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On the voluntary disclosure of redundant information [☆]

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Abstract

Why do firms engage in costly, voluntary disclosure of information which is subsumed by a later announcement? We consider a model in which the firm's manager can choose to disclose short-term information which becomes redundant later. When disclosure costs are sufficiently low, the manager discloses even if she only cares about the long-term price of the firm. Intuitively, by disclosing, she causes early investors to trade less aggressively, reducing price informativeness, which in turn increases information acquisition by late investors. The subsequent increase in acquisition more than offsets the initial decrease in price informativeness and, consequently, improves long term prices.

JEL classification: D21; D82; D83; D84; G32

Keywords: Costly voluntary disclosure; Information acquisition; Redundant information

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1. Introduction

Will voluntary disclosure of information (e.g., the existence of ongoing projects), which will be completely subsumed by a later announcement (e.g., the ultimate cash flows from the projects), have an incremental impact on a firm's stock price at the time of the later announcement? A Bayesian economist might be tempted to respond that the answer is no. After all, the early information becomes redundant at the later date.[1]

This reaction reflects a broader attitude about the impact on long-term valuations of trading on short-term information. Stiglitz (1989) considers an example of investors acquiring information today that becomes publicly available tomorrow, and argues that while there may be a private benefit to trading on such information in the short-term, there is no long-term effect on prices. In this case, what are the firm's incentives to disclose such short-term information?

Importantly, the settings we have in mind are ones in which disclosure is truly redundant. Specifically, it does not lead to feedback effects that would impact firm investment decisions, nor does it convey any information about persistent unobservable factors that could impact firm value beyond the subsequent disclosure at the later date. Trueman (1986) forcefully argues that in such a setting:

> "*the disclosure would simply advance the time at which investors learn something about the firm's earnings. The market value of the firm at the end of the period, after the actual earnings had been reported, however, would be unaffected by the forecast release (since the estimated earnings becomes irrelevant for valuation at that time).*"

While the above argument is intuitive, in this paper we show that strategic early disclosure can increase firm market value at future dates even after the disclosure becomes redundant. Our insight is that disclosure affects investors' information acquisition decisions, and consequently, influences the information environment at later dates. Disclosure directly impacts early investors' information choices, which changes the public information available to later investors (via the information revealed by short-term prices), and consequently affects their information acquisition and demand for the stock.

Model and intuition. To highlight the economic channel, we restrict attention to a stylized setting. Specifically, we study a model with two trading periods in which the firm's terminal value depends on the payoffs to a long term project (i.e., the assets-in-place of the firm) and, possibly, a short term project. If it exists, the short-term project's payoff is publicly revealed after trade in the first period and before trade in the second period. The long-term project's payoff is revealed only after trade in the second period, when it is paid out as part of the terminal value.

We assume that the manager knows with certainty whether the short-term project exists, and can disclose this information truthfully before the first round of trade by paying a cost (as in Verrecchia (1983)). The manager's objective at the disclosure stage is to maximize the long-run (second-period) market value. The short-term project's payoff is publicly revealed before the second round of trading, independently of the manager's disclosure decision, and makes the manager's disclosure completely redundant. Before trading in each round, investors choose

[1] It is worth distinguishing between "redundant" and "uninformative" disclosures. An uninformative disclosure contains no payoff relevant information at any time. A redundant disclosure is one which is not *incrementally* informative about payoffs, given the publicly available information at the later date.

whether or not to acquire costly information about the long-term project (as in Grossman and Stiglitz (1980)).

Our main result is that, provided that the cost of disclosure is not too high, the manager will voluntarily disclose the existence of the short-term project. By disclosing, the manager affects information acquisition and trading by investors. When early (first-period) investors learn that there is a short-term project, they face more uncertainty about the second-period price, which, in general, has two effects on first period price informativeness. First, increased uncertainty reduces how aggressively informed investors trade on their information, which tends to reduce price informativeness. Second, increased uncertainty can either increase or decrease the fraction of early investors who choose to acquire information.[2] However, we show that the impact of lowering trading intensity dominates and, as a result, disclosure always reduces first-period price informativeness. Once the short-term project's payoff is revealed prior to the second round of trade, the increased uncertainty about the long-term project leads more second-period investors to become informed about it. We show that the impact of more information acquisition at the later date dominates the impact of higher uncertainty in the short term. Hence, the resulting long term prices more precisely reflect the firm's true value and hence are higher on average, due to a lower risk premium.

Empirical relevance. Our analysis is particularly relevant when the projects that a firm may have undertaken are themselves subject to high uncertainty. For instance, firms that engage in multi-stage R&D investments (e.g., clinical trials for pharmaceuticals) often choose to disclose this information in early stages even though they are not required to do so, and such disclosures become redundant once the outcomes are realized. Our model implies that the impact of such disclosures on investors' information acquisition and longer-term prices is likely to be stronger when there is higher uncertainty about the ultimate payoffs from the R&D project. Consistent with our mechanism, Cookson et al. (2022) show that forward-looking, speculative disclosures are associated with a gradual, longer-term increase in average prices, a gradual increase in liquidity, and more informed trading. Moreover, they show that these effects are stronger when such disclosures are about R&D and for firms with higher idiosyncratic volatility.

A company announcing that it has placed a bid as a contractor for a project is another example where our mechanism applies. There is clearly no requirement to announce that a firm is making a bid, and the announcement is credible. The disclosure becomes redundant when the winner of the bidding process is announced. Arguably, announcements of winning a bid on a contract also fit our setting. While the SEC requires firms to disclose "material" events expeditiously, firms have leeway as to what is considered material, since it is difficult to evaluate the final impact of such contracts at the time of winning. As such, the announcement of winning a bid may increase investors' uncertainty about future payoffs.[3] After all, especially with contracts for innovative products, winning a bid for a contract does not guarantee that the firm will indeed succeed in developing and delivering. Moreover, at the time of winning the bid there could be substantial uncertainty as to the subsequent revenues the project will generate even conditional on success. A significant part of this uncertainty is likely to be resolved at the delivery stage, making the initial disclosure about winning the contract redundant at this stage.[4]

[2] As we discuss below in Section 4.3, the fact that the fraction informed can increase or decrease with residual uncertainty also holds in the single period model of Grossman and Stiglitz (1980) – see their Section II.H.

[3] In general, events that impact more than 5% of total sales or total assets are generally considered material.

[4] It is important to note that the ongoing existence of a short-term project can be effectively exogenous even if the initial investment decisions that generated the project were endogenous choices by the firm. For instance, while a phar-

Capital expenditure ("capex") forecasts by management, which are purely voluntary, are another disclosure where our mechanism could play a role. Such capex guidance often reflects medium-term projections and mentions purpose of funds, indicating what projects they are directed towards. Our model predicts that higher guidance (relative to market expectations) should be associated with new projects, and so should lead to higher uncertainty for investors in the short term. The negative cumulative abnormal return observed in the window around management capex forecast announcements (see Jayaraman and Wu (2020)) is broadly consistent with this prediction.

The rest of the paper is as follows. The next section discusses the related literature and our contribution to it. Section 3 presents the model and discusses the key assumptions. Section 4 provides the main analysis of the paper. Section 5 presents extensions of the benchmark analysis to study the effects of persistent noise trading, of long-lived investors, and of mixed-strategy disclosure, and Section 6 concludes. Unless noted otherwise, proofs and additional analysis is in the Appendix.

2. Related literature

There is an extensive literature on understanding the rationales for disclosure. Diamond (1985) shows how pre-commitment to publicly disclosing information can improve welfare by improving risk sharing and saving real resources which would otherwise be devoted to private information acquisition. In the presence of proprietary disclosure costs, sufficiently good news is disclosed and bad news is withheld (Verrecchia (1983)), and improvements in the quality of managers' information increases disclosure (Verrecchia (1990)). A similar threshold disclosure characterization exists if investors are uncertain as to whether the manager has information (Dye (1985), Jung and Kwon (1988)). Diamond and Verrecchia (1991) show disclosure changes risk for market makers, which affects their willingness to provide liquidity.[5] We contribute to this literature by introducing a distinct rationale for firms to voluntarily disclose information.

Our paper is related to the literature on earnings guidance, which focuses on the manager's incentives to influence investors' expectations about future earnings. In Trueman (1986), early voluntary disclosure that is later validated by a mandatory disclosure helps the manager signal to investors about her persistent skill in identifying optimal investment decisions.[6] In contrast, our model is designed so that at the time of the mandatory disclosure, any earlier voluntary disclosures become completely redundant. Yet, our analysis highlights a channel whereby the earlier disclosure still increases firm value at the later date.

Standard intuition suggests that greater public disclosure by firms should "crowd out" acquisition of private information by investors (e.g., Diamond (1985), Gao and Liang (2013), Colombo et al. (2014)). In these traditional settings, the firm's disclosure and investors' infor-

maceutical firm endogenously chooses how much to invest in R&D, whether a particular early-stage trial leads to viable pathway for future exploration is arguably exogenous. This is because the outcome of research is often stochastic and need not always succeed (see Banerjee and Breon-Drish (2022) for another application of this observation). Similarly, while a firm may endogenously choose whether to submit a bid, the event of whether or not the firm wins the bid is exogenous to the firm. Our analysis focuses on the whether the firm chooses to disclose the outcome of such an event after it is realized.

[5] See also surveys in Verrecchia (2001), Dye (2001) and Beyer et al. (2010).

[6] In his model, the manager's ability to predict the firm's future optimal production level is the unobservable characteristic. By releasing a forecast that is subsequently validated, the manager signals to investors that he has that skill, which improves subsequent firm investment decisions, and hence increases firm value.

mation acquisition are about the same component of fundamentals, and so additional disclosure about fundamentals discourages information acquisition in the short-term and the long-term. In contrast, our model highlights a novel mechanism through which firm disclosure about one component of fundamentals can affect information acquisition about a different component – discouraging information acquisition in the short-term but *encouraging* information acquisition in the long-term. Our results also highlight the importance of separately considering the short-term and long-term impact of disclosures on information acquisition and, consequently, price informativeness, since these can be in opposite directions.

The literature on feedback effects highlights a related complementarity between disclosure and informed trading by investors.[7] In such models, the manager chooses to strategically disclose information to encourage investors to trade more aggressively on their private information, or acquire more information. This results in more informative prices and, consequently, better informed real decisions by the manager. For instance, Goldstein and Yang (2015), Goldstein and Yang (2019) and Goldstein et al. (2020) highlight how disclosure along one dimension of fundamentals can crowd in more informed trading along another dimension. Xiong and Yang (2021) considers an oligopoly setting where firms disclose information about consumer demand to encourage investors to trade on their private information about an orthogonal component.

In contrast to this literature, the disclosure in our setting is not about the realization of cash flows per se, but the exposure to an additional source of risk (the short-term project). As such, the most closely related papers are Smith (2022), Lassak (2023) and Schneemeier (2023). Smith (2022) shows that disclosure about a firm's riskiness can induce investors to acquire more information about fundamentals. Lassak (2023) studies a setting where disclosure about cash flows can increase uncertainty and shows that the firm discloses information only if it crowds in more learning by investors. Schneemeier (2023) considers a setting in which a manager's voluntary disclosure about her signal precision can indicate greater investment in a growth opportunity, which increases payoff uncertainty and can crowd in more informed trading.

The economic mechanism in our model is distinct from this work. First, the manager's motivation for disclosure does not rely on any feedback effects: the manager does not learn from, or make investment decisions based on, the equilibrium price. Second, in our setting, the direct effect of the firm's disclosure about the short-term project is to initially *discourage* informed trading by early investors and so make short-term prices *less* informative. However, we show that through the endogenous information acquisition choices of later investors, this leads to more informative prices in the long term. Importantly, in our model, the manager's disclosure decisions reduce price informativeness in the short-term, yet there is an amplification effect in the long term, whereby long-term prices are more informative with the disclosure than without.

Our analysis also highlights the importance of considering the consequences of risk and investor risk-aversion for voluntary disclosure. A related, but distinct, motive for early disclosure can arise when investors have a preference for early resolution of uncertainty (e.g., Ai (2010), Epstein et al. (2014), Kadan and Manela (2019)). In our baseline setting, investors are assumed to have expected utility preferences, and so do not prefer early (or late) resolution of uncertainty. Two related papers are Dye and Hughes (2018), who study a how firms' voluntary disclosure decisions to risk-averse investors are affected by systematic risk, and Banerjee et al. (2023), who study how a firm's disclosure decision is affected by the presence of diversely-informed risk-averse investors. In these papers, importantly, there is only one trading date (i.e., there is

[7] See Bond et al. (2012) and Goldstein and Yang (2017) for recent surveys.

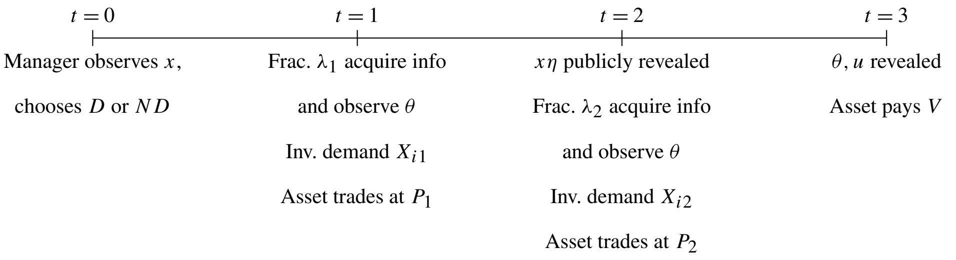


Fig. 1. Timeline of events.

no notion of redundancy), and the manager's disclosure and the information that investors are endowed with are both about the firm's total cash flow. In contrast, our analysis features endogenous information acquisition by investors over multiple periods. This multi-period setting is critical for the mechanism we focus on, because a reduction in price informativeness in the short run leads to improved price informativeness in the long run.

A different rationale for strategically increasing uncertainty to induce information collection is to reduce agency costs, as featured in Strobl (2014). He considers a static model with moral hazard and adverse selection, and shows that managerial investment behavior under the optimal contract tends to lead to increased uncertainty about output, and consequently more information collection by investors. Importantly, there is no notion of redundancy and no disclosure decision, which are crucial elements of our analysis. Moreover, there is no systematic increase in expected prices as a result of information acquisition, a key result of our paper, and investors collect information only once, so there is no notion of shifting the distribution of information collection over time, unlike the economic mechanism in our model.

3. Model

In this section, we describe the model and discuss some important assumptions. Fig. 1 summarizes the timing of events.

