Learning from Prices and the Dispersion in Beliefs

Snehal Banerjee
Northwestern University

The article develops a dynamic model that nests the rational expectations (RE) and differences of opinion (DO) approaches to study how investors use prices to update their valuations. When investors condition on prices (RE), investor disagreement is related positively to expected returns, return volatility, and market beta, but negatively to return autocorrelation. When investors do not use prices (DO), these relations are reversed. Tests of these predictions on the cross-section of stocks using analyst forecast dispersion and volume as proxies for disagreement provide empirical evidence that is consistent with investors using prices on average. (*JEL* G12, G14)

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

How do investors use the information in an asset's price to update their beliefs about its payoff? While there are many different models of how investors *might* use prices, empirically determining what they *actually do* has proved to be a challenge. The standard theoretical approach in the literature is to assume rational expectations (RE), where investors agree on the interpretation of signals and thus condition on prices efficiently to infer the private information of others. Alternate approaches, however, suggest reasons for why investors may not condition on prices. For instance, in a differences of opinion (DO) model, investors "agree to disagree" about the distribution of payoffs and signals and, therefore, may not use prices to update their beliefs.[1] Additionally, investors may not condition on prices correctly if they exhibit behavioral biases or simply do not know how to invert prices into payoff-relevant information.[2]

I would like to thank Anat Admati, Carlos Corona, Peter DeMarzo, Michael Grubb, Scott Joslin, Yaniv Konchitchki, Ilan Kremer, Stefan Nagel, Paul Pfleiderer, Matthew Spiegel (the editor), Natalya Teplitsky, Viktor Todorov, and anonymous referees. I would also like to thank seminar participants at the Stanford GSB, University of California–Berkeley, University of Washington–Seattle, Yale University, Princeton University, Columbia University, University of Chicago, Northwestern University, Duke University, Washington University, Rice University, New York University, University of Michigan–Ann Arbor, and University of Texas–Austin. Send correspondence to Snehal Banerjee, Kellogg School of Management, Northwestern University, 2001 Sheridan Rd, Evanston, IL 60201; telephone: (847) 491 8335. E-mail: snehal-banerjee@kellogg.northwestern.edu.

1 In most DO models, the beliefs of other investors are assumed to be common knowledge, so there is no need to condition on prices. In some recent models, this common knowledge assumption is relaxed, and investors may condition on prices to update their higher-order beliefs (e.g., Banerjee, Kaniel, and Kremer 2009).

2 The learning literature (see Blume, Bray, and Easley 1982 for an early survey) has found that convergence to rational expectations through learning requires investors to have an extensive structural knowledge of the

© The Author 2011. Published by Oxford University Press on behalf of The Society for Financial Studies. All rights reserved. For Permissions, please e-mail: journals.permissions@oup.com.
doi:10.1093/rfs/hhr050 Advance Access publication July 28, 2011

While these approaches differ sharply in their assumptions about how investors learn from prices, they are extremely difficult to distinguish empirically. The underlying assumptions about beliefs, information, and learning are not observable and cannot be directly tested. More importantly, the predictions of these models on observable characteristics, such as return and volume patterns, are often similar despite the differences in their assumptions. For instance, the static Hellwig (1980) RE model and the Lintner (1969) counterpart (in which investors condition only on private information), are impossible to distinguish empirically, since one can make these models observationally equivalent by setting investors' risk aversions and signal precisions appropriately. Even dynamic RE and DO models often deliver similar predictions (e.g., both types of models can generate persistence in trading volume and serial correlation in returns).[3]

The article shows that dynamic RE and DO models *can* be distinguished empirically by comparing how return-volume characteristics change with investor disagreement in equilibrium. When investors condition on prices as in the RE case, assets with higher investor disagreement have higher expected returns, higher volatility, higher market betas, and higher covariance between volume and absolute returns, but lower return autocorrelation. When investors do not condition on prices, as in the DO case, higher disagreement assets have lower expected returns, lower volatility, lower betas, and lower covariance between volume and absolute returns, but higher return autocorrelation. In either case, assets with higher disagreement have higher expected volume.

Since the model nests both the RE and DO cases and its predictions depend only on the signs, and not levels, of observable correlations, the empirical tests that distinguish the two cases are easy to detect and potentially robust to misspecification. These predictions are tested on the cross-section of stock returns and volume using proxies for disagreement based on analyst forecast dispersion and trading volume. The empirical evidence from both portfolio sorts and Fama–MacBeth regressions is broadly consistent with investors conditioning on prices, although there is substantial variation across stocks in the degree to which they do so. Moreover, firms in which investors appear to condition on prices more are larger and have higher average returns, higher market betas, higher return volatility, higher trading volume, higher market-to-book ratios, higher analyst coverage, and higher volatility in earnings.

The article develops an overlapping generations (OLG) model that generalizes the models in Hellwig (1980) and Admati (1985) and is closely related

economy and of other investors' beliefs. This level of sophistication may not be a realistic assumption. The behavioral finance literature suggests a number of cognitive biases, including overconfidence about private information and underreaction to public information, which may lead investors to not condition on prices.

[3] Noisy RE models like those of Wang (1994) and He and Wang (1995) and DO models like those of Scheinkman and Xiong (2003) and Banerjee and Kremer (2010) can generate these and other, richer patterns in return and volume dynamics.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

to the models of Spiegel (1998) and Watanabe (2008). The OLG assumption allows me to introduce dynamics while maintaining tractability. A dynamic model implies that an investor's demand for a risky asset depends not only on her beliefs about the fundamentals (i.e., future dividends), but also on her beliefs about future prices. Investors receive private information about next period's dividends and have heterogeneous beliefs about the informativeness of other investors' signals. This determines the extent to which they condition on prices. At one extreme, investors believe that others' signals contain no information at all and so rely only on their own private information when updating their beliefs; this corresponds to the DO case. At the other extreme, investors realize that others' signals are just as informative as their own and condition on the price efficiently, as in an RE equilibrium. The market-clearing condition determines equilibrium prices, and aggregate supply shocks prevent these prices from being fully revealing.

An asset's return and volume characteristics are driven by the investors' disagreement about its payoffs and their perceived risk from holding the asset. The dispersion in beliefs, or disagreement, is measured as the variance in the investors' equilibrium expectations about the asset's payoffs. The perceived risk depends on the investors' posterior variance in payoffs. The expected return and market beta of the asset are increasing in the perceived risk. Since payoffs depend on future dividends *and* future prices, the perceived risk is higher when prices are more sensitive to dividend and supply shocks, and hence more volatile. This also implies that higher perceived risk is associated with lower return autocorrelation and higher correlation between absolute returns and volume. Finally, expected volume increases with the level of disagreement.

The empirical predictions used to distinguish RE from DO follow from the difference in the relation between perceived risk and dispersion. In the DO case, perceived risk is negatively related to disagreement. Intuitively, when investors agree to disagree and have heterogeneous beliefs, being more certain about an asset's payoffs leads to more disagreement. On the other hand, when investors disagree in the RE case, they realize that others have payoff-relevant information that they do not have. This leads investors to condition on prices more aggressively, which makes aggregate demand, and hence prices, more sensitive to dividend and supply shocks. Since payoffs depend not only on future dividends, but also on future prices, higher price sensitivity to fundamental shocks implies higher perceived risk from holding the asset. This implies that disagreement and perceived risk are positively related in the RE model.

The rest of the article is organized as follows. The next section presents a brief overview of the related literature. Section 2 presents the theoretical results of the article, Section 3 presents the empirical analysis, and Section 4 concludes. Unless noted otherwise, proofs are in Appendix A.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

1. Related Literature

The current article is most closely related to Lang, Litzenberger, and Madrigal (1992). The authors develop empirical predictions about the relation between volume, price changes, and changes in average forecasts around earnings announcements to distinguish between competitive (Walrasian) and rational expectations equilibria, with and without aggregate noise. As in the current article, the empirical evidence is consistent with the noisy rational expectations equilibrium. However, since the tests are based on static models and rely on the difference in levels of regression coefficients, they are difficult to detect and more sensitive to misspecification. In contrast, the predictions in the current article are based on a dynamic model and rely on the signs of the relations between disagreement and a number of return-volume characteristics. These differences in sign are easier to detect, and arguably more robust to the underlying specifications of the model.

The current article contributes to the theoretical and empirical literature on the relation between analyst forecast dispersion and return-volume characteristics. The related papers can be summarized as follows:

1. *Relation between Expected Returns and Dispersion.*
   Negative Relation. Diether, Malloy, and Scherbina (2002) and Goetzmann and Massa (2005) document a negative relation between belief dispersion and returns, which they motivate with the optimistic pricing model in Miller (1977). Park (2005) extends the Harrison and Kreps (1978) model to argue that speculative pricing leads to the negative return-disagreement relation. The negative relation in these DO models relies crucially on the presence of short sales constraints, which prevents full revelation of information and leads to overpricing. Johnson (2004) argues that the negative relation can be explained using an option pricing result, namely that, for a levered firm, expected returns should decrease with firm-level risk, which is potentially related to dispersion in forecasts. Ang and Ciccone (2001) claim that forecast dispersion is a proxy for firm transparency, and opaque firms, which have high forecast dispersion, usually have lower returns. Zhang (2006) claims that the negative relation is due to behavioral biases and that dispersion in forecasts proxy for information uncertainty.

   Positive Relation. Qu, Starks, and Yan (2004) also argue that dispersion in analyst forecasts proxies for information uncertainty, but derive a positive relation between disagreement and returns.

   Positive and Negative Relation. Anderson, Ghysels, and Juergens (2005) develop a general equilibrium model in which dispersion in beliefs is a risk factor. They find evidence of a negative relation between expected returns and short-term dispersion, but a positive relation between returns and long-term dispersion.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

2. *Positive Relation between Trading Volume and Dispersion*. Ajinkya, Atiase, and Gift (1991) document that forecast dispersion is positively related to trading volume.
3. *Positive Relation between Volatility and Dispersion*. Ajinkya and Gift (1985) document a positive relation between volatility and disagreement, and Shalen (1993) develops a two-period rational expectations model in which dispersion is positively related to the volatility in prices and the correlation between volume and absolute price changes.
4. *Positive Relation between Return Autocorrelation and Dispersion*. Verardo (2009) finds a positive relation between forecast dispersion and autocorrelation in returns and argues that this is consistent with the overreaction and self-attribution model of Daniel, Hirshleifer, and Subrahmanyam (1998); the underreaction to public news model of Hong, Kubik, and Solomon (2000); and the parameter uncertainty model of Lewellen and Shanken (2002).

The negative return-disagreement relation in the DO case and the positive return-disagreement relation in the RE case arise naturally in a standard setup even *without* short sales constraints or behavioral biases. More generally, the degree to which investors condition on prices also affects how disagreement is related to other return-volume characteristics.

This article is more generally related to the literature on rational expectations and differences of opinion. In the RE literature, the current model is most closely related to the long-lived investor model of Wang (1994), and the overlapping generations models of Spiegel (1998), Biais, Bossaerts, and Spatt (2010), and Watanabe (2008), although the focus of these papers is different. Wang (1994) studies the relation between returns, volume, and ex-ante information asymmetry (signal noise). Spiegel (1998) studies the role of rational expectations equilibria in generating excess volatility in returns. Biais, Bossaerts, and Spatt (2010) develop a general equilibrium model and then empirically show that a price-contingent portfolio outperforms a passive indexing portfolio. Watanabe (2008) studies how correlations across asset returns are determined in a noisy rational expectations model, and looks at the trading behavior of hierarchically informed investors.

Models in which investors exhibit DO have been useful in explaining many empirical features of price and volume dynamics. These include models of speculation, bubbles, and crashes (e.g., Harrison and Kreps 1978; Hong and Stein 2003; Scheinkman and Xiong 2003; Cao and Ou-Yang 2009), volume and volume-return characteristics (e.g., Harris and Raviv 1993; Kandel and Pearson 1995), positive autocorrelation in volume (e.g., Harris and Raviv 1993; Banerjee and Kremer 2010) and positive autocorrelation in returns (e.g., Banerjee, Kaniel, and Kremer 2009). A class of dynamic DO models (e.g., Detemple and Murthy 1994; Zapatero 1998; Basak 2000; Buraschi and Jiltsov 2006) in which pricing can be done using a stochastic discount factor

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

have also been successful in matching moments of the return-volume distribution. This article adds to this literature by providing a dynamic DO model in which disagreement arises endogenously as a result of asymmetric information and formally deriving implications on the relation between dispersion and return-volume characteristics. Moreover, while the DO model in this article has a natural RE counterpart that facilitates comparison, many of the models in the existing literature do not.

2. Theoretical Analysis

2.1 Model setup and equilibrium

2.1.1 Payoffs and preferences. There are N risky assets and one risk-free asset in the economy. The risky assets pay dividends D_{t+1} in period $t+1$, given by the following process:

$$D_{t+1} = (I - \Lambda)D + \Lambda D_t + \delta_{t+1}, \text{ where } \delta_{t+1} \sim N(0, V_d). \tag{1}$$

The $N \times N$ matrix V_d is the covariance matrix of the dividend shocks, and is assumed to be positive-definite. The diagonal matrix Λ captures the serial correlation in dividends, and is assumed to be non-negative and with all its elements less than 1. The risk-free asset pays an exogenously fixed rate $r_f > 0$.[4] Let the equilibrium prices of the risky assets be denoted by the vector P_{t+1} and the dollar return on the risky assets be denoted by R_{t+1}, where

$$R_{t+1} = P_{t+1} + D_{t+1} - (1 + r_f)P_t. \tag{2}$$

At each date t, there is a continuum of investors indexed by i. Investor i in generation t is born with wealth $w_{i,t}$, and has exponential utility over her wealth $w_{i,t+1}$ in the next period. For notational simplicity, set the coefficient of risk aversion to one. Denote investor i's information set at date t by $\mathcal{F}_{i,t}$ and her equilibrium portfolio allocation in risky assets by $x_{i,t}$. Then, agent i solves the following optimization problem at date t:

$$x_{i,t} = \arg\max_x E[-\exp\{w_{i,t+1}\}|\mathcal{F}_{i,t}], \text{ where } w_{i,t+1} = w_{i,t}(1+r_f) + xR_{t+1}, \tag{3}$$

and this implies that her optimal demand $x_{i,t}$ is given by

$$x_{i,t} = \text{var}[R_{t+1}|\mathcal{F}_{i,t}]^{-1}E[R_{t+1}|\mathcal{F}_{i,t}]. \tag{4}$$

[4] The risk-free rate must be strictly greater than zero, since the risky assets pay an infinite stream of dividends with positive means. However, as is standard in these models, the risk-free rate is exogenously set since the analysis is concerned with the study of the return and volume characteristics of the risky assets (e.g., Wang 1994; He and Wang 1995; Spiegel 1998).

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

2.1.2 Information and beliefs. In period t, investor i receives a private signal $Y_{i,t}$ about next period's dividend shock of the form

$$Y_{i,t} = \delta_{t+1} + s_{i,t}, \text{ where } s_{i,t} \sim N(0, V_s). \tag{5}$$

Investors have heterogeneous priors over these signals—in particular, investor i's beliefs about the signal of investor j is given by

$$Y_{i,j,t} = \rho\delta_{t+1} + \sqrt{(1-\rho^2)}\phi_{i,t+1} + s_{j,t},$$
$$\text{where } s_{j,t} \sim N(0, V_s), \phi_{i,t+1} \sim N(0, V_d), \rho \in [0, 1] \tag{6}$$

and $\phi_{i,t+1}$ is independent of δ_{t+1}. The shock $\phi_{i,t+1}$ is assumed to have variance V_d so that investor i's beliefs about the aggregate private information $\int_j Y_{i,j,t} dj$ has variance V_d irrespective of the level of ρ. The beliefs are assumed to be symmetric for tractability and to eliminate other, potentially confounding effects that make the intuition for the model less transparent.[5]

This information structure allows us to parsimoniously model the extent to which investors believe that the signals of other investors are informative using a single parameter ρ. Moreover, the parameter ρ summarizes the degree to which investors condition on prices—when ρ is higher, each investor thinks the others' signals are more informative, and therefore puts more weight on prices when updating her beliefs. As a result, this specification not only nests the rational expectations (i.e., $\rho = 1$) and differences of opinion (i.e., $\rho = 0$) benchmarks, but also allows us to model investors who know how informative their own signals are, but misestimate the informativeness of others' signals (where $\rho \in (0, 1)$). Hence, the model nests a particular specification of "relative overconfidence"—each investor believes her private signal, though noisy, is more informative than the signals of others. This inefficient use of conditioning information need not be irrational, since investors may not condition correctly on prices if they do not know how to invert prices correctly or if they have heterogeneous priors. The specific reason for why investors do not condition on prices correctly does not change the predictions of the model, and the specification of beliefs above lets us consider a wide range of investor behavior in a tractable manner.

