would be generally consistent with the observed disappearance of anomalies.

Finally, the behavioral model has no direct prediction for weakening underreaction without additional assumptions. If investors' ability to process and interpret public information improves over time, we expect the prices to better reflect the information and underreaction to weaken over time. However, if investors are dogmatic about their (incorrect) priors, or continue to be boundedly rational, underreaction should persist and the empirical anomalies should not disappear over time.

3. Conclusion

BDM encourage researchers to consider "rational inattention" as a possible explanation for many underreaction phenomena in financial markets. The purpose of this discussion has been to formalize what "rational inattention" could mean in two different models, a "costly processing" model and an "unknowable relevance" model, and to juxtapose these rational models with a behavioral model of inattention. All three models deliver different predictions for empiricists, from the existence of price-based underreaction to the time-series of underreaction.

Like BDM, we encourage future researchers to consider inattention as a plausible mechanism to explain observed underreaction. We hope the discussion herein provides additional perspective on how one might identify an appropriate model of inattention to explain phenomenon which looks like underreaction to publicly available information.

References

Andrei, Daniel, Friedman, Henry L., Ozel, N Bugra, 2019. Economic uncertainty and investor attention. Working paper.
Ball, Ray, Brown, Philip, 1968. An empirical evaluation of accounting income numbers. J. Account. Res. 6 (2), 159–178.
Banerjee, Snehal, Kremer, Ilan, 2010. Disagreement and learning: dynamic patterns of trade. J. Finan. 65 (4), 1269–1302.
Banerjee, Snehal, Kaniel, Ron, Kremer, Ilan, 2009. Price drift as an outcome of differences in higher-order beliefs. Rev. Finan. Stud. 22 (9), 3707–3734.
Bernard, Victor, 1993. Stock price reactions to earnings announcements: a summary of recent anomalous evidence and possible explanations. In: Thaler, Richard (Ed.), Advances in Behavioral Finance. New York: Russell Sage Foundation, pp. 303–340.
Bienertova-Vasku, Julie, Zlámal, Filip, Prusa, Tomás, Novák, Jan, Mikes, Ondrej, Cupr, Pavel, Pohorolá, Aneta, Svancara, Jan, Andrýsková, Lenka, Pikhart, Hynek, 2017. Parental heights and maternal education as predictors of length/height of children at birth, age 3 and 19 years, independently on diet: the ELSPAC study. Eur. J. Clin. Nutr 71, 1193–1199.
Blankespoor, Elizabeth, deHaan, Ed, Marinovic, Ivan, 2020. Disclosure Processing Costs, Investors' Information Choice, and Equity Market Outcomes: A Review. J. Account. Econom.
Brandt, Michael W., Kishore, Runeet, Santa-Clara, Pedro, Venkatachalam, Mohan, 2007. Earnings announcements are full of surprises. Working paper.
Calluzzo, Paul, Moneta, Fabio, Topaloglu, Selim, 2019. When anomalies are publicized broadly, do institutions trade accordingly? Manag. Sci. 65 (10), 4555–4574.
Fama, Eugene F., 1998. Market efficiency, long-term returns, and behavioral finance. J. Finan. Econom. 49 (3), 283–306.
Green, Jeremiah, Hand, John R.M., Zhang, X. Frank, 2017. The characteristics that provide independent information about average us monthly stock returns. Rev. Finan. Stud. 30 (12), 4389–4436.
Grossman, Sanford J., Stiglitz, Joseph E., 1980. On the impossibility of informationally efficient markets. Am. Econom. Rev. 70 (3), 393–408.
Harris, Milton, Raviv, Artur, 1993. Differences of opinion make a horse race. Rev. Finan. Stud. 6 (3), 473–506.
Harrison, J. Michael, Kreps, David M., 1978. Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations. Q. J. Econom. 92 (2), 323–336.
Hong, Harrison, Stein, Jeremy C., 1999. A unified theory of underreaction, Momentum trading, and overreaction in asset markets. J. Finan. 54 (6), 2143–2184.
Hou, Kewei, Xue, Chen, Zhang, Lu, 2015. Digesting anomalies: an investment approach. Rev. Finan. Stud 28 (3), 650–705.
Kandel, Eugene, Pearson, Neil D., 1990. Differential interpretation of public signals and trade in speculative markets. J. Pol. Econom. 103 (4), 831–872.
McLean, David R., Pontiff, Jeffrey, 2016. Does academic research destroy stock return predictability? J. Finan. 71 (1), 5–32.
Novy-Marx, Robert, Velikov, Mihail, 2016. A taxonomy of anomalies and their trading costs. Rev. Finan. Stud. 29 (1), 104–147.
Richardson, Scott, Tuna, Irem, Wysocki, Peter, 2010. Accounting anomalies and fundamental analysis: a review of recent research advances. J. Account. Econom. 50 (2–3), 410–454.
Scheinkman, José, Xiong, Wei, 2003. Overconfidence and speculative bubbles. J. Pol. Econom. 111 (6), 1183–1220.
Subramanyam, K.R., 1996. Uncertain precision and price reactions to information. Account. Rev. 71 (1), 207–219.

[15] This is related to, but conceptually distinct from, transaction costs shrinking over time. Transaction cost reductions are another plausible explanation for a time-series reduction in underreaction.