Payoffs. There are four dates $t \in \{0, 1, 2, 3\}$ and two securities. The gross return on the risk-free security is normalized to one. The risky security is a claim to a public firm with terminal value V, which will be realized at $t = 3$. The value V is given by

$$V = \bar{V} + x\eta + \theta + u, \tag{1}$$

where $\bar{V}$ is a known constant, η, θ, and u are independently normally distributed and $x \in \{0, 1\}$ is independent with prior probability p on $x = 1$. The aggregate supply of the risky security is given by $Z_t = \bar{Z} + z_t$ where z_t are normally distributed, and are independent of each other and other random variables. We denote the date t price of the risky security by P_t, and we define $P_3 = V$.[8] We denote the mean of η by $\bar{\eta}$, normalize the means of θ, u and z_t to zero without loss of generality, and denote the variance (precision) of these shocks by $\sigma^2_{(\cdot)}$ ($\tau_{(\cdot)}$, respectively).

[8] The equality $P_3 = V$ would also follow from imposing market clearing in a date 3 trading round. Since the terminal payoff V is known at date 3 (i.e., the payoff is realized and therefore known with certainty by all investors), there would be an arbitrage opportunity at that date if $P_3 \neq V$, investors would seek to trade arbitrarily large quantities to take advantage of this opportunity, and the market would not clear. Market clearing is only possible if $P_3 = V$ – at this price all investors would be exactly indifferent to holding any given quantity of the asset.

The event where $x = 1$ corresponds to the case where the short-term project exists and $x = 0$ to the case it does not. The payoff to the short-term project $x\eta$ is publicly revealed at date $t = 2$, and the payoff $\theta + u$ is publicly revealed at date $t = 3$ when the asset pays off.

Investors. There are overlapping generations (OLG) of investors. Generation t consists of a continuum of investors, indexed by $i \in [0, 1]$ with CARA utility and risk-aversion γ. Investor i in generation t can pay a cost c to observe θ immediately before trading at date t, and submits demand schedule X_{it} to maximize her expected utility over wealth at date $t + 1$. Importantly, the price at date $t + 1$ is determined by the trading demand by investors in generation $t + 1$. With some abuse of notation, we denote investors who choose to acquire information about θ by $i = I$, those who choose to remain uninformed by $i = U$, and the fraction who choose to become informed at date t by λ_t. Let $\mathcal{F}_{It} = \sigma\left(\theta, \{P_k\}_{k \le t}\right)$ and $\mathcal{F}_{Ut} = \sigma\left(\{P_k\}_{k \le t}\right)$ denote the information sets at time t for informed and uninformed traders, respectively. We will use $\mathbb{E}_{it}$ and $\mathbb{V}_{it}$, $i \in \{I, U\}$, to denote the relevant conditional expectation and conditional variance operators.

Manager. The firm's manager knows x at date $t = 0$ and chooses whether or not to disclose it at a cost $c_D > 0$.[9] The assumption that disclosure is costly follows the large theoretical and empirical literature on disclosure, which has discussed at length costs associated with voluntary disclosures. These costs go far beyond direct costs of preparing and disseminating information. A central cost labeled "proprietary costs", first coined in Verrecchia (1983), stems from the concern that disclosure of information can damage a firm's competitive position.[10] Following this literature, we also assume that the disclosure is verifiable – a manager cannot claim that a short-term project exists when it does not, or vice versa. However, the manager can choose to remain silent. This reflects the fact that firm disclosures are usually subject to audits and scrutiny by regulators and that managers are incentivized (contractually and legally) to report information truthfully.

We emphasize that the manager does not choose the value of x itself. That is, in order to focus cleanly on the role of disclosure, as distinct from the role of endogenous investment, we assume that the existence of the short-term project has been determined at a previous date. Only outside investors are uncertain about the existence of the project, and have prior probability p that it exists. Let $d = D$ and $d = ND$ correspond to the choice of disclosure and non-disclosure, respectively. The manager optimally chooses her disclosure strategy to maximize the expected date 2 price. Let

$$U_d(x) = \mathbb{E}\left[P_2 \middle| d, x\right]$$

denote the expected price conditional on the realized value of x and the disclosure decision d. Formally, a type x manager's problem is

$$U(x) \equiv \max_{d \in D, ND} U_d(x) - c_D \mathbf{1}_{\{d=D\}}. \tag{2}$$

3.1. Discussion of assumptions

Our model is stylized for tractability. In this subsection, we provide discussion of our assumptions and how our mechanism extends to more general settings.

9 Importantly, note that the manager does not observe the realization of η until date 2.

10 See Verrecchia (2001) and Dye (2001) for reviews of the proprietary cost literature.

Interpretation of dates. It is worth noting that we do not assume the time between dates is equal. In particular, we interpret the time between dates zero and one to be extremely short (e.g., of the order of days), since P_1 reflects the price reaction to the firm's disclosure at date zero. We expect the time between dates one and two to be of the order of several months to a few years. For instance, in many R&D projects (e.g., pharmaceutical trials), the time between the initial investment (e.g., start of the trial) and the outcome of the trial is likely to be many months if not a few years. In the case of announcing a bid for a contract, we expect that such tender processes last for a few months to a couple of years. Finally, capex guidance is usually concerned with annual forecasts, and so becomes redundant within a year (when actual capex is reported). Similarly, the time horizon between dates two and three can be quite long. The time from a successful R&D investment (successful clinical trials) to completion of a commercial product (the launch of a drug) can be many months to years. Moreover, the time between winning a bid for a contract and the completion of the project can be several years.

Manager's objective. We focus on a manager who is concerned with the "long run", post-disclosure price P_2 so that the disclosure is unambiguously redundant by the time her "utility" is realized. This is the starkest setting in which to illustrate that, despite the redundancy, she may still find it optimal to engage in costly disclosure. In general, the manager's disclosure has two effects on the short-term price P_1. First, by affecting the residual risk perceived by date 1 investors, it affects their information acquisition decisions and consequently the risk premium on the stock at date 1. Second, there is a mechanical "expected cash flow" effect of the project's mean payoff, $\overline{\eta}$ – by disclosing the existence of the project, all else equal the date 1 price increases (or decreases, if negative) by $\overline{\eta}$. Hence, if the project mean, $\overline{\eta}$, is sufficiently low (or negative), disclosing the project would always tend to reduce the date 1 price. In contrast, if the project mean, $\bar{\eta}$, is sufficiently high, disclosure of the project would always tend increase the date 1 price in expectation.

In practice, vesting requirements for stock and option grants should tend to induce managers to care about maximizing longer-term share price. For instance, Gopalan et al. (2014) find that vesting periods for stock and option grants for executives cluster around 3-5 years. Indeed, one can ask within our model whether a manager who owns shares would prefer to commit (ex-ante) to sell them at date 1 or date 2. From an ex-ante perspective (i.e., before observing x), one can show that the expected date 2 price is always higher than the expected date 1 price and so she would always choose to sell at date 2. Intuitively, since investors are learning about fundamentals that do not change, and more information gets revealed over time, the date 2 risk premium is smaller than the date 1 risk premium and consequently, the expected date 2 price is higher than the expected date 1 price.[11]

In principle, one could consider a setting in which the manager seeks to maximize a weighted sum of the "short-term" and "long-term" prices, e.g., $(1-\omega)\,\mathbb{E}[P_1|d,x]+\omega\mathbb{E}[P_2|d,x]$ for $\omega \in [0,1]$, though such a setting is generally no longer analytically tractable since the manager's problem no longer reduces to maximizing date 2 information acquisition, λ_2.[12] We expect that our mechanism will be present, qualitatively, in any situation in which the manager's objective

[11] This can be seen by comparing the expected values of the asset price functions from equations (3)-(4). From the ex-ante manager's perspective, given a disclosure decision and consequent values of the equilibrium price coefficients, we have $\mathbb{E}[P_2]=A_2+x\bar{\eta}$ and $\mathbb{E}[P_1]=A_1$. The equations pinning down the equilibrium coefficients in eq. (39)-(40) in the appendix immediately yield $\mathbb{E}[P_1]=A_1<A_2+x\bar{\eta}=\mathbb{E}[P_2]$.

[12] One could also allow the manager's objective to depend on the terminal price $P_3=\bar{V}+x\eta+\theta+u$, but because this quantity is exogenous, excluding it is without loss of generality.

places sufficient weight on the long-run price, P_2, i.e., when ω is sufficiently large. Indeed, by continuity, as long as the manager's objective places sufficiently large weight on the long-run price, P_2, disclosure of the project will continue to be optimal.

Disclosures about project outcomes. It is not necessary for the payoff $x\eta$ to be perfectly publicly revealed at date 2 for the initial disclosure to become redundant and for our mechanism to operate – revelation of a noisy signal at date 2 about η in the event that the project exists would have qualitatively similar implications.

Note that in our analysis the manager does not observe the realization of η when choosing whether to disclose x, so there is no scope for her to disclose information about η itself. In principle, one could allow the disclosure of x at date 1 to be accompanied by a noisy, public signal about the realization of η of the form $s_\eta = \eta + \varepsilon_\eta$, with ε_η independently normally distributed. In this case, our results would be qualitatively unchanged – we would only have to change the notation in the analysis to replace the unconditional moments of η by their conditional counterparts (i.e., replace $\bar{\eta}$ by $\mathbb{E}\left[\eta|s_\eta\right]$ and σ_η^2 by $\mathbb{V}\left[\eta|s_\eta\right]$) in the event of disclosure. Because such a disclosure still raises investors' perceived residual risk at date 1 relative to no-disclosure, and therefore lowers price informativeness at date 1 (which increases information collection at date 2), our key mechanism still operates to incentivize disclosure.[13]

Disclosures about project riskiness. In our analysis, the variable $x \in \{0, 1\}$ corresponds to an indicator variable for whether or not the firm has a project. All of our results go through as stated for a more general setting $x \in \{x_L, x_H\}$ where $|x_L| < |x_H|$, and where we interpret x as being the firm's exposure to the risk factor that determines the short-term project's payoff. This is because the manager does not choose whether to make the investment, but only to disclose whether the firm has the project.[14] Note that this is true even when the exposure to η is negative (i.e., $x < 0$) — what matters for the investors' information acquisition decisions is the magnitude of the risk exposure i.e., $x^2\sigma_\eta^2$, but not the sign of the exposure. The sign of x affects the expectations component in prices at dates 1 and 2 — however, the impact on P_2 is the same irrespective of whether the manager discloses x at date 1 because $x\eta$ is publicly revealed before trade at date 2.

We expect our results to be qualitatively similar if x follows a more general discrete distribution with more than two states and the disclosure cost is sufficiently small. In this case, we conjecture that all managers above the lowest possible value of $|x|$ will find it optimal to disclose and the lowest possible type will refrain from disclosing. Moreover, while we expect that our mechanism is qualitatively robust to even more general distributions and cost functions, such settings are generally intractable.

In our baseline analysis, we focus on pure strategy disclosures for tractability. As we discuss in Section 4.4, when investors have CARA utility and place positive probability on more than one value of x, then there is no longer a linear equilibrium in the financial market, and it is not possible to characterize the equilibrium, or even demonstrate existence of equilibrium. In Section 5.3,

[13] Note that if the manager could choose the precision of such a signal about η, we expect her to prefer signals that are as uninformative as possible. Specifically, suppose the manager could commit to disclosing a noisy signal s_η before observing the realization of η. A more informative disclosure about the short-term payoff (η) should lead to an increase in the date 1 price informativeness, since date 1 investors face less residual risk, but a decrease in the expected date 2 price, because the high date 1 price informativeness discourages information collection at date 2. As such, if the manager wants to maximize the expected date 2 price, she should prefer not to disclose any information about the realization of the payoff η.

[14] In this setting, we would still have that disclosing x_H leads investors to face more uncertainty at date 1, which makes date 1 prices less informative, and consequently, leads to more information acquisition at date 2. As such, firms with $x = x_H$ would disclose this information, while firms with $x = x_L$ would be indifferent between disclosing and not.

we consider an alternative specification in which investors have mean-variance preference. This allows us to entertain the possibility of mixed strategy disclosure i.e., when the firm chooses a disclosure probability $r \in [0, 1]$.

Learning about long-term (θ) vs. short-term (η) payoffs. We analyze a setting in which investors can learn about the long-term component of payoffs, θ but not about the short-term payoff, η. We expect our key mechanism to survive qualitatively, and indeed to even strengthen, in a setting in which $t = 1$ investors can learn about both short-term (η) and long-term (θ) payoffs. Intuitively, disclosure about the short-term project (i.e., $x = 1$) would cause date 1 investors to re-allocate some of their information acquisition activities from θ to η: if the project is disclosed, date 1 investors have an incentive to learn about its payoff η, which comes at the expense of learning more about the long-term fundamental θ. In turn, this would lower $t = 1$ price informativeness about θ and lead investors to learn more about θ in the second period; note that because η is revealed immediately before date 2, investors do not have any incentive to learn about it privately in that period. This would ultimately lead disclosure to have an even larger positive effect on the expected date 2 price than if date 1 investors could learn only about θ.

Long-lived investors and persistent noise trading. We consider a setting with short-lived investors in order to transparently and tractably illustrate the important economic forces. In Section 5.1 below, we consider a fully dynamic version of our model, in which investors are long-lived and can acquire a signal at a time of their choosing. We show that our main result obtains for a range of parameter values: that is, disclosing higher x leads to more information acquisition at date 2, and consequently, higher expected price.

Similarly, the assumption that asset supply is i.i.d. is for simplicity. In Section 5.2 below, we show that our results are robust to correlated asset supply (i.e., $Z_t = \overline{Z} + \phi(Z_{t-1} - \overline{Z}) + z_t$ for $\phi \neq 0$). What is important is that disclosure about the existence of the project reduces price informativeness in the first trading round by making the asset riskier from the perspective of date 1 investors (and thereby inducing informed investors to trade less aggressively), which increases the value of acquiring information prior to the date 2 trading round.

4. Analysis

Our focus in this section is to show that there exists an equilibrium in which a manager with a short-term project ($x = 1$) discloses this information at date 0, while a manager without a project ($x = 0$) does not disclose. Importantly, in this equilibrium, investors at date 1 and 2 infer $x = 0$ (with probability 1) in the event that the manager does not disclose.

We shall establish this by working backwards. First, taking a disclosure $d \in \{D, ND\}$ and investor's information acquisition choices λ_1 and λ_2 as given, we solve for the equilibrium prices P_1 and P_2 in Section 4.1. Next, given a disclosure policy d, we solve for the optimal information acquisition choices at dates 1 and 2 in Section 4.2. Finally, we establish sufficient conditions under which our conjectured disclosure policy is the unique equilibrium policy in Section 4.3.