2.1.3 Aggregate supply and market clearing. The aggregate supply Z_{t+1} of the risky assets at date $t+1$ is assumed to be stochastic and of the form

$$Z_{t+1} = Z + z_{t+1}, \text{ where } z_{t+1} \sim N(0, V_z), \tag{7}$$

[5] The predictions of the model should be qualitatively unaffected if investors have signals of different quality or have different risk aversion. In the DO case ($\rho = 0$), investors will not condition on the price, and the risk premium will be determined by a risk-tolerance-weighted average of the investors' conditional variances in payoffs. In the RE case ($\rho = 1$), one would have to assume a setting either with symmetric beliefs or with hierarchical beliefs to maintain tractability and avoid the infinite regress problem. Again, the risk premium would be determined by a weighted average of investors' conditional variances. Disagreement could be analogously defined as in the symmetric case, although the difference in the quality of information might lead to another source of disagreement.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

and the market clearing condition sets the aggregate demand equal to the aggregate supply, i.e.,

$$\int_i x_{i,t} di = Z_t. \tag{8}$$

The role of the supply shocks is to prevent prices from being fully revealing in the RE model, and can be replaced by other sources of noise (e.g., endowment shocks, alternative investment opportunities) without qualitatively changing the theoretical results. The assumption that the supply shocks are i.i.d. over time is important, as it ensures that prices are not mechanically predictable over time. In particular, this implies that, in the DO equilibrium (i.e., $\rho = 0$), investors have no reason to condition on prices. In contrast, if the aggregate supply was assumed to be predictable (e.g., if it followed an auto-regressive process), then investors would condition on prices even when $\rho = 0$ in order to update their beliefs about future supply shocks and payoffs. This would confound the predictions that distinguish the RE and DO equilibria, since investors would condition on prices in both cases. From an empirical perspective, while we may expect to find persistence in supply shocks at short horizons (e.g., over days or weeks), the independence assumption is not likely to be restrictive over the monthly horizon at which the predictions are tested.

2.1.4 Factor structure. While deriving analytical solutions under arbitrary correlation structures is intractable, one can derive the theoretical predictions of the model under the assumption of a factor structure. In particular, following Watanabe (2008) and Van Nieuwerburgh and Veldkamp (2009), assume that the covariance matrices V_d, V_s, and V_z have the following spectral decomposition:

$$V_d = \Gamma W_d \Gamma', \quad V_s = \Gamma W_s \Gamma', \text{ and } \quad V_z = \Gamma W_z \Gamma', \tag{9}$$

for diagonal matrices W_d, W_s, and W_z, and a common orthogonal matrix of eigenvectors Γ. This implies that the covariance matrices V_d, V_s, and V_z commute, and allows one to consider a large range of correlation structures among the shocks d_{t+1}, s_{t+1}, and z_{t+1}. Moreover, as discussed in Section 2.4, numerical results suggest that the predictions of the model hold even under more general correlation structures. Denote the single asset parameter by the lowercase letter of its matrix counterpart; thus, asset n has dividend shock variance $\upsilon_{d,n}$, supply shock variance $\upsilon_{z,n}$, and signal noise $\upsilon_{s,n}$.

The analysis focuses on the stationary, linear equilibria of this model. Suppose investor i conjectures a linear equilibrium of the form

$$P_t = AD_t + B\bar{Y}_{i,t} + Cz_t + K, \text{ where } \bar{Y}_{i,t} = \int_j Y_{i,j,t} dj. \tag{10}$$

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

Note that, if investor i exhibits RE ($\rho = 1$), she realizes that $\bar{Y}_{i,t} = \delta_{t+1}$, but if she exhibits DO ($\rho = 0$), she believes that $\bar{Y}_{i,t}$ and δ_{t+1} are independent. The market clearing condition allows us to characterize the equilibrium of this model as follows.

Lemma 1. *Suppose prices are of the form (10), and that shocks (i.e., d_{t+1}, s_{t+1}, and z_{t+1}) share a factor structure given by (9). Then, the market clearing condition (8) implies that the price coefficients in a stationary linear equilibrium of the model are given by*

$$A = ((1+r_f)I - \Lambda)^{-1}\Lambda, \quad B = \tfrac{1}{1+r_f}(A+I)V_\delta\left(V_s^{-1} + \rho V_p^{-1}\right),$$
$$C = \tfrac{1}{1+r_f}\left(\rho(A+I)V_\delta V_p^{-1}B^{-1}C - V_R\right),$$
$$K = \tfrac{1}{r_f}[(A+I)(I-\Lambda)D - V_R Z], \tag{11}$$

where $F = B^{-1}C$ solves the following matrix equation:

$$F = -\,V_s(A+I)'\Big[I + \tfrac{1}{(1+r_f)^2}\left(V_s^{-1} + \rho V_p^{-1}\right)$$
$$\times\left(V_d + F V_z F'\right)V_\delta'(V_s^{-1} + \rho V_p^{-1})'\Big], \tag{12}$$

and where $V_p = ((1-\rho^2)V_d + F V_z F')$, $V_\delta = (V_d^{-1} + V_s^{-1} + \rho^2 V_p^{-1})^{-1}$, and

$$V_R = (A+I)V_\delta(A+I)' + B V_d B' + C V_z C'. \tag{13}$$

Existence and equilibrium selection. The above equilibrium is characterized by the solution F to a sixth-order polynomial matrix equation (12). Since the equilibrium cannot be solved in closed form, characterizing the conditions for existence is difficult. However, numerical solutions suggest that stationary linear equilibria exist for reasonable parameter values. Figure 1 shows the parameter ranges over which equilibria exist for a single asset in the DO equilibrium ($\rho = 0$) and the RE equilibrium ($\rho = 1$), respectively. Increasing the risk-free rate r, decreasing autocorrelation Λ, decreasing prior variance V_d, decreasing supply noise V_z, and decreasing signal noise V_s all increase the likelihood that an equilibrium exists. The implied condition for existence imposes a restriction on how large the total risk from the supply shock noise V_z and the prior variance V_d can be. Specifically, a large V_z or V_d increases the current risk premium V_R, which in turn increases the risk premium in the previous period—rolling back, the risk premium explodes when either source of risk is too large, and this leads to the non-existence of stationary equilibria. Moreover, for a given set of parameters, when investors condition on prices, the total risk

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

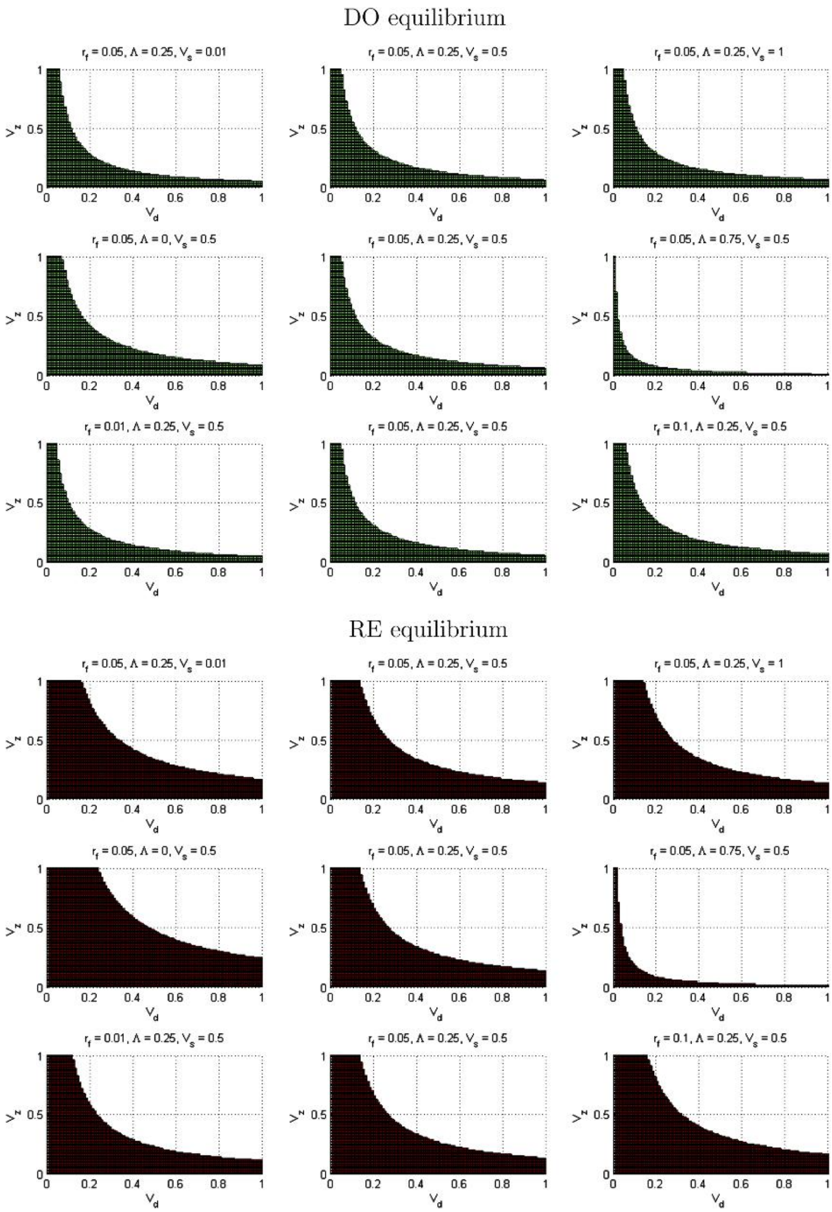

Figure 1
Existence of stationary equilibria
This figure plots parameter ranges for which stationary equilibria exist in the DO ($\rho = 0$) and RE ($\rho = 1$) equilibria. The shaded region represents the region in which an equilibrium exists. Parameter values are as in the plots. In each panel, the first row shows the effect of increasing signal noise V_s, the second row shows the effect of increasing autocorrelation in dividends Λ, and the third row shows the effect of increasing the risk-free rate r_f.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

they face decreases, and so the parameter space over which RE equilibria exist is slightly larger.

Since the model has an infinite horizon and overlapping generations, there are potentially multiple equilibria, each corresponding to a different root of F. Given the factor structure specified in (9), the matrix F has a spectral decomposition given by $F = \Gamma W_F \Gamma'$. Numerically solving the system suggests there are two real roots for each element of W_F. This is similar to the models in Spiegel (1998) and Watanabe (2008), who also show there are 2^N equilibria in an economy with N assets. These equilibria can be generally grouped into low-volatility and high-volatility equilibria based on whether one considers the less negative root or the more negative root (for each element) of W_F, respectively.

The choice of which type of equilibrium to study is somewhat arbitrary, as both types have desirable theoretical and empirical properties.[6] On the one hand, Liang (2008) argues that the high-volatility equilibrium (with the more negative roots) are more stable to perturbations in the beliefs of investors, in the sense that they can be shown to be the stationary limit of non-stationary equilibria of the model. Also, Spiegel (1998) and Watanabe (2008) demonstrate that the equilibria with the more negative roots can generate empirically relevant features of asset returns like excess volatility and correlation.

On the other hand, the low-volatility equilibria also have a number of appealing properties. First, as shown in Appendix B, the low-volatility equilibrium corresponds to the limit of the unique equilibrium of the finite horizon version of the model. Second, as Figure 2 suggests, the low-volatility equilibrium is characterized by intuitive comparative statics properties. For instance, in the low-volatility equilibrium, increasing the prior variance or the variance of supply shocks makes the equilibrium prices less informative, while the reverse is true in the high-volatility equilibrium. Finally, and most importantly, the empirical characteristics of the returns and volume in the sample appear more consistent with the low-volatility equilibrium. Given the nature of aggregate supply shocks in the model, the high-volatility equilibrium generates almost perfectly negative auto-correlation in returns and extremely high positive correlation between absolute returns and volume, neither of which is consistent with the empirical evidence.

The theoretical and empirical analysis that follows focuses on the low-volatility equilibrium. However, as discussed in Section 2.5, numerical simulations suggest that the relation between investor disagreement and return-volume characteristics can still be useful in determining whether investors condition on prices in the high-volatility equilibria.

2.2 Returns, volume, and investor disagreement

If the signal noise V_s were empirically observable, one could distinguish the RE and DO models based on the comparative statics of return and volume

6 I thank the editor for highlighting this point.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

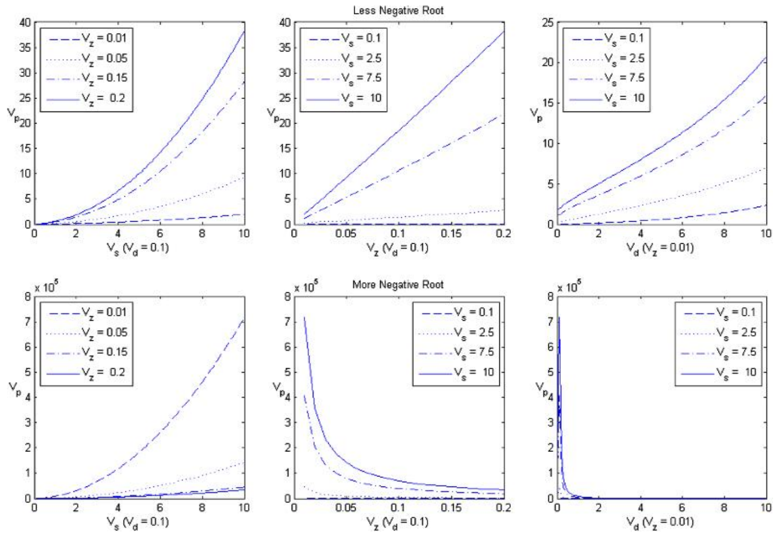

Figure 2
Comparative statics for low-volatility and high-volatility RE equilibria
The figure plots the variance in price signal V_p as a function of signal noise V_s, supply shock noise V_z, and prior variance in dividends V_d for the low- and high-volatility equilibria when $\rho = 1$. The risk-free rate r_f is set to 0.05, and autocorrelation in dividends Λ is set to 0.25. The first row shows the results for the low-volatility (less negative root) equilibrium, and the second row shows the results for the high-volatility (more negative root) equilibrium.

characteristics with respect to V_s. However, since it depends on the quality of investors' private information, V_s is usually not empirically observable. Therefore, one must derive empirical predictions based on the relation between return-volume characteristics and investor disagreement, which one could potentially proxy for empirically.

In equilibrium, investor disagreement V_μ is the cross-sectional variance in investors' posterior expectations about next period's dividends, and is given by

$$V_\mu \equiv \text{var}\left[E[D_{t+1}|\mathcal{F}_{i,t}] - \int_i E[D_{t+1}|\mathcal{F}_{i,t}]di\right] = V_\delta V_s^{-1} V_\delta'. \qquad (14)$$

Note that V_μ is the level of disagreement determined endogenously in equilibrium—it depends on the precision of the investors' private information, their prior beliefs, and the degree to which they condition on prices.

