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Feedback Effects and Systematic Risk Exposures

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ABSTRACT

We model the "feedback effect" of a firm's stock price on investment in projects exposed to a systematic risk factor, like climate risk. The stock price reflects information about both the project's cash flows and its discount rate. A cash-flow-maximizing manager treats discount rate fluctuations as "noise," but a price-maximizing manager interprets such variation as information about the project's net present value. This difference qualitatively changes how investment behavior varies with the project's risk exposure. Moreover, traditional objectives (e.g., cash flow or price maximization) need not maximize welfare because they do not correctly account for hedging and risk-sharing benefits of investment.

SINCE HAYEK (1945), IT HAS BEEN recognized that prices aggregate information that is dispersed across the economy and convey it to real decision makers. The "feedback effects" literature studies this mechanism in the context of corporate investment, emphasizing how asset prices reflect information about future investment opportunities, and how this information affects firms' production and investment decisions (see Bond, Edmans, and Goldstein (2012) and Goldstein (2023) for insightful surveys). Existing analyses focus on the extent to which prices reflect information about future cash flows and interpret noncash-flow variation in prices as noise that needs to be filtered out by decision makers.

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Yet, a fundamental tenet of capital budgeting is that firms' optimal investment decisions should depend on not only projects' expected cash flows, but also their discount rates. Moreover, a project's discount rate is driven by its loadings on systematic sources of risk and investors' aggregate preferences over, and exposures to, these risks. While a firm's manager is unlikely to directly observe these preferences and exposures, they impact investors' demands and equilibrium asset prices. This suggests that prices are crucial sources of information about discount rates for managers making investment decisions.

To study feedback effects when managers learn about discount rates from prices, we develop a model in which a firm's stock price conveys information about both future cash flows and investors' risk exposures. When the manager chooses investment to maximize expected cash flows, she interprets noncash-flow variation in prices as noise. In contrast, when the manager chooses investment to maximize the future share price, noncash-flow variation in prices conveys useful information about the project's discount rate.[1] Consequently, she no longer explicitly seeks to filter out such information and instead incorporates the information in prices on both cash flows and discount rates when making her investment decisions.

This difference has important implications for how investment in a project depends on its risk exposure. For a cash-flow-maximizing manager, an increase in a project's risk exposure makes the stock price a noisier signal about the project's expected cash flows. This makes the manager's investment decision less sensitive to the information in the price. In contrast, for a price-maximizing manager, an increase in the project's risk exposure makes the price more volatile, which, as we show, causes her conditional expectation of the project's net present value (NPV) to vary more. All else equal, this makes the investment decision more sensitive to the price, as we clarify below.

Finally, we show that traditional managerial objectives, like cash-flow or price maximization, do not generally lead to investment decisions that maximize investor welfare. Clearly, since cash-flow maximization ignores the impact of the project's risk exposure on investors' ability to "hedge" the systematic risk factor, it can lead to underinvestment or overinvestment relative to welfare maximization.[2] Price maximization leads to inefficient investment decisions for two reasons. First, while the share price does reflect information about risk exposures through the discount rate, the risk premium in price reflects the disutility that the risk of a *marginal* share of the stock imposes on an investor. Welfare, however, depends on an investor's disutility from bearing the risk of her entire share holdings. Second, the price does not account for the fact that investing in a risk-exposed project makes the stock a better instrument for risk-sharing across investors, which increases welfare.

[1] As we discuss below, the project's risk exposure is known to both the manager and investors, but the stock price conveys information about the associated factor risk premium to the manager.

[2] In what follows, our terminology explicitly distinguishes between *hedging* and *risk-sharing*. The former refers to investors' desire to buy more (less) of assets that pay out more (less) during adverse systematic factor outcomes. The latter refers to investors' ability to share and reallocate *differential* exposures to systematic risk by trading a risk-exposed security.

Model and Intuition. Our analysis applies quite broadly to investment in risky projects when market feedback plays an important role. A particularly salient application is to climate-sensitive investment, and so, we use this setting to describe our model's economic forces and predictions. A firm's manager decides whether to invest in a project that is exposed to a systematic climate risk factor. The firm's stock is traded by risk-averse investors who are informed about the project's expected cash flows and have heterogeneous exposures to climate risk. The price aggregates not only investors' information about cash flows, but also their dispersed exposures toward the project's climate risk exposure, or "greenness." A "green" ("brown") project is defined as one that pays higher (lower) cash flows when climate outcomes are worse, while a "neutral" project's cash flows are uncorrelated with climate outcomes. As such, green projects are negatively exposed to the climate risk factor, while brown projects are positively exposed to this factor.[3]

For example, consider a consumer electronics firm deciding whether to invest in electric vehicle (EV) technology, such as batteries or semiconductors. Such green investment is negatively exposed to climate risk. For instance, shifts in regulatory policy in response to climate change may lead to more favorable treatment of EVs relative to traditional vehicles.[4] Thus, the firm's price and the information it conveys to the manager depend in part on the fact that such investments are likely to perform better when aggregate climate outcomes are worse.[5]

We compare two managerial objectives. First, in line with the existing feedback literature, we consider the case in which the manager chooses investment to maximize expected cash flows. In this case, we show that a higher (absolute) exposure to climate risk shocks makes the price a noisier signal about cash flows, which, in turn, makes the manager's investment decision *less* sensitive to the price. As a result, for ex-ante unattractive projects (i.e., projects with negative ex-ante net expected cash flows), the manager is less likely to invest in green (or brown) projects than in climate-neutral projects.

Second, we consider the case in which the manager's objective is to maximize the firm's expected stock price. In this case, she invests only when the stock price is sufficiently high, because this implies that the project's NPV,

[3] Our definitions of "green" versus "brown" projects are consistent with the empirical literature (e.g., Engle et al. (2020), Bolton and Kacperczyk (2021)), as we discuss in Section I.A. Moreover, there is substantial evidence that investors have time-varying exposures to climate risk that affect their demands for green and brown stocks and, in turn, these stocks' discount rates (e.g., Choi, Gao, and Jiang (2020), Pástor, Stambaugh, and Taylor (2022), Bolton and Kacperczyk (2023)).

[4] For example, Panasonic, historically associated with consumer electronics, is now also a leading manufacturer of rechargeable batteries for EV companies. Such investments are likely to benefit from regulatory changes that provide tax subsidies to encourage the purchase of EVs, which is an example of climate transition risk (e.g., Giglio, Kelly, and Stroebel (2021)).

[5] Consistent with managers responding to the information that prices contain about cash flows and discount rates, empirical evidence shows that firms' investment in climate-exposed projects often responds to changes in their stock prices, even when driven by shocks to investor demand for green exposure rather than cash flow news (e.g., Li et al. (2020), Bai et al. (2021), Briere and Ramelli (2021)).

conditional on the price information, is positive. In effect, when conditioning on the price, she learns about both investors' cash flow information *and* their aggregate risk exposure, which drives the project's discount rate. In fact, we show that price aggregates these two types of information in an efficient manner from the manager's perspective, in that she makes the same investment decision that she would if she observed them separately.[6]

Once again, when the project has a greater absolute exposure to climate risk, the firm's price is a noisier signal of cash flows. Yet, in stark contrast to cash-flow maximization, this causes her investment decision to become *more* sensitive to the price. This is because the price signal, and consequently, the manager's conditional expectation of the project's NPV, is more volatile. For an ex-ante unattractive project, this increased volatility increases the likelihood that the project will have a positive conditional NPV and as a result *increases* the likelihood of investment.[7]

An increase in the project's climate exposure also affects its expected NPV: greener projects provide a hedge against bad climate outcomes and thus, all else equal, carry lower discount rates. The overall effect of a project's climate exposure on the likelihood of investment trades off the impact of these channels. In fact, when the ex-ante uncertainty over the aggregate demand for a climate hedge is sufficiently high, the effect of climate exposure on the volatility of a project's NPV dominates its effect on its expected NPV. This implies, for example, that the manager may be more likely to invest in brown projects that are ex-ante unattractive than in comparable neutral projects.

Welfare. Differences in managerial objectives also have important implications for investor welfare. We first consider a benchmark in which all investors have identical exposures to the climate risk factor. In this case, maximizing cash flows clearly does not align with maximizing shareholder welfare because it ignores the impact of investment on investors' aggregate climate exposure—for example, it leads to underinvestment in green projects. More surprisingly, we show that the price-maximizing investment rule also does not align with the welfare-maximizing price-contingent investment rule as long as the firm is not arbitrarily small (i.e., as long as the investment decision has an effect on aggregate exposures). Analogous to the intuition of Spence (1975) in the context of a monopolist's choice of product quality, this is because the price reflects the marginal disutility from bearing the risk of the last outstanding share, while welfare depends on the average disutility from bearing the risk of all outstanding shares. Because the marginal disutility of the last share is higher than the average disutility of all shares, the price-maximizing rule tends to

[6] This establishes an equivalence between our setting, where the manager infers their project's discount rate and cash-flow information from prices, and traditional production-based asset pricing models, where the manager is assumed to exogenously know these two types of information (e.g., Cochrane (1991)).

[7] Intuitively, the manager's investment decision is a real options problem, and higher volatility in the project's NPV increases the likelihood of exercise for an "out-of-the-money" option (ex-ante unattractive project) but decreases the likelihood of exercise for an "in-the-money" option (ex-ante attractive project).

underinvest. Finally, we show that for brown projects, price and cash-flow incentives can be balanced through appropriate weighting to induce the manager to maximize investor welfare, while for green projects this may not be possible.

We next consider the general setting in which investors have heterogeneous exposures to climate risk. This is a realistic feature: investors' climate exposures differ with age, geography, and adaptability (Giglio, Kelly, and Stroebel (2021)), and, as evidenced by the swath of actively traded climate-based exchange-traded funds (ETFs), investors appear to use financial markets as a means to hedge and share such risk exposures.[8] For example, an investor who lives in coastal California is more exposed to climate risk due to rising sea levels, and therefore, has a different demand for green stocks than an investor who lives in central Kansas.[9] In such settings, a firm's investment in a climate-sensitive project has an additional impact on welfare because it allows investors to use the firm's stock to help share risk: all else equal, both investors are better off when the Kansas investor sells some shares of a firm that invests in green EV projects to the California investor. However, the welfare improvement as a result of this "risk-sharing" channel is not captured by the stock price, which reflects investors' disutility of risk of a marginal share of the stock and not the heterogeneity in their exposures.

This implies that, even when the per-capita endowment of shares is negligible (so that the investment decision does not affect aggregate risk), both price maximization and cash-flow maximization lead to underinvestment relative to welfare maximization. Moreover, while feedback necessarily increases the firm's expected cash flows or share price (depending on the manager's objective), we show that it can decrease investor welfare.[10] Intuitively, without feedback, the manager would always invest in an ex-ante attractive project, whereas with feedback, she would not invest in such a project if the equilibrium price were sufficiently low. This lower investment increases welfare due to higher valuations, but decreases welfare due to the risk-sharing channel. When investors' exposures to climate risk are sufficiently diverse or per-capita ownership of the firm is sufficiently small, the latter effect dominates and welfare is higher without feedback than with. In such settings, our analysis suggests that providing additional incentives for managers to invest in green projects (e.g., by linking their compensation to climate scores) can increase

[8] There is ample evidence that investors use financial assets to attempt to hedge and share climate risks—see, for example, Ilhan (2020), Krueger, Sautner, and Starks (2020), Ilhan et al. (2023), Giglio, Kelly, and Stroebel (2021), and our discussion in Section I.A. Moreover, total assets under management in sustainability-focused funds roughly doubled from Q4 2019 to Q3 2022, concurrent with over 200 sustainability fund launches per year (see Morningstar's "Global sustainable fund flows: Q3 2022 in review").

[9] Consistent with this, Ilhan (2020) documents that households with differential exposures to sea-level rise have different participation in equity markets and consequently different portfolios.

[10] For simplicity, we assume that investors do not have access to other securities that let them share climate risks. However, we expect that similar results would arise if the market for trading climate risk shocks is imperfect. As we discuss in Section I.A, this is consistent with the empirical evidence that suggests that investors have different exposures to climate risk and find this risk difficult to hedge.

investor welfare even though it may lead to lower valuations and lower future profitability.

Overview. The rest of the paper is organized as follows. Section I introduces the model and discusses key assumptions. Section II characterizes the equilibrium under cash-flow maximization and price maximization. Section III presents our results on investor welfare. Section IV discusses the related literature. Section V concludes. Proofs of our results are in the Appendix and additional analysis is presented in the Internet Appendix.[11]

I. Model

We consider a model of feedback effects where the investment is exposed to a systematic risk. We present the model in the context of climate risk as it is a significant and direct application, but as we discuss in the conclusion, our analysis has other applications.

Payoffs. There are four dates $t \in \{1, 2, 3, 4\}$ and two securities. The risk-free security is normalized to the numeraire. A share of the risky security is a claim to terminal per-share cash flows V generated by the firm at date four, and trades on dates one and three at prices P_1 and P_3, respectively.

Investors. There is a continuum of investors, indexed by $i \in [0, 1]$, with constant absolute risk aversion (CARA) utility over terminal wealth with risk-aversion γ. Investor i has initial endowment of n shares of the risky asset and $z_i = Z + \zeta_i$ units of exposure to a nontradeable source of income that has payoff of $-\eta_C$, where $Z \sim N\left(\mu_Z, \tau_Z^{-1}\right)$, $\zeta_i \sim N\left(0, \tau_\zeta^{-1}\right)$, and $\eta_C \sim N\left(0, \tau_\eta^{-1}\right)$ are independent of each other and all other random variables.[12] Investor i chooses trades X_{it}, $t \in \{1, 3\}$ to maximize her expected utility over terminal wealth, which is given by

$$W_i = (n + X_{i1} + X_{i3})V - X_{i3}P_3 - X_{i1}P_1 - z_i\eta_C. \tag{1}$$

We interpret η_C as climate risk shocks, which reduce investor wealth and in turn utility.[13] Furthermore, Z captures investors' aggregate exposure to climate risk shocks, and μ_Z is the average exposure to climate risk. The natural restriction for this interpretation is $\mu_Z > 0$, which implies that shocks to the climate (i.e., positive innovations to η_C) have, in expectation, a negative impact on the average investor. In our analysis, we focus on this restriction to clearly distinguish between projects that are positively versus negatively exposed to the climate.

[11] The Internet Appendix is available in the online version of the article on *The Journal of Finance* website.

[12] We let $\tau_{(.)}$ denote the unconditional precision and $\sigma^2_{(.)}$ the unconditional variance of all random variables.

[13] While, for concreteness, we refer to η_C as a nontradeable payoff, we could equivalently interpret it as a nonmonetary climate shock to which investors are differentially exposed and thus that affects their utility directly.

We further require the parameter restriction $1 > \gamma^2 \frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)$ to ensure that the unconditional expected utility is finite. Intuitively, if this condition is violated, the climate payoffs $z_i \eta_C$ are sufficiently uncertain ex-ante that the expected utility diverges to $-\infty$. This is a natural condition that arises when characterizing ex-ante expected utility in any CARA-Normal model in which traders have random endowments, and therefore, the unconditional distribution of wealth involves a product of normally distributed random variables.[14] We summarize these restrictions in the following assumption, which we maintain throughout our analysis.

ASSUMPTION 1: *(i) The average exposure to climate risk μ_Z is positive, that is, $\mu_Z > 0$.*

(ii) Uncertainty about overall climate payoffs is sufficiently small, that is, $1 > \gamma^2 \frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)$.