4.1. Financial market equilibrium

For given disclosure and information choices, in such an equilibrium, the financial market equilibrium either places probability 1 on $x = 1$ (in the event that the $x = 1$ manager discloses) or places probability 1 on $x = 0$ (in the event that the $x = 0$ manager discloses or either manager does not disclose and is inferred to be the $x = 0$ type). Hence, the derivation of the financial market equilibrium follows from the standard “conjecture and verify” approach. Fix the fraction

λ_t of investors in generation t who acquire information about fundamentals θ. We conjecture that prices are of the form:

$$P_1(d) = A_1(d) + B_1(d)\, s_{p1}, \quad \text{and} \tag{3}$$

$$P_2(x,d) = A_2(d) + B_2(d)\, s_{p2} + C_2(d)\, s_{p1} + x\eta, \tag{4}$$

where the price signal $s_{pt} \equiv \theta + b_t(d)\, z_t$ for $t \in \{1,2\}$. Note that the equilibrium price coefficients $\{A_t, B_t, C_t, b_t\}$ depend on the manager's date zero disclosure decision d. However, in what follows, we will suppress this dependence for notational convenience unless necessary. The above conjecture implies that the date t price provides a noisy, linear signal s_{pt} about fundamentals θ to the uninformed investors of that generation. Moreover, the uninformed investors at date 2 can condition on the date 1 price to infer s_{p1}. This implies that the conditional beliefs of an uninformed investor at date $t = 1$ are given by:

$$\mathbb{E}_{U1}[\theta] = \frac{\tau_{p1} s_{p1}}{\tau_\theta + \tau_{p1}}, \quad \mathbb{V}_{U1}[\theta] = \frac{1}{\tau_\theta + \tau_{p1}} \equiv \frac{1}{\tau_{U1}}, \quad \text{where } \tau_{p1} \equiv \tau_z / b_1^2. \tag{5}$$

Similarly, the conditional beliefs of an uninformed investor at date $t = 2$ are given by:

$$\mathbb{E}_{U2}[\theta] = \frac{\tau_{p1} s_{p1} + \tau_{p2} s_{p2}}{\tau_\theta + \tau_{p1} + \tau_{p2}}, \quad \mathbb{V}_{U2}[\theta] = \frac{1}{\tau_\theta + \tau_{p1} + \tau_{p2}} \equiv \frac{1}{\tau_{U2}}, \quad \text{where } \tau_{p2} \equiv \tau_z / b_2^2. \tag{6}$$

Note that investor i in generation t chooses optimal demand X_{it} to maximize CARA utility over wealth at date $t + 1$ i.e.,

$$X_{it} \equiv \arg\max_X \mathbb{E}_{it}[-e^{-\gamma\{W_t + X(P_{t+1} - P_t)\}}] \tag{7}$$

$$= \frac{\mathbb{E}_{it}[P_{t+1}] - P_t}{\gamma \mathbb{V}_{it}[P_{t+1}]} \tag{8}$$

where the date 3 price is $P_3 = V$. This implies that the optimal demand for date 2 informed and uninformed investors are given by

$$X_{I2} = \frac{1}{\gamma} \frac{\bar{V} + x\eta + \theta - P_2}{1/\tau_u}, \quad \text{and} \quad X_{U2} = \frac{1}{\gamma} \frac{\bar{V} + x\eta + \mathbb{E}_{U2}[\theta] - P_2}{1/\tau_u + 1/\tau_{U2}}, \tag{9}$$

respectively. The market clearing condition at date 2 is:

$$\lambda_2 X_{I2} + (1 - \lambda_2) X_{U2} = \bar{Z} + z_2. \tag{10}$$

Re-arranging terms, we see that the market clearing price verifies the conjecture in (3). Similarly, the optimal demand for date 1 informed and uninformed investors are given by

$$\begin{aligned} X_{I1} &= \frac{1}{\gamma} \frac{A_2 + B_2\theta + C_2 s_{p1} + x\bar{\eta} - P_1}{x^2/\tau_\eta + B_2^2/\tau_{p2}}, \quad \text{and} \\ X_{U1} &= \frac{1}{\gamma} \frac{A_2 + B_2 \mathbb{E}_{U1}[\theta] + C_2 s_{p1} + x\bar{\eta} - P_1}{x^2/\tau_\eta + B_2^2/\tau_{p2} + B_2^2/\tau_{U1}}, \end{aligned} \tag{11}$$

respectively, where investors understand $x = 1$ if $d = D$ and $x = 0$ if $d = ND$. Again, the date 1 market clearing condition, which is given by:

$$\lambda_1 X_{I1} + (1 - \lambda_1) X_{U1} = \bar{Z} + z_1, \tag{12}$$

implies that the market clearing price verifies the conjecture in (3)-(4).

The following result characterizes the financial market equilibrium.

Lemma 1. *Fix the fraction of informed at each date i.e., $\lambda_1, \lambda_2 \in [0, 1]$. There exists an equilibrium in which date 1 and 2 equilibrium prices are given by (3)-(4), where the price signals $s_{pt} \equiv \theta + b_t z_t$,*

$$b_2 = -\frac{\gamma}{\lambda_2 \tau_u}, \quad b_1 = -\frac{\gamma}{B_2 \lambda_1} \left(\frac{B_2^2 \gamma^2}{\lambda_2^2 \tau_u^2 \tau_z} + \frac{x^2}{\tau_\eta} \right), \tag{13}$$

and the price coefficients A_1, A_2, B_1, B_2 and C_2 are characterized in the appendix.

4.2. Information acquisition choices

Given the characterization of the financial market equilibrium in the previous section, one can characterize the optimal information acquisition choices for generation t investors by comparing their expected utility with and without private information.

Let $\mathbb{E}_{t^-}[\cdot]$ refer to the expectation of generation t investors before they have acquired any information or observed the date t price. Then, the expected utility from acquiring information is given by:

$$U_{I,t} \equiv \mathbb{E}_{t^-}[\mathbb{E}_{It}[-e^{-\gamma\{W_t + X_{It}(P_{t+1} - P_t) - c\}}]], \tag{14}$$

while the expected utility from not acquiring information is given by:

$$U_{U,t} \equiv \mathbb{E}_{t^-}[\mathbb{E}_{Ut}[-e^{-\gamma\{W_t + X_{Ut}(P_{t+1} - P_t)\}}]]. \tag{15}$$

Standard calculations show that the relative expected utility can be expressed as:

$$\Gamma_t(\lambda_1, \lambda_2) \equiv \frac{U_{I,t}}{U_{U,t}} = e^{\gamma c} \sqrt{\frac{\mathbb{V}_{It}[P_{t+1}]}{\mathbb{V}_{Ut}[P_{t+1}]}} \tag{16}$$

just as in Grossman and Stiglitz (1980). Note that if $\Gamma_t(\lambda_t = 1) < 1$, then all investors in generation t choose to become informed (i.e., $\lambda_t = 1$), while if $\Gamma_t(\lambda_t = 0) > 1$, then no investors acquire information (i.e., $\lambda_t = 0$).

As is standard in the literature, in what follows we focus on equilibria featuring "interior" information choices i.e., $\lambda_1, \lambda_2 \in (0, 1)$ to keep the analysis transparent. In Appendix B.1, we characterize conditions under which such interior equilibria obtain. Consistent with intuition, the information equilibrium is interior when information costs neither "too high" (so that no investors acquire information) nor "too low" (so that all investors acquire information). Moreover, to ensure that date 1 information choices are interior when the firm has a short-term project (i.e., $x = 1$), the prior uncertainty about this projects payoff must not be too high (i.e., τ_η cannot be too low), because otherwise, no investors acquire information.

4.3. Disclosure decision

We begin by showing that the expected date 2 price is increasing in λ_2.

Lemma 2. *For a fixed x, the expected date 2 price $\mathbb{E}[P_2]$ is an increasing function of the fraction of investors who acquire information at date 2 (i.e., λ_2).*

Proof. Note that

$$\mathbb{E}[P_2] = \bar{V} - \frac{\gamma}{\frac{\lambda_2}{\mathbb{V}_{I2}[P_3]} + \frac{(1-\lambda_2)}{\mathbb{V}_{U2}[P_3]}} \bar{Z} + x\bar{\eta}. \tag{17}$$

Furthermore, when λ_2 is interior, it is pinned down by the information acquisition condition $\Gamma_2(\lambda_2) = 1$, which implies

$$\mathbb{V}_{U2}[P_3] = e^{2\gamma c}\mathbb{V}_{I2}[P_3] \tag{18}$$

Hence, the weighted average precision can be expressed as:

$$\frac{\lambda_2}{\mathbb{V}_{I2}[P_3]} + \frac{(1-\lambda_2)}{\mathbb{V}_{U2}[P_3]} = \frac{1}{\mathbb{V}_{I2}[P_3]}\left(\lambda_2\left(1 - e^{-2c\gamma}\right) + e^{-2c\gamma}\right). \tag{19}$$

Since $\mathbb{V}_{I2}[P_3] = 1/\tau_u$, implies that the expected price $\mathbb{E}[P_2]$ is an increasing function of λ_2. □

The above result highlights a key feature of our setting: in equilibrium, the expected price increases in the fraction of investors informed at date 2. Intuitively, as more investors become informed, the equilibrium price becomes more informative about fundamentals. This implies that the weighted average precision increases, which in turn implies that the risk premium (price discount) is lower.[15]

There are two notable features in the above analysis. First, the equilibrium posterior variance of uninformed and informed investors are proportional (i.e., equation (18) holds). This is an implication of equilibrium in information acquisition — an investor must be indifferent between paying the cost to acquire information and remaining uninformed — and arises generally in models with fixed costs of information (e.g., Grossman and Stiglitz (1980) and related models).

Second, the posterior variance of informed investors at date 2 ($\mathbb{V}_{I2}[P_3]$) does not depend on the fraction of informed investors λ_2. This is because informed investors observe a perfect signal about fundamentals (θ) if they choose to acquire information and consequently have nothing to learn from the price. However, this is not critical for the relation between expected price and the fraction λ_2 of informed investors, and similar results hold when informed investors observe a signal with noise.

Next, we show that the fraction of investors who acquire information at date 2 (i.e., λ_2) is higher with disclosure ($d = D$) than not ($d = ND$).

Lemma 3. *The date 1 price is less informative (i.e., the precision of the date-1 price signal, τ_{p1}, is lower) and date 2 information acquisition is higher (i.e., the date-2 fraction of informed investors, λ_2, is higher) when the manager discloses.*

This result is intuitive. When $d = D$, investors infer that $x = 1$, and so face higher unlearnable uncertainty at date 1. This increase in uncertainty has two effects. First, the direct effect is to lead informed investors to trade less aggressively on their information which tends to make the date 1 price less informative — this is apparent from the expression for date 1 demands in Equation (11).

Second, the indirect effect of an increase in uncertainty is to change the amount of information acquisition at date 1 (i.e., λ_1 may be higher or lower). There are two forces that operate in

[15] This is driven by the assumption that investors have CARA utility, and so the risk-premium reflects their posterior uncertainty about payoffs.

different directions on the fraction informed when unlearnable uncertainty increases. First, because risk-averse traders anticipate trading less aggressively on their signals when uncertainty is higher, it makes acquiring information about the learnable component less valuable. This tends to reduce the fraction informed. On the other hand, this reduction in trading aggressiveness tends to reduce the informativeness of the price-signal and so encourage more traders to acquire private information.

Despite the non-monotonic effect on λ_1, the lemma establishes that the date 1 price is always less informative when unlearnable uncertainty is higher. Intuitively, this is because the direct impact via trading aggressiveness dominates any potential increase in λ_1. In turn, this implies that prior to acquiring information, date 2 investors have a higher conditional variance about fundamentals θ, which leads to more information acquisition prior to the date 2 trading round.

To gain some intuition for this result, it is illustrative to consider the impact of residual uncertainty on price informativeness in the static model of Grossman and Stiglitz (1980). Denote the terminal payoff by $\theta + u$ in their single period model. The (interior) equilibrium fraction of informed investors is pinned down by the indifference condition $e^{\gamma c} = \sqrt{\frac{\frac{1}{\tau_u}+\frac{1}{\tau_U}}{\frac{1}{\tau_u}}}$, where $\tau_U = \tau_\theta + \tau_p$ is the uninformed conditional precision of θ and $\tau_p = \frac{\lambda^2 \tau_u^2 \tau_z}{\gamma^2}$ is the precision of the price-signal. This implies that the equilibrium λ can be expressed as $\lambda = \frac{\gamma}{\sqrt{\tau_u \tau_z}} \sqrt{\frac{1}{e^{2\gamma c}-1} - \frac{\tau_\theta}{\tau_u}}$, and so is hump-shaped in τ_u. This is analogous to our result that λ_1 may be higher or lower as a result of increased uncertainty due to the manager's disclosure.

The overall impact of residual uncertainty on price informativeness can be decomposed as:

$$\frac{d\tau_p}{d\tau_u} = \frac{2\tau_u \tau_z \lambda^2}{\gamma^2} \left(\underbrace{1}_{\text{direct}} + \underbrace{\frac{\partial \lambda/\lambda}{\partial \tau_u/\tau_u}}_{\text{indirect}} \right) \tag{20}$$

where the first term reflects the direct effect due to the impact on trading aggressiveness and the second term, which is the elasticity of the fraction informed λ with respect to τ_u, reflects the indirect effect due to the impact on the fraction of investors who choose to become informed. Because the elasticity of λ with respect to τ_u is always bounded below by -1 i.e.,

$$\frac{\partial \lambda/\lambda}{\partial \tau_u/\tau_u} = \frac{\tau_u}{2\left(\tau_u - \tau_\theta\left(e^{2c\gamma}-1\right)\right)} - 1 > -1, \tag{21}$$

the direct impact always dominates the indirect impact.[16] As a result, higher residual uncertainty (lower τ_u) always decreases price informativeness (lower τ_p).

We are now ready to establish the existence of the conjectured equilibrium.

Proposition 1. *Suppose the cost of disclosure $c_D > 0$ is strictly less than the increase in the firm's risk premium without disclosure vs. with disclosure, $\frac{\gamma}{\frac{\lambda_2(ND)}{\mathbb{V}_{I2}[P_3]}+\frac{(1-\lambda_2(ND))}{\mathbb{V}_{U2}[P_3]}}\bar{Z} - \frac{\gamma}{\frac{\lambda_2(D)}{\mathbb{V}_{I2}[P_3]}+\frac{(1-\lambda_2(D))}{\mathbb{V}_{U2}[P_3]}}\bar{Z}$. Then, there exists an equilibrium in which a manager with a short-term project (i.e., $x = 1$) discloses this information (i.e., chooses $d = D$), but a manager without the project (i.e., $x = 0$)*

[16] One can see this more directly by noting that, in equilibrium, $\tau_p = \frac{\tau_u}{e^{2c\gamma}-1} - \tau_\theta$, which always increasing in τ_u.

does not disclose (i.e., $d = ND$). Moreover, the expected long term price $\mathbb{E}[P_2]$ is higher with disclosure than without.