In equilibrium, the traded volume is the cross-sectional average, across investors, of the absolute change in their positions over time:

$$\mathcal{V}_{t+1} \equiv \int_i |x_{i,t+1} - x_{i,t}|di = \int_i |z_{t+1} - z_t - F^{-1}(s_{i,t+1} - s_{i,t})|di. \qquad (15)$$

Dollar returns and rates of return. Since prices and dividends are normally distributed, I follow the literature in considering dollar (or price) returns in the

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

theoretical analysis, which are defined as

$$R_{t+1} \equiv P_{t+1} + D_{t+1} - (1 + r_f)P_t. \tag{16}$$

This ensures that returns are also normally distributed, and allows me to derive analytical expressions for the distribution of returns in closed form. However, the empirical analysis of the model's predictions uses rates of return that are defined as

$$r_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t} - 1. \tag{17}$$

The rates of return are given by ratios of normally distributed random variables, which implies that analytically proving results about their distributional properties is not tractable. As discussed in Section 2.4, numerical simulations suggest that the theoretical predictions derived about dollar returns also hold when using rates of return.

Lemma 2 characterizes the return-volume moments used in the empirical predictions.

Lemma 2. *Suppose volume is of the form (15) and returns are of the form (16). Then, the moments for the return-volume distribution are given by*

1. $E[R_{t+1}] = V_R Z$
2. $var[R_{t+1}] = BV_dB' + ((A+I) - (1+r_f)B)V_d((A+I) - (1+r_f)B)' + (1 + (1+r_f)^2)CV_zC'$
3. $cov[R_{t+2}, R_{t+1}] = ((A+I) - (1+r_f)B)V_dB' - (1+r_f)CV_zC'$
4. $E[\mathcal{V}_{t+1}] = \sqrt{\frac{4}{\pi} diag(V_z + F^{-1}V_s(F^{-1})')}$
5. $cov[\mathcal{V}_{t+1}, |R_{t+1}|] = \Psi((2+r_f)diag(CV_z)) = \Psi((2+r_f)diag(-CV_z))$

where Ψ is a function, symmetric around zero, defined in Appendix A.

There are a few immediate implications for the return-volume characteristics. For instance, volume is independent of dividend shocks and the autocorrelation in volume is constant since investors are symmetrically informed and the information is short-lived. By allowing for persistent information or a hierarchical information structure, one can generate correlation between volume and dividend shocks, and non-trivial autocorrelation patterns in volume (e.g., Wang 1994), but this makes the model less tractable and the intuition behind the results less clear.

The expected returns are proportional to the perceived risk per unit of the asset, which is given by the posterior variance in returns V_R. Return volatility is positively related to the posterior variance in returns, and return autocorrelation is negatively related to it.[7] Intuitively, prices are more sensitive to supply shocks when risk is high, and this makes prices more volatile and more strongly

[7] This is especially apparent as we take the limit of the above expressions when $r_f \to 0$.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

mean-reverting. There are two sources of volume: supply shocks and informed trade. Informed trade increases in the posterior dispersion in beliefs (V_μ) but decreases in the perceived risk (V_R). As a result, volume increases in disagreement and decreases in the posterior variance in returns. Finally, note that the only common components between returns and volume are supply shocks—this is because dividend shocks do not affect the volume, and investor disagreement does not affect prices (while the average beliefs do). Hence, the higher the perceived risk V_R, the higher the sensitivity of prices to supply shocks, and the higher the covariance between absolute returns and volume.

While the distribution of the rates of return in (17) are not easy to characterize analytically, one can show that they satisfy a conditional CAPM relation, as described in Lemma 3.

Lemma 3. *Suppose the rates of return are of the form (17). Then, expected rates of return satisfy a conditional CAPM relation with respect to the information set of the average investor, i.e.,*

$$\bar{E}_t[r_{j,t+1} - r_f] = \beta_{j,t}\bar{E}_t[r_{M,t+1} - r_f], \tag{18}$$

where $r_{j,t+1}$ is the rate of return on asset j, $r_{M,t+1}$ is the rate of return on the market portfolio, and $\beta_{j,t}$ is the market beta of asset j. Moreover, $\beta_{j,t}$ is increasing in its perceived risk when the price of the market portfolio is positive.

This conditional CAPM characterization is analogous to the ones in Biais, Bossaerts, and Spatt (2010) and Van Nieuwerburgh and Veldkamp (2009), and holds under the average beliefs across investors. One can interpret this characterization as being the conditional CAPM relation in a representative investor economy in which the representative investor's beliefs coincide with the average beliefs in the original economy.[8] In the empirical section, the analysis follows Lewellen and Nagel (2006) and uses short-horizon, rolling window estimates of market beta to proxy for the conditional beta $\beta_{j,t}$.

2.3 Model predictions

The main predictions of the model that can be used to distinguish the RE and DO cases are summarized in Proposition 1.

Proposition 1. *Suppose volume is given by (15) and returns are given by (16). Consider two assets that differ only in the level of their signal noise ($\upsilon_{s,n}$).*

1. *If investors exhibit differences of opinion (i.e., $\rho = 0$), the asset with higher investor disagreement will have lower expected returns, lower return volatility, higher serial covariance in returns, and lower covariance between absolute returns and volume.*

[8] The representative investor's beliefs need not coincide with any individual investor's beliefs in the original economy. For a more detailed analysis of this issue, see Biais, Bossaerts, and Spatt (2010).

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

2. *If investors exhibit rational expectations (i.e.,* $\rho = 1$*), the asset with higher investor disagreement will have higher expected returns, higher return volatility, lower serial covariance in returns, and higher covariance between absolute returns and volume.*

In both cases, the asset with higher investor disagreement will have higher expected volume.

These results are driven by how disagreement and risk change with changes in ρ and signal noise V_s. To begin with, note that, for any given level of ρ, the equilibrium disagreement across investors first increases and then decreases in signal noise. When signals are very noisy, investors do not put too much weight on their private signals and hence do not disagree much. When signals are very precise, investors put a lot of weight on their signals but there is little dispersion in these signals. For intermediate levels of signal precision, investors put enough weight on their signals and there is enough dispersion to generate disagreement in equilibrium. Figure 3 shows an instance of this for the case of a single asset.

While the relation between disagreement and signal noise is the same for different levels of ρ, the relation between perceived risk and signal noise is different. To see why, first note that perceived risk can be decomposed into two components,

$$V_R = \text{var}[R_{t+1}|\mathcal{F}_{i,t}] = \underbrace{\text{var}[(A+I)D_{t+1}|\mathcal{F}_{i,t}]}_{\text{Dividend Risk}} + \underbrace{\text{var}[B\bar{Y}_{i,t+1} + Cz_{t+1}|\mathcal{F}_{i,t}]}_{\text{Price Risk}}. \tag{19}$$

The first component, called *dividend risk*, increases in the signal noise V_s and decreases in ρ. As in a static model, this risk is the posterior variance of the predictable component of payoffs. When the information is better (i.e., V_s is lower), or when investors are using the information in prices more efficiently (i.e., ρ is higher), the predictable dividend risk is lower. Moreover, note that, since this is the only source of risk in a static model, static DO and RE models cannot be distinguished along this dimension.

The second component of perceived risk is *price risk*. This corresponds to the variance of the unpredictable component of returns. Since investors at time t have no information about the mean signal or the supply shock at time $t+1$, the conditional variance of these terms is the same as the unconditional variance, and so can be expressed as

$$\text{var}\left[B\bar{Y}_{i,t+1} + Cz_{t+1}|\mathcal{F}_{i,t}\right] = B(\rho^2 V_d + V_p)B', \text{ where } B \propto V_\delta(V_s^{-1} + \rho V_p^{-1}). \tag{20}$$

When investors do not condition on prices much (i.e., $\rho = 0$), price risk is given by BV_pB', which decreases and then increases in signal noise. This is

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

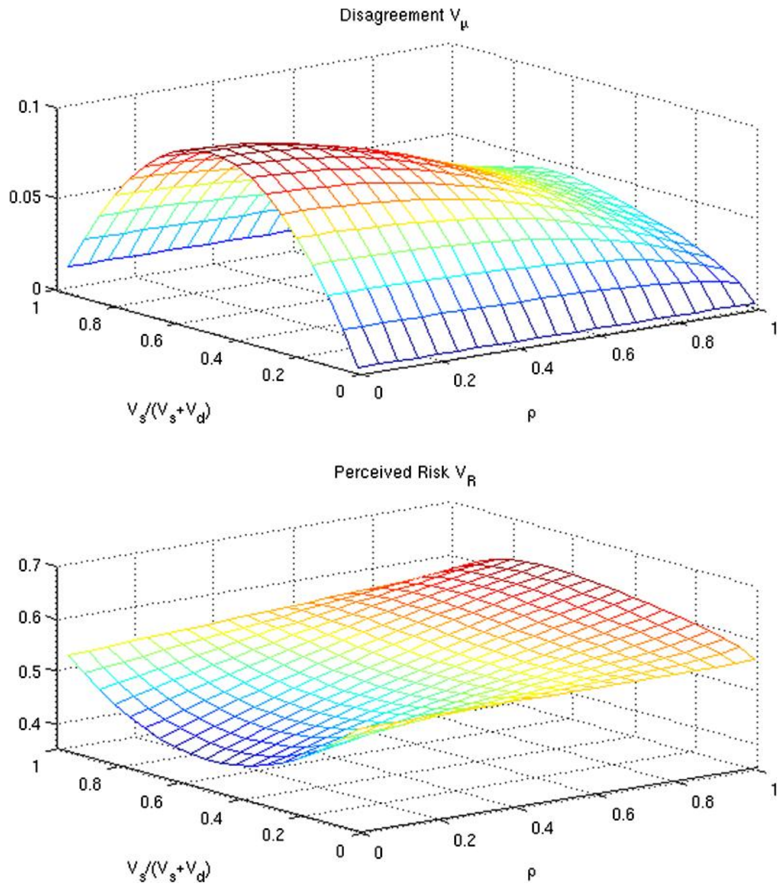

Figure 3
The relation between perceived risk and investor disagreement
The figure plots perceived risk V_R and equilibrium disagreement V_μ as a function of the signal noise ratio (i.e., $V_s/(V_s + V_d)$) and ρ for a model calibrated to match the rates of return on the market portfolio.

because an increase in signal noise has two competing effects. First, an increase in noise V_s leads to a decrease in the price sensitivity B to the aggregate signal, since for $\rho = 0$, B is linear in $V_\delta V_s^{-1}$. This is intuitive since investors trade less aggressively on their private information when signal noise is higher. Second, an increase in V_s also leads to an increase in V_p since investors have less informative signals about future fundamentals. Moreover, the first effect dominates when signal noise is low but the second effect dominates when signal noise is high.[9] As a result, when investors do not condition much on prices, price risk first decreases and then increases in signal noise.

[9] The derivative of price risk is given by $2BV_p \frac{\partial B}{\partial V_s} + B^2 \frac{\partial V_p}{\partial V_s}$. In the low-volatility equilibrium, $\frac{\partial V_p}{\partial V_s}$ is relatively small for low V_s and so the derivative is dominated by the first term for low V_s and the second term for high V_s. As we shall see in Section 2.5 , the opposite is true in the high-volatility equilibrium when $\frac{\partial V_p}{\partial V_s}$ is relatively large.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

When investors condition on prices (ρ is larger), price risk increases and then decreases with signal noise. While conditioning on prices reduces the variance of the predictable component of perceived risk, it also increases the sensitivity of prices to dividend and supply shocks. All else equal, a higher weight on the price signal when updating (i.e., higher $V_\delta V_p^{-1}$) implies a higher price sensitivity B to the aggregate signal. For a given ρ, the weight that investors put on prices increases and then decreases in signal noise. When signal noise is low, the price is very informative and acts as a substitute to the private signal. As a result, the weight investors put on prices in updating their beliefs increases in signal noise. However, when signal noise is high, prices do not contain much information either, and so the weight on prices decreases with signal noise. This implies that, when investors condition on prices a lot, price risk first increases and then decreases with signal noise. Figure 3 shows an instance of the effect of ρ on the perceived risk V_R. When ρ is small, V_R is a U-shaped function of V_s, while when ρ is large, V_R is a hump-shaped function of V_s.

The intuition for the change in the relation between disagreement and perceived risk as ρ changes is as follows. When ρ is low, investors agree to disagree more, and disagreement is largest when investors are most certain about total payoffs. As a result, perceived risk and disagreement are negatively related. As ρ increases, investors condition more on prices to infer the information of others. In this case, when disagreement is high, each investor conditions more heavily on prices since these contain information she does not have. However, since everyone conditions on prices more heavily, this makes prices more sensitive to dividend and supply shocks. This increases the volatility in prices which, in turn, increases the total perceived risk. As a result, perceived risk and disagreement are positively related.

Given the results in Lemma 2 and Proposition 1, we also have the following corollary:

Corollary 1. *Suppose volume is given by (15) and returns are given by (16). If expected volume and return volatility are negatively related, then investors exhibit differences of opinion (i.e., $\rho = 0$).*

This follows from the observation that expected volume increases in disagreement for either equilibrium, but return volatility decreases in disagreement only in the DO equilibrium. Note that the converse is not true. If investors exhibit differences of opinion, volume and return volatility need not be negatively correlated. This is because both volume and volatility increase in supply shock variance, which induces a positive correlation between the two. All else equal, Corollary 1 suggests that investors in assets with negative volume-volatility correlation are more likely to exhibit DO.

Finally, given the relation between V_μ and V_R and the result in Lemma 3, we have the following result:

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

Corollary 2. *Suppose rates of return are given by (17) and the market portfolio has a positive price, and consider two assets that differ only in the level of their signal noise ($\upsilon_{s,n}$). If investors exhibit differences of opinion (i.e., $\rho = 0$), the asset with higher investor disagreement will have lower market beta. If investors exhibit rational expectations (i.e., $\rho = 1$), the asset with higher investor disagreement will have higher market beta.*

Since the market beta of an asset is increasing in its perceived risk, the above result follows from the relation between perceived risk and disagreement in the RE and DO cases. Unlike the case of dollar returns, it is difficult to analytically derive predictions about the rates of return in the model. However, as discussed in Section 2.4, numerical simulations suggest that the predictions in Proposition 1 also hold when using rates of return.

2.4 Robustness and discussion of theoretical results

In this section, I use numerical simulations to study how robust the model's predictions are to some of the specific assumptions made for tractability in the theoretical analysis. The parameters used for these simulations are presented in Table 1 and are reasonable in the sense that they picked to match the sample moments of monthly returns for the three Fama–French factors on average.

2.4.1 Continuity in the parameter ρ. While the empirical predictions strictly rely on the extreme cases of DO ($\rho = 0$) and RE ($\rho = 1$), the model's predictions are "smooth" in ρ. Figures 3 and 4 provide numerical evidence of

Table 1
Parameter values for numerical simulations

Moment	MKT	SMB	HML
Mean (Data)	0.008838	0.0046737	0.0079522
Mean (Model)	0.008838	0.004674	0.007952
Variance (Data)	0.0019707	0.001046	0.0009499
Variance (Model)	0.001971	0.001046	0.000950
Parameters	**MTRF**	**SMB**	**HML**
V_d	0.393974	1.364135	0.341628
V_z	0.128104	0.018961	0.058051
Λ	0.128552	0.010997	0.031661
D	0.224241	0.294439	0.264939
Z	0.175857	0.018800	0.337853

The table reports the parameter values used in the numerical simulations in Figures 3 through 5, which are picked to match the monthly returns on the Fama–French three factors over the period January 1983 to December 2008. In particular, when $\rho = 0.5$ and $\frac{V_s}{V_s+V_d} = 1$ for the chosen parameter values, the model generates rates of returns whose moments match the returns of three Fama–French factors generated by adding the risk-free rate to each factor's excess return. It is useful to convert excess returns to returns in this way to calibrate the model. The correlation in the three factors is accounted for by fitting the parameters to match an orthogonal rotation of these factors, and then the simulated returns are rotated back. The (monthly) risk-free rate for the sample and the model is 0.00404.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

this for a single-asset model calibrated to match the moments of the market portfolio return. As can be seen from Figure 3, when investors believe that others' signals are not very informative (ρ is small), the correlation between risk and disagreement is negative, as in the DO model. From Figure 4, we see that, for low ρ, disagreement is negatively related to expected returns, variance in returns, and covariance between absolute returns and volume, and positively related to the autocorrelation in returns. In this case, while investors do condition on prices, the weight they put on prices is not very large. The resulting increase in the price volatility is not large enough to offset the decrease in perceived risk about dividends. However, when investor beliefs about the informativeness of others' signals is higher (i.e., ρ is large), then the weight on prices is higher, and this increases the price volatility effect. As a result, for large ρ, the correlation between risk and disagreement is positive, as in the RE model. Figure 4 shows that the predictions about the return-volume characteristics for large ρ are similar to those for the RE case. Finally, the continuity of the model's predictions in ρ appear to hold generally under a wide range of parameter specifications, including those that match the return characteristics for the size and market-to-book Fama–French factors.