The firm. The firm generates cash flows per share, $A \sim N\left(\mu_A, \tau_A^{-1}\right)$, from assets in place. In addition, the firm's manager decides whether to invest in a new project. The investment decision is binary and denoted by $k \in \{0, 1\}$. The firm's cash flow per share, given an investment choice k, equals

$$V(k) = A + k\Big(\theta + \alpha \eta_C + \sqrt{1-\alpha^2}\,\eta_I - c\Big), \tag{2}$$

where $\theta \sim N\left(\mu_\theta, \tau_\theta^{-1}\right)$ and $\eta_C, \eta_I \sim N\left(0, \tau_\eta^{-1}\right)$ are independent of each other and other random variables, $\alpha \in [-1, 1]$, and $c \geq 0$. The component θ reflects the learnable component of cash flows for the investment opportunity, η_C reflects shocks to the "climate" component of cash flows, and η_I reflects shocks to the "idiosyncratic" component of cash flows. The cost of investment is c, which is assumed to be nonnegative.

The parameter α captures the extent to which the project's cash flows are correlated with climate risk shocks. When $\alpha = 0$, the new project's cash flows are uncorrelated with climate risk and therefore are not useful for hedging—we refer to such projects as "neutral" projects. When $\alpha > 0$, the project's cash flows are *higher* when climate outcomes are worse (η_C is higher)—we refer to these projects as "green" projects. This increase in cash flows may be due to higher demand for the product (e.g., EVs) or regulatory changes (e.g., higher taxes on greenhouse gas emissions) driven by adverse changes in the climate. Analogously, when $\alpha < 0$, the project's cash flows are *lower* when climate outcomes are worse—we refer to these projects as "brown" projects.[15]

[14] See, for instance, Assumption 1.1 in Rahi (1996), Assumption 1 in Marín and Rahi (1999), equation (1.2) in Vayanos and Wang (2012), and equation (8) in Bond and Garcia (2022), among others.

[15] Note that since positive realizations of η_C shocks increase marginal utility, green projects are *negatively* exposed to climate risk, while brown projects are *positively* exposed. While there is some disagreement in the literature regarding how different types of stocks' returns correlate with climate outcomes (e.g., Giglio, Kelly, and Stroebel (2021)), our definitions of "green" and "brown" projects correspond to how they are classified by the empirical literature (e.g., Bolton and

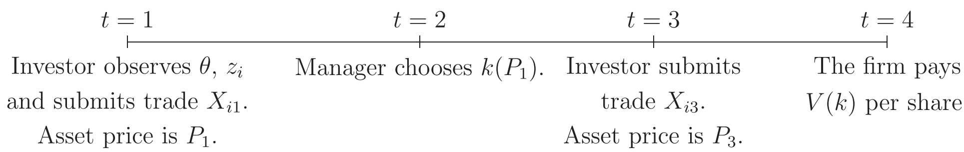

Figure 1. Timeline of events.

Information and timing of events. Figure 1 summarizes the timing of events. At date 1, all investors observe θ perfectly. Let $\mathcal{F}_{i1} = \sigma(\theta, z_i, P_1)$ and $\mathcal{F}_{i3} = \sigma(\theta, z_i, P_1, P_3, k)$ denote investor i's information set at the trading stages, with expectation, covariance, and variance operators $\mathbb{E}_{it}[\cdot]$, $\mathbb{C}_{it}[\cdot]$, and $\mathbb{V}_{it}[\cdot]$, respectively. Then, investor i chooses trade X_i to maximize her expected utility,

$$\mathcal{W}_i \equiv \sup_{x \in \mathbb{R}} \mathbb{E}_{i1}\left[-e^{-\gamma W_i}\right]. \tag{3}$$

The date 1 price is determined by the market-clearing condition

$$\int_i X_{i1} di = 0. \tag{4}$$

At date 2, the manager chooses investment k given her information. Importantly, the manager does not observe θ directly, but can condition on the information in the stock price P_1. Hence, her information set is $\mathcal{F}_m = \sigma(P_1)$. We consider two natural objectives for the manager. A *cash-flow-maximizing* manager chooses investment to maximize her conditional expectation of the terminal cash flow,

$$k(P_1) = \arg\max_k \mathbb{E}[V|\mathcal{F}_m], \tag{5}$$

while a *price-maximizing* manager chooses investment to maximize her conditional expectation of the date 3 price,

$$k(P_1) = \arg\max_k \mathbb{E}[P_3|\mathcal{F}_m]. \tag{6}$$

As we discuss below, these objectives lead to different investment rules and differ in their effect on investor welfare.

The date 3 price is again determined by the market-clearing condition (4), evaluated at the $t = 3$ trades X_{i3} that maximize investor expected utilities at that date. Note, however, that since the manager's investment decision is perfectly anticipated by investors at date 1, and there are no additional shocks or information, we show that *in equilibrium* the date 3 price is equal to the date

Kacperczyk (2021) and Hsu, Li, and Tsou (2023)). Specifically, as we shall see, green stocks carry a price premium, while brown stocks carry a discount, as a result of their exposure to climate risk. For tractability, we abstract from other sources of systematic risk and focus only on exposure to climate risk.

1 price. At date 4, the firm's terminal cash flows per share V are realized and paid to the investors.

Equilibrium. An equilibrium consists of trades $\{X_{i1}, X_{i3}\}$, prices $\{P_1, P_3\}$, and an investment rule $k(P_1)$ such that (i) the trades X_{it} maximize investor i's expected utility, given her information $\mathcal{F}_{it}$ and the investment rule $k\,(P_1)$, (ii) the investment rule $k(P_1)$ satisfies (5) or (6), and (iii) the equilibrium prices $\{P_1, P_3\}$ are determined by market clearing at dates 1 and 3, respectively.

A. Discussion of Assumptions

The manager's objective. We consider two possible objectives for the manager: cash-flow maximization and price maximization. The former corresponds to the benchmark in the existing feedback effects literature and speaks to the incentives created by compensation linked to earnings and other accounting-based performance metrics that are widely used in practice (e.g., Guay, Kepler, and Tsui (2019), Li and Wang (2016), and Bettis et al. (2018)). The latter corresponds to maximizing the project's risk-adjusted NPV in our setting and speaks to the incentives created by equity compensation. This benchmark is consistent with prior work that builds on the investment capital asset pricing model (CAPM) and q-theory of investment, which typically assumes that the firm invests to maximize its market capitalization (e.g., Cochrane (1991), Liu, Whited, and Zhang (2009)).

Considering the benchmarks separately allows us to provide a sharp comparison of the impact of feedback on investment decisions under these different objectives. Moreover, as we discuss further in Section III, we show that neither objective alone necessarily maximizes welfare, even though in some settings, a combination of the two can be used to do so.

Two trading dates. The assumption of two trading dates is for the sake of exposition. The second trading date does not play a role when the manager maximizes cash flows. When she maximizes price, we expect the results to be the same in a setting without date 3, but in which the manager commits to an investment schedule $k(P)$ to maximize the date 1 price. In this case, the manager commits to exactly the same investment schedule as we characterize because, as we show in Section II.B, this investment schedule solves the relaxed problem of maximizing the expected price by choosing an investment rule that is an arbitrary function of θ and Z.[16]

Homogeneous investor information. Since our primary focus is on *managerial* learning from prices, we shut down *investor* learning from prices by assuming that all investors share a common signal about fundamentals. The assumption simplifies the analysis and ensures that the financial market equilibrium does not exhibit multiplicity of the type studied by Ganguli and Yang (2009). Moreover, this assumption ensures that the traditional Hirshleifer (1971) effect does not arise in our setting, in contrast to results from existing literature (e.g., Marín and Rahi (2000), Dow and Rahi (2003)). Finally, we have confirmed

[16] We thank the editor and a referee for highlighting this equivalence.

that our main results are qualitatively similar when investors have private signals and learn from the price.

Assets in place and divestment decisions. The presence of assets in place is not qualitatively important for our results but aids tractability by ensuring that the firm's cash flows remain uncertain in the absence of investment. Moreover, the assumption that assets in place are uncorrelated with climate risk is made for expositional clarity and can be relaxed.[17] Since the investment decision is binary, one can equivalently apply our analysis to study divestment decisions. For instance, a firm with $k = 1$ and $\alpha < 0$ has an existing negative climate exposure (e.g., a traditional car manufacturer). In this case, a decision of $k = 0$ corresponds to divesting brown technology, or equivalently, investing in green technology to mitigate the firm's existing exposure (e.g., by transitioning to EV technology).

Aggregate demand for hedging. The assumption that Z is stochastic reflects the feature that investors' concern about, and desire to hedge, climate vary over time. For instance, one can interpret news that suggests climate change is accelerating as an increase in Z. This assumption is further consistent with the empirical evidence that aggregate demand for climate hedges varies over time and with economic conditions. For instance, Bolton and Kacperczyk (2021) show that the pricing of carbon transition risk varies across countries and has risen over time. Moreover, Choi, Gao, and Jiang (2020) show that the price premium applied to green versus brown stocks varies with weather patterns, and Alekseev et al. (2021) show that weather patterns influence mutual fund demand for climate-exposed stocks. As we discuss below, this variation generates changes in the discount rate that the manager applies to the project when making her investment decision.

Discrete investment, market incompleteness, and hedging ability. Our model is one of incomplete markets. The firm's investment decision endogenously changes the completeness of the market by allowing investors to trade the climate risk factor (we refer to this as the risk-sharing channel; see Section III). The starkness of this result is a consequence of discrete investment choice, but the economic mechanism arises more generally. Under a continuous investment choice, as the firm invests more, its cash flows are more sensitive to the risk that investors seek to hedge versus the assets in place. All else equal, this makes it less costly for investors to hedge their exposures using the stock, in the sense that they are exposed to less extraneous risk.[18]

A potential concern is that this channel would disappear if markets were complete and investors could trade η_C directly. In practice, markets appear to be far from complete: investors have different exposures to climate risk due to differences in their demographic characteristics and risk preferences

[17] For instance, if A is positively correlated with η_C, one can decompose A as $A = \lambda\eta_C + \varepsilon_A$ for $\lambda > 0$ and $\mathbb{C}(\varepsilon_A, \eta_C) = 0$. In this case, the investment decision still changes the overall exposure of the firm to climate risk (i.e., λ with no investment versus $\lambda + \alpha$ with investment) and the economic forces underlying our analysis continue to operate.

[18] An earlier version of the paper considered more general investment decisions and found that the key economic forces that drive our results obtain in this more general setting.

(e.g., Ilhan et al. (2023)), and they find this risk difficult to hedge (e.g., Krueger, Sautner, and Starks (2020), Pástor, Stambaugh, and Taylor (2021), Giglio, Kelly, and Stroebel (2021)). Indeed, Engle et al. (2020) find that a dynamic equity portfolio optimized to hedge climate risk is at most 30% correlated with news about such risk.[19]

Another potential concern is that the investment decision of a single firm will not have a meaningful effect on market completeness. A multifirm model with discount rate variation and feedback effects is not analytically tractable. However, we expect the impact of climate investment on market completeness to aggregate across firms and thus continue to be relevant in such a setting. That is, one can interpret our model as that of a representative firm in an industry or sector with correlated shocks to profitability and climate exposures. In practice, we expect that correlated investment choices (e.g., several automakers investing in EV technology) should affect investors' ability to hedge climate risk. Moreover, since stock prices do not fully reflect the risk-sharing benefit of climate-sensitive investment, our observation that managers fail to internalize this welfare externality would continue to hold in a multifirm economy.

II. Equilibrium

In general, solving for an equilibrium with feedback effects is complicated by the fact that the asset price must simultaneously clear the market, be consistent with manager and investor beliefs, and be consistent with the anticipated real investment decision. We focus on equilibria of the following form.

DEFINITION 1: A *threshold equilibrium* is one in which:

(i) the price at both dates depends on the underlying random variables through a linear statistic, $s_p = \theta + \frac{1}{\beta}Z$, where β is an endogenous constant,

(ii) the price takes an identical piecewise linear form at both dates,

$$P_3 = P_1 = \begin{cases} A_1 + B_1 s_p & \text{when } s_p > \bar{s} \\ A_0 & \text{when } s_p \le \bar{s} \end{cases}, \tag{7}$$

where the price coefficients A_0, A_1, and B_1 and the threshold $\bar{s}$ are endogenous, and

(iii) the manager invests in the project if and only if $P_1(s_p) \neq P_1(\bar{s})$, that is, the share price is not equal to the constant no-investment price.

This type of equilibrium has an intuitive structure and several desirable properties. First, the equilibrium price is a generalized linear function of

[19] The multidimensional nature of climate risk may also contribute to market incompleteness. Different types of investments may be necessary to hedge the various dimensions of climate risk. For instance, green energy may serve as a hedge of carbon-transition risk, while green real estate may better hedge the potential for sea-level rise.

fundamentals—it depends on θ and Z only through a linear statistic, $s_p = \theta + \frac{1}{\beta}Z$. Second, there is a price level $P_1(\bar{s})$ that reveals to the manager that the market anticipates she will not invest, and, consistent with this, she finds it optimal not to invest. Thus, the price naturally is piecewise linear in s_p, increasing in s_p when the manager invests, and constant when she does not. These properties ensure that the analysis is tractable and facilitate comparison with existing work.

As is common in feedback effects models, in general there can exist multiple equilibria with each characterized by a different investment policy. For instance, if the project is ex-ante sufficiently unprofitable, there is an equilibrium in which investors do not trade on their information and the manager relies on her ex-ante optimal choice, which is not to invest. These equilibria are sustained only because the price does not reveal any information when the market expects the manager not to invest. We focus on the equilibrium with the lowest threshold $\bar{s}$, that is, with the most investment. This equilibrium is the natural one as it would be the unique equilibrium if the price always revealed s_p, which would arise, for instance, if the firm's assets in place were correlated with the payoff on the project. This is a common feature of feedback effects models—Dow, Goldstein, and Guembel (2017) follow a similar approach in choosing among equilibria, selecting the most informative equilibrium (see, e.g., the discussion immediately following their Lemma 1). See also Dow and Gorton (1997), who consider another feedback setting that generally features multiple equilibria.[20]

In the Appendix, we formally solve the model by working backwards. We sketch the approach here. Given an investment decision $k \in \{0, 1\}$ at date 2, investor i's beliefs about the asset payoff at $t = 3$ are conditionally normal, with

$$\mathbb{E}_{i3}\big[V(k)\big] = \mu_A + k(\theta - c),\ \mathbb{C}_{i3}\big(V(k), \eta_C\big) = \frac{k\alpha}{\tau_\eta},\ \text{and}\ \mathbb{V}_{i3}\big(V(k)\big) = \frac{1}{\tau_A} + \frac{k^2}{\tau_\eta}, \quad (8)$$

and hence her optimal trade is

$$X_{i3} = \frac{\mathbb{E}_{i3}\big[V(k)\big] + \gamma \mathbb{C}_{i3}\big(V(k), \eta_C\big) z_i - P_3}{\gamma \mathbb{V}_{i3}\big(V(k)\big)} - (n + X_{i1}). \quad (9)$$

In turn, market clearing implies

$$P_3 = \mu_A - \frac{\gamma}{\tau_A} n + k\left(\theta - c - \frac{\gamma}{\tau_\eta}(n - \alpha Z)\right). \quad (10)$$

[20] Note that if investors in our model also learned noisy information from the equilibrium price (e.g., if they received heterogeneous private signals), then there would be a further potential source of nonuniqueness, even holding fixed the manager's investment policy. As Pálvölgyi and Venter (2015) show, in standard static, noisy rational expectations models, investor learning from prices generally leads to a continuum of discontinuous equilibria in the financial market. Characterizing such equilibria in a version of our model with heterogeneous information would be an interesting problem for future work but is beyond the scope of the current paper.

This immediately implies that, in any equilibrium, regardless of the manager's objective function, we must have $s_p = \theta + \frac{\gamma}{\tau_\eta}\alpha Z$, or equivalently, $\beta = \frac{\tau_\eta}{\gamma\alpha}$.