Proof. To establish that the conjectured equilibrium is in fact an equilibrium, it suffices to show that each manager type prefers to play her conjectured strategy given the strategy of the other type. Consider first the manager without a project ($x = 0$). Taking the $x = 1$ manager's disclosure strategy as given, then if the $x = 0$ manager follows her conjectured strategy and does not disclose she is inferred to be the low type. If she deviates and discloses $x = 0$ then she is still identified as the low type and also pays the disclosure cost. Hence, playing her conjectured strategy is optimal.

Consider now the $x = 1$ manager. Taking the $x = 0$ manager's strategy of non-disclosure as given, we need to establish that the $x = 1$ manager prefers to disclose. Supposing that she instead deviates and refrains from disclosing, then in the conjectured equilibrium the market will infer her to be an $x = 0$ type. The payoff from disclosure is

$$U_D(1) - c_D = \mathbb{E}[P_2(1, D)] - c_D \tag{22}$$

while the payoff from non-disclosure (given that $x\eta$ is publicly revealed at date two) is: $U_{ND}(1) = \mathbb{E}[P_2(1, ND)]$. The incremental benefit from disclosing versus not disclosing is

$$(U_D(1) - c_D) - U_{ND}(1) = \mathbb{E}[P_2(1, D)] - \mathbb{E}[P_2(1, ND)] - c_D. \tag{23}$$

Since $\lambda_2(D) > \lambda_2(ND)$ by Lemma 3 and since $\mathbb{E}[P_2]$ is increasing in λ_2 by Lemma 2, we have that

$$\mathbb{E}[P_2(1, D)] - \mathbb{E}[P_2(1, ND)] > 0, \tag{24}$$

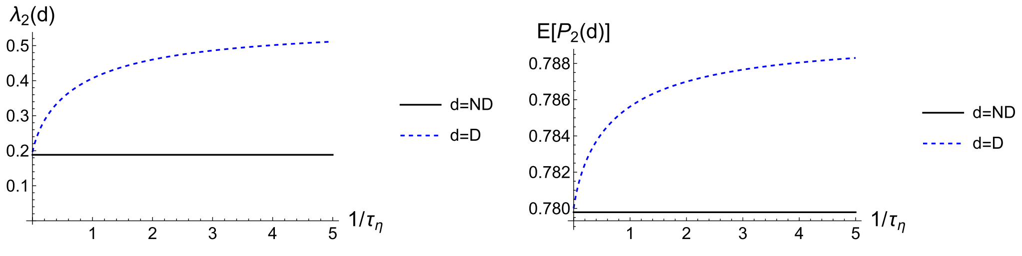
which establishes the result about the expected price. Moreover, this implies that as long as the cost of disclosure c_D is strictly less than this difference in expected prices, which corresponds to the increase in the date-2 risk premium if the firm does not disclose vs. does disclose,

$$\mathbb{E}[P_2(1, D)] - \mathbb{E}[P_2(1, ND)] = \frac{\gamma}{\frac{\lambda_2(ND)}{\mathbb{V}_{I2}[P_3]} + \frac{(1-\lambda_2(ND))}{\mathbb{V}_{U2}[P_3]}}\bar{Z} - \frac{\gamma}{\frac{\lambda_2(D)}{\mathbb{V}_{I2}[P_3]} + \frac{(1-\lambda_2(D))}{\mathbb{V}_{U2}[P_3]}}\bar{Z},$$

then it is optimal for the $x = 1$ manager to disclose. □

As discussed above, the key mechanism that causes disclosure to increase the expected price at date 2, $\mathbb{E}[P_2]$, is that the presence of the project increases the risk faced by investors at date 1. Consequently, they trade less aggressively, which decreases price informativeness and therefore incentivizes information collection at date 2. Fig. 2 provides an illustration of this channel. Panel (a) plots the equilibrium fraction of investors who acquire information at date 2, and panel (b) plots the equilibrium expected date 2 price $\mathbb{E}[P_2]$, in the event of disclosure (i.e., $d = D$, dashed) versus no-disclosure (i.e., $d = ND$, solid), as a function of the variance of the project payoff, $1/\tau_\eta$. Note that the effect of disclosure on date 2 information acquisition and, consequently, on the expected date 2 price is higher when the project is risker. Intuitively, this is because the riskier the project is, the more that disclosing its presence harms date 1 price informativeness and therefore increases the value of acquiring information at date 2. It follows that the incentive to disclose is stronger for riskier projects.

The figure plots the fraction of informed investors at date 2 (i.e., λ_2) and the expected date 2 price as a function of the variance of the project payoff $1/\tau_\eta$ for disclosure (i.e., $d = D$, dashed) versus no-disclosure (i.e., $d = ND$, solid). The other parameter values are as follows: $\bar{V} = 1$, $\gamma = 0.2$, $c = 0.3$, $\tau_\theta = 1$, $\tau_u = 1$, $\tau_z = 1$, $\bar{\eta} = 0$.



(a) λ_2 versus $1/\tau_\eta$ (b) $\mathbb{E}[P_2]$ versus $1/\tau_\eta$

Fig. 2. λ_2 and $\mathbb{E}[P_2]$ as a function of the project variance, $1/\tau_\eta$.

4.4. Equilibrium uniqueness

Proposition 1 establishes the existence of an equilibrium of our conjectured form. However, it does not speak to the existence of other equilibria in which, e.g., both types do not disclose with positive probability. We will show below that within the class of equilibria in which the financial market is linear, the equilibrium we characterize is unique. To do so, we must entertain the possibility of managers following mixed disclosure strategies, so it is helpful to make explicit the dependence of expected P_2 on the market's belief about the manager's type. Hence, let $U_{ND}(x; q) = \mathbb{E}[P_2|d = ND, x]$ denote the expected price as a function of x in the event that the manager does not disclose and the market assigns probability q to $x = 1$ in the event of no disclosure. Suppose that in the case that both types disclose with probability one, and hence the conditional probability given nondisclosure is not defined by Bayes rule, the market assigns off-equilibrium belief $q_{OFF} = 0$. That is, in the event of off-equilibrium non-disclosure the market assigns the manager the lowest type. The following Lemma establishes that we can rule out any equilibria with interior λ in which an $x = 0$ manager discloses with positive probability.

Lemma 4. *There do not exist equilibria in which an $x = 0$ manager discloses with strictly positive probability.*

Intuitively, in any conjectured equilibrium in which an $x = 0$ manager discloses, she can make herself strictly better off by refraining from disclosing, saving the disclosure cost, and, at worst, still being perfectly identified at an $x = 0$ type. Owing to Lemma 4, the only remaining candidate equilibria (with interior λ's) are those in which the $x = 0$ manager never discloses and the $x = 1$ manager discloses with probability r_1 that is strictly less than one, $r_1 \in [0, 1)$.

When the $x = 1$ manager mixes between disclosure and not, then when the market observes no disclosure, traders at date 1 perceive the future asset price as following a normal mixture distribution and there does not exist a linear equilibrium in the financial market, which we record in the following Proposition.

Proposition 2. *There do not exist equilibria in which an $x = 1$ manager discloses with probability less than one and the financial market equilibrium is linear.*

Hence, there are no equilibria in which the manager follows a mixed disclosure strategy (or never discloses) and asset prices are linear functions of the underlying shocks.

While we have focused on linear equilibria, it is unclear whether or not any noisy rational expectations equilibrium outside of the linear class even exists in the case of mixed strategy disclosure when investors have CARA utility. Our setting does not meet any known conditions for characterizing equilibria outside of the linear class, such as the “exponential family” condition of Breon-Drish (2015). Characterizing rational expectations equilibria in settings where payoffs follow more general distributions (e.g., normal mixture distributions, as would be the case with mixed strategy disclosure) is a difficult and long-standing open problem, and is beyond the scope of this paper. Intuitively, with CARA utility and uncertainty about whether the firm has a project (i.e., whether $x = 0$ or 1), an investor's demand is no longer “additively separable” in her signal and a function of the price. This implies that inference from the market clearing price, as required by rational expectations, is no longer tractable. Fully numerical approaches, such as that proposed by Bernardo and Judd (2000) are also difficult to implement in our setting, which features multiple interlinked trading rounds, learning from a sequence of prices (for the date 2 traders), and multiple rounds of information acquisition

In order to address the underlying issue of mixed strategy disclosure, in Section 5.3, we consider an alternative specification in which investors have mean-variance preferences. In this case, the financial market equilibrium always remains linear, even if the manager follows a mixed disclosure strategy, and can be characterized for any given set of parameters, which in turn, allows us to numerically analyze both pure strategy and mixed strategy equilibria.

While we are unable to provide a full characterization of mixed strategy equilibria in our baseline model with CARA utility, under an additional economically natural continuity assumption on the (admittedly endogenous) expected price in the event of nondisclosure, the following Proposition rules out mixed strategies when disclosure costs are sufficiently low and the prior probability p is sufficiently low.

Proposition 3. *Suppose that $U_{ND}(1; q)$ is continuous in q at $q = 0$. Then if the disclosure cost c_D and the prior probability p that $x = 1$ are sufficiently small, there does not exist an equilibrium in which an $x = 1$ manager discloses with probability $r_1 < 1$.*

Intuitively, when p is very low, if an $x = 1$ manager does not disclose the market assigns probability close to zero that she is the $x = 1$ type. We know from Proposition 1 that, as long as costs are sufficiently small, if the market assigns probability *equal to* zero that $x = 1$, then an $x = 1$ manager finds it optimal to disclose and thereby identify herself to the market. Hence, under continuity of non-disclosure expected utility U_{ND} at $q = 0$, it remains optimal for the $x = 1$ manager to disclose when this probability is positive but small.

5. Extensions and robustness

The details of the analysis for these extensions are presented in Appendix B.

5.1. Long lived investors

In Appendix B.2 we show our results generalize for a range of parameter values in a setting in which investors are long-lived and can acquire information at either date 1 or 2. Specifically, we assume that the asset payoff and supply dynamics are the same as in the benchmark setting.

However, in contrast to the assumption of short-lived investors in the benchmark setting, we assume that there is a unit mass of long-lived investors with CARA (γ) utility over $t = 3$ wealth who participate at both trading dates. Each investor i can pay a cost to observe θ immediately before trading at the date of her choosing (i.e., at $t = 1^-$ or $t = 2^-$).[17]

In order to sustain an equilibrium with information acquisition at $t = 2^-$ we assume time-dependent information costs c_t with $c_1 > c_2$. This is because the gross value of information decreases deterministically in time. If c was constant in time, then in any conjectured interior equilibrium with nonzero acquisition at the second date, we necessarily have a profitable deviation for any investor who is currently acquiring at $t = 2^-$ to instead acquire at $t = 1^-$, pay the same cost, and yet obtain strictly higher expected utility.

Investors submit demand schedules $X_{it}, t \in \{1, 2\}$ to maximize expected utility over terminal wealth. We subscript quantities associated with an investor informed at $t = 1^-$ by $I1$, at $t = 2^-$ by $I2$, and those who choose to remain uninformed by $U1$ and $U2$. The fraction of investors who are informed at date t is denoted by λ_t.

As in the earlier analysis, we conjecture and verify that there exists a financial market equilibrium in which prices are of the form:

$$P_1 = A_1 + B_1 s_{p1}, \quad \text{and} \quad P_2 = A_2 + B_2 s_{p2} + C_2 s_{p1} + x\eta, \tag{25}$$

where the $s_{pt} \equiv \theta + b_t(Z_t - \bar{Z})$ for $t \in \{1, 2\}$. Relative to the benchmark analysis in Section 4, there are two notable changes. First, the optimal demand for investors at date $t = 1$ reflects a dynamic hedging demand. Specifically, while the optimal demand for investor i at date 2 is given by

$$X_{i1} = \frac{\mathbb{E}_{i2}[V] - P_2}{\gamma \mathbb{V}_{i2}[P_3]}, \tag{26}$$

as before, we show that the optimal demand for investor i at date 1 can be expressed as:

$$X_{i1} = \frac{1}{\gamma} \frac{\mathbb{E}_{i1}[P_2 - P_1 - \beta_{i1}(V - P_2)]}{\mathbb{V}_{i1}(P_2 - P_1 - \beta_{i1}(V - P_2))} \tag{27}$$

where $\beta_{i1} = \frac{\mathbb{C}_{i1}(V - P_2, P_2 - P_1)}{\mathbb{V}_{i1}(V - P_2)}$ is the conditional regression coefficient of $P_2 - P_1$ on $V - P_2$ given investor i's information set. Using these demands, we can solve for the equilibrium price coefficients by imposing market clearing at both dates.

Second, while the date $t = 2^-$ information equilibrium condition (for an interior equilibrium) is given by

$$\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U2}(V)}} e^{\gamma c_2} = 1, \tag{28}$$

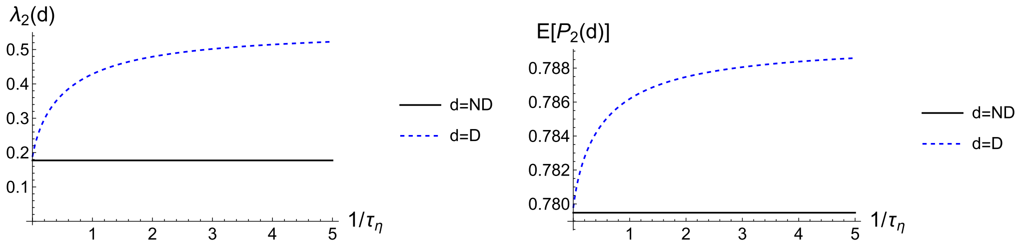
as in the benchmark analysis, the date $t = 1^-$ information equilibrium condition reduces to

$$e^{\gamma(c_1 - c_2)} \sqrt{\frac{\mathbb{V}_{I1}(P_2 - P_1 - \beta_{I1}(V - P_2))}{\mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V - P_2))}} = 1. \tag{29}$$

This reflects the fact that investors have the option to wait until date $t = 2^-$ to acquire information, and so will only acquire information at date 1 (i.e., pay a cost c_1 instead of c_2), if the

[17] Since the information available to investors is the same in either period, investors will not choose to acquire information in both periods.

The figure plots the fraction of informed investors at date 2 (i.e., λ_2) and the expected date 2 price as a function of the variance of the project payoff $1/\tau_\eta$ for disclosure (i.e., $d = D$, dashed) versus no-disclosure (i.e., $d = ND$, solid). The other parameter values are as follows: $\bar{V} = 1$, $\gamma = 0.2$, $c_1 = 0.7$, $c_2 = 0.3$, $\tau_\theta = 1$, $\tau_u = 1$, $\tau_z = 1$, $\bar{\eta} = 0$.