2.4.2 Rates of return. As mentioned earlier, since prices and dividends are normally distributed, rates of return are given by ratios of normally distributed random variables. As a result, it is not analytically tractable to prove results about the distributional properties of rates of return and how they change with disagreement. However, one can show numerically that the empirical predictions of the model are robust to using rates of return instead of dollar returns. Figure 5 plots moments of simulated rates of return as functions of signal noise (i.e., V_s) and the degree to which investors condition on price (i.e., ρ). As can be seen, expected return, variance in returns, and correlation between absolute returns and volume are U-shaped functions of signal noise when investors exhibit DO (i.e., $\rho = 0$), but hump-shaped functions of signal noise when investors exhibit RE (i.e., $\rho = 1$). Similarly, autocorrelation in returns is hump-shaped when ρ is zero, but U-shaped when ρ is one. Finally, as equilibrium disagreement does not depend on whether dollar returns or rates of returns are used, it is a hump-shaped function of signal noise for both DO and RE. These numerical simulations suggest that the predictions of the model from the last subsection should hold even if rates of return are used instead of dollar returns.

2.4.3 Factor structure of shocks. Numerically solving the model suggests that these predictions are robust to relaxing the assumptions about the factor structure in (9) assumed about the shocks in the model (i.e., d_{t+1}, s_{t+1}, and z_{t+1}). Figure 6 plots the coefficients from regressing the difference in expected returns on the difference in levels of disagreement for each model. The negative relation between perceived risk and disagreement in the DO model and the positive relation in the RE model persists even when dividend shocks,

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

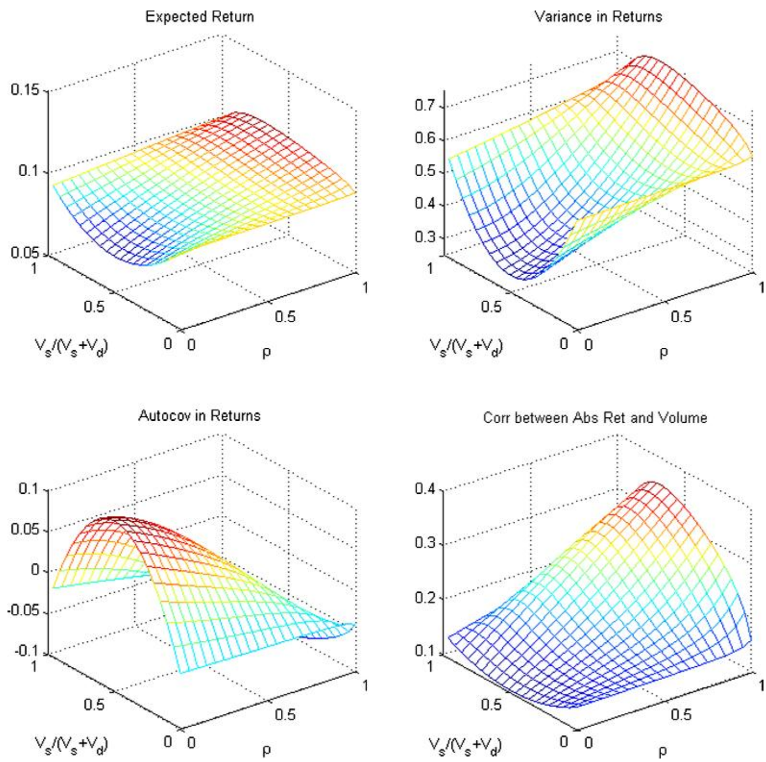

Figure 4
Model implied return-volume characteristics using dollar returns
The figure plots model implied disagreement and return-volume characteristics (i.e., average returns, variance in returns, return autocovariance, and covariance between absolute returns and volume) as a function of the signal noise ratio (i.e., $V_s/(V_s+V_d)$) and ρ for a model calibrated to match the rates of return on the market portfolio.

supply shocks, and shocks to private signals exhibit a correlation structure that is more general than those permitted by (9). Hence, while analytical proofs for these relations are difficult to obtain, the model's predictions appear robust to allowing for correlation across assets.

2.4.4 Normal-exponential model with OLG.

The assumptions of overlapping generations and normal payoffs–exponential preferences are made for tractability. The normal-exponential framework lets me consider a tractable model in which investors can condition on prices, and the OLG assumption allows me to introduce dynamics without making the model overly complicated. For instance, while the finite horizon model in Appendix B also generates the same empirical predictions, the recursive nature of the price coefficients makes it difficult to prove the comparative statics results analytically. Similarly, in fully dynamic models (e.g., Wang 1994; He and Wang 1995), the effects of

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

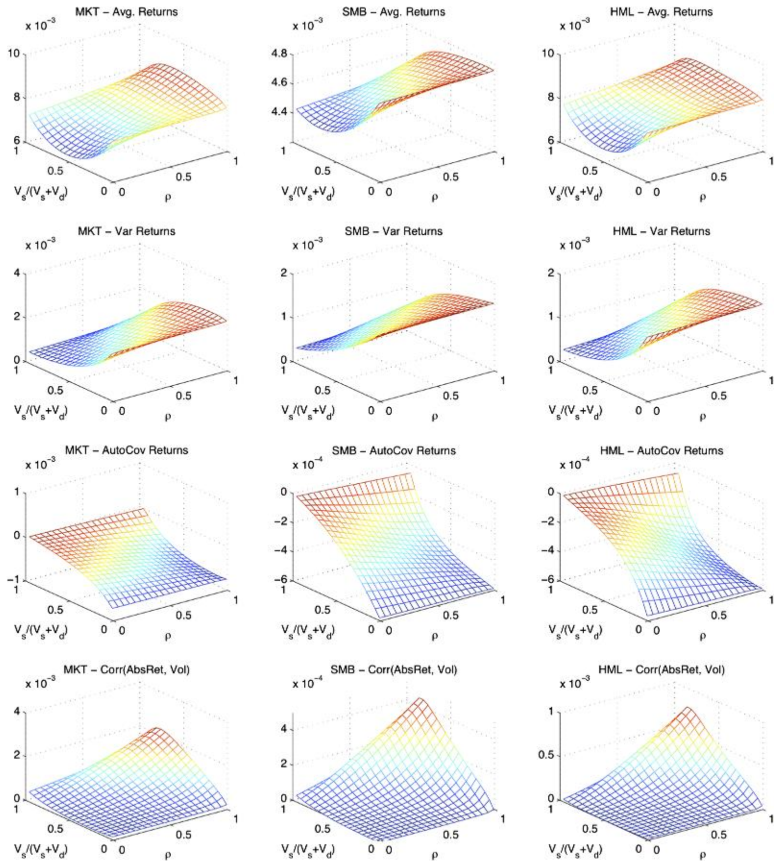

Figure 5
Model implied return-volume characteristics using rates of return
The figure plots average rates of return, variance in returns, return autocorrelation, and covariance between absolute returns and volume as a function of the signal noise ratio (i.e., $V_S/(V_S+V_d)$) and ρ for models calibrated to match the rates of return on the Fama–French factors (i.e., *MKT*, *SMB*, and *HML*).

conditioning on prices are confounded by other model features like hedging demands and asymmetric updating due to hierarchical beliefs. Moreover, the empirical predictions used to test the model do not rely on the "fit" of the model, but on comparative statics results derived above, which are potentially robust to the specific functional form assumptions.

2.4.5 Asymmetric information/beliefs. An alternative approach to nesting the RE and DO models would be in an economy in which some investors condition on prices efficiently while others do not condition on prices at all. In such a specification, the fraction of RE investors would be the parameter analogous to ρ. While this alternative model produces qualitatively similar predictions to

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

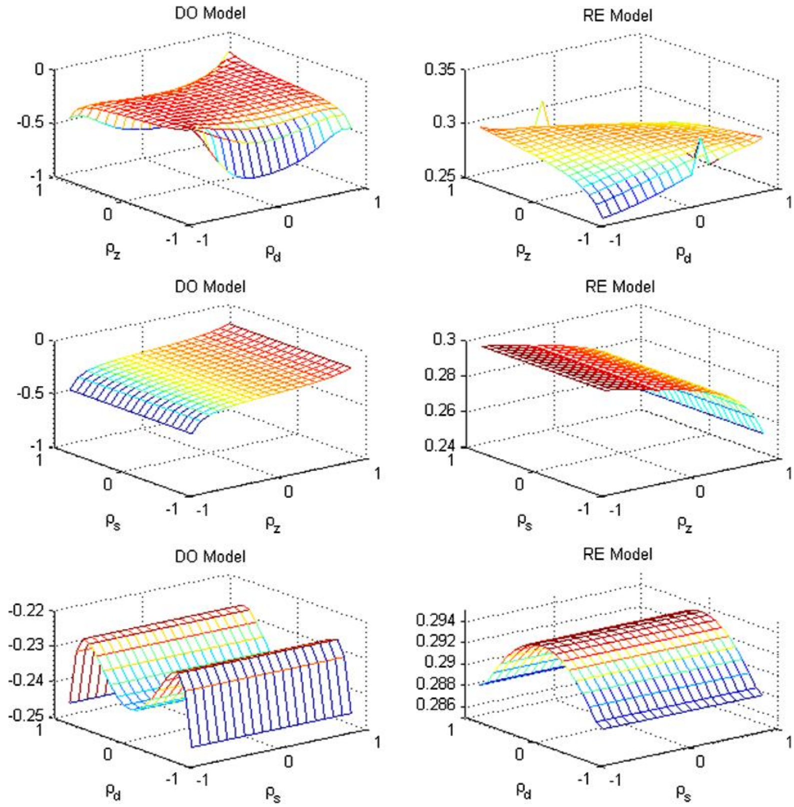

Figure 6
Model predictions with general correlation structures
The figure plots the regression coefficients of the difference in expected returns on the difference in disagreement in the 2-risky asset DO ($\rho = 0$) and RE ($\rho = 1$) equilibria as a function of the correlation in dividend shocks (ρ_d), supply shocks (ρ_z), and private signals (ρ_s). When not plotted, these correlations are set to 0. Other model parameters are as follows: $D = (1, 1)'$, $Z = (1, 1)'$, $\Lambda_1 = \Lambda_2 = 0$, $\upsilon_{d,n} = 0.1$, $\upsilon_{z,n} = 0.01$, and the ratio of the $\upsilon_{s,n}$'s of one asset to the other varies from 0.1 to 10.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

those of the current model, the lack of symmetry in investors' beliefs makes the analysis less tractable.

The underlying features of the model that drive the empirical predictions are (i) the non-monotonic relation between signal noise and dispersion in beliefs; and (ii) the trade-off between lower dividend risk and higher future price volatility when investors condition on prices to update their beliefs. Models with alternative specifications for preferences of information structures (e.g., investors with hierarchical information, models with naïve investors who do not condition on prices, and sophisticated investors who do) that also generate these effects will have similar empirical predictions. However, the predictions of the current model, in which disagreement arises endogenously as a result of heterogeneous information, may be different than those of a model in which the disagreement is modeled essentially as an exogenous process (e.g., Detemple and Murthy 1994; Zapatero 1998; Basak 2000; Buraschi and Jiltsov 2006). Moreover, while this alternate class of models offers many theoretical and empirical advantages, it is not easily nested in a model in which investors condition on prices to update their beliefs.[10]

2.5 High-volatility equilibria

As discussed in Section 2.1, since the model has an infinite horizon and overlapping generations, it admits multiple equilibria. Recall that the high-volatility equilibrium is characterized by the more negative root F of Equation (12) and corresponds to more noisy prices (i.e., higher V_p). As in Spiegel (1998), Liang (2008), and Watanabe (2008), these equilibria correspond to self-fulfilling beliefs in which investors perceive prices to be noisy and hence perceive risk to be high. Investors demand larger compensation for holding the risky asset, which implies that prices are more sensitive to supply shocks and, therefore, more noisy.

While the empirical predictions of the model described in Section 2.3 are derived in the low-volatility equilibria, numerical simulations suggest that the relation between disagreement and return-volume characteristics can also be useful in the high-volatility equilibrium to determine whether investors condition on prices. For instance, Figure 7 plots the high-volatility equilibrium of a single-asset model for which the parameters are chosen to match the mean and variance of the market return. The numerical analysis suggests that the empirical predictions are reversed for the high-volatility equilibrium. In particular, when investors do not condition on prices (i.e., $\rho = 0$), disagreement is positively related to expected returns, variance in returns, and the covariance between absolute returns and volume, but negatively related to autocorrelation in returns. When investors do condition on prices (i.e., $\rho = 1$), these relations have the opposite signs.

As in the low-volatility equilibrium, Lemma 2 holds in the high-volatility equilibrium and the empirical predictions are driven by the relation between disagreement and risk. Specifically, the above predictions follow from the positive relation between disagreement and perceived risk in the high-volatility DO equilibrium and the negative relation between disagreement between risk in the high-volatility RE equilibrium. These results, in turn, follow from how disagreement and risk change with changes in signal noise V_s. In both the high-volatility DO and RE equilibria, disagreement first increases and then decreases with signal noise. As in the low-volatility equilibria, there is low disagreement when signals are very noisy (investors put little weight on their signals) or very precise (there is little dispersion in the signals).

[10] Theoretically, pricing in these heterogeneous belief models can be represented with a stochastic discount factor. This makes pricing of general classes of securities feasible. Empirically, these models have been useful in fitting return and volume characteristics of aggregate data.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

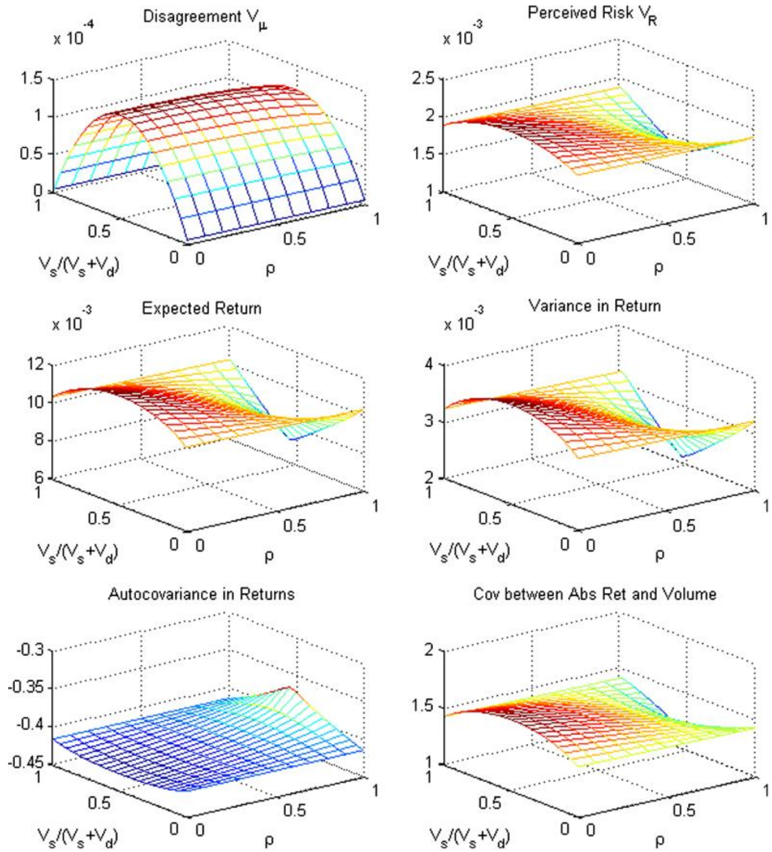

Figure 7
Model predictions in high-volatility equilibrium
The figure plots average rates of return, variance in returns, return autocovariance, and covariance between absolute returns and volume as a function of the signal noise ratio (i.e., $V_s/(V_s + V_d)$) and ρ in the high-volatility equilibrium. The model's parameters are selected to match the mean and variance of the market return. In particular, the parameters are set to $r_f = 0.004$, $V_d = 0.000548$, $V_z = 381.03$, $\Lambda = 0.0001$, $D = 355.68$, and $Z = 5.5$. The mean and variance in market returns in the data are 0.00883 and 0.00197, respectively. The mean and variance in returns from the model are 0.00873 and 0.00264, respectively.