At date 2, the manager chooses whether to invest to maximize her objective, given her information set $\mathcal{F}_m = \sigma(P_1)$. Below we characterize the equilibrium under cash-flow maximization and price maximization separately. As we will see, these equilibria differ only in the threshold price $P_1(\bar{s})$ above which the manager chooses to invest.

A. Cash-Flow Maximization

The manager's conditional expectation of cash flows, given s_p, is

$$\mathbb{E}\big[V(k)|s_p\big] = \mu_A + k\big(\mathbb{E}\big[\theta|s_p\big] - c\big), \text{ where} \tag{11}$$

$$\mathbb{E}\big[\theta|s_p\big] = \frac{\tau_\theta\mu_\theta + \tau_p\Big(s_p - \frac{\gamma}{\tau_\eta}\alpha\mu_Z\Big)}{\tau_\theta + \tau_p}, \text{ and } \tau_p = \tau_Z\left(\frac{\tau_\eta}{\gamma\alpha}\right)^2. \tag{12}$$

This implies that if the manager were directly able to observe the signal s_p in all states of the world, her optimal investment rule would be

$$k = \begin{cases} 1 & \text{when} \quad \mathbb{E}[\theta|s_p] > c \\ 0 & \text{when} \quad \mathbb{E}[\theta|s_p] \le c. \end{cases} \tag{13}$$

The manager cannot always observe s_p because the price does not vary with s_p when the market expects her not to invest. However, in the threshold equilibrium with the most investment, this creates no additional difficulty for the manager, because the price is a sufficient statistic for s_p in making her investment decision. In this equilibrium, the investment threshold, which we refer to as $\bar{s}_C$, satisfies $\mathbb{E}[\theta|s_p = \bar{s}_C] = c$, so that the manager is indifferent between investing and not investing when $s_p = \bar{s}_C$. Applying (11), we obtain

$$\bar{s}_C = c - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p} + \frac{\gamma}{\tau_\eta}\alpha\mu_Z. \tag{14}$$

Given the conjectured price function, if the manager observes $P_1 = A_0$, she infers that with probability one $s_p \le \bar{s}_C$, and hence chooses not to invest. If she observes any $P_1 \ne A_0$, she infers the realized value of s_p, necessarily strictly greater than $\bar{s}_C$, and so she chooses to invest. Thus, in equilibrium, she is able to implement the same investment rule almost everywhere that she would if she directly observed s_p.

Finally, stepping back to $t = 1$, note that the manager's investment decision is a deterministic function of P_1. Thus, investors can anticipate the manager's investment decision by observing the date 1 price. In turn, *in equilibrium* investors can perfectly anticipate P_3 and therefore the equilibrium price at $t = 1$ must satisfy $P_1 = P_3$ for the market to clear. Following this reasoning, the next proposition characterizes the threshold equilibrium with maximum investment.

PROPOSITION 1: *Suppose that the manager maximizes expected cash flows. In the investment-maximizing threshold equilibrium, equilibrium prices are*

$$P_1 = P_3 = \mu_A - \frac{\gamma n}{\tau_A} + k\left(s_p - c - \frac{\gamma}{\tau_\eta} n\right), \tag{15}$$

and the manager's investment decision is

$$k = \mathbf{1}\left\{P_1 \neq \mu_A - \tfrac{\gamma}{\tau_A} n\right\}, \tag{16}$$

where $s_p \equiv \theta + \frac{\gamma}{\tau_\eta}\alpha Z$, $\tau_p \equiv \left(\frac{\tau_\eta}{\gamma\alpha}\right)^2 \tau_Z$, *and* $\bar{s}_C \equiv c - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p} + \frac{\gamma}{\tau_\eta}\alpha\mu_Z$.

It is worth noting that investors' beliefs about the asset payoff remain normal given their information set in all states of the world, since the manager's investment decision is determined by the date 1 price P_1. This ensures that the equilibrium is tractable.

For a cash-flow-maximizing manager, discount rate variation (i.e., shocks to Z) adds noise to the information about θ that is relevant for her investment decision. Proposition 1 clarifies how the project's greenness affects the manager's inference about cash flows from the price. First, an increase in the project's sensitivity to climate risk (i.e., higher $|\alpha|$) makes the price less informative about cash flows (i.e., decreases forecasting price efficiency [FPE])—this is apparent from the expression for τ_p. Second, since $\mu_Z > 0$, an increase in greenness α leads to a higher threshold $\bar{s}_C$. Intuitively, since a green project provides a hedge to investors, the price P_1 is higher on average, and this leads to a positive bias in the price signal s_p. Since the manager wants to learn about cash flows (θ), she corrects for this bias in her threshold.

Together, these effects reflect that for a cash-flow-maximizing manager, an increase in climate sensitivity makes the price a noisier and more biased signal. As we show in the next subsection, this is no longer the case when the manager chooses investment to maximize the expected price.

B. Price Maximization

We can follow similar steps to derive the equilibrium when the manager maximizes the firm's stock price. Recall that the date 3 market-clearing price can be expressed as

$$P_3 = \begin{cases} \mu_A - \frac{\gamma}{\tau_A} n & \text{when } k = 0 \\ \mu_A - \frac{\gamma}{\tau_A} n + k\left(s_p - c - \frac{\gamma}{\tau_\eta} n\right) & \text{when } k = 1 \end{cases}. \tag{17}$$

This implies that if the manager observed s_p in all states, she would invest when $s_p > c + \frac{\gamma}{\tau_\eta} n$. Similar to the cash-flow-maximization case, in the equilibrium with maximum investment, the price reveals s_p whenever knowing the value of s_p would lead the manager to invest. As a result, she is able to implement the same investment rule that she would if she could directly observe s_p, and so, the investment threshold satisfies

$$\bar{s} = \bar{s}_P = c + \frac{\gamma}{\tau_\eta} n. \tag{18}$$

Finally, the manager's investment decision is again known given the price at date 1, so that no new information arrives between dates 1 and 3, and P_1 and P_3 must be equal. The following proposition formally establishes these results.

PROPOSITION 2: *Suppose that the manager maximizes the expected date 3 price. In the investment-maximizing threshold equilibrium, equilibrium prices are*

$$P_1 = P_3 = \mu_A - \frac{\gamma}{\tau_A} n + k\left(s_p - c - \frac{\gamma}{\tau_\eta} n\right), \tag{19}$$

and the manager's investment decision is

$$k = \mathbf{1}\left\{P_1 \neq \mu_A - \tfrac{\gamma}{\tau_A} n\right\} = \mathbf{1}\{s_p > \bar{s}_P\}, \tag{20}$$

where $s_p = \theta + \frac{\gamma}{\tau_\eta}\alpha Z$ *and* $\bar{s}_P \equiv c + \frac{\gamma}{\tau_\eta} n$.

The manager's optimal investment takes the form of a NPV rule, whereby she invests if and only the statistic

$$NPV \equiv s_p - \bar{s} = \underbrace{\theta - c}_{\text{cash flows}} \; \underbrace{- \tfrac{\gamma}{\tau_\eta}(n - \alpha Z)}_{\text{discount rate}} \tag{21}$$

is greater than zero. The first term, $\theta - c$, reflects the expected cash flows from the project net of investment costs—this captures the "cash-flow news" contained in the price. The second term, $-\frac{\gamma}{\tau_\eta}(n - \alpha Z)$, reflects a discount due to the risk premium investors demand for holding shares of the stock. We refer to this as "discount rate news" because it reflects variation in the project's impact on price that is driven by factors other than its expected cash flows. Consistent with intuition, the discount is higher (the NPV is lower) when the firm is larger (i.e., n is higher) because investors have to bear more aggregate risk. Moreover, the discount is lower (higher) for green (brown) projects when $Z > 0$.[21] This is because green projects reduce investors' exposure to (negative) climate shocks, while brown projects exacerbate it.

While the cash-flow and discount rate news in prices are not separately observable to the manager, they both factor into her decision of whether to invest

[21] It is possible that $Z < 0$ in our model, so that brown projects are priced at a premium. However, the probability of this outcome can be made arbitrarily small by setting μ_Z and τ_Z appropriately.

because they both influence how the project will impact the date 3 price. In principle, this implies that the manager must learn about both from the date 1 price, that is, she must separately compute $\mathbb{E}[\theta|P_1]$ and $\mathbb{E}[Z|P_1]$. However, her inference problem takes a transparent form in our setting because the price signal that she conditions on and the objective she intends to maximize put the same (relative) weights on θ and Z. In particular, the equilibrium date 1 and date 3 prices put the same weights on θ and Z. This implies that the manager does not need to *separately* update on θ and Z to determine whether investment will lead to a higher price. Instead, she can directly infer the relevant combination $\theta + \frac{\gamma}{\tau_\eta}\alpha Z$ from the date 1 price.[22]

Note that this simplification of the manager's learning problem in the case of price maximization is a derived result, not an assumption. In Section III of the Internet Appendix, we show that this result extends to the case in which investors are endowed with dispersed, private noisy signals about θ and learn about θ from prices, similar to Hellwig (1980). The reason is that, in this setting, the date 1 and 3 prices continue to place the same weights on cash flow and discount rate news. However, this result needs not arise when the date 1 price puts different relative weights on θ and Z than the manager's objective does. For instance, the simplification does not obtain if the manager maximizes a combination of expected cash flows (or earnings) and expected price.

Similarly, if a public signal about η_C becomes available before trade at date 3 but after the date 2 investment decision, then the relative weights on θ and Z will differ across the two dates. We focus on the simpler specification without a public signal in our model because it is a natural benchmark that transparently illustrates the main economic mechanisms that result from the manager learning about discount rates from the price. We expect similar forces to apply in richer settings, although the analysis would be less transparent.

The above also clarifies that while feedback plays an important role in the equilibrium, the equilibrium of our specific setting turns out to be identical to one in which the manager directly observes θ and Z. As such, our analysis highlights an important equivalence between a class of feedback effects models in which the manager maximizes the expected price of the firm and traditional, production-based asset pricing models in which the manager is assumed to exogenously know the profitability and discount rate of the project she is considering. To the extent that, in practice, managers rely on prices to learn about discount rate information, our analysis of the price-maximization benchmark provides a microfoundation for the latter class of models. In particular, if investors' exposures to climate risk, which affect the project's discount rate, are heterogeneous across investors and privately known, then prices provide a natural source of such information to managers.

[22] One may be able to capture similar forces with a single trading date if the manager could simultaneously commit to a real investment schedule $k(P)$ to maximize the equilibrium price P at the same time that investors trade.

C. Probability of Investment

In this section, we compare how feedback from prices affects investment decisions under the two managerial objectives. The results below generate testable predictions that relate managerial incentives, the "greenness" of a project, and the probability of investment. We begin by characterizing the likelihood of investment with cash-flow maximization.

PROPOSITION 3: *Suppose that the manager maximizes $\mathbb{E}[V|\mathcal{F}_m]$. In equilibrium, the unconditional probability of investment is given by*

$$\Pr\left(s_p > \bar{s}_C\right) = \Phi\left(\frac{\mathbb{E}\left[s_p\right] - \bar{s}_C}{\sqrt{\mathbb{V}\left[s_p\right]}}\right), \tag{22}$$

where $\mathbb{E}\left[s_p\right] = \mu_\theta + \frac{\gamma}{\tau_\eta}\alpha\mu_Z$, $\mathbb{V}\left[s_p\right] = \frac{1}{\tau_\theta} + \frac{1}{\tau_p}$, $\tau_p = \left(\frac{\tau_\eta}{\gamma\alpha}\right)^2\tau_Z$, and $\Phi(\cdot)$ denotes the cumulative distribution function (CDF) of a standard normal random variable. The probability of investment:

(i) increases with ex-ante profitability $\mu_\theta - c$,

(ii) does not depend on firm size n or the average climate risk exposure μ_Z,

(iii) increases with τ_θ and $|\alpha|$ and decreases with τ_Z if the project is ex-ante profitable (i.e., $\mu_\theta - c > 0$), and

(iv) decreases with τ_θ and $|\alpha|$ and increases with τ_Z if the project is ex-ante unprofitable (i.e., $\mu_\theta - c < 0$).

Consistent with intuition, the probability of investment increases with the ex-ante profitability $\mu_\theta - c$ of the project. Moreover, since the manager's objective is to maximize expected cash flows, the firm's systematic risk (e.g., n) and investors' aggregate exposure to climate risk (i.e., μ_Z) do not affect the likelihood of investment.

The above also clarifies that, for a cash-flow-maximizing manager, variation in the project's risk premium, as captured by $\frac{\gamma}{\tau_\eta}\alpha Z$, generates noise in her price signal. To see this more explicitly, note that if such a manager were to directly observe θ as opposed to the price signal s_p, she would invest if and only if $\theta > c$. In this case, the probability of investment can be expressed as

$$\Pr(\theta > c) = \Phi\left(\frac{\mu_\theta - c}{\sqrt{\mathbb{V}[\theta]}}\right).$$

This depends on the project's ex-ante profitability $\mu_\theta - c$ and prior precision τ_θ in the same manner as when the manager observes only s_p, but is independent of the variation in the project's risk premium (as driven by α and τ_Z).

In contrast, when the manager relies on (noisy) price information about cash flows, the likelihood of investment depends on this premium. In fact, a higher exposure to climate risk (i.e., higher $|\alpha|$) serves to make the price a noisier signal about cash flows, and so, the manager is more likely to invest in line with her prior beliefs. This implies that for ex-ante profitable projects (i.e., if

$\mu_\theta - c > 0$), the manager is more likely to invest in climate-exposed projects than in climate-neutral ones. On the other hand, for unprofitable projects, an increase in climate exposure leads to a decrease in the likelihood of investment.

The results above are largely consistent with traditional feedback effects models in which the manager maximizes expected cash flows and so treats noncash-flow variation in prices as noise. As we show next, this is no longer the case when the manager maximizes the share price.

PROPOSITION 4: *Suppose the manager maximizes $\mathbb{E}[P_3|\mathcal{F}_m]$. In equilibrium, the unconditional probability of investment is given by*

$$\Pr\left(s_p > \bar{s}_P\right) = \Phi\left(\frac{\mathbb{E}\left[s_p\right] - \bar{s}_P}{\sqrt{\mathbb{V}\left[s_p\right]}}\right). \tag{23}$$

The probability of investment:

(i) increases with ex-ante profitability $\mu_\theta - c$,

(ii) decreases with firm size n,

(iii) increases with μ_Z for green firms (i.e., $\alpha > 0$) but decreases with μ_Z for brown firms (i.e., $\alpha < 0$),

(iv) increases with τ_θ and τ_Z if and only if $\mathbb{E}[s_p] - \bar{s}_P = \mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta} > 0$, and

(v) decreases with greenness α if and only if $\left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} - \frac{\tau_\eta\tau_Z}{\gamma\alpha}\frac{1}{\tau_\theta}\mu_Z\right)\operatorname{sgn}(\alpha) > 0$.

Consistent with intuition, Proposition 4 establishes that the probability of investment increases in the expected NPV of the project $\mathbb{E}[s_p] - \bar{s}_P$ and decreases (increases) with the variance of the price signal $\mathbb{V}[s_p]$ when $\mathbb{E}[s_p] - \bar{s}_P > 0$ ($\mathbb{E}[s_p] - \bar{s}_P < 0$). This directly implies parts (i) to (iv) of the proposition. From equation (21), we know that the expected NPV increases with expected profitability $\mu_\theta - c$, decreases with firm size n, and increases with μ_Z if and only if $\alpha > 0$, which implies parts (i) to (iii). Similarly, part (iv) follows because an increase in τ_θ or τ_Z leads to a reduction in the variance of the price signal $\mathbb{V}[s_p]$, which leads to more investment when the expected NPV is positive (i.e., $\mathbb{E}[s_p] - \bar{s}_P > 0$) but less investment when it is negative.