(a) λ_2 versus $1/\tau_\eta$ (b) $\mathbb{E}[P_2]$ versus $1/\tau_\eta$

Fig. 3. λ_2 and $\mathbb{E}[P_2]$ as a function of the project variance, $1/\tau_\eta$, with long-lived investors.

reduction in the variance of the orthogonal part of the date 1 return (i.e., the part of return $P_2 - P_1$ that is conditionally independent of $V - P_2$) is sufficiently large relative to the incremental information cost $c_1 - c_2$. Together, equations (28) and (29) pin down the equilibrium fractions of informed traders, λ_1 and λ_2, in any interior equilibrium.

Note that Lemma 2 applies directly in this setting, since the date 2 price has the same functional form as in the benchmark model and the date $t = 2^-$ information condition is the same. Hence, the expected date 2 price increases with disclosure if and only if the date 2 fraction of informed traders λ_2 increases with disclosure. While an analytical proof of Lemma 3 is not tractable, we can numerically show that the key result obtains i.e., $\lambda_2(D) > \lambda_2(ND)$ in this setting for a range of parameter values. As a result, the analog of Proposition (1) applies even when investors are not myopic and can choose when to acquire information. Fig. 3 provides an illustration of these results. As in our benchmark analysis, both the fraction of informed investors (Panel (a)) and the expected price at date 2 (Panel (b)) are higher with disclosure than without, and the effect of disclosure on both the fraction informed and the expected price is stronger when the project is riskier ($1/\tau_\eta$ is higher).

5.2. Persistent aggregate supply shocks

In Appendix B.3, we consider an extension of our benchmark model that retains the assumption of short-lived investors but extends the setting to allow for persistence in the supply shocks. Specifically, suppose that the aggregate supply of the stock follows an AR(1) process i.e.,

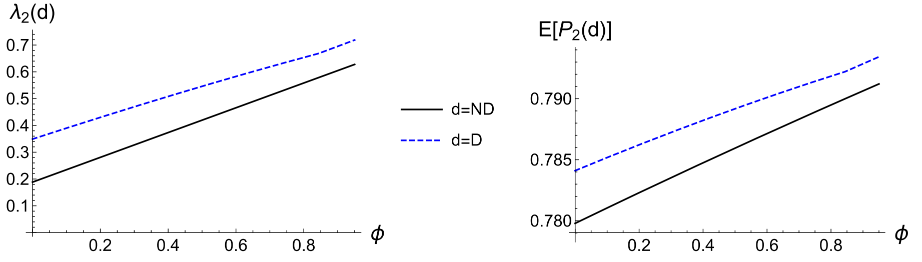
$$Z_t = \overline{Z} + \phi(Z_{t-1} - \overline{Z}) + z_t; \qquad Z_0 \equiv \bar{Z}, \tag{30}$$

for $\phi \in (0, 1)$ and z_t independently normally distributed with precisions τ_{zt}. This nests our benchmark setting as the special case in which $\phi = 0$ and $\tau_{z1} = \tau_{z2} = \tau_z$.

We conjecture and verify that there exists a financial market equilibrium in which prices are of the form:

$$P_1 = A_1 + B_1 s_{p1}, \quad \text{and} \quad P_2 = A_2 + B_2 s_{p2} + C_2 s_{p1} + x\eta, \tag{31}$$

where the $s_{pt} \equiv \theta + b_t(Z_t - \bar{Z})$ for $t \in \{1, 2\}$. While much of the analysis follows from that in the benchmark model, a key difference is that the price signals s_{p1} and s_{p2} are now correlated, The figure plots the fraction of informed investors at date 2 (i.e., λ_2) and the expected date 2 price as a function of ϕ, for disclosure (i.e., $d = D$, dashed) versus no-disclosure (i.e., $d = ND$, solid). The other parameter values are as follows: $\bar{V} = 1$, $\gamma = 0.2$, $c_1 = c_2 = 0.3$, $\tau_\theta = 1$, $\tau_u = 1$, $\tau_{z1} = \tau_{z2} = 1$, $\tau_\eta = 2$, $\bar{\eta} = 0$.



(a) λ_2 versus ϕ (b) $\mathbb{E}[P_2]$ versus ϕ

Fig. 4. λ_2 and $\mathbb{E}[P_2]$ as a function of persistence ϕ of supply shocks.

which affects the date 1 investors' beliefs about date 2 prices, and date 2 investors' beliefs about the terminal payoff. The explicit calculations are provided in Appendix B.

Given these differences, however, the functional forms for demand functions and the information conditions are analogous to those in Section 4, and a version of Lemma 1 obtains. Moreover, Lemma 2 applies directly in this setting. Although we expect the analog to Lemma 3 to hold, analytically establishing this result is intractable. However, we can demonstrate numerically that the result obtains i.e.,

$$\lambda_2(D) \geq \lambda_2(ND), \tag{32}$$

for a wide range of parameters, and consequently, Proposition 1 applies in this setting.

Fig. 4 provides an illustration. Panel (a) plots the fraction of investors who acquire information at date 2^-, and panel (b) plots the expected price at date 2. As in our benchmark model, disclosure leads to more information acquisition and higher expected prices. Notably, the fraction of investors who acquire information at date 2 tends to increase with ϕ. Intuitively, this is because when noise is persistent, the date 2 price is less (incrementally) informative given the public information, since the noise in the date 1 and date 2 signals are correlated. As a result, the incremental value of acquiring information is higher in this case. Similar results hold in the long-lived investor setting of Section 5.1 above, with persistent supply shocks.[18]

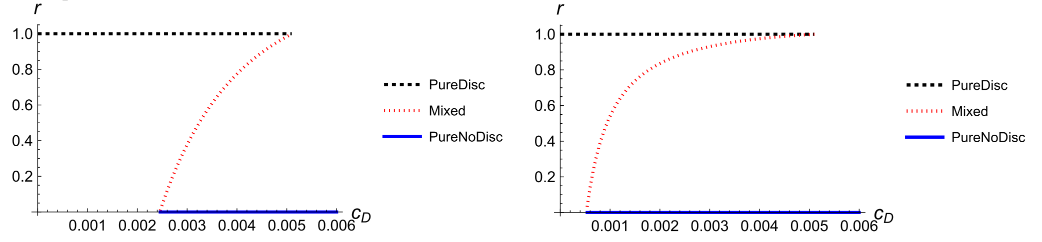
5.3. *Mean-variance preferences and mixed disclosure equilibria*

In Appendix B.4, we consider a variant of our model in which investors have mean-variance preferences.[19] In addition to establishing the robustness of our results with respect to the CARA utility specification, this extension allows us to consider the possibility that $x = 1$ firm mixes between disclosing or not, and to entertain the possibility of equilibria in which firms never

[18] The setting in Appendix B.2 already incorporates both long-lived investors and potentially persistent shocks.

[19] We thank an anonymous referee for suggesting mean-variance preferences as a more tractable setting in which to study mixed strategy equilibria.

The figure plots the equilibrium disclosure probability $r \in [0, 1]$ for a firm that has the short-term project ($x = 1$) as a function of the disclosure cost c_D. The other parameter values are as follows: $\bar{V} = 1$, $\gamma = 0.2$, $c = 0.3$, $\tau_\theta = 1$, $\tau_u = 1$, $\tau_z = 1$, $\tau_\eta = 1$, $\bar{\eta} = 0$, $\overline{Z} = 1$. The low prior probability case corresponds to $p = 0.25$ and the high prior probability case corresponds to $p = 0.75$.



(a) low prior prob ($p = 0.25$) (b) high prior prob ($p = 0.75$)

Fig. 5. Equilibrium disclosure probability r for the $x = 1$ firm as a function of the disclosure cost c_D.

disclose. Specifically, we assume that investor i at date t chooses $X_{i,t}$ to maximize an explicit mean variance objective[20]:

$$X_{i,t} \equiv \arg\max_{x \in \mathbb{R}} \mathbb{E}_{it}\left[W_t + x\left(P_{t+1} - P_t\right)\right] - \frac{\gamma}{2}\mathbb{V}_{it}\left[W_t + x\left(P_{t+1} - P_t\right)\right]. \tag{33}$$

Moreover, we allow the manager to mix between disclosing and not: she chooses a disclosure probability $r \in [0, 1]$, subject to disclosure cost $c_D > 0$.[21] As we discuss above in Section 4.4, even numerically characterizing such an equilibrium is infeasible when investors have CARA utility. However, as we show in the appendix, when investors have mean-variance preferences, we are able to characterize the financial market and information acquisition equilibrium up to a set of polynomial equations (as in the baseline model), which we can easily solve numerically for a given set of parameters.

Using this numerical approach, we explore how the nature of equilibria changes as a function of the disclosure cost c_D. Note that the $x = 0$ firm always discloses with probability zero in any equilibrium since she can always make herself better of by not disclosing, saving on the disclosure cost, and being assigned a (weakly) positive probability of being the $x = 1$ type. Fig. 5 plots the equilibrium disclosure probability r for the $x = 1$ type firm as a function of disclosure costs c_D. Consistent with Proposition 1, when the disclosure cost is sufficiently low, there only exists a pure strategy equilibrium in which the $x = 1$ manager always discloses (dashed line). Moreover, it is naturally the case that when the disclosure cost is sufficiently high, there only exists a pure strategy equilibrium in which the manager never discloses (solid line). For "intermediate" costs, both these pure strategy equilibria exist and, in addition, there exists a mixed strategy equilibrium (dotted line) in which the $x = 1$ manager discloses with probability $r \in (0, 1)$.

[20] Note that assuming mean-variance preferences is generally *not* equivalent to assuming expected CARA utility. If asset payoffs are conditionally Gaussian, then a CARA investor's portfolio problem happens to reduce to a mean-variance problem. However, if a CARA investor perceives payoffs as non-Gaussian (as they necessarily do if the manager follows a mixed disclosure strategy), then her portfolio problem generally does *not* reduce to a mean-variance problem.

[21] The cost is incurred conditional on disclosing i.e., if she ultimately discloses, whether under a pure or mixed strategy, she pays cost c_D, while if she does not disclose, she pays no cost.

These results are broadly consistent with the economic intuition from our benchmark analysis. Note that in any equilibrium, the $x = 1$ manager is (weakly) more likely to disclose than the $x = 0$ manager. Moreover, our numerical exploration suggests that, regardless of the precise value of disclosure cost c_D, there always exists a pure strategy equilibrium of some form (e.g., either one with disclosure, one without disclosure, or both), while a mixed strategy equilibrium can only be sustained for an intermediate range of disclosure costs.

6. Concluding remarks

We propose a novel rationale for voluntary disclosure, by studying how voluntary, costly disclosure affects information acquisition in a dynamic model of trading. We show that a manager finds it optimal to disclose information that becomes redundant at a later date, even if she intends to maximize long-term share prices. By disclosing information about the presence of a short-term risky project, the manager increases perceived risk and reduces price informativeness in early periods. Once the payoffs of this short-term project are revealed, later investors acquire information more aggressively than they would have if the manager had not disclosed the project earlier. We show that increased information acquisition by later investors can dominate the short-term increase in uncertainty, and lead to long-term prices that are more informative and higher on average. Furthermore, the impact of disclosure on investors' information acquisition and longer-term prices is likely to be larger for firms in industries where cashflows of typical projects are more uncertain and dispersed. Because our findings rely upon the manager caring about long-term prices as opposed to the short-term prices, a further prediction of our model is that the pervasiveness of voluntary disclosures stemming from our mechanism should be positively associated with the degree of the CEO's long-term orientation.[22]

Our analysis suggests a number of natural extensions. For instance, it would be interesting to consider the strategic timing of voluntary disclosure (as in Guttman et al. (2014)) in a setting with dynamic information acquisition. It might also be interesting to study how voluntary disclosure is affected by dynamic information acquisition in the presence of real investment and feedback effects. It would also be interesting to study how our analysis changes if managers can engage in cheap talk or costly lying. Finally, one could endogenize the manager's objective as part of an optimal contracting problem and study disclosure policies in the resulting equilibrium. We leave these questions for future work.

Declaration of competing interest

None.

Data availability

No data was used for the research described in the article.

[22] Long-term orientation can be proxied, for example, by the executive pay duration measure of Gopalan et al. (2014), which is defined as the weighted average of the vesting periods of the different components of executive pay.

Appendix A. Proofs

A.1. Proof of Lemma 1

We can solve for the coefficients b_1 and b_2 by observing that $s_{p,t}$ is a linear transformation of the informed residual demand $\lambda_t X_{It} - z_t$. This implies:

$$b_2 = -\frac{\gamma}{\lambda_2 \tau_u}, \quad b_1 = -\frac{\gamma}{\lambda_1} \frac{x^2/\tau_\eta + B_2^2/\tau_{p2}}{B_2}. \tag{34}$$

Next, we solve for the price coefficients by imposing market clearing and matching coefficients. Specifically, note that B_2 is given by:

$$B_2 = \frac{\lambda_2 (\tau_{U1} + \tau_u) + \tau_{p2}}{\tau_{U1} + \lambda_2 \tau_u + \tau_{p2}}, \tag{35}$$

where $\tau_{U1} = \tau_\theta + \tau_{p1}$, and $\tau_{pt} = \tau_z / b_t^2$. Substituting, this implies that B_2 is a solution to $H(B_2) = 0$, where

$$H(B_2) = \frac{\lambda_2 \left(\gamma^2 \left(\frac{B_2^2 \lambda_1^2 \tau_\eta^2 \tau_z^3}{\left(\frac{B_2^2 \gamma^3 \tau_\eta}{\lambda_2^2 \tau_u^2} + \gamma x^2 \tau_z \right)^2} + \tau_\theta + \tau_u \right) + \lambda_2 \tau_u^2 \tau_z \right)}{\gamma^2 \left(\frac{B_2^2 \lambda_1^2 \tau_\eta^2 \tau_z^3}{\left(\frac{B_2^2 \gamma^3 \tau_\eta}{\lambda_2^2 \tau_u^2} + \gamma x^2 \tau_z \right)^2} + \tau_\theta + \lambda_2 \tau_u \right) + \lambda_2^2 \tau_u^2 \tau_z} - B_2 \tag{36}$$

Note that

$$\begin{aligned} H(0) &= \frac{\lambda_2 \left(\gamma^2 (\tau_\theta + \tau_u) + \lambda_2 \tau_u^2 \tau_z \right)}{\gamma^2 (\tau_\theta + \lambda_2 \tau_u) + \lambda_2^2 \tau_u^2 \tau_z} > 0 \\ H(1) &= \frac{(\lambda_2 - 1)\left(\tau_\theta \left(\gamma^3 \tau_\eta + \gamma \lambda_2^2 x^2 \tau_u^2 \tau_z \right)^2 + \lambda_1^2 \lambda_2^4 \tau_\eta^2 \tau_u^4 \tau_z^3 \right)}{\tau_\theta \left(\gamma^3 \tau_\eta + \gamma \lambda_2^2 x^2 \tau_u^2 \tau_z \right)^2 + 2\gamma^2 \lambda_2^3 x^2 \tau_\eta \tau_u^3 \tau_z \left(\gamma^2 + \lambda_2 \tau_u \tau_z \right) + \lambda_2^5 x^4 \tau_u^5 \tau_z^2 \left(\gamma^2 + \lambda_2 \tau_u \tau_z \right) + \lambda_2 \tau_\eta^2 \tau_u \left(\gamma^6 + \gamma^4 \lambda_2 \tau_u \tau_z + \lambda_1^2 \lambda_2^3 \tau_u^3 \tau_z^3 \right)} \\ &\leq 0, \end{aligned}$$

since $\lambda_2 \leq 1$, which implies there exists a solution to $H(B_2) = 0$ for $B_2 \in (0, 1)$. While we have been unable to prove analytically that there is a unique solution of $H(B_2) = 0$, we have verified numerically over an extensive range of parameter values that there is only one solution for each given set of parameters.