However, perceived risk behaves differently as a function of signal noise in the high-volatility DO and RE equilibria. As in Equation (19), perceived risk can be decomposed into two components: *dividend risk*, which is the posterior variance of the predictable component of payoffs, and *price risk*, which is the variance of the unpredictable component. Moreover, as in the low-volatility equilibria, dividend risk in both the RE and DO high-volatility equilibria are decreasing in signal noise, since noisier private signals lead to more uncertainty about next period's dividends.

In contrast to the low-volatility equilibrium, price risk first increases and then decreases with signal noise in a high-volatility DO equilibrium.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

An increase in signal noise leads to more uncertainty about future fundamentals (which increases price risk), but leads investors to trade less aggressively (which makes prices less sensitive to fundamentals and hence decreases price risk). However, in a high-volatility equilibrium, since investors perceive risk to be high, they do not trade very aggressively even when private signals are very precise.[11] This implies that, when signals are precise, the first effect dominates the second, and price risk increases with signal noise. On the other hand, when signal noise is high, uncertainty about future fundamentals is already high, and so the second effect dominates—price risk decreases with signal noise. As a result, price risk first increases and then decreases with signal noise in the high-volatility DO equilibrium.

In the high-volatility RE equilibrium, price risk first decreases and then increases in signal noise. When private signals are very precise, an increase in signal noise leads investors to condition more on prices. However, in a high-volatility equilibrium, since investors perceive prices to be noisy and risk to be high, they do not condition very heavily on prices or trade very aggressively. As a result, the decrease in uncertainty due to conditioning on prices offsets the increased sensitivity of prices to fundamental shocks (as a result of more aggressive trading), and price risk decreases in signal noise when private signals are precise. When signal noise is high, prices are also uninformative and investors do not condition on them very heavily; hence, price risk increases with signal noise. This implies that price risk decreases and then increases with signal noise in the high-volatility RE equilibrium.

The predictions in Section 2.3 and the empirical analysis that follows focus on the low-volatility equilibrium. As mentioned in Section 2.1, this is partly because the empirical distribution of returns and volume in the sample appear more consistent with the low-volatility equilibrium. The numerical analysis in this section suggests that similar empirical analysis may also be useful in determining how investors condition on prices during periods of high volatility and uncertainty (e.g., during crises and periods of contagion). However, a complete theoretical and empirical analysis of the high-volatility equilibria is beyond the scope of this article and is left for future work.

3. Empirical Analysis

In order to test the predictions of the model, one needs to empirically proxy for disagreement. The dispersion in analyst forecasts is both intuitively appealing and popular in the literature as a proxy for investor disagreement (e.g., Diether, Malloy, and Scherbina 2002). However, some caveats must be kept in mind when using this variable. First, since analyst forecast dispersion is in

[11] Recall that the derivative of price risk is given by $2BV_P\frac{\partial B}{\partial V_s}+B^2\frac{\partial V_P}{\partial V_s}$, and $B\in[0,1]$. In the high-volatility equilibrium, $\frac{\partial V_P}{\partial V_s}$ is relatively large for low V_s and so the derivative of price risk is dominated by the second term for low V_s (when B is large) and the first term for high V_s (when B is small).

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

terms of dollars per share, one must control for a measure of size per share to avoid a mechanical relation between size and dispersion (firms that have higher earnings per share have higher dispersion). While the literature suggests a number of variables that can be used to scale analyst dispersion (e.g., absolute lagged earnings, lagged price, absolute mean earnings forecast), these variables potentially have a strong effect on the measure of dispersion and often lead to opposite implications for the joint distribution of returns, volume, and dispersion.[12] To avoid favoring one model over the other, I use the unadjusted analyst forecast dispersion (denoted by *AFD*), dispersion scaled by absolute median forecast (denoted by *AFD*/*ME*), and dispersion scaled by lagged price (denoted by *AFD*/*P*) as proxies for disagreement.

In addition to being a proxy for disagreement, analyst forecast dispersion may also reflect uncertainty about earnings (e.g., Barron, Kim, Lim, and Stevens 1998; Doukas, Kim, and Pantzalis 2006). Moreover, there is evidence of biases in analyst forecasts, including overoptimism (e.g., Hong and Stein 2003), underreaction to public information and overconfidence in private information (e.g., Abarbanell and Bernard 1992), and herding and anti-herding of forecasts (e.g., Hong, Kubik, and Solomon 2000; Bernhardt, Campello, and Kutsoati 2006). This potentially raises concerns about the use of analyst forecast dispersion as a proxy for investor disagreement. To alleviate some of these concerns, I also use trading volume (as measured by average turnover) as another proxy for disagreement. While analyst forecast dispersion and trading volume are both noisy proxies for investor disagreement, they do not suffer from the same criticisms. Hence, the evidence based on all the proxies, especially if consistent across all of them, is arguably more convincing than relying on each proxy individually.

Finally, even though the model's predictions are based on monotonic relations between disagreement and return-volume characteristics, these relations are extremely nonlinear. To ensure that the empirical analysis is robust to misspecification, both portfolio sorts and Fama–MacBeth regressions are used to study how return-volume characteristics change with disagreement. The empirical evidence is consistent across the two approaches and so is more convincing taken together.

3.1 Data selection and summary statistics

Daily return and volume data from CRSP are used to compute monthly estimates of return and volume characteristics for firms. As in the previous literature, log turnover is used as a measure of volume and realized variance is used as a measure for return volatility. Dimson (1979) betas are calculated using

[12] For instance, Diether, Malloy, and Scherbina (2002) use absolute mean earnings estimate and find a negative relation between expected returns and disagreement. In contrast, Qu, Starks, and Yan (2004) use lagged price and find a positive relation.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

daily returns to compute monthly estimates of market beta for each stock.[13] The disagreement proxies are based on the dispersion in analyst forecasts of annual earnings per share from IBES.[14] Firm-specific data used to compute book value of assets and the debt to equity ratio are from Compustat. In order to have enough firms in each cross-section, and to make the results more comparable with the existing literature, the sample covers January 1983 to September 2007. Observations in which the share price is lower than \$5 or the number of analysts is lower than 2 are excluded. Observations in which the scaling variable is zero are also dropped.

Table 2 reports the summary statistics. There are 297 months of data, and the average number of firms per cross-section is over 2,000. There is a selection bias in the sample, as it is restricted to the IBES firms with at least two analysts following them. This subset of firms is large and has average monthly returns of about 1.7 percent. All the measures of belief dispersion appear to be left skewed, which indicates that the empirical results are not dominated by the high-disagreement firms. Finally, Table 2 also provides the pairwise correlations between the disagreement proxies and market value of equity (*Size*) and market-to-book ratio. With the exception of *AFD* and *AFD*/*P*, the dispersion proxies are not strongly correlated with each other. None of the proxies are strongly correlated with size, but *VOL* is positively correlated with the market-to-book ratio.

3.2 Return-volume characteristics across dispersion measures

Proposition 1 and Corollary 2 predict that, when investors exhibit rational expectations, higher disagreement is associated with higher expected returns, higher volatility, higher market beta, lower autocorrelation in returns, and higher correlation between absolute returns and volume. When investors do not condition on prices, and exhibit differences of opinion, the reverse is true. These predictions are tested using two approaches: portfolio sorts and Fama–MacBeth regressions.

3.2.1 Portfolio sorts. Each month, firms are sorted into quintiles based on proxies of belief dispersion and average return-volume characteristics are calculated for each quintile. Table 3 reports the time-series averages of the return-volume characteristics for each quintile to see how they change on average

---

[13] As in Lewellen and Nagel (2006), the beta coefficient on lags 2–4 are constrained to be the same in order to reduce the number of estimated parameters. Hence, the beta estimate at time t on stock i is $\beta_{i,t} = b_{i,0} + b_{i,1} + b_{i,2}$, where

$$r_{i,t} - r_{f,t} = b_{i,j,0} r_{M,t} + b_{i,j,1} r_{M,t-1} + b_{i,j,2}[(r_{M,t-2} + r_{M,t-3} + r_{M,t-4})/3] + \varepsilon_{i,t}.$$

[14] The summary file for the data unadjusted for stock splits, now available through WRDS, is used to avoid the rounding error documented by Diether, Malloy, and Scherbina (2002) and others. The use of annual earnings forecasts is to maximize the sample size.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

Table 2
Summary statistics

	Mean	Median	5%	95%	Std. Dev.	*N* obs
Avg. Returns	0.017	0.015	−0.188	0.229	0.135	712137
Realized Variance	0.020	0.010	0.002	0.072	0.046	712137
Log Turnover (*VOL*)	1.494	1.381	0.459	2.872	0.739	712137
C(Abs Ret, Vol)	0.292	0.305	−0.135	0.666	0.241	712133
Autocorr in Returns	−0.002	−0.002	−0.360	0.359	0.215	711265
Market Beta	1.094	1.011	−1.747	4.324	2.111	712137
AFD	0.653	0.040	0	0.409	32.768	712137
AFD/ME	0.114	0.036	0	0.340	0.741	648375
AFD/P	0.060	0.002	0	0.027	3.922	712136
Size	2958	498	51	10438	13137	712136
Market / Book	2.006	1.391	0.889	5.117	2.360	706744
Debt / Equity	1.664	0.234	0	2.609	42.582	662252
Earnings / Share (EPS)	1.257	0.824	−1.328	3.336	314.833	709901
Coverage	8.630	5.917	2	24.189	7.288	712137
Std. Dev. EPS	1.428	0.149	0	1.246	87.684	693169

	AFD	*AFD/ME*	*AFD/P*	*VOL*
AFD/ME	0.014			
AFD/P	0.915	0.012		
VOL	−0.006	0.012	−0.006	
Size	0.001	−0.019	−0.003	0.019
Market / Book	0.005	−0.012	0.004	0.229

The table shows the summary statistics of the variables. The return and volume statistics are based on monthly aggregates of daily returns and volumes. The table reports summary statistics for average returns, realized variance in returns, average log turnover (*VOL*), analyst forecast dispersion (AFD), analyst forecast dispersion scaled by median estimate (*AFD/ME*), analyst forecast dispersion scaled by lagged price (*AFD/P*), market value of equity in millions (Size), debt-to-equity ratio, market-to-book ratio, earnings per share (EPS), analyst coverage, and standard deviation in EPS (calculated using the last 8 quarters). The data are from January 1983 to September 2007.

across quintiles. In Table 4, the same analysis is repeated but with return-volume characteristics that are adjusted for size and market-to-book ratios.[15] Specifically, stocks are sorted into 5 × 5 portfolios based on size and market-to-book each month and average return-volume characteristics are calculated for each of these 25 portfolios. For each stock-month observation, the return-volume characteristic is adjusted by subtracting the relevant portfolio average for that month. Then, stocks are sorted into quintiles based on proxies of belief dispersion and the time-series average characteristics for each quintile are reported in Table 4.

Across both Tables 3 and 4, the evidence is more consistent with investors exhibiting RE than with investors exhibiting DO. Moreover, while the estimates across the two tables are similar in sign and magnitude, they appear to be more statistically significant for the size and M/B adjusted specification in Table 4. Across all four proxies, return volatility (as measured by realized variance), market beta, and the correlation between absolute returns and volume increase with disagreement, and the difference between the lowest and highest quintiles is statistically significant. The difference in return

15 I thank the referee for suggesting this exercise.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

Table 3
Return and volume characteristics by disagreement quintiles

Avg. Returns	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5-Q1	t-stat
AFD	0.0234	0.0177	0.0155	0.0139	0.0114	−0.0120	−2.62
AFD/ME	0.0188	0.0149	0.0154	0.0151	0.0148	−0.0040	−0.93
AFD/P	0.0202	0.0151	0.0154	0.0145	0.0175	−0.0028	−0.59
VOL	0.0087	0.0123	0.0155	0.0190	0.0272	0.0185	3.38
Real. Variance	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5-Q1	t-stat
AFD	0.0189	0.0171	0.0172	0.0179	0.0217	0.0027	2.35
AFD/ME	0.0140	0.0128	0.0152	0.0177	0.0222	0.0083	9.07
AFD/P	0.0159	0.0152	0.0167	0.0188	0.0263	0.0104	8.50
VOL	0.0112	0.0120	0.0145	0.0200	0.0353	0.0241	14.35
Market Beta	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5-Q1	t-stat
AFD	1.0493	1.0398	1.0758	1.0905	1.1946	0.1454	10.99
AFD/ME	0.9882	1.0057	1.0595	1.1069	1.1443	0.1561	12.26
AFD/P	1.0280	1.0305	1.0727	1.1054	1.2149	0.1869	13.47
VOL	0.7972	0.9849	1.0939	1.2180	1.3573	0.5601	30.97
Auto Corr in Rets	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5-Q1	t-stat
AFD	−0.0022	−0.0019	−0.0018	−0.0011	−0.0001	0.0021	0.84
AFD/ME	−0.0015	−0.0008	−0.0017	−0.0005	−0.0017	−0.0003	−0.10
AFD/P	−0.0025	−0.0008	−0.0021	−0.0008	−0.0006	0.0019	0.74
VOL	0.0009	0.0008	−0.0016	−0.0022	−0.0047	−0.0056	−2.10
Corr AbsRet & Vol	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5-Q1	t-stat
AFD	0.2833	0.2819	0.2794	0.2857	0.3063	0.0230	7.12
AFD/ME	0.2732	0.2719	0.2824	0.2912	0.3000	0.0268	7.80
AFD/P	0.2906	0.2790	0.2780	0.2850	0.3037	0.0131	3.78
VOL	0.2192	0.2388	0.2741	0.3180	0.3862	0.1669	43.64

The table reports the time-series average of return-volume characteristics of portfolios formed by sorting stocks into quintiles based on disagreement proxies every month. The differences between the averages for the highest and lowest quintiles (and the t-stat) are also reported. The proxies for disagreement are analyst forecast dispersion (AFD), forecast dispersion scaled by absolute median estimate (*AFD/ME*), forecast dispersion scaled by lagged price (*AFD/P*), and average volume (*VOL*).

autocorrelation across high- and low-disagreement firms is only statistically significant for the *VOL* proxy, and in this case, return autocorrelation decreases with disagreement.

However, the evidence on the relation between average returns and disagreement is more mixed. Average returns decrease with disagreement for *AFD* but increase with disagreement for *VOL* (the relation is not statistically significant for the other two proxies). This mixed evidence with respect to average returns is all the more interesting given the unambiguously positive relation between market beta and disagreement.