Part (v) of Proposition 4 shows that the project's sensitivity to the risk factor, α, has a nuanced impact on the likelihood that the manager invests. An increase in α has two, potentially offsetting, effects. First, an increase in α increases the expected NPV $\mathbb{E}[s_p] - \bar{s}_P$ because it reduces the on-average discount due to climate risk. Since the manager's objective is to maximize the share price, this implies that all else equal, investment is likelier in green projects than brown projects. We refer to this as the "expected NPV" channel.

Second, an increase in the magnitude of the project's climate exposure $|\alpha|$ increases the variance of the price signal $\mathbb{V}[s_p]$, which, in turn, makes the conditional NPV of the project more variable. All else equal, this makes it more

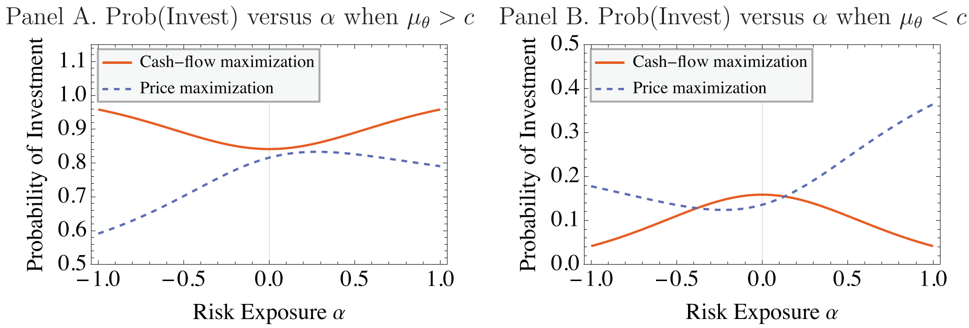

Figure 2. Probability of investment. This figure compares the probability that the firm invests as a function of α and μ_Z under the cash-flow and price-maximization benchmarks. Unless otherwise mentioned, the parameters employed are: $\tau_\theta = \tau_\eta = \tau_A = \gamma = 1,\ \tau_Z = \mu_Z = 0.5$, and $n = 0.1$. Panel A depicts results for projects that have positive ex-ante NPV (i.e., $\mu_\theta > c$), while Panel B considers projects that have negative ex-ante NPV (i.e., $\mu_\theta < c$). In the solid (dashed) lines, we consider price maximization (cash-flow maximization). In Panel A (B), we set $\mu_\theta - c = 1$ ($\mu_\theta - c = -1$), which implies that the project is ex-ante desirable (undesirable) in both the cash-flow and price-maximization cases, that is, $\forall\alpha \in [-1, 1]$, $\mathbb{E}\left[s_p\right] > \bar{s}_P$ and $\mathbb{E}\left[s_p\right] > \bar{s}_C$ ($\mathbb{E}\left[s_p\right] < \bar{s}_P$ and $\mathbb{E}\left[s_p\right] < \bar{s}_C$). (Color figure can be viewed at wileyonlinelibrary.com)

likely that a project with negative expected NPV will be desirable ex-post (i.e., increases the likelihood that the investment option will be "in the money"), and hence increases the likelihood of investment of such a project. Similarly, it reduces the likelihood that a project with positive expected NPV will be ex-post desirable, and thus decreases the likelihood of investment in such a project. We refer to this as the "variance of NPV" channel. The overall effect of α depends on the relative magnitude of these two channels.

As Figure 2 illustrates, this is in sharp contrast to the case in which the manager maximizes cash flows. The figure compares the probability of investment as a function of climate exposure α for the two managerial objectives. Consistent with the above results, for ex-ante profitable projects (i.e., $\mu_\theta > c$), an increase in $|\alpha|$ leads to more investment under cash-flow maximization but can lead to less investment under price maximization—see Panel A. In contrast, for ex-ante unprofitable projects (i.e., $\mu_\theta < c$), Panel B illustrates that the opposite results hold.

The characterization of the equilibrium thresholds under the two managerial objectives immediately gives us the following result.

COROLLARY 1: *Cash-flow maximization leads to more investment than price maximization (i.e., $\bar{s}_P > \bar{s}_C$) if and only if*

$$\frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z) > -\frac{\tau_\theta}{\tau_p}(\mu_\theta - c). \tag{24}$$

In particular, cash-flow maximization leads to "overinvestment" relative to price maximization when the project is expected to be highly profitable ex-ante (i.e., $\mu_\theta - c$ is sufficiently high), investors' expected climate exposures are small (i.e., μ_Z is low), or the project is sufficiently brown (i.e., α is small or negative).[23]

More generally, the price- and cash-flow-maximization benchmarks can be thought of as lying on the opposite end of a spectrum. While we focus on these two extremes to simplify the exposition and clarify the intuition for our results, we expect that in practice a manager's decision reflects a weighted average of both considerations. Importantly, our analysis implies that when the manager focuses more on prices and less on cash flows, she will treat prices as less noisy signals and place more weight on them when making investment decisions.

Our results also imply that how shareholders or regulators can incentivize managers to pursue green investment depends on the ex-ante desirability of the projects. For ex-ante unprofitable projects (i.e., $\mu_\theta < c$), tilting the manager's incentives toward price maximization (e.g., by proving more short-term, price-sensitive compensation) increases the likelihood of investing in green projects. This is likely to apply to speculative investments in green technology, which may be ex-ante unprofitable on a purely cash-flow basis. In contrast, for ex-ante profitable projects (i.e., $\mu_\theta > c$), making compensation more sensitive to accounting-based measures of expected cash flows (e.g., earnings) tilts the manager towards cash-flow maximization and consequently increases investment in green projects.

III. Welfare

In this section, we explore the relationship between feedback, investment, and investor welfare. We begin in Section III.A by characterizing the channels through which investment affects investor welfare. In Section III.B, we consider the special case in which investors have homogeneous climate exposures (i.e., $\tau_\zeta \to \infty$). This allows us to explicitly characterize the welfare-maximizing price-contingent investment rule and compare it to price-maximizing and cash-flow-maximizing rules. In Section III.C, we reintroduce heterogeneity in risk exposures and show how the manager's use of the information in price may harm investor welfare.

A. The Impact of Investment on Welfare

Existing models of feedback effects focus on the impact that feedback has on a firm's expected cash flows. In many such models, investors are risk-neutral so that maximizing expected cash flows aligns with welfare maximization.[24] However, in our model, investor risk aversion implies that investment has

[23] Note that when investors are risk-neutral (i.e., $\gamma = 0$), $\bar{s}_P = \bar{s}_C = c$ and so the investment rules coincide.

[24] Section IV discusses notable exceptions, like Dow and Rahi (2003).

multiple, potentially offsetting, effects on investor welfare, due to the riskiness of the project and the stock's usefulness as a hedge.

Because investors are ex-ante symmetric, the ex-ante expected utility of an arbitrary investor is an unambiguous measure of welfare,

$$\mathcal{W} \equiv \mathbb{E}\left[-e^{-\gamma W_i(k(s_p))}\right] \tag{25}$$

$$= \Pr(k=1)\mathbb{E}\left[-e^{-\gamma W_i(1)}|k=1\right] + \Pr(k=0)\mathbb{E}\left[-e^{-\gamma W_i(0)}|k=0\right], \tag{26}$$

where

$$W_i(k) = \begin{cases} X_i(V(1)-P) + nV(1) - z_i\eta_C & k=1 \\ nV(0) - z_i\eta_C & k=0 \end{cases}. \tag{27}$$

Proposition I.A10 in Section I of the Internet Appendix characterizes this expression in closed form. However, to understand the relevant economic forces, it is helpful to study the simpler special case in which investment is *fixed* at arbitrary level k, in which case the model reduces to a standard unconditionally linear-normal form. In this case, we have

$$\mathbb{E}\left[-e^{-\gamma W_i(k)}\right] = -e^{-\gamma\, CE(k)}, \tag{28}$$

where the certainty equivalent $CE(k)$ can be expressed, after grouping terms, as

$$CE(k) = \underbrace{\mathbb{E}[V(k)]n}_{\text{Cash flow channel}} \underbrace{-\frac{\gamma}{2}\left(\frac{1}{\tau_A} + k^2\left(\frac{1}{\tau_\theta} + \frac{1-\alpha^2}{\tau_\eta}\right)\right)n^2}_{\text{Nonclimate risk channel}}$$
$$\underbrace{-\frac{\gamma}{2}\frac{1}{\tau_\eta}(\mu_Z - k\alpha n)^2}_{\text{Climate risk channel}}(1+\Gamma) - \frac{1}{\gamma}\log(D(k)), \tag{29}$$

where

$$D(k) = \underbrace{\sqrt{\frac{\frac{1}{\tau_A} + k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A} + k^2\left(\frac{1}{\tau_\eta} + \frac{1}{\beta^2(\tau_Z+\tau_\zeta)}\right)}}}_{\text{Value of information}} \sqrt{\frac{\Gamma}{\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)}} \tag{30}$$

and

$$\Gamma(k) \equiv \frac{\gamma^2 \frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)}{1 - \gamma^2 \frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)\left(1 - \underbrace{\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)}{\frac{1}{\tau_A} + k^2\left(\frac{1}{\beta^2(\tau_Z+\tau_\zeta)} + \frac{1}{\tau_\eta}\right)} \times \left(\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}}\right)^2}_{\text{Risk-sharing channel}}\right)}. \tag{31}$$

We label all five terms in these expressions that depend on the investment choice k and we discuss them in turn.

- The *cash flow channel* reflects that investment k affects the investor's expected wealth via their ownership of n shares. Investment increases (decreases) welfare through this channel when the project's expected cash flows are positive (negative).
- The *nonclimate risk channel* reflects that the investment k increases investors' exposure to nonclimate risks via the θ and η_I shocks.
- The *climate risk channel* captures the fact that the investment k affects investors' aggregate exposure to climate shocks. The average investor's total climate exposure is given by $\mu_Z - k\alpha n$, which reflects both the direct exposure and the exposure through ownership of the stock. When the direct exposure is sufficiently large (i.e., $\mu_Z > n$), investment in green projects ($\alpha > 0$) decreases aggregate climate exposure and hence increases welfare, while investment in brown projects ($\alpha < 0$) increases aggregate climate exposure and decreases welfare. This channel is further scaled by the term $1 + \Gamma$, which reflects *uncertainty* about the exposure to climate risk. When investors' total exposure to climate risk $Z + \zeta_i$ is constant (i.e., τ_Z, $\tau_\zeta \to \infty$), we have $\Gamma = 0$. However, when investors face uncertainty about their exposure from either source, we have $\Gamma > 0$, which amplifies the disutility of climate exposure.
- The *risk-sharing channel* reflects that the project enables investors to share their idiosyncratic exposures to climate risk, ζ_i, by trading the stock. All else equal, investment improves welfare through this channel. By sharing risk, investors reduce the dispersion in their climate exposures, reducing the effect of uncertainty about exposures, Γ.
  The overall amount of risk-sharing reflects both the effectiveness of the stock as a hedging instrument (i.e., the correlation of the stock return with climate risk), and the proportion of climate exposures that are shared (i.e.,

the proportion of climate exposures that are idiosyncratic, ζ_i),

$$\text{Risk-sharing channel} = \underbrace{\frac{k^2\alpha^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}}_{\substack{\text{Hedging effectiveness of stock}\\ =\text{Corr}^2(V-P,\eta_C|z_i)}} \times \underbrace{\left(\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^2}_{\%\text{ shareable climate exposure}}. \tag{32}$$

- The *value of information channel* captures the fact that investors' information about cash flows renders the stock more useful in hedging. Observing θ increases the conditional correlation between the stock's payoff and η_C. Moreover, this effect is relevant only when the project is undertaken, and so disappears when $k = 0$. This takes the familiar form of the ratio of investors' conditional variance of the asset return with and without conditioning on θ.[25]

Importantly, when the manager chooses investment to maximize the expected price, she fails to appropriately account for the impact of her decision on the other components of welfare, as we discuss next.

B. Homogeneous Risk Exposures

We begin with the special case of our model in which all investors have homogeneous exposures to climate risk. In this case, we can explicitly characterize the welfare-maximizing price-contingent investment rule, as we show in the following proposition.

PROPOSITION 5: *Suppose that investors have identical exposures to climate risk (i.e., $\tau_\zeta \to \infty$). Then, the welfare-maximizing investment policy is*

$$\arg\max_{k\in\{0,1\}} \mathcal{W}(k;\theta,Z) = \arg\max_{k\in\{0,1\}} \mathcal{W}(k;s_p) = \mathbf{1}\{s_p > \bar{s}_W\}, \tag{33}$$

where $\bar{s}_W \equiv c + \frac{1}{2}\frac{\gamma}{\tau_\eta}n$. Moreover,

[25] In our model, investors are endowed with information. However, this term still captures the improvement in utility as a result of observing θ relative to being uninformed. Specifically, given fixed k, this ratio can be represented as $\frac{\mathbb{V}(V|\theta,z_i,P)}{\mathbb{V}(V-P|z_i)}$, which reflects the proportional improvement in expected utility from conditioning on θ, z_i, and P versus z_i alone. The welfare expressions in Bond and Garcia (2022) include a similar term, which they further decompose into a product of the classic value of cash-flow information, $\frac{\mathbb{V}(V|\theta,z_i,P)}{\mathbb{V}(V|z_i,P)}$, and the value of providing versus demanding liquidity (i.e., using a price-contingent schedule versus not), $\frac{\mathbb{V}(V|z_i,P)}{\mathbb{V}(V-P|z_i)}$. Because these effects are not a primary focus of our analysis, we choose to concisely represent them in a single term.

(i) *cash-flow maximization leads to underinvestment relative to welfare maximization if and only if*

$$\bar{s}_C - \bar{s}_W = \frac{\gamma}{\tau_\eta}\alpha\mu_Z - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p} - \frac{1}{2}\frac{\gamma}{\tau_\eta}n > 0, \tag{34}$$

but overinvestment otherwise, and

(ii) *price maximization always leads to underinvestment relative to welfare maximization, since* $\bar{s}_P - \bar{s}_W = \frac{1}{2}\frac{\gamma}{\tau_\eta}n$.

As we show in the Appendix, the expressions for welfare simplify considerably when investors have homogeneous exposures to climate risk because there is no trade in equilibrium. Consequently, neither the risk-sharing channel nor the value of information channel are operational. In this case, we show that the welfare-maximizing investment rule, conditional on full information (i.e., knowledge of θ and Z), depends on θ and Z only through the linear combination s_p and thus coincides with the welfare-maximizing s_p-contingent (i.e., price-contingent) investment rule.[26] Importantly, a fully-informed manager would optimally put the same weights on θ and Z as the equilibrium price does if her objective were to maximize welfare.

However, as Proposition 5 clarifies, the cash-flow-maximizing investment rule does not maximize welfare. In particular, cash-flow maximization tends to lead to underinvestment in green projects but overinvestment in brown projects that are ex-ante profitable, relative to welfare maximization. Using the investment thresholds, we can characterize the values of θ for which the manager invests in each case. Specifically, for a given Z, the above implies that under welfare maximization, the manager invests if and only if

$$\theta > c + \frac{1}{2}\frac{\gamma}{\tau_\eta}n - \frac{\gamma}{\tau_\eta}\alpha Z \equiv \theta_W^*, \tag{35}$$

while under cash-flow maximization the manager invests if and only if

$$\theta > c + \frac{\gamma}{\tau_\eta}\alpha(\mu_Z - Z) - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p} \equiv \theta_c. \tag{36}$$

As a result, cash-flow maximization with noisy (price) information leads to underinvestment relative to welfare maximization if and only if $\theta_c - \theta_W^* > 0$.

To gain intuition into what drives this difference, recall that if the manager perfectly observes θ and maximizes cash flows, she invests if and only if $\theta > c$.