Moreover, letting $\psi(B_2) \equiv \frac{B_2^2 \lambda_1^2 \tau_\eta^2 \tau_z^3}{\left(\frac{B_2^2 \gamma^3 \tau_\eta}{\lambda_2^2 \tau_u^2} + \gamma x^2 \tau_z \right)^2}$, note that we can express

$$H'(B_2) = -1 - \frac{\gamma^2 \lambda_2 \tau_u \left(\gamma^2 (1 - \lambda_2) + \lambda_2 \tau_u \tau_z (1 - \lambda_2 \tau_z) \right)}{\left(\gamma^2 \tau_\theta + \gamma^2 \psi + \lambda_2 \tau_u \left(\gamma^2 + \lambda_2 \tau_u \tau_z^2 \right) \right)^2} \times \psi'(B_2) \tag{37}$$

where

$$\psi'(B_2) = \frac{2 B_2 \lambda_1^2 \lambda_2^4 \tau_\eta^2 \tau_u^4 \tau_z^3 \left(\lambda_2^2 x^2 \tau_u^2 \tau_z - B_2^2 \gamma^2 \tau_\eta \right)}{\gamma^2 \left(B_2^2 \gamma^2 \tau_\eta + \lambda_2^2 x^2 \tau_u^2 \tau_z \right)^3}. \tag{38}$$

Given B_2, we can solve for (b_1, b_2), and then solve for the other coefficients using the following system:

$$A_2 = \bar{V} - \frac{\gamma \bar{Z}\left(\gamma^2\left(\frac{\tau_z}{b_1^2} + \tau_\theta + \tau_u\right) + \lambda_2^2 \tau_u^2 \tau_z\right)}{\gamma^2 \tau_u \left(\frac{\tau_z}{b_1^2} + \tau_\theta + \lambda_2 \tau_u\right) + \lambda_2^2 \tau_u^3 \tau_z} \tag{39}$$

$$A_1 = A_2 - \frac{\gamma \bar{Z}}{\frac{1-\lambda_1}{B_2^2\left(\frac{1}{\frac{\tau_z}{b_1^2}+\tau_\theta} + \frac{b_2^2}{\tau_z}\right) + \frac{x^2}{\tau_\eta}} + \frac{\lambda_1 \tau_\eta \tau_z}{b_2^2 B_2^2 \tau_\eta + x^2 \tau_z}} + x\bar{\eta} \tag{40}$$

$$C_2 = \frac{b_2^2 (1-\lambda_2)\tau_z}{b_1^2\left(b_2^2\left(\frac{\tau_z}{b_1^2} + \tau_\theta + \lambda_2 \tau_u\right) + \tau_z\right)} \tag{41}$$

$$B_1 = \frac{b_1^2 \tau_\theta (B_2 \lambda_1 + C_2)\left(b_2^2 B_2^2 \tau_\eta + x^2 \tau_z\right) + (B_2 + C_2)\tau_z \left(B_2^2\left(b_1^2 \lambda_1 + b_2^2\right)\tau_\eta + x^2 \tau_z\right)}{B_2^2 \tau_\eta \left(b_2^2\left(b_1^2 \tau_\theta + \tau_z\right) + b_1^2 \lambda_1 \tau_z\right) + x^2 \tau_z \left(b_1^2 \tau_\theta + \tau_z\right)}. \quad \square \tag{42}$$

A.2. Proof of Lemma 3

We first establish the claim about the fraction informed, λ_2, and then show that it implies the claim about the precision τ_{p1}. For an interior λ_1, λ_2, note that the information equilibrium conditions imply

$$W = \frac{\mathbb{V}_{Ut}[\mathbb{E}_{It}[P_{t+1}]]}{\mathbb{V}_{It}[P_{t+1}]} \tag{43}$$

where $W \equiv e^{2\gamma c} - 1$. Substituting in explicitly, the information equilibrium conditions for dates $t = 2$ and $t = 1$ are given by

$$W = \frac{1/\tau_{U2}}{1/\tau_u}, \quad \text{and} \quad W = \frac{B_2^2/\tau_{U1} + x^2/\tau_\eta}{B_2^2/\tau_{p2} + x^2/\tau_\eta}, \tag{44}$$

respectively. Plugging in $\tau_{U2} = \tau_{U1} + \tau_{p2}$ and rearranging terms gives:

$$\tau_e = W\left(\tau_{U1} + \tau_{p2}\right) \tag{45}$$

$$\frac{B_2^2}{\left(B_2^2/\tau_{p2} + x^2/\tau_\eta\right)} = W \tau_{U1} \tag{46}$$

Next, recall that since B_2 is given by

$$B_2 = \frac{\lambda_2(\tau_{U1} + \tau_u) + \tau_{p2}}{\tau_{U1} + \lambda_2 \tau_u + \tau_{p2}} \tag{47}$$

we can substitute in to express it as

$$B_2 = \frac{\lambda_2 \tau_u (1 + W) + W \tau_{p2}(1 - \lambda_2)}{(1 + \lambda_2 W)\tau_u} \tag{48}$$

$$= \frac{\lambda_2 \left(\gamma^2(W+1) + (1-\lambda_2)\,\lambda_2 \tau_z W \tau_u\right)}{\gamma^2\,(\lambda_2 W + 1)} \tag{49}$$

Combining (45) and (46), plugging in (49), and rearranging, characterizes the equilibrium relation between λ_2 and x in any interior equilibrium:

$$\frac{x^2}{\tau_\eta} = \frac{B_2^2}{\tau_{p2}} \frac{(W+1)\,\tau_{p2} - \tau_u}{\tau_u - W\tau_{p2}} \tag{50}$$

$$= \frac{\lambda_2^2 \left(\gamma^2\,(1+W) + (1-\lambda_2)\,\lambda_2 W \tau_u \tau_z\right)^2}{\gamma^4\,(1+\lambda_2 W)^2} \frac{(W+1)\,\tau_{p2} - \tau_u}{\tau_{p2}\left(\tau_u - W\tau_{p2}\right)} \tag{51}$$

$$= \frac{\left(\gamma^2(W+1) + (1-\lambda_2)\,\lambda_2 W \tau_u \tau_z\right)^2 \left(\gamma^2 - \lambda_2^2(W+1)\tau_u\tau_z\right)}{\gamma^2 \tau_u^2\,(\lambda_2 W+1)^2 \tau_z \left(\lambda_2^2 W \tau_u \tau_z - \gamma^2\right)} \equiv G\,(\lambda_2) \tag{52}$$

where the last line defines the function $G(\lambda_2)$ in order to condense notation. Note that $G\,(0) = -\frac{\gamma^2(W+1)^2}{\tau_u^2\tau_z} < 0$ and $G\,(1) = -\frac{\gamma^2\left(\gamma^2-(W+1)\tau_u\tau_z\right)}{\tau_u^2\tau_z\left(\gamma^2-W\tau_u\tau_z\right)}$. For the equilibrium $\lambda_2 \in (0,1)$ to exist, we need to have: $G\,(1) > \frac{x^2}{\tau_\eta} > 0 > G\,(0)$, which is equivalent to restricting

$$\frac{\gamma^2}{(1+W)\,\tau_z} < \tau_u < \frac{\gamma^2}{W\tau_z} \tag{53}$$

$$\Leftrightarrow \gamma^2 > W\tau_u\tau_z, \tag{54}$$

$$\gamma^2 < (1+W)\,\tau_u\tau_z \tag{55}$$

Moreover, tedious algebra establishes that

$$G\,(\lambda_2) = \frac{\left(\frac{W+1}{W} - \frac{\gamma^2}{\lambda_2^2 W\tau_u\tau_z}\right)\left(1 - \frac{(1-\lambda_2)\left(\frac{1}{W} - \frac{\lambda_2^2\tau_u\tau_z}{\gamma^2}\right)}{\lambda_2 + \frac{1}{W}}\right)^2}{\tau_u\left(\frac{1}{W} - \frac{\lambda_2^2\tau_u\tau_z}{\gamma^2}\right)} \equiv g_1\,(\lambda_2)\,g_2\,(\lambda_2)\,g_3\,(\lambda_2) \tag{56}$$

By the implicit function theorem, in order to show that λ_2 is increasing in x^2, we need to show that $G(\lambda_2)$ above is increasing in λ_2. Now,

$$g_3\,(\lambda_2) = \frac{1}{\tau_u\left(\frac{1}{W} - \frac{\lambda_2^2\tau_u\tau_z}{\gamma^2}\right)} > 0 \tag{57}$$

from (53), and $g_3\,(\lambda_2)$ is increasing in λ_2. Next,

$$g_2\,(\lambda_2) = \left(1 - \frac{(1-\lambda_2)\left(\frac{1}{W} - \frac{\lambda_2^2\tau_u\tau_z}{\gamma^2}\right)}{\lambda_2 + \frac{1}{W}}\right)^2 > 0 \tag{58}$$

and since $\frac{1}{W} - \frac{\lambda_2^2\tau_u\tau_z}{\gamma^2} > 0$, $g_2\,(\lambda_2)$ is increasing in λ_2. Finally, $g_1\,(\lambda_2)$ is increasing in λ_2. Moreover, it must be positive in an interior equilibrium since $G\,(\lambda_2) = \frac{x^2}{\tau_\eta} \geq 0$ and we already know that g_2 and g_3 are positive. This implies

$$\frac{dG}{d\lambda_2} = g_1' g_2 g_3 + g_1 g_2' g_3 + g_1 g_2 g_3' > 0. \tag{59}$$

To summarize this shows that $G(\lambda_2)$ is increasing in λ_2. Since $G(\lambda_2) = \frac{x^2}{\tau_\eta}$, this implies that the equilibrium λ_2 is increasing in x^2 as long as λ_1 and λ_2 are interior.

Now consider the $t = 1$ price-signal precision τ_{p1}. Once again using the $t = 2$ information equilibrium condition, we can write λ_2 explicitly in any interior equilibrium:

$$W = \frac{1/\tau_{U2}}{1/\tau_u} \tag{60}$$

$$\Rightarrow e^{2\gamma c} - 1 = \frac{\tau_u}{\tau_\theta + \tau_{p1} + \tau_z \left(\frac{\lambda_2 \tau_u}{\gamma}\right)^2} \tag{61}$$

$$\Rightarrow \lambda_2 = \gamma \frac{1}{\sqrt{\tau_u \tau_z}} \left(\frac{1}{e^{2\gamma c} - 1} - \frac{\tau_\theta + \tau_{p1}}{\tau_u} \right)^{1/2} \tag{62}$$

the only term on the right-hand side of eq. (62) that depends on the manager's disclosure x is the $t = 1$ price-signal precision τ_{p1} and we have already shown that λ_2 increases in x^2. Hence, it must be the case that τ_{p1} decreases in x^2. □

A.3. *Proof of Lemma 4*

Suppose to the contrary that there exists an interior equilibrium in which an $x = 0$ manager discloses with probability $r_0 \in (0, 1]$, an $x = 1$ manager discloses with probability $r_1 \in [0, 1]$, and we do not have $r_0 = r_1 = 1$ (we consider this case separately below). In such an equilibrium, the market assigns probability

$$q(r_0, r_1) = \frac{p(1 - r_1)}{(1 - p)(1 - r_0) + p(1 - r_1)}$$

that $x = 1$ in the event of no disclosure. Consider first the case $r_1 = 1$. In this case the market assigns probability $q = 0$ and therefore the expected price for an $x = 0$ manager is identical whether she discloses or not, $U_{ND}(0; q) = U_D(0)$. Hence,

$$U_D(0) - c_D < U_{ND}(0; q),$$

which implies that the $x = 0$ manager strictly prefers not disclosing. Consider next the case in which $r_1 \in [0, 1)$. In this case, because the $x = 1$ manager does not disclose with positive probability, then we know that she is either indifferent (in the case $r_1 \in (0, 1)$) or strictly prefers not disclosing (in the case $r_1 = 0$), which implies:

$$\begin{aligned} & U_D(1) - c_D - U_{ND}(1; q) \leq 0 \\ \Rightarrow\ & U_D(0) - c_D - U_{ND}(0; q) < 0 \end{aligned}$$

where the second line follows from Lemma 2, which establishes that $U_D(1) > U_D(0)$, and the observation that $U_{ND}(1; q) = U_{ND}(0; q)$, since the expected non-disclosure price only depends on the market's beliefs q and not the realized value of x. This implies that the $x = 0$ manager strictly prefers not disclosing. Finally, consider the case in which $r_0 = r_1 = 1$. In this case, Bayes rule does not pin down the probability that the market assigns to $x = 1$ in event of nondisclosure. Given off-equilibrium belief $q_{OFF} = 0$, we again have $U_{ND}(0; q_{OFF}) = U_D(0)$ and hence

$$U_D(0) - c_D < U_{ND}(0; q)$$

so that the $x = 0$ manager strictly prefers not disclosing. $\square$

A.4. Proof of Proposition 2

Suppose that there exists an equilibrium in which the manager follows a disclosure strategy $r_0 = 0$, $r_1 \in [0, 1)$ and asset prices in the event of nondisclosure $d = ND$ are linear functions of fundamentals

$$P_1(ND) = A_1(ND) + B_1(ND)\, s_{p1}, \quad \text{and}$$
$$P_2(x, ND) = A_2(ND) + B_2(ND)\, s_{p2} + C_2(ND)\, s_{p1} + x\eta$$

where the price signals $s_{pt} \equiv \theta + b_t(ND)\, z_t$ for $t \in \{1, 2\}$, and we define the precisions $\tau_{pt} \equiv \tau_z / b_t^2$. In the analysis that follows we will suppress the explicit dependence of the coefficients on the event of nondisclosure in order to reduce clutter. Let $\mathcal{F}_{It} = \sigma\left(d, \theta, \{P_k\}_{k \le t}\right) = \sigma\left(d, \theta, \left\{s_{pk}\right\}_{k \le t}\right)$ and $\mathcal{F}_{Ut} = \sigma\left(d, \{P_k\}_{k \le t}\right) = \sigma\left(d, \left\{s_{pk}\right\}_{k \le t}\right)$ denote the information sets at time t for informed and uninformed investors, respectively, with conditional expectation and variance operators $\mathbb{E}_{it}$ and $\mathbb{V}_{it}$, $i \in \{I, U\}$.