To better understand this apparent discrepancy in the relation between average returns and disagreement across the proxies, Table 5 presents results from double sorts on disagreement and size, and disagreement and market-to-book, respectively. For each disagreement proxy, I calculate the average return for size/market-to-book and disagreement quintiles for each month, and then report the time-series difference between high- and low-disagreement quintiles

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

Table 4
Adjusted return and volume characteristics by disagreement quintiles

Avg. Returns	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5-Q1	t-stat
AFD	0.00159	−0.00018	−0.00006	−0.00024	−0.00178	−0.00337	−3.35
AFD/ME	0.00085	−0.00047	0.00002	0.00062	0.00088	0.00003	0.03
AFD/P	0.00037	−0.00168	−0.00031	−0.00026	0.00190	0.00153	1.34
VOL	−0.00527	−0.00177	0.00041	0.00156	0.00507	0.01033	4.97
Real. Variance	**Q1 (low)**	**Q2**	**Q3**	**Q4**	**Q5 (high)**	**Q5-Q1**	**t-stat**
AFD	−0.00234	−0.00158	−0.00061	0.00058	0.00403	0.00638	22.75
AFD/ME	−0.00432	−0.00376	−0.00213	−0.00021	0.00328	0.00760	26.75
AFD/P	−0.00322	−0.00256	−0.00102	0.00064	0.00615	0.00937	30.07
VOL	−0.00654	−0.00496	−0.00283	0.00085	0.01349	0.02003	26.99
Market Beta	**Q1 (low)**	**Q2**	**Q3**	**Q4**	**Q5 (high)**	**Q5-Q1**	**t-stat**
AFD	−0.05974	−0.05449	−0.00917	0.00969	0.11324	0.17299	16.37
AFD/ME	−0.10360	−0.07909	−0.02851	0.02392	0.06750	0.17110	16.03
AFD/P	−0.07543	−0.06708	−0.01570	0.02349	0.13473	0.21016	19.45
VOL	−0.25393	−0.08622	0.00689	0.11070	0.22234	0.47627	34.22
Auto Corr in Rets	**Q1 (low)**	**Q2**	**Q3**	**Q4**	**Q5 (high)**	**Q5-Q1**	**t-stat**
AFD	−0.00046	−0.00024	−0.00049	−0.00005	0.00089	0.00135	1.44
AFD/ME	−0.00008	0.00049	−0.00024	0.00064	−0.00051	−0.00043	−0.44
AFD/P	−0.00057	0.00072	−0.00083	0.00017	0.00051	0.00108	1.21
VOL	0.00211	0.00179	−0.00002	−0.00087	−0.00301	−0.00512	−4.37
Corr AbsRet & Vol	**Q1 (low)**	**Q2**	**Q3**	**Q4**	**Q5 (high)**	**Q5-Q1**	**t-stat**
AFD	−0.01148	−0.00949	−0.00680	0.00292	0.02562	0.03710	36.07
AFD/ME	−0.01992	−0.01883	−0.00548	0.00785	0.02269	0.04261	37.41
AFD/P	−0.01201	−0.01329	−0.00755	0.00497	0.02788	0.03989	39.59
VOL	−0.05175	−0.04337	−0.01384	0.02534	0.08361	0.13536	74.13

The table reports the time-series average of adjusted return-volume characteristics of portfolios formed by sorting stocks into quintiles based on disagreement proxies every month. The differences between the averages for the highest and lowest quintiles (and the t-stat) are also reported. The proxies for disagreement are analyst forecast dispersion (AFD), forecast dispersion scaled by absolute median estimate (*AFD/ME*), forecast dispersion scaled by lagged price (*AFD/P*), and average volume (*VOL*). Each month, stocks are first sorted into 5 × 5 portfolios based on market-to-book and size, and the equal-weighted average characteristics for each portfolio are calculated. These average characteristics are subtracted from the return-volume characteristics of each stock to yield the adjusted return-volume characteristics that are reported in the table.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

for each size/market-to-book quintile. This provides a summary of how the relations between disagreement and average returns change as we move from small firms to big firms and from value firms to growth firms.

The relation between disagreement and average returns is consistent with the evidence in Tables 3 and 4 when looking across size portfolios. Average returns are positively (and statistically significantly) related to disagreement as measured by *VOL*, but negatively related to *AFD*, *AFD/ME*, and *AFD/P*. However, when sorting on market-to-book, average returns appear to be positively related to disagreement in the highest M/B quintile (except for *AFD*, which is not statistically significant), but not significantly related to disagreement in the lowest M/B quintile. In other words, higher disagreement is associated with higher returns for growth stocks but there is no statistically significant relation between disagreement and returns for value stocks. Since *VOL* is positively correlated

Table 5
Difference in average returns between high- and low-disagreement stocks across size and M/B quintiles

Size Quintiles	Q1 (small)	Q2	Q3	Q4	Q5 (big)
AFD	−0.00834	−0.01222	−0.01441	−0.01337	−0.01
t-stat	−1.65	−2.30	−2.90	−10.84	−2.44
AFD/ME	−0.00345	−0.0088	−0.00637	−0.00545	−0.00291
t-stat	−0.73	−1.83	−1.37	−5.66	−0.72
AFD/P	−0.00062	−0.00852	−0.01144	−0.00954	−0.00668
t-stat	−0.12	−1.60	−2.24	−7.13	−1.51
VOL	0.05438	0.0237	0.01202	0.00691	0.00596
t-stat	8.83	3.85	2.15	4.25	1.19
M/B Quintiles	**Q1 (value)**	**Q2**	**Q3**	**Q4**	**Q5 (growth)**
AFD	−0.0055	−0.00519	−0.00168	−0.00363	−0.00214
t-stat	−1.20	−1.32	−0.38	−0.69	−0.32
AFD/ME	−0.00365	−0.00358	0.00305	0.00687	0.01247
t-stat	−0.84	−0.91	0.70	1.37	2.02
AFD/P	−0.00279	0.00101	0.00919	0.01274	0.01653
t-stat	−0.62	0.24	1.95	2.32	2.40
VOL	−0.00477	−0.00243	0.00211	0.0125	0.03097
t-stat	−0.86	−0.48	0.39	2.21	4.92

The table reports the difference in the time-series average return between high- and low-disagreement portfolios across size and market-to-book quintiles. The returns are formed by sorting stocks into quintiles based on disagreement and size/market-to-book ratios and calculating the average return every month. The proxies for disagreement are analyst forecast dispersion (AFD), forecast dispersion scaled by absolute median estimate (*AFD/ME*), forecast dispersion scaled by lagged price (*AFD/P*), and average volume (*VOL*).

with market-to-book (see Table 2) while the analyst forecast dispersion proxies are not, this helps explain the apparently conflicting evidence about the return-disagreement relation from Table 3. Moreover, it suggests that investors in value firms may condition on prices differently compared to those in growth firms.

3.2.2 Fama–MacBeth regressions. The above analysis is complemented with Fama–MacBeth regressions, which serve as a robustness check to the analysis using portfolio sorts. Each month, the following cross-sectional regression is run for each return-volume characteristic as the dependent variable and for each disagreement proxy:

$$\text{Dep Var}_{i,t} = a_0 + a_1 \text{ disagree}_{i,t} + a_2 \text{ size}_{i,t} + a_3 \text{ market/book}_{i,t} + a_4 \text{ ann month}_{i,t} + \varepsilon_{i,t}. \qquad (21)$$

The control variables are (log) market size, the market to book ratio, and an indicator variable for whether the current month is a month in which earnings are announced.[16]

The results in Table 6 are generally consistent with the evidence in Tables 3 and 4. The disagreement coefficients for realized variance, market beta, and

16 I thank the referee for suggesting a control for the announcement month.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

Table 6
Fama–MacBeth regressions of return-volume characteristics on disagreement

AFD	Adj. R^2	disp	t-stat	size	t-stat	M/B	t-stat	ann	t-stat
Avg. Returns	0.034	−0.001	−1.20	−0.004	−6.03	0.007	12.49	0.002	1.31
Real. Variance	0.117	0.000	1.74	−0.004	−21.39	0.003	14.46	0.004	7.50
Market Beta	0.007	0.032	1.66	−0.006	−1.45	0.031	5.92	0.007	0.30
Corr AbsRet & Vol	0.036	0.000	0.40	−0.003	−5.86	0.008	12.49	0.042	11.06
Auto Corr in Rets	0.009	0.001	0.66	0.000	−0.05	−0.001	−2.19	−0.005	−2.12
AFD/ME	Adj. R^2	disp	t-stat	size	t-stat	M/B	t-stat	ann	t-stat
Avg. Returns	0.007	0.032	1.66	−0.006	−1.45	0.031	5.92	0.002	1.40
Real. Variance	0.126	0.003	12.76	−0.003	−21.50	0.002	13.81	0.004	7.13
Market Beta	0.007	0.051	3.93	0.002	0.46	0.029	5.34	−0.017	−0.82
Corr AbsRet & Vol	0.024	0.017	13.01	0.011	12.85	0.016	16.89	0.042	10.92
Auto Corr in Rets	0.009	−0.001	−0.74	0.000	−0.02	−0.001	−1.97	−0.006	−2.36
AFD/P	Adj. R^2	disp	t-stat	size	t-stat	M/B	t-stat	ann	t-stat
Avg. Returns	0.035	0.001	0.24	−0.004	−6.03	0.007	12.51	0.002	1.35
Real. Variance	0.118	0.010	2.79	−0.004	−21.22	0.003	14.45	0.004	7.50
Market Beta	0.007	0.571	1.85	−0.005	−1.31	0.031	5.90	0.006	0.28
Corr AbsRet & Vol	0.022	0.063	2.74	0.010	11.58	0.014	16.43	0.042	11.04
Auto Corr in Rets	0.009	0.011	0.47	0.000	−0.05	−0.001	−2.19	−0.005	−2.14
VOL	Adj. R^2	disp	t-stat	size	t-stat	M/B	t-stat	ann	t-stat
Avg. Returns	0.059	0.006	2.97	−0.003	−5.71	0.007	13.20	0.002	1.21
Real. Variance	0.233	0.014	12.20	−0.005	−19.97	0.001	11.55	0.003	6.74
Market Beta	0.023	0.300	18.68	−0.013	−3.42	0.003	0.79	−0.015	−0.66
Corr AbsRet & Vol	0.082	0.091	48.37	0.007	8.90	0.006	10.42	0.035	9.58
Auto Corr in Rets	0.010	−0.004	−3.61	0.000	0.04	−0.001	−1.63	−0.005	−2.05

The table shows the coefficients from Fama–MacBeth regressions of return and volume characteristics on the four disagreement proxies and three control variables. The proxies for disagreement are analyst forecast dispersion (AFD), forecast dispersion scaled by absolute median estimate (*AFD/ME*), forecast dispersion scaled by lagged price (*AFD/P*), and average volume (*VOL*). The three control variables are log size, market-to-book ratio, and an indicator variable for whether the current month is the month in which earnings are announced (ann). Adjusted R^2's and Newey–West standard errors with three lags are reported.

correlation between absolute returns and volume are positive for all disagreement proxies, and often statistically significant. As in the case with portfolio sorts, the coefficient on return autocorrelation is negative and statistically significant for the *VOL* proxy, but not statistically significant otherwise. Finally, average returns are positively (and statistically significantly) related to the *VOL* proxy, but not significantly related to the other proxies.

The coefficient on the size control variable suggests that small firms have higher average returns and higher volatility in returns. The coefficient on the market-to-book ratio implies that growth firms have higher, more volatile returns, higher market betas, higher correlation between absolute returns and volume, and more negative serial correlation in returns. Finally, in months when earnings are announced, stocks exhibit higher volatility, lower serial correlation in returns, and higher correlation between absolute returns and volume, on average. While the coefficients on the control variables are generally statistically significant, the coefficients on the disagreement proxies suggest that

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

the results in Tables 3 and 4 are robust to changes in specification and the introduction of control variables.

3.3 The degree of conditioning on prices

The empirical evidence from the cross-sectional analysis is generally more consistent with the RE equilibrium than the DO equilibrium. However, these results are not useful in distinguishing firms in which investors condition on prices efficiently from firms in which investors rely on prices less. Moreover, the results depend on the proxy for belief dispersion and different proxies may lead to conflicting evidence (as in the case for average returns).

Corollary 1 suggests a way to empirically characterize the degree to which investors condition on prices, without relying on a proxy for disagreement. In particular, recall that firms with a negative correlation between volume and volatility are more likely to have investors with differences of opinion. All else equal, firms with a high positive correlation between volume and volatility are more likely to have investors who condition on prices and exhibit rational expectations. As a result, by sorting firms into deciles based on the correlation between volume and volatility, and comparing the firms at the two extremes, I characterize the difference in firms that are more likely to have investors that exhibit DO versus those that exhibit RE. Table 7 shows that stocks in which investors condition on prices less (i.e., the decile of stocks with the lowest correlation between volume and volatility) have lower average returns, lower volatility in returns, and lower turnover. They are also smaller in size and have lower market-to-book ratios, higher leverage, lower analyst coverage, and lower standard deviation in earnings.

The differences in these firm characteristics between the extreme decile are generally statistically significant, and often economically important. For instance, the difference in average returns between the two extreme deciles is 1.47% per month, and the difference in market-to-book ratio and debt-to-equity ratio are 0.5 and −0.9, respectively. The evidence on the market-to-book ratio appears consistent with the pattern in the relation between average returns and disagreement uncovered in Table 5. Investors in value stocks appear to condition on prices less and exhibit DO. The analyst coverage for the lowest decile firms is nearly two analysts lower than those in the highest decile. Somewhat surprisingly, there does not seem to be a significant difference in disagreement (as measured by analyst forecast dispersion) across the groups, although this is not inconsistent with the model.

Since the model makes no predictions about the levels of return-volume characteristics in the RE and DO cases, the empirical results in Table 7 are outside the model and intended to be descriptive. However, the results are often consistent with the intuition from the model. For instance, when investors condition on prices more (ρ is higher), prices are more sensitive to shocks in fundamentals and aggregate supply. This leads to higher perceived risk, which in turn leads to higher returns, market betas, and higher volatility in returns.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

Table 7
Characteristics of firms sorted on the degree to which investors condition on prices

Decile	Corr(V,T)	Avg. Ret	R Var.	Mkt Beta	Log Turn.	AC in Rets	Corr(AR,T)
1	−0.140	0.0012	0.0114	0.909	1.149	0.0003	−0.133
2	0.028	0.0056	0.0122	0.974	1.229	−0.0003	0.037
3	0.124	0.0100	0.0133	1.031	1.300	−0.0002	0.131
4	0.201	0.0128	0.0147	1.068	1.367	−0.0008	0.205
5	0.269	0.0173	0.0160	1.101	1.425	−0.0022	0.271
6	0.333	0.0209	0.0177	1.124	1.479	−0.0026	0.332
7	0.397	0.0255	0.0200	1.145	1.536	−0.0024	0.393
8	0.464	0.0282	0.0229	1.179	1.593	−0.0014	0.459
9	0.541	0.0280	0.0262	1.184	1.642	−0.0027	0.533
10	0.661	0.0159	0.0316	1.187	1.647	−0.0014	0.645
RE–DO		0.0147	0.0201	0.278	0.498	−0.0016	0.778
t-stat		3.28	15.1	17.78	16.77	−0.59	248.35

Decile	Size	M/B	D/E	EPS	*AFD*	Coverage	Stdev. EPS
1	1638.65	1.62	2.02	2.22	0.97	7.68	1.15
2	2149.36	1.7	1.56	1.54	0.45	8.34	0.94
3	2499.52	1.78	1.66	1.26	0.51	8.7	0.96
4	2641.38	1.85	1.68	1.38	0.81	8.98	1.07
5	2775.39	1.91	1.67	2.44	0.55	9.16	1.23
6	2984.95	1.97	1.52	1.63	0.67	9.25	2.25
7	3145.67	2.03	1.43	1.84	0.69	9.43	1.49
8	3233.54	2.09	1.33	0.4	0.65	9.47	1.25
9	3304.14	2.12	1.18	2.01	0.75	9.61	1.49
10	3464.17	2.12	1.12	0.64	1.09	9.72	1.91
RE–DO	1825.52	0.5	−0.9	−1.58	0.12	2.04	0.77
t-stat	11.87	14.81	−3.53	−0.67	0.48	16.1	2.28

The table reports the time-series average of characteristics of portfolios formed by sorting stocks every month into deciles based on the degree to which investors condition on prices. Stocks are sorted every month based on the correlation between realized variance and turnover (i.e., Corr(V,T)). Negative/low-correlation firms are more likely to have investors who do not condition on prices (DO), while high-correlation firms are more likely to have investors who do condition on prices (RE). The characteristics reported are average return, realized variance, log turnover, autocorrelation in returns, correlation between absolute returns and turnover (i.e., Corr(AR,T)), market value of equity (Size), market-to-book ratio (M/B), debt-to-equity ratio (D/E), earnings per share (EPS), analyst forecast dispersion (AFD), analyst coverage, and standard deviation in EPS (calculated using the last 8 quarters). The difference in the extreme deciles and associated t-statistic is also reported.