[26] To streamline the presentation and derivation, we formulate the investment rule as s_p-contingent. However, as in the baseline model, it can be implemented as a price-contingent rule. Intuitively, with probability 1, the equilibrium price reveals s_p when investors anticipate that the investment is undertaken and does not reveal s_p otherwise. This allows one to directly map the s_p-contingent investment rule to an equivalent price-contingent rule under which the manager does not invest if she observes a price realization that anticipates no investment, $P_1 = \mu_A - \frac{\gamma}{\tau_A}n$, and invests otherwise.

Thus, the distortion in investment decisions stems from two factors: (i) the misalignment between the manager's preferences and those of investors, even when the manager is fully informed, and (ii) the noise in the information that price conveys about cash flows to the manager. Specifically, note that

$$\theta_c - \theta_W^* = \underbrace{c - \theta_W^*}_{\substack{\text{impact of different}\\ \text{preferences}}} + \underbrace{\theta_c - c}_{\substack{\text{impact of noisy}\\ \text{price information}}}, \tag{37}$$

where

$$c - \theta_W^* = \frac{\gamma}{\tau_\eta}\left(\alpha Z - \frac{1}{2}n\right) \quad \text{and} \quad \theta_c - c = \frac{\gamma}{\tau_\eta}\alpha(\mu_Z - Z) - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p}. \tag{38}$$

When $c - \theta_W^* > 0$, which holds when αZ is large relative to n, the difference in the manager's preference pushes her to underinvest relative to the welfare-maximizing level. Intuitively, the manager disregards both the welfare impact of the investment's climate exposure (determined by its greenness α and the aggregate climate exposure Z), and its nonclimate risk (determined by n). Taken together, this pushes her to underinvest when the project is sufficiently green and investors' climate exposures are large, and to overinvest otherwise.

Next, when $\theta_c - c > 0$, noisy learning from prices pushes toward underinvestment. Specifically, relative to the full-information cash-flow-maximizing benchmark, noisy learning drives the manager to overinvest in ex-ante attractive projects (i.e., $\mu_\theta - c > 0$) but underinvest in ex-ante unattractive projects (i.e., $\mu_\theta - c < 0$). This is because the manager continues to weight her prior beliefs when deciding whether to invest. Moreover, when the climate risk premium is higher than expected (i.e., $Z > \mu_Z$), noisy learning leads to overinvestment in green projects and underinvestment in brown projects, relative to the full-information cash-flow-maximizing benchmark.

Interestingly, these two sources of distortion partly offset each other via their dependence on Z. The noisy price information wedge $\theta_c - c$ reflects the fact that a cash-flow-maximizing manager would like to ignore Z when choosing investment but cannot do so because she can only condition on the noisy price signal s_p, which is driven in part by αZ. However, this is beneficial from a welfare perspective because it implies that the manager is more likely to invest when αZ is higher, which is when the investment is more valuable as a climate hedge (as captured in the difference in preference wedge $c - \theta_W^*$).

Somewhat surprisingly, we find that the price-maximizing rule also leads to underinvestment relative to the welfare-maximizing rule whenever $n > 0$. Note that in this case, there is no distortion from noisy information—as previously discussed, the date 1 price puts the same weights on θ and Z, as does her objective. The difference between the two thresholds stems from the fact that, while welfare depends on the average risk borne by investors, the price reflects the marginal disutility from the risk of holding the last outstanding share of the firm. Since the marginal disutility from holding the last share is higher than the average disutility from bearing the risk of all shares that investors hold, the price-maximizing rule leads to underinvestment relative to the welfare-maximizing rule. However, it is worth noting that this difference disappears when the per-capita endowment of shares per firm becomes arbitrarily small (i.e., $n \to 0$).

The point that decisions based on prices may be socially suboptimal because prices reflect marginal and not average valuations was first raised by Spence (1975) in the context of a monopolist's choice of product quality. He shows that the chosen quality may be too high or too low from the perspective of social welfare. The intuition for this result is analogous to ours: quality is chosen based on information contained in a good's price, which reflects the valuation of the marginal consumer, while welfare depends on information about the average consumer.[27]

Proposition 5 also implies that, in this benchmark, one can implement the welfare-maximizing investment rule by inducing the manager to maximize a weighted average of cash flows and the date 3 price, as summarized by the following result.

PROPOSITION 6: *Suppose that the manager maximizes a weighted average of expected price and expected cash flows,*

$$k(P_1) = \arg\max_k \delta\, \mathbb{E}[P_3|P_1] + (1-\delta)\mathbb{E}[V|P_1], \tag{39}$$

where

$$\delta = \frac{\frac{\tau_\theta}{\tau_\theta+\tau_p}(\mu_\theta - c) - \frac{\tau_p}{\tau_\theta+\tau_p}\frac{\gamma}{\tau_\eta}\left(\alpha\mu_Z - \frac{1}{2}n\right)}{\frac{\tau_\theta}{\tau_\theta+\tau_p}(\mu_\theta - c) - \frac{\tau_p}{\tau_\theta+\tau_p}\frac{\gamma}{\tau_\eta}\left(\alpha\mu_Z - \frac{1}{2}n\right) + \frac{1}{2}\gamma\frac{1}{\tau_\eta}n}. \tag{40}$$

Then, in the maximum-investment threshold equilibrium, the manager invests if and only if $s_p > \bar{s}_W$. When the project is unexposed to the climate ($\alpha = 0$), we have that $\delta = \frac{1}{2}$. Moreover, $\delta \in (0,1)$ if and only if $\frac{\tau_\theta}{\tau_\theta+\tau_p}(\mu_\theta - c) > \frac{\tau_p}{\tau_\theta+\tau_p}\frac{\gamma}{\tau_\eta}\left(\alpha\mu_Z - \frac{1}{2}n\right)$.

Recall that price maximization leads to underinvestment relative to welfare maximization, but cash-flow maximization can lead to overinvestment for brown ($\alpha < 0$), ex-ante profitable ($\mu_\theta > c$), projects. In such cases, the above result implies that there exists a $\delta \in (0,1)$ such that a weighted-average objective of the form in (39) leads the manager to follow a welfare-maximizing investment rule. In particular, by incentivizing the manager to maximize a weighted average of expected price and expected cash flows, where δ is set as in (40), investors can ensure that the manager's investment rule maximizes ex-ante welfare.

[27] An analogous difference is also highlighted by Levit, Malenko, and Maug (2022), who show that while prices are determined by the valuation of the marginal investor, valuation is determined by the valuation of the average (posttrade) shareholder in their setting. Bernhardt, Liu, and Marquez (2018) highlight a similar difference in the context of takeovers.

However, the above result also implies that such compensation schemes may not be appropriate when the manager is considering whether to invest in green projects. For instance, consider a green project with $\mu_\theta = c$. If $2\alpha\mu_Z > n > \alpha\mu_Z$, Proposition 6 shows that welfare maximization requires the manager to place a negative weight on the price (i.e., $\delta < 0$). However, if $n < \frac{2\alpha\mu_Z\tau_\eta^2\tau_z}{\alpha^2\gamma^2\tau_\theta + 2\tau_\eta^2\tau_z}$, welfare maximization requires the manager to place a negative weight on cash flows (i.e., $\delta > 1$). Intuitively, both price maximization and cash-flow maximization lead to underinvestment, so the welfare-maximizing combination puts *negative* weight on the objective that leads to greater underinvestment. However, such negative sensitivity to prices or earnings is difficult to implement in practice. Moreover, traditional compensation schemes that put positive weights on prices and earnings-based incentives may actually lead to lower investor welfare relative to exclusively focusing on one type of objective or the other.

C. Heterogeneous Risk Exposures

The previous discussion illustrates that even when investors have identical climate exposures, neither cash-flow maximization nor price maximization are generally equivalent to welfare maximization. These differences are further amplified when investors have heterogeneous climate exposures.

When the manager maximizes expected cash flows, she does not account for either the nonclimate risk channel or the climate risk channel. Heterogeneity in investors' climate exposures amplifies the effect that her neglect of the climate risk channel has on welfare. Intuitively, as can be seen from the expression for the certainty equivalent in (29), this heterogeneity amplifies the disutility that climate risk creates. Specifically, one can show that the amplification factor Γ increases in τ_ζ^{-1}, and so, the project's impact on welfare via aggregate climate risk rises with τ_ζ^{-1}.

To gain intuition for the price-maximization case, note that the share price $P(k)$ can be expressed as

$$P(k) = \mathbb{E}_i[V] + \gamma Z \mathbb{C}_i[V, \eta_C] - \gamma n \mathbb{V}_i[V]. \tag{41}$$

This expression reveals that the price reflects the aggregate climate exposure, Z, but does not reflect the diversity in climate exposures (i.e., τ_ζ^{-1}), which determines the gains from sharing climate risk (i.e., the risk-sharing channel). Similarly, the price does not reflect the value of information channel because it does not capture the additional hedging benefit that investors gain from having observed θ in the event that the manager invests (i.e., when $k = 1$). Because each of these channels improves welfare, this implies that a price-maximizing manager tends to underinvest in climate-sensitive projects relative to a welfare-maximizing rule. Finally, to reiterate, heterogeneity in exposures, as captured by τ_ζ^{-1}, amplifies the welfare effect of the climate risk channel. Thus, the price also does not fully account for the climate risk channel, leading to underinvestment in green projects, which reduce aggregate climate risk, and overinvestment in brown projects, which increase it.

While we are not able to analytically characterize the welfare-maximizing s_p-dependent investment rule in the general heterogeneous exposures case, we can establish that if the firm is arbitrarily small (i.e., $n \to 0$), then the welfare-maximizing rule is to *always* invest. We record this in the following proposition.

PROPOSITION 7: *Suppose that the share endowment is zero ($n = 0$) and exposures are heterogeneous $\frac{1}{\tau_\zeta} > 0$. Then, the welfare-maximizing s_p-dependent investment policy is to always invest, that is,*

$$\arg\max_{k \in \{0,1\}} \mathcal{W}\big(k; s_p\big) = 1. \tag{42}$$

Hence, both cash-flow maximization and price maximization lead to underinvestment relative to welfare maximization in any states in which they lead the manager to not invest.

The intuition for this result is straightforward. When the firm is in zero supply, investment affects welfare only through the risk-sharing and value of information channels. Moreover, this implies that, irrespective of the information revealed by s_p, investors strictly prefer that the firm takes the project, so that the firm's shares are useful for sharing risk. This result further clarifies that traditional managerial incentives can be misaligned relative to welfare maximization, even if the investment decision has no effect on aggregate expected cash flows or aggregate risk, when investors have heterogeneous risk exposures. The effect of investment on risk-sharing can be sufficient to lead investment to be socially suboptimal.

The misalignment between the manager's objective and investor welfare implies that feedback from prices need not always improve welfare. To formalize this intuition, we compare investor welfare to a benchmark in which the manager ignores the information in price and instead chooses to maximize the *ex-ante expectation* of cash flows or the share price. In this case, the manager invests if and only if the unconditional expectation of the price signal s_p exceeds the corresponding threshold $\bar{s} \in \{\bar{s}_C, \bar{s}_P\}$.

The next proposition characterizes sufficient conditions under which feedback reduces welfare.

PROPOSITION 8: *Suppose that the no-feedback investment policy is $k = 1$ (i.e., $\mathbb{E}[s_p] > \bar{s}$ for the relevant threshold $\bar{s} \in \{\bar{s}_C, \bar{s}_P\}$) and the project is exposed to climate risk (i.e., $\alpha \neq 0$). Then feedback reduces welfare if*

(i) n is sufficiently small, or

(ii) gains from risk-sharing are sufficiently large (i.e., τ_ζ is sufficiently small).

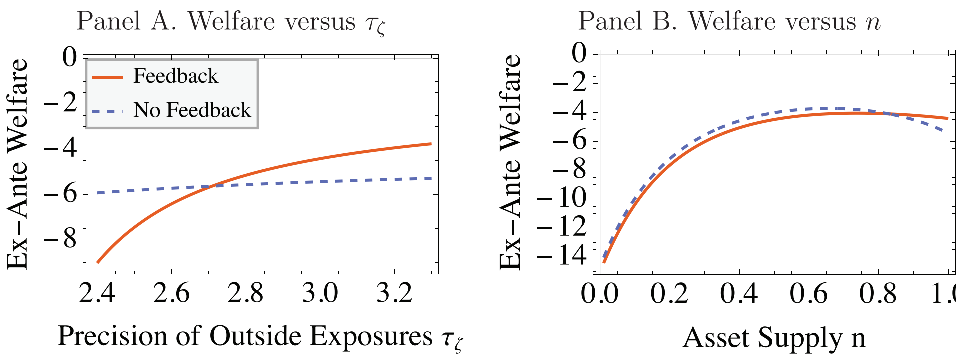

Figure 3. Ex-ante welfare: Feedback versus no feedback. This figure plots ex-ante welfare (i.e., ex-ante expected utility) as a function of τ_ζ and n for a project with positive expected NPV. Unless otherwise mentioned, the parameters employed are: $\tau_\theta = 0.5,\ \tau_\zeta = 3,\ \tau_Z = 2,\ \mu_A = 0,\ \tau_A = 5$, and $\mu_\theta = c = \tau_\eta = \gamma = \mu_Z = n = \alpha = 1$. These parameters ensure that the expected NPV of the project is positive. (Color figure can be viewed at wileyonlinelibrary.com)

Figure 3 illustrates these results for the case in which the manager maximizes the price. The intuition is as follows.[28] When the ex-ante NPV of the project is positive, in the no-feedback benchmark, the manager always invests. In contrast, given feedback, the firm does not invest for $s_p \leq \bar{s}_P$. On the one hand, feedback improves the expected price of the stock, which tends to improve welfare through the cash-flow channel. On the other hand, because it leads to no investment in some states, feedback reduces the ability of investors to use the stock as a hedge, and so reduces welfare via the risk-sharing channel. It also affects the aggregate exposure to nonclimate and climate risk (with the direction depending on the sign of α).

When the per-capita endowment of shares n is small, the cash flow and nonclimate risk channels are relatively small. Moreover, the firm's investment decision has a small effect on the aggregate climate exposure, and so, the climate risk channel is muted. However, the risk-sharing channel remains important since it is unaffected by n: regardless of the firm's size, its stock remains a useful hedge in the event of investment. Consequently, the risk-sharing channel dominates, and investors are better off with a rule that always invests, yielding hedging benefits in all states of the world. Analogously, when τ_ζ is small, investors' exposures to the climate are highly diverse, so that the ability to share risk provides them with large welfare gains. Hence, the risk-sharing channel dominates in the limit, once again leading investors to prefer an investment rule that ensures that the asset is always useful for hedging.

It is worth noting that the manager's use of price information *always* increases the firm's expected cash flows (share price) under cash-flow maximization (price maximization): any additional information that she infers from the price can only improve investment efficiency as measured by her objective

[28] The economic intuition for cash-flow maximization is analogous.

function. As a result, Proposition 8 implies that an increase in investment efficiency need not align with an improvement in investor welfare.

D. Implications for Managerial Compensation

Our welfare results speak to the recent debate on the effectiveness of the use of climate-risk metrics in executive compensation. On the one hand, there has been a rapid increase in the use of such measures. Edmans (June 27, 2021) cites that "51% of large U.S. companies and 45% of leading U.K. firms use ESG metrics in their incentive plans," and Hill (November 14, 2021) cites a survey conducted by Deloitte in September 2021 that suggests "24 per cent of companies polled expected to link their long-term incentive plans for executives to net zero or climate measures over the next two years." [29]

On the other hand, there is ample skepticism about the effectiveness of such incentives. In addition to issues around the measurement and monitoring of such objectives and the possibility of unintended consequences, Edmans (June 27, 2021) argues that incentivizing environmental, social, and governance (ESG) performance may not necessarily lead to better financial performance. Instead, he advocates for the use of long-term stock-based compensation, arguing that "[s]ince material ESG factors ultimately improve the long-term stock price, this holds CEOs accountable for material ESG issues – even if they aren't directly measurable."