In an equilibrium of the posited form, all investors at the date 1 trading round assign probability

$$q(r_0 = 0, r_1) = \frac{p(1 - r_1)}{(1 - p) + p(1 - r_1)} \in (p, 1)$$

that the firm has a project, $x = 1$. Consider the problem of an arbitrary informed investor at date 1

$$\max_X \mathbb{E}_{I1}\left[-e^{-\gamma X (P_2 - P_1)}\right]$$

Computing the expected utility in the objective function yields

$$\begin{aligned}
&\mathbb{E}_{I1}\left[-e^{-\gamma X(P_2(x) - P_1)}\right] \\
&= q\mathbb{E}_{I1}\left[-e^{-\gamma X(P_2(x) - P_1)}\middle| x = 1\right] + (1 - q)\,\mathbb{E}_{I1}\left[-e^{-\gamma X(P_2(x) - P_1)}\middle| x = 0\right] \\
&= -qe^{-\gamma X\left(\mathbb{E}_{11}[P_2(1)] - P_1\right) + \frac{1}{2}\gamma^2 X^2 \mathbb{V}_{I1}(P_2(1))} - (1 - q)\, e^{-\gamma X\left(\mathbb{E}_{11}[P_2(0)] - P_1\right) + \frac{1}{2}\gamma^2 X^2 \mathbb{V}_{I1}(P_2(0))} \\
&= -qe^{-\gamma X\left(A_2 + B_2\theta + C_2 s_{p1} + \bar{\eta} - P_1\right) + \frac{1}{2}\gamma^2 X^2\left(1/\tau_\eta + B_2^2/\tau_{p2}\right)} \\
&\quad - (1 - q)\, e^{-\gamma X\left(A_2 + B_2\theta + C_2 s_{p1} - P_1\right) + \frac{1}{2}\gamma^2 X^2 B_2^2/\tau_{p2}}
\end{aligned}$$

where the first equality uses the law of iterated expectations to condition down on x and the second equality uses the fact that, given x, the second period price is conditionally Normally distributed under the informed investor information set, and the final equality plugs in for the conditional means and variances. The investor's maximization problem is strictly concave and defined for demands X on the entire real line. Hence, there is a unique optimal demand X_{I1}, for which the FOC is necessary and sufficient:

$$
\begin{aligned}
0 = q &\left(A_2 + B_2\theta + C_2 s_{p1} + \bar{\eta} - P_1 - \gamma X_{I1}\left(\tfrac{1}{\tau_\eta} + \tfrac{B_2^2}{\tau_{p2}}\right)\right) \\
&\qquad \times e^{-\gamma X_{I1}\left(A_2 + B_2\theta + C_2 s_{p1} + \bar{\eta} - P_1\right) + \frac{1}{2}\gamma^2 X_{I1}^2\left(1/\tau_\eta + B_2^2/\tau_{p2}\right)} \\
&+ (1-q)\left(A_2 + B_2\theta + C_2 s_{p1} - P_1 - \gamma X_{I1}\tfrac{B_2^2}{\tau_{p2}}\right) \\
&\qquad \times e^{-\gamma X_{I1}\left(A_2 + B_2\theta + C_2 s_{p1} - P_1\right) + \frac{1}{2}\gamma^2 X_{I1}^2 B_2^2/\tau_{p2}} \\
= q &\left(A_2 + B_2\theta + C_2\frac{P_1 - A_1}{B_1} + \bar{\eta} - P_1 - \gamma X_{I1}\left(\tfrac{1}{\tau_\eta} + \tfrac{B_2^2}{\tau_{p2}}\right)\right) e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}} \\
&+ (1-q)\left(A_2 + B_2\theta + C_2\frac{P_1 - A_1}{B_1} - P_1 - \gamma X_{I1}\tfrac{B_2^2}{\tau_{p2}}\right)
\end{aligned}
\tag{63}
$$

where the second equality divides out terms that are common across the two exponentials and plugs in explicitly for s_{p1} in terms of the price P_1 using the initial functional form conjecture. Equation (63) does not have a closed form solution, but it uniquely characterizes the informed demand function $X_{I1}(\theta, P_1)$.

Similarly, consider the problem of an arbitrary uninformed investor at date 1

$$
\max_X \mathbb{E}_{U1}\left[-e^{-\gamma X(P_2 - P_1)}\right].
$$

Under the conjectured price functions and resulting uninformed information set, for the date 1 uninformed investors, the fundamental θ is conditionally normally distributed with conditional mean and variance

$$
\mathbb{E}_{U1}[\theta] = \frac{\tau_{p1}s_{p1}}{\tau_\theta + \tau_{p1}} = \frac{\tau_{p1}}{\tau_\theta + \tau_{p1}}\frac{P_1 - A_1}{B_1}, \quad \mathbb{V}_{U1}[\theta] = \frac{1}{\tau_\theta + \tau_{p1}} \equiv \frac{1}{\tau_{U1}}. \tag{64}
$$

Hence, computing the expected utility in the objective function yields

$$
\begin{aligned}
&\mathbb{E}_{U1}\left[-e^{-\gamma X(P_2(x) - P_1)}\right] \\
&= q\mathbb{E}_{U1}\left[-e^{-\gamma X(P_2(x) - P_1)}\middle| x = 1\right] + (1-q)\,\mathbb{E}_{U1}\left[-e^{-\gamma X(P_2(x) - P_1)}\middle| x = 0\right] \\
&= -qe^{-\gamma X\left(\mathbb{E}_{U1}[P_2(1)] - P_1\right) + \frac{1}{2}\gamma^2 X^2 \mathbb{V}_{U1}(P_2(1))} - (1-q)\,e^{-\gamma X\left(\mathbb{E}_{U1}[P_2(0)] - P_1\right) + \frac{1}{2}\gamma^2 X^2 \mathbb{V}_{U1}(P_2(0))} \\
&= -qe^{-\gamma X\left(A_2 + B_2\mathbb{E}_{U1}[\theta] + C_2 s_{p1} + \bar{\eta} - P_1\right) + \frac{1}{2}\gamma^2 X^2\left(1/\tau_\eta + B_2^2/\tau_{p2} + B_2^2/\tau_{U1}\right)} \\
&\qquad - (1-q)\,e^{-\gamma X\left(A_2 + B_2\mathbb{E}_{U1}[\theta] + C_2 s_{p1} - P_1\right) + \frac{1}{2}\gamma^2 X^2\left(B_2^2/\tau_{p2} + B_2^2/\tau_{U1}\right)}
\end{aligned}
$$

where the first equality uses the law of iterated expectations to condition down on x and the second equality uses the fact that, given x, the second period price is conditionally Normally distributed under the uninformed investor information set, and the final equality plugs in for the conditional means and variances. As for an informed investor, an uninformed investor's maximization problem is strictly concave and defined for demands X on the entire real line. Hence, there is a unique optimal demand X_{U1}, for which the FOC is necessary and sufficient:

$$
\begin{aligned}
0 = q &\left(A_2 + B_2\mathbb{E}_{U1}[\theta] + C_2 s_{p1} + \bar{\eta} - P_1 - \gamma X_{U1}\left(\tfrac{1}{\tau_\eta} + \tfrac{B_2^2}{\tau_{p2}} + \tfrac{B_2^2}{\tau_{U1}}\right)\right) \\
&\qquad \times e^{-\gamma X_{U1}\left(A_2 + B_2\mathbb{E}_{U1}[\theta] + C_2 s_{p1} + \bar{\eta} - P_1\right) + \frac{1}{2}\gamma^2 X_{U1}^2\left(1/\tau_\eta + B_2^2/\tau_{p2} + B_2^2/\tau_{U1}\right)}
\end{aligned}
$$

$$
\begin{aligned}
&+(1-q)\left(A_2+B_2\mathbb{E}_{U1}[\theta]+C_2 s_{p1}-P_1-\gamma X_{U1}\left(\tfrac{B_2^2}{\tau_{p2}}+\tfrac{B_2^2}{\tau_{U1}}\right)\right)\\
&\qquad\times e^{-\gamma X_{U1}\left(A_2+B_2\mathbb{E}_{U1}[\theta]+C_2 s_{p1}-P_1\right)+\frac{1}{2}\gamma^2 X_{U1}^2\left(B_2^2/\tau_{p2}+B_2^2/\tau_{U1}\right)}\\
&=q\left(A_2+\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{P_1-A_1}{B_1}+\bar{\eta}-P_1\right.\\
&\qquad\left.-\gamma X_{U1}\left(\tfrac{1}{\tau_\eta}+\tfrac{B_2^2}{\tau_{p2}}+\tfrac{B_2^2}{\tau_{U1}}\right)\right)e^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}\\
&\quad+(1-q)\left(A_2+\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{P_1-A_1}{B_1}-P_1-\gamma X_{U1}\left(\tfrac{B_2^2}{\tau_{p2}}+\tfrac{B_2^2}{\tau_{U1}}\right)\right)
\end{aligned}
\tag{65}
$$

where the second equality divides out terms that are common across the two exponentials and plugs in explicitly for $\mathbb{E}_{U1}[\theta]$ and s_{p1} in terms of the price P_1 using eq. (64) and the initial functional form conjecture. Equation (65) does not have a closed form solution, but it uniquely characterizes the uninformed demand function $X_{U1}(P_1)$.

With the optimal demand functions pinned down, the market clearing condition requires

$$
\lambda_1 X_{I1}(\theta,P_1)+(1-\lambda_1)X_{U1}(P_1)=\bar{Z}+z_1. \tag{66}
$$

The demand functions characterized by eqs. (63) and (65), and the market clearing condition (66) fully characterize the $t=1$ equilibrium price. In order for our conjecture that P_1 is linear to be consistent, it must be the case that the P_1 that satisfies this set of equilibrium conditions is a linear function of the form $P_1=A_1+B_1 s_{p1}=A_1+B_1(\theta+b_1 z_1)$. We will proceed by enforcing the initial conjecture that the price is a linear function of this form and showing that this leads to a contradiction.

Specifically, we will show that a linear P_1 so defined has non-constant derivative, which contradicts linearity. By the implicit function theorem, the demand functions characterized in eqs. (63) and (65) are continuously differentiable in their arguments, and it therefore follows from another application of the implicit function theorem that the equilibrium price defined by eq. (66) is a continuously differentiable function of the underlying random variables θ and z_1. Differentiating the market clearing condition totally yields

$$
\frac{\partial}{\partial z_1}P_1=\frac{1}{\lambda_1\frac{\partial}{\partial P_1}X_{I1}(\theta,P_1)+(1-\lambda_1)\frac{\partial}{\partial P_1}X_{U1}(P_1)}. \tag{67}
$$

Furthermore, computing the partial derivative of the informed demand function with respect to P_1 using the implicit function theorem on eqs. (63) yields

$$
\frac{\partial}{\partial P_1}X_{I1}(\theta,P_1)=-K_{I1}^{-1} \tag{68}
$$

where

$$
K_{I1}=\frac{q\gamma\left(\frac{1}{\tau_\eta}+\frac{B_2^2}{\tau_{p2}}\right)e^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}+(1-q)\gamma\frac{B_2^2}{\tau_{p2}}}{\left(1-\frac{C_2}{B_1}\right)\left(qe^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}+(1-q)\right)}
$$

$$+\frac{q\left(A_2+B_2\theta+C_2\frac{P_1-A_1}{B_1}+\bar{\eta}-P_1-\gamma X_{I1}\left(\frac{1}{\tau_\eta}+\frac{B_2^2}{\tau_{p2}}\right)\right)\left(\gamma\bar{\eta}-\gamma^2 X_{I1}\frac{1}{\tau_\eta}\right)e^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}}{\left(1-\frac{C_2}{B_1}\right)\left(qe^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}+(1-q)\right)}$$

$$=\frac{q\gamma\left(\frac{1}{\tau_\eta}+\frac{B_2^2}{\tau_{p2}}\right)e^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}+(1-q)\gamma\frac{B_2^2}{\tau_{p2}}}{\left(1-\frac{C_2}{B_1}\right)\left(qe^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}+(1-q)\right)}$$

$$+\gamma\frac{\left(\bar{\eta}-\gamma X_{I1}\frac{1}{\tau_\eta}\right)^2}{1-\frac{C_2}{B_1}}\frac{qe^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}}{qe^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}+(1-q)}\left(1-\frac{qe^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}}{qe^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}+(1-q)}\right)$$

where the final equality uses the FOC from eq. (63) to substitute

$$\left(A_2+B_2\theta+C_2\frac{P_1-A_1}{B_1}-P_1-\gamma X_{I1}\frac{B_2^2}{\tau_{p2}}\right)$$

$$=-\left(\bar{\eta}-\gamma X_{I1}\frac{1}{\tau_\eta}\right)\frac{qe^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}}{qe^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2\frac{1}{\tau_\eta}}+(1-q)}$$

in the second term and simplifies the resulting expression. Similarly, using the implicit function theorem to compute the partial derivative of uninformed demand using eq. (65) gives

$$\frac{\partial}{\partial P_1}X_{U1}(P_1)=-K_{U1}^{-1} \tag{69}$$

where

$$K_{U1}=\frac{q\gamma\left(\frac{1}{\tau_\eta}+\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)e^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)\gamma\left(\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)}{\left(1-\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{1}{B_1}\right)\left(qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)\right)} \tag{70}$$

$$+\frac{q\left(A_2+\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{P_1-A_1}{B_1}+\bar{\eta}-P_1-\gamma X_{U1}\left(\frac{1}{\tau_\eta}+\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)\right)\left(\gamma\bar{\eta}-\gamma^2 X_{U1}\frac{1}{\tau_\eta}\right)e^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}}{\left(1-\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{1}{B_1}\right)\left(qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)\right)}$$

$$=\frac{q\gamma\left(\frac{1}{\tau_\eta}+\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)e^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)\gamma\left(\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)}{\left(1-\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{1}{B_1}\right)\left(qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)\right)} \tag{71}$$

$$+\gamma\frac{\left(\bar{\eta}-\gamma X_{U1}\frac{1}{\tau_\eta}\right)^2}{1-\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{1}{B_1}}\frac{qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}}{qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)}\left(1-\frac{qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}}{qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)}\right) \tag{72}$$