One might also expect market-to-book ratios, which are another measure of how sensitive prices are to fundamental shocks, to be higher in this case. Finally, it is more likely that sophisticated investors (e.g., institutional investors), who condition on prices, trade in large stocks with higher turnover and higher analyst coverage.

3.4 Discussion

Perhaps not surprisingly, the empirical evidence suggests that, on average, investors condition on prices. The true behavior of investors is likely to be neither as efficient as in an RE equilibrium nor as inefficient as in a pure DO equilibrium, but somewhere in between. Moreover, there is significant variation across firms and over time in how investors use prices to update their beliefs. As a consequence, results in the existing literature must be interpreted carefully.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

The analysis also suggests that, when testing models, it is useful to derive and test multiple predictions of each model. As previously mentioned, much of the existing literature focuses on the relation between expected returns and investor disagreement. By deriving and testing predictions on additional return-volume characteristics, one is able to better distinguish among possible alternatives. For instance, the empirical evidence from the relation between disagreement and average returns or return autocorrelation does not individually provide conclusive evidence for either the RE and DO cases. However, the evidence across all the return-volume characteristics and all the disagreement proxies, when taken together, provides stronger evidence for the RE case.

Finally, the predictions of the model provide a new empirical characterization of firms in which investors exhibit RE or DO. In particular, one can proxy for the degree to which investors condition on prices using correlations between observable characteristics. These proxies could be used to revisit some of the existing empirical literature on rational expectations, differences of opinion, behavioral biases, and investor sophistication, and complement other, less direct proxies such as firm size or institutional ownership. For instance, by focusing on firms in which investors condition on prices less, one should expect to find greater effects of behavioral biases, especially if other firm characteristics (e.g., high return volatility) make limits to arbitrage more binding.

4. Conclusion

The article develops a dynamic, heterogeneous beliefs framework that nests the classic RE and DO approaches to study how investors use the information in prices to update their beliefs. When investors condition only on their private information, disagreement is negatively related to expected returns, volatility, and covariance between volume and absolute returns, but positively related to return autocorrelation. However, when investors condition on prices efficiently, these predictions are reversed. These predictions are tested using the cross-section of stocks and disagreement proxies based on analyst forecast dispersion and volume. The empirical evidence is more consistent with investors exhibiting rational expectations, although there is substantial variation in the extent to which investors condition on prices.

The model also provides a novel measure of investor sophistication based on the degree to which investors condition on prices without having to rely on proxies of belief dispersion. Under the model's assumptions, a negative correlation between trading volume and return volatility implies that investors exhibit differences of opinion. Sorting stocks based on this correlation and comparing the extreme deciles, I find that sophisticated investors who condition on prices (exhibit RE) are more likely to trade in stocks that are larger in size and have higher average returns, market betas, volatility, trading volume, market-to-book ratios, analyst coverage, and volatility in earnings.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

While useful in generating sharp empirical predictions and clarifying intuition, the model is stylized. It would be interesting to consider a broader class of models, with more general information structures, and see if empirical predictions on observables can be derived to distinguish them. Another interesting line of research would be to endogenize the degree to which investors condition on prices in a setting with uncertainty to examine how investors learn to use prices to update their beliefs.

Appendix A—Proofs

Proof of Lemma 1. Under the assumption that the shocks admit a factor structure given by (9), we know that the covariance matrices V_d, V_s, and V_z can be represented as

$$V_d = \Gamma W_d \Gamma', V_s = \Gamma W_s \Gamma', \text{ and } \quad V_z = \Gamma W_z \Gamma'. \tag{A1}$$

Conjecture that the price coefficients B and C also have the same factor structure, i.e., they can be represented as

$$B = \Gamma W_B \Gamma' \text{ and } C = \Gamma W_C \Gamma'. \tag{A2}$$

Denote investor i's beliefs about next period's dividend shocks by

$$\mu_{i,t} = E[\delta_{t+1}|\mathcal{F}_{i,t}], \text{ and } V_\delta = \text{var}[\delta_{t+1}|\mathcal{F}_{i,t}]. \tag{A3}$$

These beliefs depend on the conjectures that investors have about the informativeness of others' signals. In particular, since investor i's beliefs about investor j's signals are given by (6), and their conjecture about prices is given by (10), Bayesian updating leads to beliefs about $\tilde{\delta}_{t+1}$ given by

$$V_\delta \equiv \text{var}[\delta_{t+1}|\mathcal{F}_{i,t}] = (V_d^{-1} + V_s^{-1} + \rho^2 V_p^{-1})^{-1} \tag{A4}$$

$$\mu_{i,t} \equiv E[\delta_{t+1}|\mathcal{F}_{i,t}] = V_\delta[V_s^{-1} Y_{i,t} + \rho V_p^{-1} B^{-1}(P_t - K - AD_t)], \tag{A5}$$

where $F = B^{-1}C$ and let $V_p \equiv ((1-\rho^2)V_d + F V_z F')$. Since the investors have symmetric information sets, the posterior variance about dividends is the same for all investors. This implies that beliefs about returns are given by

$$E[R_{t+1}|\mathcal{F}_{i,t}] = (A+I)((I-\Lambda)D + \Lambda D_t + \mu_{i,t}) + K - (1+r_f)P_t \tag{A6}$$

$$\text{var}[R_{t+1}|\mathcal{F}_{i,t}] = (A+I)V_\delta(A+I)' + B V_d B' + C V_z C' \equiv V_R. \tag{A7}$$

Substituting these beliefs in the market clearing condition implies that

$$(1+r_f)P_t = (A+I)((I-\Lambda)D + \Lambda D_t + \bar{\mu}_t) + K - V_R(Z + z_t),$$

$$\text{where } \bar{\mu}_t = \int_i \mu_{i,t} di.$$

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

The expressions for A, B, and K follow from matching terms. Specifically,

$$A(1+r_f) = (A+I)\Lambda \tag{A8}$$

$$B(1+r_f) = (A+I)V_\delta(V_s^{-1} + \rho V_p^{-1}) \tag{A9}$$

$$C(1+r_f) = \rho(A+I)V_\delta V_p^{-1} B^{-1} C - V_R \tag{A10}$$

$$K(1+r_f) = (A+I)(I-\Lambda)D + K - V_R Z \tag{A11}$$

$$V_R = (A+I)V_\delta(A+I)' + BV_d B' + CV_z C'. \tag{A12}$$

To characterize the equation for F, note that $V_R = (A+I)V_\delta(A+I)' + B(V_d + FV_z F')B'$, which implies

$$F = -V_s(A+I)'\Big[I + \tfrac{1}{(1+r_f)^2}(V_s^{-1} + \rho V_p^{-1})(V_d + FV_z F')V_\delta'(V_s^{-1} + \rho V_p^{-1})'\Big].$$

Moreover, given the above characterization of the price coefficients, the conjecture about the spectral decomposition of B and C is correct, and the matrices V_R, V_δ, and V_p also share the same common matrix of eigenvectors, Γ. ■

Proof of Lemma 2. The expressions follow from using the properties of half-normal distributions. Specifically, if y_1 and y_2 are normally distributed with variance σ_1^2 and σ_2^2 and covariance $\sigma_{1,2}$, then

$$E[|y_i|] = \sqrt{\frac{2}{\pi}\sigma_{y_i}^2}, \qquad \text{var}[|y_i|] = \frac{\pi - 2}{\pi}\sigma_{y_i}^2,$$

$$\text{cov}(|y_1|, |y_2|) = \Psi(\sigma_{1,2}) = \frac{2\sigma_1\sigma_2}{\pi - 2} \times \left((1-\rho)^{3/2} - 1 + \rho^2\sqrt{1-\rho^2} + |\rho| \arctan\left(|\rho|/\sqrt{1-\rho^2}\right)\right),$$

where $\rho = \frac{\sigma_{1,2}}{\sqrt{\sigma_1^2\sigma_2^2}}$ is the correlation between y_1 and y_2. ■

Proof of Lemma 3. Denote the payoff matrix by $F_{t+1} = P_{t+1} + D_{t+1}$, and let $\bar{E}_t$ denote the average belief across all investors at date t. Then, the market payoff is given by

$$F_{m,t+1} = F_{t+1}' Z_t = \sum_{k=1}^{N} F_{k,t+1} Z_{k,t}, \tag{A13}$$

and the rate of return on asset j is given by $r_{j,t+1} = \frac{F_{j,t+1}}{P_{j,t}} - 1$. Rearrange the price of asset j as

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

$$P_{j,t}(1+r_f) = \bar{E}_t[F_{j,t+1}] - \sum_{k=1}^{N} \mathrm{cov}_t(F_{j,t+1}, F_{k,t+1}) Z_{k,t}$$
$$= \bar{E}_t[F_{j,t+1}] - \mathrm{cov}_t(F_{j,t+1}, F_{m,t+1}), \tag{A14}$$

which implies that

$$\bar{E}_t[r_{j,t+1} - r_f] = \bar{E}_t\left[\frac{F_{j,t+1}}{P_{j,t}} - (1+r_f)\right]$$

$$= \frac{1}{P_{j,t}} \mathrm{cov}_t\left(F_{j,t+1}, F_{m,t+1}\right) = P_{m,t}\mathrm{cov}_t(r_{j,t+1}, r_{m,t+1}).$$

For the market portfolio, this implies that $\bar{E}_t[r_{m,t+1} - r_f] = P_{m,t}\mathrm{var}_t(r_{m,t+1})$. This implies a conditional CAPM relation, i.e.,

$$\bar{E}_t[r_{j,t+1} - r_f] = \beta_{j,t}\bar{E}_t[r_{m,t+1} - r_f], \text{ where } \beta_{j,t} = \frac{\mathrm{cov}_t(r_{j,t+1}, r_{m,t+1})}{\mathrm{var}_t(r_{m,t+1})}. \tag{A15}$$

Finally, note that when $P_{m,t} > 0$, $\beta_{j,t} =$ is increasing in $\mathrm{cov}_t(F_{j,t+1}, F_{m,t+1})$, which is an increasing function of the jth row of V_R, a measure of asset j's perceived risk. ∎

Proof of Proposition 1. From the proof of Lemma 1, we know that, given the factor structure in (9), the price coefficients B and C and the covariance matrices V_R, V_δ, and V_p share the same common matrix of eigenvectors Γ. Similarly, V_μ also shares the same factor structure. For a covariance matrix V_X, denote the spectral decomposition by $V_X = \Gamma W_X \Gamma'$, where W_X is a diagonal matrix. Denote the element of V_X and W_X corresponding to asset n by $\upsilon_{X,n}$ and $w_{X,n}$, respectively, and denote the (i, j)th element of Γ by $g_{i,j}$. Using this notation, we have

$$\upsilon_{x,n} = \sum_{i=1}^{N} g_{n,i}^2 w_{x,i}. \tag{A16}$$

This implies that, in order to show that $\upsilon_{x,n}$ and $\upsilon_{y,n}$ are positively (negatively) related, it is enough to show that $w_{x,n}$ and $w_{y,n}$ are positively (negatively, respectively) related. Finally, denote asset-specific price coefficients by the lowercase letters (i.e., a, b, and c) $f = c/b$, $\alpha = 1 + a$ and $R_f = 1 + r_f$.

To establish the results in the proposition, we consider the effect of $w_{s,n}$ on the following expressions that drive disagreement and return-volume moments:

- Disagreement is proportional to $w_{\delta,n}^2 / w_{s,n}$.
- Expected returns are proportional to $w_{R,n}$.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

- Return variance and autocovariance: $\mathrm{var}[R_{t+1}] \propto w_{R,n} + \kappa$, and $\mathrm{cov}[R_{t+1}, R_{t+2}] \propto -\frac{\kappa}{R_f}$, where $\kappa = R_f^2 w_{R,n} - (R_f^2 - 1)\alpha^2 w_{\delta,n} - \alpha^2 w_{d,n}$.
- Expected volume: $E[\mathcal{V}_{t+1}]$ are increasing in $w_{z,n} + w_{s,n}/f^2$.
- Covariance between volume and abs. returns: $\mathrm{cov}[\mathcal{V}_{t+1}, |R_{t+1}|]$ is increasing in $-cw_{z,n}$.

DO case ($\rho = 0$)

1. In the DO case, disagreement is proportional to $w_{\mu,n} = \frac{1}{w_{s,n}}\left(\frac{w_{d,n}w_{s,n}}{w_{d,n}+w_{s,n}}\right)^2$, which implies $\frac{\partial w_{\mu,n}}{\partial w_{s,n}} = \frac{w_{d,n}^2(w_{d,n}-w_{s,n})}{(w_{s,n}+w_{d,n})^2}$ Hence, disagreement first increases and then decreases in signal noise.
2. Posterior variance in returns is given by
$$w_{R,n} = \frac{R_f^2}{2w_{z,n}} - \frac{R_f}{w_{z,n}}\left(\frac{R_f^2}{4} - w_{z,n}\xi_n\right)^{1/2},$$
$$\text{where } \xi_n = \frac{\alpha^2 w_{d,n}w_{s,n}}{w_{d,n}+w_{s,n}}\left(1 + \frac{w_{d,n}^2}{w_{s,n}(w_{d,n}+w_{s,n})R_f^2}\right).$$
This implies that $\frac{\partial w_{R,n}}{\partial w_{s,n}} \propto \frac{\partial \xi_n}{\partial w_{s,n}} = \frac{\alpha^2 w_{d,n}^2}{R_f^2(w_{s,n}+w_{d,n})^3}\Big((w_{s,n} + w_{d,n})R_f^2 - 2w_{d,n}\Big)$. Hence, expected returns decrease and then increase in signal noise. Moreover, the trough coincides with the hump in $w_{\mu,n}$ when $r_f \to 0$.
3. Return volatility and autocorrelation are driven by $\kappa = R_f^2 w_{R,n} - (R_f^2 - 1)\alpha^2 w_{\delta,n} - \alpha^2 w_{d,n}$. This implies that
$$\frac{\partial \kappa}{\partial w_{s,n}} = \frac{\alpha^2 w_{d,n}^2}{(w_{d,n}+w_{s,n})^3}\left(\frac{R_f(R_f^2 w_{s,n} + w_{d,n}(R_f^2-2))}{\sqrt{R_f^2-4w_{z,n}\xi}} - (R_f^2 - 1)(w_{d,n} + w_{s,n})\right),$$
which is negative when $w_{s,n} = 0$, and positive when $w_{s,n} \to \infty$, and switches sign once. Hence, volatility in returns decreases and then increases in $w_{s,n}$, but serial covariance in returns increases and then decreases.
4. Expected volume is driven by $\kappa_2 = w_{z,n} + \frac{w_{s,n}}{f^2} \Rightarrow \frac{\partial \kappa_2}{\partial w_{s,n}} = \frac{1}{f^2}\left(1 - 2\frac{w_{s,n}}{f}\frac{\partial f}{\partial w_{s,n}}\right)$, which implies that volume increases and then decreases with $w_{s,n}$.
5. Covariance between absolute returns and volume is driven by $-cw_{z,n}$, which implies that this decreases and then increases with $w_{s,n}$.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

RE case ($\rho = 1$)
In the RE case, we know that $w_{R,n} = -\alpha f w_{\delta,n}/w_{s,n}$, where f solves the following expression:

$$f + w_{s,n}\alpha + \frac{\alpha w_{d,n}(w_{d,n}+w_{p,n})(w_{s,n}+w_{p,n})^2}{R_f^2 w_{p,n}(w_{d,n}w_{s,n}+w_{d,n}w_{p,n}+w_{p,n}w_{s,n})} = 0. \tag{A17}$$

Since we pick the less negative root of (A17), we know that, at $w_{s,n} = 0$, we have

$$f = \frac{-R_f^2+\sqrt{R_f^4-4\alpha^2 w_{d,n}w_{z,n}}}{2\alpha w_{z,n}}, \qquad \frac{\partial f}{\partial w_{s,n}} = -\frac{R_f^2}{\alpha w_{d,n}w_{z,n}} - \frac{\alpha R_f^2}{\sqrt{R_f^4-4\alpha^2 w_{d,n}w_{z,n}}} < 0.$$

Moreover, when $w_{s,n} \to \infty$, we can use implicit differentiation on (A17) to show that $\frac{\partial f}{\partial w_{s,n}} \to 0$. Finally, note that $\frac{1}{w_{p,n}}\frac{\partial w_{p,n}}{\partial w_{s,n}} = \frac{2}{f}\frac{\partial f}{\partial w_{s,n}} > 0$.