Our analysis suggests that this may not be true because the stock price (even in the long term) does not fully account for the benefit of investing in climate-exposed projects. As such, providing additional incentives based on climate metrics (e.g., bonuses linked to climate targets) can improve overall investor welfare. This is despite the fact that such incentives may decrease stock prices and future profitability on average by leading to inefficient overinvestment (from the perspective of a price-maximization or cash-flow maximization objective) in green projects. Yet, when investors have diverse climate risk exposures and find it difficult to hedge these exposures, such incentives improve their ability to hedge risks and consequently can improve overall welfare.

IV. Related Literature

Our paper adds to the literature on feedback effects (see Bond, Edmans, and Goldstein (2012) and Goldstein (2023) for recent surveys and early work by Khanna, Slezak, and Bradley (1994), Subrahmanyam and Titman (2001), and others). In contrast to our setting, much of this literature focuses on economies in which (i) investors are risk-neutral or the stock price is set by a risk-neutral market maker, (ii) the noise in prices arises due to noise traders with unmodeled utility functions, and (iii) the manager's investment choice

[29] More broadly, Edmans, Gosling, and Jenter (2023) find that over 50% of surveyed directors and investors report that offering variable pay to the CEO is useful to help "motivate the CEO to improve outcomes other than long-term shareholder value."

maximizes the firm's expected terminal cash flow. As a result, such models are not well suited to study how discount rate variation affects investment decisions or how feedback affects investor welfare.[30] To our knowledge, our paper is the first to develop a model of feedback effects in which managers learn about not only cash flows but also discount rates from prices, even though prior work alludes to this channel (Diamond (1967)).

Bond, Edmans, and Goldstein (2012) highlight the important distinction between FPE, which measures how well prices predict future cash flows, and revelatory price efficiency (RPE), which captures how useful prices are for real investment decisions. While in many settings, more informative prices lead to better investment decisions, a key takeaway of their analysis is that, in some cases, RPE may be low even when FPE is high. Our analysis provides an instance in which the opposite is true: with price maximization, we show that feedback raises investment efficiency and so RPE is high even through FPE may be low since prices are noisy signals of cash flows.

The most closely related papers in this literature are Dow and Rahi (2003), Hapnes (2020), and Gervais and Strobl (2021). Dow and Rahi (2003) explore how increases in informed trading affect investment efficiency and risk-sharing in a setting in which investors are risk-averse but prices are set by a risk-neutral market maker. They argue that investment efficiency always improves with more informed trading, but risk-sharing may either worsen due to the Hirshleifer (1971) effect or improve when information decreases uncertainty over the component of the asset's payoffs that is unrelated to the component that investors wish to hedge. Hapnes (2020) characterizes managerial investment behavior and investor information acquisition in a Grossman and Stiglitz (1980)-type model with feedback; however, the analysis does not study the effect of feedback on welfare. Gervais and Strobl (2021) consider the impact of informed, active money management on investment decisions in a setting with feedback. They study how the gross and net performance of the actively managed fund compares with the market portfolio and study how the presence of an informed money manager affects welfare.

We view our analysis as complementary. We focus on how investment in a project affects the risk exposure of a firm's cash flows, which, in turn, affects how useful the stock is for hedging. This highlights a novel channel through which feedback affects welfare: intuitively, firms' investment decisions *endogenously* affect the degree of market completeness in the economy.[31] Also, since investors are identically informed in our analysis, the traditional Hirshleifer

[30] See Diamond and Verrecchia (1981), Wang (1994), Schneider (2009), Ganguli and Yang (2009), Manzano and Vives (2011), and Bond and Garcia (2022) for models in which noise is driven by hedging needs as in our model. Existing feedback models with risk-neutral pricing include Dow, Goldstein, and Guembel (2017), Davis and Gondhi (2019), and Goldstein, Schneemeier, and Yang (2020).

[31] This also distinguishes our analysis from Marín and Rahi (1999, 2000) and Eckwert and Zilcha (2003), who consider how exogenous differences in market completeness influence investor welfare.

(1971) effect is turned off, which allows us to clearly distinguish our novel channel from earlier work.[32]

Our focus on welfare is also complementary to recent work by Bond and Garcia (2022), who show that while indexing may reduce price efficiency, it improves retail investor welfare due to improvements in risk-sharing. Bond and Garcia (2022) also make substantial progress on characterizing welfare in CARA-Normal settings, which we leverage in our derivations. Tension between notions of firm profitability and welfare also appears in Goldstein and Yang (2022), who show that improvements in price informativeness increase producer profits due to better-informed real investment, but may harm welfare by destroying risk-sharing opportunities, similar to the Hirshleifer (1971) effect. Similar to our findings, other papers studying discrete investment choice also emphasize the importance of the firm's "default" investment decision in the absence of feedback.[33] Our analysis complements this earlier work by identifying a novel tension between managerial investment choices and welfare that is driven by how investment affects the ability of investors to use the stock to hedge risk.

Our paper is also related to the growing theoretical literature on ESG investing and climate risk.[34] Our work is most closely related to Pástor, Stambaugh, and Taylor (2021) and Goldstein et al. (2021). Pástor, Stambaugh, and Taylor (2021) show that green assets have lower costs of capital because investors enjoy holding them and they hedge climate risk. Goldstein et al. (2021) consider a model in which traditional and green investors are informed about a firm's financial and ESG output, and demonstrate that this can lead to multiple equilibria. Our setting generates distinct predictions for green investment decisions and welfare by incorporating the feedback effect and considering green investment's impact on risk-sharing efficiency.

The production-based asset pricing literature beginning with Cochrane (1991) also considers how variation in firms' discount rates affects the relationship between investment, expected cash flows, and expected returns. This work assumes that a manager knows not only her project's risk factor loadings,

[32] While the Hirshleifer (1971) effect and our risk-sharing channel both affect the ability of investors to share risk, the two mechanisms are distinct. The Hirshleifer (1971) effect refers to the phenomenon whereby the introduction of public information destroys risk-sharing opportunities. In contrast, our risk-sharing channel captures the fact that endogenous investment decisions can affect the effective completeness of the market by directly changing the risk exposures of traded securities.

[33] For instance, Dow, Goldstein, and Guembel (2017) show that investors' equilibrium information acquisition hinges on whether the firm defaults to a risky or a riskless project. Davis and Gondhi (2019) show that complementarity in learning depends on both the default investment decision and the correlation between the investment and assets in place. Goldstein, Schneemeier, and Yang (2020) study information acquisition in a feedback model with multiple sources of uncertainty. They show that investors seek to acquire the same information as management for positive NPV projects, but different information for negative NPV projects.

[34] Additional studies include Heinkel, Kraus, and Zechner (2001), Friedman and Heinle (2016), Chowdhry, Davies, and Waters (2019), Oehmke and Opp (2020), Pedersen, Fitzgibbons, and Pomorski (2021), and Jagannathan et al. (2023).

but also the conditional risk premia associated with these factors. However, in practice, factor risk premia depend on dispersed information (e.g., investors' risk exposures and preferences) and thus are difficult for managers to observe directly. Instead, prices are a crucial source of information about discount rates. Our analysis explores the implications of such managerial learning on investment decisions and investor welfare.

V. Conclusions

In this paper, we develop a model of informational feedback effects in which a firm's investment alters its exposure to an aggregate risk, and we discuss its application to climate-exposed investment. When a firm invests in a project that is exposed to climate risk, it affects how useful the asset is as a hedge for climate risk. As a result, the firm's stock price reflects information about investors' climate exposures and the project's expected cash flows, which are both relevant to the manager's investment choice. We show that this has novel implications for how a project's greenness affects the likelihood of investment, conditional expected returns and future profitability. Moreover, we show that because the price does not fully reflect the welfare externality generated by investment in climate-sensitive projects, price-maximization tends to lead to underinvestment in green projects.

In addition to climate-exposed investments, our model's predictions on investment and managerial incentives apply broadly to investments that are exposed to systematic risks with variable risk premia. For instance, investments that are exposed to commodity prices may serve as inflation hedges and thus may have discount rates that vary with investors' aggregate inflation concerns. Moreover, investments in emerging markets are exposed to aggregate demand in those markets and so are likely to have discount rates that vary with uncertainty over this demand. Our model's implications for feedback's impact on welfare also apply more generally, whenever the market is incomplete with respect to the investment's risk exposure.

A notable contribution of our analysis is to provide a tractable feedback effects framework with investor risk-aversion and priced risk factors. Immediate extensions include generalizations to the structure of cash flows and information. For instance, allowing for both public and private information signals would enable future research to assess the merits of disclosure regarding firms' climate risk exposures. Similarly, introducing multiple dimensions of fundamentals as in Goldstein and Yang (2019) and Goldstein, Schneemeier, and Yang (2020) could enable future work to assess how climate-exposed investments interact with the other risks that firms face. Finally, it may be interesting to consider how dynamics and multiple traded assets influence managers' ability to infer discount rate information from prices.

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Appendix: Proofs

A. Proof of Proposition 1

We first establish the existence of the stated equilibrium. We then argue that, among all threshold equilibria, the stated equilibrium involves the most investment. Begin by conjecturing an equilibrium of the form posited in the text. That is, suppose that there is a random variable of the form $s_p = \theta + \frac{1}{\beta}Z$ and a threshold $\bar{s} \in \mathbb{R}$ such that the asset prices at the two trading dates are identical and take the form

$$P_1 = P_3 = \begin{cases} A_1 + B_1 s_p & s_p > \bar{s} \\ A_0 & s_p \leq \bar{s} \end{cases}. \tag{A.1}$$

We can now derive the equilibrium, and confirm the above conjecture, by working backwards. At date $t = 3$, investors can observe the actual investment decision made at $t = 2$. Hence, they perceive the asset payoff as conditionally normally distributed with conditional moments

$$\mathbb{E}_{i3}[V(k)] = \mathbb{E}_{i3}[A + k(\theta + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I - c)] = \mu_A + k(\theta - c), \tag{A.2}$$

$$\mathbb{C}_{i3}(V(k), \eta_C) = k\alpha\frac{1}{\tau_\eta}, \tag{A.3}$$

$$\mathbb{V}_{i3}(V(k)) = \frac{1}{\tau_A} + k^2\frac{1}{\tau_\eta}. \tag{A.4}$$

An arbitrary investor i solves the following static optimization problem at this date:

$$\max_{x \in \mathbb{R}} \mathbb{E}_{i3}[-e^{-\gamma W_{i4}}]. \tag{A.5}$$

Given her demand x, the investor's terminal wealth W_{i4} is

$$W_{i4} = (n + X_{i1} + x)V - xP_3 - X_{i1}P_1 - z_i\eta_C, \tag{A.6}$$

where X_{i1}, the trade from the $t = 1$ trading round, is taken as given.

Applying well-known results for CARA utility, this problem leads to a standard mean-variance demand function:

$$X_{i3} = \frac{\mathbb{E}_{i3}[V(k)] + \gamma\mathbb{C}_{i3}(V(k), \eta_C)z_i - P_3}{\gamma\mathbb{V}_{i3}(V(k))} - (n + X_{i1}). \tag{A.7}$$

Plugging in for the conditional moments from above and enforcing market clearing yields the equilibrium price

$$P_3 = \mu_A + k(\theta - c) + \gamma k\alpha\frac{1}{\tau_\eta}Z - \gamma\left(\frac{1}{\tau_A} + k^2\frac{1}{\tau_\eta}\right)n \tag{A.8}$$

$$= \mu_A - \gamma \tfrac{1}{\tau_A} n + k\Big(\theta + \gamma\alpha \tfrac{1}{\tau_\eta} Z - c - \gamma \tfrac{1}{\tau_\eta} n\Big), \tag{A.9}$$

where the second line collects terms and uses the fact that $k \in \{0, 1\}$ implies $k = k^2$ to simplify. Hence, to be consistent with our initial conjecture, the endogenous signal s_p must have coefficient $\frac{1}{\beta} = \frac{\gamma\alpha}{\tau_\eta}$ on Z. To be consistent with our conjecture, the price coefficients must satisfy

$$A_0 = \mu_A - \gamma \tfrac{1}{\tau_A} n, \tag{A.10}$$

$$A_1 = \mu_A - \gamma \tfrac{1}{\tau_A} n - c - \gamma \tfrac{1}{\tau_\eta} n, \tag{A.11}$$

$$B_1 = 1. \tag{A.12}$$

Stepping back to $t = 2$, the manager's problem is to solve

$$\max_{k \in \{0,1\}} \mathbb{E}[V(k)|P_1], \tag{A.13}$$

where she can condition on the first-period price, P_1. The optimal investment is therefore

$$k = \begin{cases} 1 & \mathbb{E}[\theta|P_1] > c \\ 0 & \mathbb{E}[\theta|P_1] \le c \end{cases}. \tag{A.14}$$

Now, let $\bar{s}_C$ denote the level of s_p such that the manager would be indifferent to investing and not investing if she observed s_p, that is,

$$\mathbb{E}[\theta|s_p = \bar{s}_C] - c = 0.$$

Because $\mathbb{E}[\theta|s_p] = \mu_\theta + \frac{\tau_p}{\tau_\theta + \tau_p}\Big(s_p - \mu_\theta - \frac{1}{\beta}\mu_Z\Big)$, with $\tau_p \equiv \beta^2 \tau_Z$, we have

$$\begin{aligned} \mathbb{E}[\theta|s_p = \bar{s}_C] - c = 0 \Leftrightarrow \bar{s}_C &= \mu_\theta + \frac{\gamma\alpha}{\tau_\eta}\mu_Z - \frac{\tau_\theta + \tau_p}{\tau_p}(\mu_\theta - c) \\ &= c - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p} + \frac{\gamma}{\tau_\eta}\alpha\mu_Z. \end{aligned}$$

We claim that the threshold $\bar{s} = \bar{s}_C$ is consistent with our conjectured equilibrium, that is, the manager invests if and only if $s_p > \bar{s}_C$. Under such a threshold, when the manager observes A_0, she knows s_p lies below $\bar{s}_C$ with probability 1, and so infers that it is suboptimal to invest. In contrast, when she observes $P_1 \neq A_0$, she infers s_p and knows that s_p lies above $\bar{s}_C$, and so chooses to invest. Thus, the investment threshold $\bar{s}_C$ is indeed consistent with conjectured form of equilibrium.

Stepping back to $t = 1$, the problem of an arbitrary investor is

$$\max_{x\in\mathbb{R}} \mathbb{E}_{i1}[-e^{-\gamma W_{i4}}],$$

where her terminal wealth is

$$W_{i4} = (n + x + X_{i3})V - X_{i3}P_3 - xP_1 - z_i\eta_C$$

and where the optimal $t = 3$ demand X_{i3} was derived above. Given the functional form for P_3, the realization of P_3 is perfectly anticipated under the investor's information set $\mathcal{F}_{i1} = \sigma(\theta, z_i, P_1)$. Hence, to rule out arbitrage, the price must satisfy $P_1 = P_3$, and consequently, all investors are indifferent to trading at $t = 1$ at this equilibrium price. Thus, we have now shown that the equilibrium stated in the proposition exists.