1. Disagreement is given by

$$w_{\mu,n} = \frac{w_{\delta,n}^2}{w_{s,n}} \Rightarrow \frac{\partial w_{\mu,n}}{\partial w_{s,n}} = \frac{w_{\delta,n}}{w_{s,n}}\left(2\frac{\partial w_{\delta,n}}{\partial w_{s,n}} - \frac{w_{\delta,n}}{w_{s,n}}\right), \text{ where}$$

$$\frac{\partial w_{\delta,n}}{\partial w_{s,n}} = \left(\frac{w_{\delta,n}}{w_{s,n}}\right)^2\left(1 + \frac{w_{s,n}^2}{w_{p,n}^2}\frac{\partial w_{p,n}}{\partial w_{s,n}}\right).$$

When $w_{s,n} = 0$, we have $\frac{w_{\delta,n}}{w_{s,n}} = 1$ and $\frac{\partial w_{\delta,n}}{\partial w_{s,n}} = 1$, and this implies $\frac{\partial w_{\mu,n}}{\partial w_{s,n}} > 0$. When $w_{s,n} \to \infty$, we know that $\frac{w_{\delta,n}}{w_{s,n}} \to 0$, and so beyond some $w_{s,n}$,

$$2\frac{\partial w_{\delta,n}}{\partial w_{s,n}} - \frac{w_{\delta,n}}{w_{s,n}} < \frac{w_{\delta,n}}{w_{s,n}}\left(2\frac{w_{\delta,n}}{w_{s,n}}\left(1 + \frac{1}{\alpha^2 w_{p,n}}\frac{\partial w_{p,n}}{\partial w_{s,n}}\right) - 1\right) < 0,$$

which implies that $\frac{\partial w_{\mu,n}}{\partial w_{s,n}} < 0$. Also, when the derivative is zero, we know that

$$2\frac{\partial w_{\delta,n}}{\partial w_{s,n}} - \frac{w_{\delta,n}}{w_{s,n}} = \frac{w_{\delta,n}}{w_{s,n}}\left(2\frac{w_{\delta,n}}{w_{s,n}}\left(1 + \frac{w_{s,n}^2}{w_{p,n}^2}\frac{\partial w_{p,n}}{\partial w_{s,n}}\right) - 1\right) = 0.$$

Note that this expression is greater than zero when $w_{s,n}$ is slightly smaller, and less than zero when $w_{s,n}$ is slightly bigger. Hence, disagreement increases and then decreases with $w_{s,n}$.

2. Posterior variance in returns is given by

$$w_{R,n} = -\alpha f w_{\delta,n}/w_{s,n} \Rightarrow \frac{\partial w_{R,n}}{\partial w_{s,n}}$$
$$= \alpha\left(-\frac{\partial f}{\partial w_{s,n}}\frac{w_{\delta,n}}{w_{s,n}} - \frac{f}{w_{s,n}}\left(\frac{w_{\delta,n}}{w_{s,n}} - \frac{\partial w_{\delta,n}}{\partial w_{s,n}}\right)\right).$$

This implies that, when $w_{s,n} = 0$, $\frac{\partial w_{R,n}}{\partial w_{s,n}} > 0$. The limit result in (i) implies that, beyond some $w_{s,n}$, $\frac{\partial w_{R,n}}{\partial w_{s,n}} < 0$. Hence, expected returns increase and then decrease in $w_{s,n}$.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

3. Return volatility and autocorrelation are driven by $\kappa = R_f^2 w_{R,n} - (R_f^2 - 1)\alpha^2 w_{\delta,n} - \alpha^2 w_{d,n}$. This implies

$$\frac{\partial \kappa}{\partial w_{s,n}} = R_f^2 \alpha \left(-\frac{\partial f}{\partial w_{s,n}} \frac{w_{\delta,n}}{w_{s,n}} - \frac{f}{w_{s,n}} \left(\frac{w_{\delta,n}}{w_{s,n}} - \frac{\partial w_{\delta,n}}{\partial w_{s,n}} \right) \right) - \alpha^2 (R_f^2 - 1) \frac{\partial w_{\delta,n}}{\partial w_{s,n}}.$$

When $w_{s,n} = 0$, $\frac{\partial \kappa}{\partial w_{s,n}} = -R_f^2 \alpha \frac{\partial f}{\partial w_{s,n}} - \alpha^2 (R_f^2 - 1)$, which implies $\frac{\partial \kappa}{\partial w_{s,n}} > 0$. When $w_{s,n} \to \infty$, using the results above, $\frac{\partial \kappa}{\partial w_{s,n}} < 0$. Hence, return volatility increases and then decreases in $w_{s,n}$, while serial covariance in returns decreases and then increases in $w_{s,n}$.

4. Expected volume and variance in volume increase and then decrease with $w_{s,n}$ as in the DO model.

5. Covariance between absolute returns and volume is driven by

$$-c = -bf \Rightarrow -\frac{\partial c}{\partial w_{s,n}} = -\left(b \frac{\partial f}{\partial w_{s,n}} - f \frac{\alpha}{w_{d,n} R_f} \frac{\partial w_{\delta,n}}{\partial w_{s,n}} \right).$$

When $w_{s,n} \to \infty, -\frac{\partial c}{\partial w_{s,n}} \to -\left(-f \frac{\alpha}{w_{d,n} R_f} \frac{\partial w_{\delta,n}}{\partial w_{s,n}} \right) < 0.$

When $w_{s,n} = 0$, $-\frac{\partial c}{\partial w_{s,n}} = -\frac{1}{R_f} \left(\frac{\partial f}{\partial w_{s,n}} - f \frac{\alpha}{w_{d,n}} \right)$

$$- \frac{1}{R_f} \left(- \frac{\alpha R_f^2}{\sqrt{R_f^4 - 4\alpha^2 w_{d,n} w_{z,n}}} - \frac{(2-\alpha) r_f + \alpha \sqrt{R_f^4 - 4\alpha^2 w_{d,n} w_{z,n}}}{2\alpha w_{d,n} w_{z,n}} \right),$$

which is greater than 0 when α is small enough. Hence, the covariance between absolute returns and volume increases and then decreases in $w_{s,n}$.

Thus, when disagreement increases due to a change in signal noise, expected returns, volatility, and covariance between absolute returns and volume decrease for the DO case but increase for the RE case. On the other hand, autocorrelation increases for the DO case but decreases for the RE case. Volume and expected volume increase in disagreement for both models. One can show analytically in the DO case, and numerically in the RE case, that as $r_f \to 0$ and $\lambda \to 0$, the peaks and troughs in the return-volume moments (as a function of signal noise) line up with the peak in disagreement. ■

Appendix B—Finite Horizon Equilibria

In the finite horizon model, the price coefficients are time dependent. The posterior variance of returns at time t is given by

$$\begin{aligned} V_{R,T-1} &= \mathrm{var}[D_T | \mathcal{F}_{i,T-1}] = V_{T-1} \\ V_{R,t} &= (A_{t+1} + I) V_{\delta,t} (A_{t+1} + I)' + B_{t+1} V_d B_{t+1}' \\ &\quad + C_{t+1} V_z C_{t+1}', \text{ where } t < T - 1. \end{aligned}$$

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

The posterior variance in beliefs is given by $V_{\delta,t} = (V_d^{-1} + V_s^{-1} + \rho^2 V_{p,t}^{-1})^{-1}$, where $V_{p,t} = (1-\rho^2)V_d + F_t V_z F_t'$ and $F_t = B_t^{-1} C_t$. The time $T-1$ price coefficients are the same as the static model coefficients, and so are unique. For earlier periods, the B_t and C_t coefficients are recursively defined, and are also uniquely determined. In particular, F_t satisfies

$$(A_{t+1} + I)(V_d^{-1} + V_s^{-1} + \rho^2 V_{p,t}^{-1})^{-1} \left(V_s^{-1} F_t + (A_{t+1} + I)' \right)$$
$$+ B_{t+1}(V_d + F_t V_z F_t') B_{t+1}' = 0,$$

which is a cubic equation in F_t. This equation has two imaginary roots and one real root, and so the equilibrium is uniquely determined.

References

Abarbanell, J., and V. Bernard. 1992. Tests of Analysts' Overreaction/underreaction to Earnings Information as an Explanation for Anomalous Stock Price Behavior. *Journal of Finance* 47:1181–207.

Admati, A. 1985. A Noisy Rational Expectations Equilibrium for Multi-asset Securities Markets. *Econometrica* 53:629–57.

Ajinkya, B., R. Atiase, and M. Gift. 1991. Volume of Trading and the Dispersion in Financial Analysts' Earnings Forecasts. *Accounting Review* 66:389–401.

Ajinkya, B., and M. Gift. 1985. Dispersion of Financial Analysts' Earnings Forecasts and the (Option Model) Implied Standard Deviations of Stock Returns. *Journal of Finance* 40:1353–65.

Anderson, E., E. Ghysels, and J. Juergens. 2005. Do Heterogeneous Beliefs Matter for Asset Pricing? *Review of Financial Studies* 18:875–924.

Ang, J. S., and S. J. Ciccone. 2001. Analyst Forecasts and Stock Returns. SSRN eLibrary.

Banerjee, S., R. Kaniel, and I. Kremer. 2009. Price Drift as an Outcome of Differences in Higher-order Beliefs. *Review of Financial Studies* 22:3707–34.

Banerjee, S., and I. Kremer. 2010. Disagreement and Learning: Dynamic Patterns of Trade. *Journal of Finance* 65:1269–302.

Barron, O., O. Kim, S. Lim, and D. Stevens. 1998. Using Analysts' Forecasts to Measure Properties of Analysts' Information Environment. *Accounting Review* 73:421–33.

Basak, S. 2000. A Model of Dynamic Equilibrium Asset Pricing with Heterogeneous Beliefs and Extraneous Risk. *Journal of Economic Dynamics and Control* 24:63–95.

Bernhardt, D., M. Campello, and E. Kutsoati. 2006. Who Herds? *Journal of Financial Economics* 80:657–75.

Biais, B., P. Bossaerts, and C. Spatt. 2010. Equilibrium Asset Pricing and Portfolio Choice under Asymmetric Information. *Review of Financial Studies* 23:1503–43.

Blume, L., M. M. Bray, and D. Easley. 1982. Introduction to the Stability of Rational Expectations Equilibrium. *Journal of Economic Theory* 26:313–17.

Buraschi, A., and A. Jiltsov. 2006. Model Uncertainty and Option Markets with Heterogeneous Beliefs. *Journal of Finance* 61:2841–97.

Cao, H. H., and H. Ou-Yang. 2009. Differences of Opinion of Public Information and Speculative Trading in Stocks and Options. *Review of Financial Studies* 22:299–335.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam. 1998. Investor Psychology and Security Market Under- and Overreactions. *Journal of Finance* 53(6):1839–85.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

Detemple, J., and S. Murthy. 1994. Intertemporal Asset Pricing with Heterogeneous Beliefs. *Journal of Economic Theory* 62:294–320.

Diether, K., C. Malloy, and A. Scherbina. 2002. Differences of Opinion and the Cross-section of Stock Returns. *Journal of Finance* 57:2113–41.

Dimson, E. 1979. Risk Measurement When Shares Are Subject to Infrequent Trading. *Journal of Financial Economics* 7:197–226.

Doukas, J., C. Kim, and C. Pantzalis. 2006. Divergence of Opinion and Equity Returns. *Journal of Financial and Quantitative Analysis* 41:573–606.

Goetzmann, W., and M. Massa. 2005. Dispersion of Opinion and Stock Returns. *Journal of Financial Markets* 8:324–49.

Harris, M., and A. Raviv. 1993. Differences of Opinion Make a Horse Race. *Review of Financial Studies* 6:473–506.

Harrison, J. M., and D. M. Kreps. 1978. Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations. *Quarterly Journal of Economics* 92:323–36.

He, H., and J. Wang. 1995. Differential Information and Dynamic Behavior of Stock Trading Volume. *Review of Financial Studies* 6:919–72.

Hellwig, M. 1980. On the Aggregation of Information in Competitive Markets. *Journal of Economic Theory* 22:477–98.

Hong, H., J. Kubik, and A. Solomon. 2000. Security Analysts' Career Concerns and Herding of Earnings Forecasts. *Rand Journal of Economics* 31:121–44.

Hong, H., and J. Stein. 2003. Differences of Opinion, Short-sales Constraints, and Market Crashes. *Review of Financial Studies* 16:487–525.

Johnson, T. 2004. Forecast Dispersion and the Cross-section of Expected Returns. *Journal of Finance* 59:1957–78.

Kandel, E., and N. D. Pearson. 1995. Differential Interpretation of Public Signals and Trade in Speculative Markets. *Journal of Political Economy* 103:831–72.

Lang, L., R. Litzenberger, and V. Madrigal. 1992. Testing Financial Market Equilibrium under Asymmetric Information. *Journal of Political Economy* 100:317–48.

Lewellen, J., and S. Nagel. 2006. The Conditional CAPM Does Not Explain Asset-pricing Anomalies. *Journal of Financial Economics* 82:289–314.

Lewellen, J., and J. Shanken. 2002. Learning, Asset-pricing Tests, and Market Efficiency. *Journal of Finance* 57:1113–45.

Liang, T. 2008. Asset Pricing Implications of Self-fulfilling Rational Expectations. Working Paper, Cornell University.

Lintner, J. 1969. The Aggregation of Investor's Diverse Judgments and Preferences in Purely Competitive Security Markets. *Journal of Financial and Quantitative Analysis* 4:347–400.

Miller, E. 1977. Risk, Uncertainty, and Divergence of Opinion. *Journal of Finance* 32(4):1151–68.

Park, C. 2005. Stock Return Predictability and the Dispersion in Earnings Forecasts. *Journal of Business* 78:2351–76.

Qu, S., L. Starks, and H. Yan. 2004. Risk, Dispersion of Analyst Forecasts, and Stock Returns. Working Paper, University of Texas at Austin.

Scheinkman, J. A., and W. Xiong. 2003. Overconfidence and Speculative Bubbles. *Journal of Political Economy* 111:1183–219.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011

Shalen, C. T. 1993. Volume, Volatility, and the Dispersion of Beliefs. *Review of Financial Studies* 6:405–34.

Spiegel, M. 1998. Stock Price Volatility in a Multiple Security Overlapping Generations Model. *Review of Financial Studies* 11:419–47.

Van Nieuwerburgh, S., and L. Veldkamp. 2009. Information Acquisition and Under-diversification. *Review of Economic Studies* 77:779–805.

Verardo, M. 2009. Heterogeneous Beliefs and Momentum Profits. *Journal of Financial and Quantitative Analysis* 44:795–822.

Wang, J. 1994. A Model of Competitive Stock Trading Volume. *Journal of Political Economy* 102:127–68.

Watanabe, M. 2008. Price Volatility and Investor Behavior in an Overlapping Generations Model with Information Asymmetry. *Journal of Finance* 63:229–72.

Zapatero, F. 1998. Effects of Financial Innovations on Market Volatility When Beliefs Are Heterogeneous. *Journal of Economic Dynamics and Control* 22:597–626.

Zhang, X. 2006. Information Uncertainty and Stock Returns. *Journal of Finance* 61:105–37.

Downloaded from rfs.oxfordjournals.org by guest on August 13, 2011