Finally, we argue that this equilibrium maximizes investment over all possible threshold equilibria. Suppose by contradiction that there were an equilibrium with a lower investment threshold $\bar{s} < \bar{s}_C$. Then, the date 1 price would reveal s_p to the manager for $s_p \in (\bar{s}, \bar{s}_C)$ and the manager would invest for such s_p. However, by the definition of $\bar{s}_C$, investment reduces expected cash flows in this region, and so, the manager could improve the expected cash flows by deviating to not investing when she observes s_p in this region. This contradicts the purported existence of an equilibrium with $\bar{s} < \bar{s}_C$ and hence establishes the claim. □

B. *Proof of Proposition 2*

The proof proceeds similarly to the proof of Proposition 1 above. Again, we begin by conjecturing asset prices in the two trading dates that take the form in (A.1). At date $t = 3$, investors can observe the actual investment decision made at $t = 2$. Hence, the date 3 equilibrium, given the manager's investment choice, follows exactly as in the previous proof: they perceive the asset payoff as conditionally normally distributed with conditional moments as in equations (A.2) to (A.4), their optimal demands take the form in (A.7), the date 3 price takes the form in (A.9), and the endogenous signal s_p again must have coefficient $\frac{1}{\beta} = \frac{\gamma\alpha}{\tau_\eta}$ on Z.

Stepping back to $t = 2$, the manager's problem is now to solve

$$\max_{k\in\{0,1\}} \mathbb{E}[P_3|P_1], \tag{A.15}$$

where she can condition on the first-period asset price, P_1. Using the expression for P_3 derived in the first step of the proof, the manager's problem reduces to

$$\max_{k\in\{0,1\}} k\mathbb{E}\left[s_p - c - \gamma\frac{1}{\tau_\eta}n\middle|P_1\right]. \tag{A.16}$$

The optimal investment is therefore

$$k = \begin{cases} 1 & \mathbb{E}[s_p|P_1] > c + \gamma \frac{1}{\tau_\eta} n \\ 0 & \mathbb{E}[s_p|P_1] \le c + \gamma \frac{1}{\tau_\eta} n \end{cases}. \tag{A.17}$$

Note that the threshold $\bar{s}_P$, defined by $\bar{s}_P \equiv c + \gamma \frac{1}{\tau_\eta} n$, is the value such that if the manager always observed s_p, she would invest if and only if $s_p > \bar{s}_P$. We claim that this threshold is consistent with our conjectured equilibrium. To see this, note that if the manager observes $P_1 = A_0$, she infers $s_p \le \bar{s}_P$, and so, she chooses not to invest. In contrast, if she observes any $P_1 \neq A_0$, she infers the realized value of s_p, necessarily strictly greater than $\bar{s}_P$, and therefore finds it optimal to invest. Hence, the investment threshold $\bar{s}_P$ is indeed consistent with our initial conjecture.

Stepping back to $t = 1$, as in the prior proof, since the manager's investment decision is a function of P_1, investors can anticipate k given the price. Thus, they can perfectly anticipate the date 3 price, and, to rule out arbitrage, the price must satisfy $P_1 = P_3$. Consequently, all investors are indifferent to trading at $t = 1$ at this equilibrium price. This completes the construction of the equilibrium.

This equilibrium maximizes investment over all possible threshold equilibria. Suppose by contradiction that there were an equilibrium with a lower investment threshold $\bar{s} < \bar{s}_P$. Then, the price would reveal s_p to the manager for $s_p \in (\bar{s}, \bar{s}_P)$. Moreover, by the definition of $\bar{s}_P$, investment lowers price on this region, and so, the manager prefers to deviate to not investing when observing s_p in this region. □

C. *Proof of Proposition 3*

The probability of investment is given by

$$\Pr\left(s_p > \bar{s}_C\right) = 1 - \Phi\left(\frac{\bar{s}_C - \mathbb{E}[s_p]}{\mathbb{V}[s_p]}\right) = \Phi\left(\frac{\mathbb{E}\left[s_p\right] - \bar{s}_C}{\sqrt{\mathbb{V}\left[s_p\right]}}\right) \tag{A.18}$$

$$= \Phi\left(\frac{\tau_\theta\left(\frac{1}{\tau_\theta} + \frac{1}{\tau_p}\right)(\mu_\theta - c)}{\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}}\right) \tag{A.19}$$

$$= \Phi\left(\tau_\theta \sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}(\mu_\theta - c)\right). \tag{A.20}$$

Recalling that $\tau_p = \beta^2 \tau_Z = \left(\frac{\tau_\eta}{\gamma\alpha}\right)^2 \tau_Z$, direct inspection immediately yields the claimed results. □

D. Proof of Proposition 4

The probability of investment is given by

$$\Pr\left(s_p > \bar{s}\right) = 1 - \Phi\left(\frac{\bar{s}_P - \mathbb{E}[s_p]}{\mathbb{V}[s_p]}\right) = \Phi\left(\frac{\mathbb{E}\left[s_p\right] - \bar{s}_P}{\sqrt{\mathbb{V}\left[s_p\right]}}\right) \tag{A.21}$$

$$= \Phi\left(\frac{\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right). \tag{A.22}$$

This immediately implies that the probability of investment is increasing in $\mu_\theta - c$, decreasing in n, and increasing in μ_Z. Moreover, for any arbitrary parameter b, after applying the monotonic transformation $\Phi^{-1}(\cdot)$ and using the definition $NPV = \theta - c - \gamma \frac{1}{\tau_\eta}(n - \alpha Z)$ from the text to condense notation, we have

$$\frac{\partial}{\partial b}\Pr\left(s_p > \bar{s}\right) \propto \frac{\partial}{\partial b}\frac{\mu_\theta - c - \dfrac{\gamma n}{\tau_\eta} + \dfrac{\alpha\gamma\mu_Z}{\tau_\eta}}{\sqrt{\dfrac{1}{\tau_\theta} + \left(\dfrac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} \tag{A.23}$$

$$= \frac{\partial}{\partial b}\frac{\mathbb{E}[NPV]}{\sqrt{\mathbb{V}(NPV)}} \tag{A.24}$$

$$= \frac{\sqrt{\mathbb{V}(NPV)}\,\frac{\partial}{\partial b}\mathbb{E}[NPV] - \mathbb{E}[NPV]\frac{\partial}{\partial b}\sqrt{\mathbb{V}(NPV)}}{\mathbb{V}(NPV)} \tag{A.25}$$

$$= \frac{|\mathbb{E}[NPV]|}{\sqrt{\mathbb{V}(NPV)}}\left(\frac{\frac{\partial}{\partial b}\mathbb{E}[NPV]}{|\mathbb{E}[NPV]|} - \operatorname{sgn}(\mathbb{E}[NPV])\frac{\frac{\partial}{\partial b}\sqrt{\mathbb{V}(NPV)}}{\sqrt{\mathbb{V}(NPV)}}\right) \tag{A.26}$$

$$= \frac{|\mathbb{E}[NPV]|}{\sqrt{\mathbb{V}(NPV)}}\left(\frac{\frac{\partial}{\partial b}\mathbb{E}[NPV]}{|\mathbb{E}[NPV]|} - \frac{1}{2}\operatorname{sgn}(\mathbb{E}[NPV])\frac{\frac{\partial}{\partial b}\mathbb{V}(NPV)}{\mathbb{V}(NPV)}\right). \tag{A.27}$$

For α, we have

$$\frac{\partial}{\partial\alpha}\Pr\left(s_p > \bar{s}\right) \propto \left(\frac{\partial}{\partial\alpha}\mathbb{E}[NPV] - \frac{1}{2}\mathbb{E}[NPV]\frac{\frac{\partial}{\partial\alpha}\mathbb{V}(NPV)}{\mathbb{V}(NPV)}\right) \tag{A.28}$$

$$= \frac{\gamma\mu_Z}{\tau_\eta} - \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta}\right)\frac{\frac{1}{\alpha}\left(\dfrac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\dfrac{1}{\tau_\theta} + \left(\dfrac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} \tag{A.29}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\frac{\gamma\mu_Z}{\tau_\eta}\frac{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}-\left(\mu_\theta-c-\frac{\gamma n}{\tau_\eta}+\frac{\alpha\gamma\mu_Z}{\tau_\eta}\right)\right) \tag{A.30}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\frac{\alpha\gamma\mu_Z}{\tau_\eta}\frac{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}-\left(\mu_\theta-c-\frac{\gamma n}{\tau_\eta}+\frac{\alpha\gamma\mu_Z}{\tau_\eta}\right)\right) \tag{A.31}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\frac{\alpha\gamma\mu_Z}{\tau_\eta}\left(1+\frac{\frac{1}{\tau_\theta}}{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\right)-\left(\mu_\theta-c-\frac{\gamma n}{\tau_\eta}+\frac{\alpha\gamma\mu_Z}{\tau_\eta}\right)\right) \tag{A.32}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\left(\frac{\frac{1}{\tau_\theta}}{\left(\frac{\alpha\gamma}{\tau_\eta}\right)\frac{1}{\tau_Z}}\right)\mu_Z-\left(\mu_\theta-c-\frac{\gamma n}{\tau_\eta}\right)\right) \tag{A.33}$$

$$= -\frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\mu_\theta-c-\frac{\gamma n}{\tau_\eta}-\tfrac{\tau_\eta\tau_Z}{\alpha\gamma}\tfrac{1}{\tau_\theta}\mu_Z\right), \tag{A.34}$$

which implies

$$\frac{\partial}{\partial\alpha}\Pr\left(s_p>\bar{s}\right)<0 \Leftrightarrow \operatorname{sgn}(\alpha)\left(\mu_\theta-c-\frac{\gamma n}{\tau_\eta}-\tfrac{\tau_\eta\tau_Z}{\alpha\gamma}\tfrac{1}{\tau_\theta}\mu_Z\right)>0. \tag{A.35}$$

Moreover, note that because the parameters $\tau \in \{\tau_Z, \tau_\theta\}$ do not enter the expected NPV and increases in these τ strictly decrease the variance of the NPV, we have

$$\frac{\partial}{\partial\tau_Z}\Pr\left(s_p>\bar{s}\right), \frac{\partial}{\partial\tau_\theta}\Pr\left(s_p\geq\bar{s}\right) \propto -\tfrac{1}{2}\operatorname{sgn}\left(\mathbb{E}[NPV]\right)\frac{\frac{\partial}{\partial\tau}\mathbb{V}(NPV)}{\mathbb{V}(NPV)} \tag{A.36}$$

$$\propto \operatorname{sgn}\left(\mathbb{E}[NPV]\right), \tag{A.37}$$

so that the dependence is pinned down by the sign of the expected NPV, which immediately establishes the claimed result. □

E. Proof of Proposition 5

To establish the claim on the welfare-maximizing policy, we compute the conditional expected utility of an arbitrary trader given (θ, Z), show that the investment rule that maximizes this utility is the one stated in the proposition, and note that this rule depends on (θ, Z) only through the linear combination $s_p = \theta + \frac{\gamma\alpha}{\tau_\eta} Z$. Hence, it can be implemented as an s_p-contingent (i.e., price-contingent) rule.

The conditional expected utility given (θ, Z) follows from the expression for investors' expected utility in equation (I.A16), after recognizing that in the case of homogeneous exposures, we have that the nontradeable endowment is $z_i = Z$ for all i, and consequently, the (θ, Z) information set is precisely the investor information set. However, we can derive the expression more directly by noting that in the case of homogeneous exposures, there is no trade in equilibrium and so investors simply hold their initial endowment of n shares. Hence, the realized utility of an arbitrary trader is

$$-e^{-\gamma(nV - Z\eta_C)} = -e^{-\gamma n(A + k(\theta + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I) - c) + \gamma Z \eta_C}. \tag{A.38}$$

The conditional expectation of this expression given (θ, Z) is

$$\mathcal{W}(k;\theta, Z) \equiv \mathbb{E}\left[-e^{-\gamma n(A + k(\theta + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I) - c) + \gamma Z \eta_C} \middle| \theta, Z \right] \tag{A.39}$$

$$= -e^{-\gamma(\mu_A + k(\theta - c))n + \frac{1}{2}\gamma^2\left(\frac{1}{\tau_A} + k^2\frac{1}{\tau_\eta}\right)n^2 - \gamma^2 kn\frac{\alpha}{\tau_\eta}Z + \frac{1}{2}\gamma^2\frac{1}{\tau_\eta}Z^2} \tag{A.40}$$

$$= -e^{-\gamma\left(\mu_A + k\left(\theta + \frac{\gamma\alpha}{\tau_\eta}Z - c\right)\right)n + \frac{1}{2}\gamma^2\left(\frac{1}{\tau_A} + k^2\frac{1}{\tau_\eta}\right)n^2 + \frac{1}{2}\gamma^2\frac{1}{\tau_\eta}Z^2} \tag{A.41}$$

$$= -e^{-\gamma(\mu_A + k(s_p - c))n + \frac{1}{2}\gamma^2\left(\frac{1}{\tau_A} + k^2\frac{1}{\tau_\eta}\right)n^2 + \frac{1}{2}\gamma^2\frac{1}{\tau_\eta}Z^2}, \tag{A.42}$$

where the second line uses standard expressions for computing the expectation of exponential-quadratic forms of normally distributed random variables, the third line groups terms, and the final line recognizes that $s_p = \theta + \frac{\gamma\alpha}{\tau_\eta} Z$.

Now, since we have $k^2 = k$ for $k \in \{0, 1\}$, it follows that the investment $k \in \{0, 1\}$ that maximizes $\mathcal{W}(k;\theta, Z)$ is

$$\begin{aligned} k(\cdot) &= \arg\max_{k\in\{0,1\}} \left(\gamma\left(\mu_A + k(s_p - c)\right)n - \frac{1}{2}\gamma^2\left(\frac{1}{\tau_A} + k\frac{1}{\tau_\eta}\right)n^2 - \frac{1}{2}\gamma^2\frac{1}{\tau_\eta}Z^2 \right) \\ &= \mathbf{1}\left\{ s_p - c - \frac{1}{2}\gamma\frac{1}{\tau_\eta}n > 0 \right\}. \end{aligned}$$

Defining $\bar{s}_W \equiv c + \frac{1}{2}\frac{\gamma}{\tau_\eta}n$ delivers the investment rule in the proposition. Because this rule depends on (θ, Z) only through s_p, it can be implemented as an s_p-contingent rule.

Furthermore, using the expressions for the price-maximizing threshold, $\bar{s} = c + \frac{\gamma}{\tau_\eta}n$, from Proposition 2, and the cash-flow-maximizing threshold, $\bar{s}_C = c - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p} + \frac{\gamma}{\tau_\eta}\alpha\mu_Z$, from equation (14), yields the expressions for $\bar{s} - \bar{s}_W$ and $\bar{s}_C - \bar{s}_W$. The claims about over- and underinvestment are immediate given the signs of these expressions. □

F. Proof of Proposition 6

Given our expressions for P_1 and P_3 in a threshold equilibrium, we can write

$$\delta\mathbb{E}[P_3|P_1] + (1-\delta)\mathbb{E}[V|P_1] \tag{A.43}$$

$$= \begin{array}{l}\delta\Big(\mu_A - \gamma\frac{1}{\tau_A}n + k(s_p - c - \gamma\frac{1}{\tau_\eta}n)\Big)+ \\ (1-\delta)\Big[\mu_A + k\Big(\mu_\theta - c + \frac{\beta^2\tau_Z}{\tau_\theta+\beta^2\tau_Z}(s_p - \mathbb{E}[s_p])\Big)\Big].\end{array} \tag{A.44}$$

The $k \in \{0, 1\}$ that maximizes this expression is

$$k(s_p) = \mathbf{1}\left\{\delta\left(s_p - c - \gamma\frac{1}{\tau_\eta}n\right) + (1-\delta)\left(\mu_\theta - c + \frac{\tau_p}{\tau_\theta+\tau_p}(s_p - \mathbb{E}[s_p])\right) > 0\right\} \tag{A.45}$$

$$= \mathbf{1}\left\{s_p > \frac{1}{\delta + (1-\delta)\frac{\tau_p}{\tau_\theta+\tau_p}}\left(\delta\left(c + \gamma\frac{1}{\tau_\eta}n\right) + (1-\delta)\left(c - \mu_\theta + \frac{\tau_p}{\tau_\theta+\tau_p}\mathbb{E}[s_p]\right)\right)\right\}. \tag{A.46}$$

Setting the threshold in this expression equal to $\bar{s}_W = c + \frac{1}{2}\gamma\frac{1}{\tau_\eta}n$ and solving for δ yields

$$\delta = \frac{\mu_\theta - c - \frac{\tau_p}{\tau_\theta+\tau_p}\mathbb{E}[s_p] + \frac{\tau_p}{\tau_\theta+\tau_p}\bar{s}_W}{\mu_\theta + \gamma\frac{1}{\tau_\eta}n - \frac{\tau_p}{\tau_\theta+\tau_p}\mathbb{E}[s_p] - \frac{\tau_\theta}{\tau_\theta+\tau_p}\bar{s}_W}$$

$$= \frac{\mu_\theta - c - \frac{\tau_p}{\tau_\theta+\tau_p}\left(\mu_\theta + \frac{1}{\beta}\mu_Z\right) + \frac{\tau_p}{\tau_\theta+\tau_p}\left(c + \frac{1}{2}\gamma\frac{1}{\tau_\eta}n\right)}{\mu_\theta + \gamma\frac{1}{\tau_\eta}n - \frac{\tau_p}{\tau_\theta+\tau_p}\left(\mu_\theta + \frac{1}{\beta}\mu_Z\right) - \frac{\tau_\theta}{\tau_\theta+\tau_p}\left(c + \frac{1}{2}\gamma\frac{1}{\tau_\eta}n\right)}$$

$$= \frac{\frac{\tau_\theta}{\tau_\theta+\tau_p}(\mu_\theta - c) - \frac{\tau_p}{\tau_\theta+\tau_p}\left(\frac{1}{\beta}\mu_Z - \frac{1}{2}\gamma\frac{1}{\tau_\eta}n\right)}{\frac{\tau_\theta}{\tau_\theta+\tau_p}(\mu_\theta - c) - \frac{\tau_p}{\tau_\theta+\tau_p}\left(\frac{1}{\beta}\mu_Z - \frac{1}{2}\gamma\frac{1}{\tau_\eta}n\right) + \frac{1}{2}\gamma\frac{1}{\tau_\eta}n}.$$

After substituting in $\frac{1}{\beta} = \frac{\gamma\alpha}{\tau_\eta}$ and $\tau_p = \beta^2\tau_Z$, this matches the expression in the proposition. If $\alpha = 0$, then $\frac{1}{\beta} \to 0$ and $\tau_p \to \infty$, and thus, this expression reduces to

$$\delta = \frac{\frac{1}{2}\gamma\frac{1}{\tau_\eta}n}{\frac{1}{2}\gamma\frac{1}{\tau_\eta}n + \frac{1}{2}\gamma\frac{1}{\tau_\eta}n} = \frac{1}{2}, \tag{A.47}$$

as claimed. More generally, note that one can express $\delta = \frac{\omega}{\omega+\upsilon}$, where $\omega = \frac{\tau_\theta}{\tau_\theta+\tau_p}(\mu_\theta - c) - \frac{\tau_p}{\tau_\theta+\tau_p}\left(\frac{1}{\beta}\mu_Z - \frac{1}{2}\gamma\frac{1}{\tau_\eta}n\right)$ and $\upsilon = \frac{1}{2}\gamma\frac{1}{\tau_\eta}n > 0$. This implies $\delta \in (0, 1)$ if and only if $\omega > 0$. □

G. Proof of Proposition 7

Consider the conditional welfare expression from Proposition I.A9 for an arbitrary investment k and price signal realization s_p. We will show that this expression is strictly increasing in k for $k \in [0, 1]$, from which it follows that the welfare-maximizing investment is $k = 1$.

For $n = 0$, the conditional welfare is

$$\mathcal{W}(k; s_p) = -D(k; s_p)\exp\{Q(k; s_p)\}, \tag{A.48}$$

where

$$Q(k; s_p) = \frac{1}{2}\gamma^2 \frac{1}{\tau_\eta}\mathbb{E}_p^2[Z]\left(1 + \Gamma(k; s_p)\right),$$

and where the determinant term D and the function Γ do not depend on n and are as given in Proposition I.A9:

$$D(k; s_p) = \sqrt{\frac{\frac{1}{\tau_A} + k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A} + k^2\left(\mathbb{V}_p(\theta|z_i) + \frac{1}{\tau_\eta}\right)}}\sqrt{\frac{\Gamma(k; s_p)}{\gamma^2\frac{1}{\tau_\eta}\left(\mathbb{V}_p(Z) + \frac{1}{\tau_\zeta}\right)}}$$

$$\Gamma(k; s_p) = \gamma^2\frac{1}{\tau_\eta}\left(\mathbb{V}_p(Z) + \frac{1}{\tau_\zeta}\right)$$
$$\times\left(1 - \gamma^2\frac{1}{\tau_\eta}\left(\mathbb{V}_p(Z) + \frac{1}{\tau_\zeta}\right)\left(1 - \frac{k^2\alpha^2\frac{1}{\tau_\eta}\left(\frac{\frac{1}{\tau_\zeta}}{\mathbb{V}_p(Z)+\frac{1}{\tau_\zeta}}\right)^2}{\frac{1}{\tau_A} + k^2\left(\mathbb{V}_p(\theta|z_i) + \frac{1}{\tau_\eta}\right)}\right)\right)^{-1}.$$

Because of the negative sign in front of the expression in equation (A.48), to show that conditional expected utility increases in k, it suffices to show that the functions Q and D are decreasing functions of k for $k \in [0, 1]$. Moreover, because k enters these expressions only in terms of k^2, it suffices to characterize their behavior as functions of k^2 for $k^2 \in [0, 1]$.

Consider first the function Γ, which appears in both Q and D. We have

$$\frac{\partial}{\partial k^2}\frac{k^2\alpha^2\frac{1}{\tau_\eta}\left(\frac{\frac{1}{\tau_\zeta}}{\mathbb{V}_p(Z)+\frac{1}{\tau_\zeta}}\right)^2}{\frac{1}{\tau_A} + k^2\left(\mathbb{V}_p(\theta|z_i) + \frac{1}{\tau_\eta}\right)} \tag{A.49}$$

$$= \frac{\frac{1}{\tau_A}\alpha^2\frac{1}{\tau_\eta}\left(\frac{\frac{1}{\tau_\zeta}}{\mathbb{V}_p(Z)+\frac{1}{\tau_\zeta}}\right)^2}{\left(\frac{1}{\tau_A} + k^2\left(\mathbb{V}_p(\theta|z_i) + \frac{1}{\tau_\eta}\right)\right)^2}, \tag{A.50}$$

which is strictly positive when $\frac{1}{\tau_\zeta} > 0$, from which it follows that Γ is *decreasing* in k^2, and consequently, Q is decreasing in k^2.

Considering D, it remains only to show that the term $\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\mathbb{V}_p(\theta|z_i)+\frac{1}{\tau_\eta}\right)}$ is decreasing since we have already shown that Γ is decreasing. We have

$$\frac{\partial}{\partial k^2}\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\mathbb{V}_p(\theta|z_i)+\frac{1}{\tau_\eta}\right)} = \frac{-\frac{1}{\tau_A}\mathbb{V}_p(\theta|z_i)}{\left(\frac{1}{\tau_A}+k^2\left(\mathbb{V}_p(\theta|z_i)+\frac{1}{\tau_\eta}\right)\right)^2}, \tag{A.51}$$

which is strictly negative when $\frac{1}{\tau_\zeta} > 0$. Hence, we have verified that investors are strictly better off when $k=1$ than $k=0$ for a given price signal realization s_p. Because this holds for all realizations of s_p, it follows that the ex-ante welfare-maximizing policy is to always invest. □

H. *Proof of Proposition 8*

Consider either threshold $\bar{s} \in \{\bar{s}_C, \bar{s}_P\}$. If the unconditional policy is to invest (i.e., $\mathbb{E}[s_p] - \bar{s} > 0$), then a manager who does not condition on price optimally invests in all states of the world, leading to "no feedback" investment $k_{NF} \equiv 1$. Hence, to establish that feedback reduces welfare, it suffices to show that welfare is higher with $k_{NF} = 1$ than with the threshold policy $k(s_p) = \mathbf{1}_{\{s_p > \bar{s}\}}$.

The small n limit in the proposition follows immediately from Proposition 7 and continuity of the expected utility in n since Proposition 7 establishes that the welfare-maximizing investment policy for $n = 0$ is for the manager to always invest. Hence, because feedback causes the manager to *not* invest with strictly positive probability, feedback strictly reduces ex-ante welfare.

The τ_ζ limit is easier to establish using the unconditional welfare expression in Proposition I.A10 directly. Establishing the limit as $\tau_\zeta \downarrow$, is equivalent to establishing the limit as $1/\tau_\zeta \uparrow$. For unconditional expected utility to exist, we must have $\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}} - \gamma^2\frac{1}{\tau_\eta} > 0 \Leftrightarrow 0 \leq \frac{1}{\tau_\zeta} < \frac{\tau_\eta}{\gamma^2} - \frac{1}{\tau_Z}$. Hence, the relevant limit is $\frac{1}{\tau_\zeta} \uparrow \frac{\tau_\eta}{\gamma^2} - \frac{1}{\tau_Z}$. Using the unconditional welfare from Proposition I.A10, welfare under the no-feedback investment level $k_{NF} = 1$ is higher than under the feedback policy $k(s_p) = \mathbf{1}_{\{s_p > \bar{s}\}}$ if and only if

$$\begin{aligned} -D(1)\exp\{Q(1)\} > &-\Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\{Q(0)\} \\ &-\left(1-\Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)\right)D(1)\exp\{Q(1)\} \\ \Leftrightarrow \Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\{Q(0)\} > &\,\Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)D(1)\exp\{Q(1)\}. \end{aligned}$$

Hence, to establish the claimed result, it suffices to show

$$\lim_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\left\{Q(0)\right\} > \lim_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)D(1)\exp\left\{Q(1)\right\}.$$

We will show this by establishing that $\lim_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)$ $\exp\left\{Q(0)\right\}=\infty$, while $\lim_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)D(1)\exp\left\{Q(1)\right\}<\infty$.

Letting $a=\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}$ to reduce clutter, note first that

$$\begin{aligned}\lim_{\frac{1}{\tau_\zeta}\uparrow a}\Gamma(k) &= \lim_{\frac{1}{\tau_\zeta}\uparrow a}\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)\\ &\quad\times\left(1-\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)\left(1-\frac{k^2\alpha^2\frac{1}{\tau_\eta}\left(\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\eta}+\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}\right)}\right)\right)^{-1}\\ &= \begin{cases}\infty & k=0\\ \text{Finite} & k=1\end{cases},\end{aligned}$$

where the finite limit in the $k=1$ case relies on the assumption $\alpha\neq 0$.

It follows that we have

$$\begin{aligned}\lim_{\frac{1}{\tau_\zeta}\uparrow a}D(k) &= \lim_{\frac{1}{\tau_\zeta}\uparrow a}\sqrt{\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\eta}+\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}\right)}}\sqrt{\frac{\Gamma(k)}{\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)}}\\ &= \begin{cases}\infty & k=0\\ \text{Finite} & k=1\end{cases}.\end{aligned}$$

Similarly,

$$\begin{aligned}\lim_{\frac{1}{\tau_\zeta}\uparrow a}Q(k) &= \lim_{\frac{1}{\tau_\zeta}\uparrow a}\left\{-\gamma\mathbb{E}_p[V]n+\frac{1}{2}\gamma^2\left(\frac{1}{\tau_A}+k^2\left(\mathbb{V}_p(\theta)+\frac{1-\alpha^2}{\tau_\eta}\right)\right)n^2\right.\\ &\quad\left.+\frac{1}{2}\gamma^2\frac{1}{\tau_\eta}\left(\mathbb{E}_p[Z]-k\alpha n-\gamma k\mathbb{C}_p(Z,\theta)n\right)^2\left(1+\Gamma(k;s_p)\right)\right\}\\ &= \begin{cases}\infty & k=0\\ \text{Finite} & k=1\end{cases}.\end{aligned}$$

Because the function Φ is bounded, together these results imply that

$$\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right) D(1)\exp\{Q(1)\} < \infty$$

as claimed.

It remains to show that $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right) D(0)\exp\{Q(0)\} = \infty$. Considering $\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}$, if $1/\beta = 0$, then $\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}$ is constant in τ_ζ and we are done. Considering $1/\beta \neq 0$, we have

$$\begin{aligned}
\lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}} &= \lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{m(0)}{\sqrt{v(0)}} \\
&= \lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{\gamma\mathbb{C}(s_p, V(0))n - \gamma^2\mathbb{C}(s_p, Z)\frac{1}{\tau_\eta}\mu_Z(1+\Gamma(0))}{\sqrt{\mathbb{V}(s_p)+\gamma^2\mathbb{C}^2(s_p, Z)\frac{1}{\tau_\eta}(1+\Gamma(0))}} \\
&= \lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{-\gamma^2\frac{1}{\beta}\mathbb{V}(Z)\frac{1}{\tau_\eta}\mu_Z(1+\Gamma(0))}{\sqrt{\mathbb{V}(s_p)+\gamma^2\mathbb{C}^2(s_p, Z)\frac{1}{\tau_\eta}(1+\Gamma(0))}} \\
&= \begin{cases} -\infty & \frac{1}{\beta} > 0 \\ \infty & \frac{1}{\beta} < 0 \end{cases},
\end{aligned}$$

where we use the fact that $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Gamma(0) = \infty$.

If $1/\beta < 0$, the proof is complete, since $Q(0) \to \infty, D(0) \to \infty$ and in this case, $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right) > 0$, so that $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right) D(0)\exp\{Q(0)\} = \infty$. If $1/\beta > 0$, then $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right) = 0$, so the limit is still indeterminate. Write $\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right) D(0)\exp\{Q(0)\}$ as

$$\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right) D(0)\exp\{Q(0)\} = \frac{\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)}{\frac{1}{D(0)}\exp\{-Q(0)\}}$$

and note that the relevant limit ultimately depends on the relative rate at which the various terms grow as $x \equiv \left(1-\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)\right)^{-1}$ approaches ∞, so that we can write

$$\lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)}{\frac{1}{D(0)}\exp\{-Q(0)\}} = \lim_{x\to\infty} \frac{\Phi(-\sqrt{x})}{\frac{1}{\sqrt{x}}\exp\{-x\}},$$

where we have used the fact that $\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}$ and $D(0)$ grow at order $\sqrt{x}$ with x and Q grows at order x with x. Using L'Hospital's rule yields

$$\begin{aligned}\lim_{x\to\infty}\frac{\Phi\left(-\sqrt{x}\right)}{\frac{1}{\sqrt{x}}\exp\{-x\}} &= \lim_{x\to\infty}\frac{-\frac{1}{2}x^{-1/2}\phi\left(-\sqrt{x}\right)}{-\frac{1}{\sqrt{x}}\exp\{-x\}-\frac{1}{2}x^{-3/2}\exp\{-x\}} \\ &= \lim_{x\to\infty}\frac{\phi\left(-\sqrt{x}\right)}{2\exp\{-x\}+x^{-1}\exp\{-x\}} \\ &= \lim_{x\to\infty}\frac{\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}x\right\}}{2\exp\{-x\}+x^{-1}\exp\{-x\}} \\ &= \infty,\end{aligned}$$

which establishes $\lim_{\frac{1}{\tau_\zeta}\uparrow a}\frac{\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)}{\frac{1}{D(0)}\exp\{-Q(0)\}}=\infty$ and completes the proof. □

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Supporting Information

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Appendix S1: Internet Appendix.
Replication Code.