Now, if the equilibrium price is linear $P_1 = A_1 + B_1(\theta + b_1 z_1)$, then there is a continuum of (θ, z_1) values at which the informed investors perceive the asset as having zero risk premium and, as a consequence of their FOC (eq. (63)) have an equilibrium demand of zero shares. Define this set of fundamentals

$$M = \left\{(\theta, z_1) : A_2 + B_2\theta + C_2 \frac{P_1 - A_1}{B_1} + q\bar{\eta} - P_1 = 0\right\}$$
$$= \{(\theta, z_1) : A_2 + B_2\theta + C_2 (\theta + b_1 z_1) + q\bar{\eta} - (A_1 + B_1(\theta + b_1 z_1)) = 0\},$$

pick any point $(t, \zeta) \in M$, and let $\hat{p} = P_1(t, \zeta)$ denote the associated price. At such a realization of fundamentals, we have that $X_{I1} = 0$ and consequently eq. (68) yields that

$$\left.\frac{\partial}{\partial P_1}\right|_{P_1=\hat{p}} X_{I1}(\theta, P_1) = -\left(\frac{q\gamma\left(\frac{1}{\tau_\eta} + \frac{B_2^2}{\tau_{p2}}\right) + (1-q)\gamma\frac{B_2^2}{\tau_{p2}}}{\left(1 - \frac{C_2}{B_1}\right)} + \frac{\gamma q(1-q)\bar{\eta}^2}{\left(1 - \frac{C_2}{B_1}\right)}\right)^{-1}.$$

Note that $\left.\frac{\partial}{\partial P_1}\right|_{P_1=\hat{p}} X_{I1}(\theta, P_1) \equiv G_{I1}$ is constant with respect to values of $(t, \zeta) \in M$.

Similarly, by the market clearing condition, since informed demand satisfies $X_{I1} = 0$, the equilibrium uninformed demand must be $X_{U1} = \frac{\bar{Z}+\zeta}{1-\lambda_1}$, and consequently eq. (70) yields that the derivative of the uninformed demand function, evaluated at $\hat{p}$ is pinned down by

$$\left.K_{U1}\right|_{P_1=\hat{p}} = \frac{q\gamma\left(\frac{1}{\tau_\eta} + \frac{B_2^2}{\tau_{p2}} + \frac{B_2^2}{\tau_{U1}}\right)e^{-\gamma\frac{\bar{Z}+z_1}{1-\lambda_1}\bar{\eta} + \frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}} + (1-q)\gamma\left(\frac{B_2^2}{\tau_{p2}} + \frac{B_2^2}{\tau_{U1}}\right)}{\left(1 - \left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}} + C_2\right)\frac{1}{B_1}\right)\left(qe^{-\gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\bar{\eta} + \frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}} + (1-q)\right)} \tag{73}$$
$$+ \gamma\frac{\left(\bar{\eta} - \gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\frac{1}{\tau_\eta}\right)^2}{1 - \left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}} + C_2\right)\frac{1}{B_1}}\frac{qe^{-\gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\bar{\eta} + \frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}}}{qe^{-\gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\bar{\eta} + \frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}} + (1-q)}$$
$$\times\left(1 - \frac{qe^{-\gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\bar{\eta} + \frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}}}{qe^{-\gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\bar{\eta} + \frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}} + (1-q)}\right).$$

Note that, since $q \in (0, 1)$, K_{U1} is a nontrivial function of the particular value of ζ, so that by eq. (69), we have $\left.\frac{\partial}{\partial P_1}\right|_{P_1=\hat{p}} X_{U1}(P_1) = G_{U1}(\zeta)$ for non-constant function $G_{U1}(\zeta) \equiv -\left(\left.K_{U1}\right|_{P_1=\hat{p}}\right)^{-1}$. Finally, returning to eq. (67), we have that the partial derivative of P_1 with respect to z_1, evaluated at the point $\theta = t, z_1 = \zeta$ is

$$\left.\frac{\partial}{\partial z_1}\right|_{\theta=t, z_1=\zeta} P_1 = \frac{1}{\lambda_1 G_{I1} + (1-\lambda_1)G_{U1}(\zeta)}.$$

Because the partial derivative depends on the particular realization $z_1 = \zeta$, it is not constant and therefore the function P_1 cannot be linear. This is a contradiction and completes the proof. □

A.5. *Proof of Proposition 3*

We know from Proposition 1 that for sufficiently small $c_D > 0$ a high-type manager strictly prefers disclosing to not disclosing and being assigned probability 0 of being the $x = 1$ type:

$$U_{ND}(1; 0) < U_D(1) - c_D \Rightarrow U_D(1) - c_D - U_{ND}(1; 0) > 0 \tag{74}$$

Fix such a sufficiently small c_D and suppose that there also exists an equilibrium in which the $x = 1$ manager discloses with probability $r_1 < 1$ and the $x = 0$ manager never discloses, $r_0 = 0$. In such an equilibrium, the market assigns probability

$$q(0, r_1) = \frac{p(1 - r_1)}{(1 - p) + p(1 - r_1)} < p$$

that $x = 1$ in the event of no disclosure. Under the assumed continuity of U_{ND}, for every $\varepsilon > 0$ there exists $q_\varepsilon > 0$ such that for $q \in [0, q_\varepsilon)$ we have

$$-\varepsilon < U_{ND}(1, q) - U_{ND}(1, 0) < \varepsilon.$$

Now, pick any ε such that $0 < \varepsilon < U_D(1) - c_D - U_{ND}(1; 0)$, which is guaranteed to exist owing to eq. (74). For any $q \in [0, q_\varepsilon)$ we have

$$\begin{aligned} U_{ND}(1; q) &< U_{ND}(1; q) + \varepsilon \\ &< U_D(1) - c_D \end{aligned}$$

where the first line follows from the continuity of U_{ND} and the second line follows from the choice of ε. Because $q(0, r_1) < p$ for any value of r_1, this implies that as long as $p < q_\varepsilon$ we have

$$U_{ND}(1; q(0, r_1)) < U_D(1) - c_D$$

which implies that the $x = 1$ manager strictly prefers to disclose, which is a contradiction. □

Appendix B. Additional analysis

B.1. *Conditions for interior equilibria*

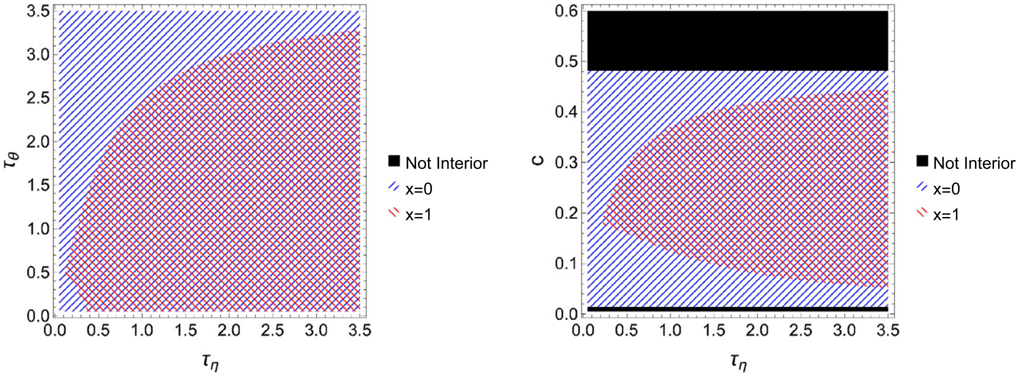
We begin with a characterization of conditions under which interior information equilibria obtain.

Lemma 5. *Fix $x \in \{0, 1\}$. If there exist $\lambda_1 \in (0, 1)$ and $\lambda_2 \in (0, 1)$ that solve the following system of two equations, where the coefficients $B_2(\lambda_1, \lambda_2)$ and $b_1(\lambda_1, \lambda_2)$ are as defined in Lemma 1, then there exists an interior equilibrium in the information market.*

$$\frac{\frac{B_2^2(\lambda_1,\lambda_2)}{\tau_\theta+\tau_z/b_1^2(\lambda_1,\lambda_2)}}{\frac{B_2^2(\lambda_1,\lambda_2)}{\tau_z\left(\frac{\lambda_2\tau_u}{\gamma}\right)^2}+\frac{x^2}{\tau_\eta}} = e^{2\gamma c} - 1 \tag{75}$$

$$\frac{\tau_u}{\tau_\theta + \frac{\tau_z}{b_1^2(\lambda_1,\lambda)} + \tau_z\left(\frac{\lambda_2\tau_u}{\gamma}\right)^2} = e^{2\gamma c} - 1 \tag{76}$$

The figures plot the region of the parameter space in which $\lambda_1, \lambda_2 \in (0, 1)$ for $x = 0$ and $x = 1$. Unless specified, the other parameters are set to $\gamma = 0.5$, $c = 0.2$, $\tau_\theta = 1$, $\tau_u = 1$, $\tau_\eta = 1$ and $\tau_z = 1$.



(a) τ_θ versus τ_η (b) τ_η versus c

Fig. 6. Parameter regions in which $\lambda_t \in (0, 1)$.

Proof. In an interior equilibrium, λ_1 and λ_2 are characterized by the conditions $\Gamma_t(\lambda_1, \lambda_2) = 1$ for $t \in \{1, 2\}$, where Γ_t is defined in eq. (44). Plugging in to the $t = 1$ condition and rearranging yields

$$\frac{\mathbb{V}_{U1}[P_2]}{\mathbb{V}_{I1}[P_2]} = e^{2\gamma c} \Leftrightarrow \frac{B_2^2 b_2^2 \frac{1}{\tau_z} + \frac{x^2}{\tau_\eta} + B_2^2 \mathbb{V}_{U1}(\theta)}{B_2^2 b_2^2 \frac{1}{\tau_z} + \frac{x^2}{\tau_\eta}} = e^{2\gamma c} \Leftrightarrow \frac{\frac{B_2^2}{\tau_\theta + \tau_z / b_1^2}}{B_2^2 \left(\frac{\gamma}{\lambda_2 \tau_u}\right)^2 \frac{1}{\tau_z} + \frac{x^2}{\tau_\eta}} = e^{2\gamma c} - 1$$

where the first equivalence follows from substituting the price function from eq. (3), and the second equivalence follows from rearranging and substituting in for the equilibrium values of $\mathbb{V}_{U1}(\theta)$ and b_2. Similarly, plugging in to the $t = 2$ condition and rearranging yields

$$\frac{\mathbb{V}_{U2}[V]}{\mathbb{V}_{I2}[V]} = e^{2\gamma c} \Leftrightarrow \frac{\frac{1}{\tau_u} + \frac{1}{\tau_{U2}}}{\frac{1}{\tau_u}} = e^{2\gamma c} \Leftrightarrow \frac{\tau_u}{\tau_\theta + \frac{\tau_z}{b_1^2} + \tau_z \left(\frac{\lambda_2 \tau_u}{\gamma}\right)^2} = e^{2\gamma c} - 1$$

where the first equivalence follows from substituting in for the variances in terms of precision, and the second equivalence follows from rearranging and substituting in the equilibrium value of τ_{U2}. $\square$

Due to the highly nonlinear nature of the information market equilibrium conditions, the equilibrium λ_t are not generally available in closed-form and it is difficult to pin down analytical conditions on primitives that ensure that the equilibrium is interior. However, it is straightforward to numerically solve for equilibrium and check whether the conditions in Lemma 5 are satisfied.

Fig. 6 provides illustrations of regions of the parameter space in which λ_t's are interior. Panel (a) illustrates how the region varies with the prior precisions of the long-term project, τ_θ, and the short-term project, τ_η. For the displayed parameter region, the equilibrium is always interior for $x = 0$. Naturally, when the short-term project does not exist ($x = 0$), the region does not vary with τ_η. When the short-term project exists, the region of interior equilibria is smaller because, when

τ_η is sufficiently small (i.e., the short-term project is sufficiently risky) then no investors acquire information at $t = 1$ ($\lambda_1 = 0$). On the other hand, when τ_η grows without bound and the short-term project becomes risk-less, the $x = 1$ equilibrium is isomorphic to the $x = 0$ equilibrium (in which the project does not exist) and therefore the interior regions must coincide.

Panel (b) illustrates how the region of interior equilibria varies with the cost of information c and the prior precision of the short-term project, τ_η. Again, we see that when $x = 0$, the value of τ_η naturally has no effect on the equilibrium. For both $x = 0$ and $x = 1$, an interior equilibrium (if one exists) holds for an intermediate region of costs. If the cost is "too high" then investors do not acquire information in either period ($\lambda_1 = \lambda_2 = 0$), while if the cost is "too low", then all investors acquire information in at least one of the periods (either $\lambda_0 = 1$ or $\lambda_1 = 1$). On the other hand, for any fixed cost c, in the $x = 1$ case, we again have $\lambda_1 \to 0$ as τ_η shrinks, while the equilibria coincide when τ_η becomes sufficiently large.

B.2. Dynamic model with persistent supply shocks

The setup follows the benchmark described in Section 3 with two differences.

- Each investor is long lived and can acquire information at the date of her choosing (i.e., at $t = 1^-$ or $t = 2^-$). As discussed in Section 5.1, to sustain an equilibrium with information acquisition at $t = 2^-$ we must have time-dependent information costs c_t, with $c_1 > c_2$.
- The aggregate supply of the risky security is Z_t, $t \in \{1, 2\}$, which follows

$$Z_t = \bar{Z} + \phi(Z_{t-1} - \overline{Z}) + z_t,$$

where $z_t \sim N(0, \tau_{zt})$ are normally distributed, independent of each other and other random variables, and we normalize $Z_0 \equiv \overline{Z}$. The special case $\phi = 0$, $\tau_{z1} = \tau_{z2} = \tau_z$ corresponds to the supply dynamics in the benchmark model and the particular dynamic extension discussed in Section 5.1.

As in the baseline model, we search for an equilibrium in which the $x = 1$ manager always discloses and the $x = 0$ manager never discloses. We solve the model by working backwards. Specifically, Section B.2.1 characterizes the equilibrium prices at dates 1 and 2, given investors' information acquisition choices. Section B.2.2 characterizes the equilibrium information acquisition choices at each date, and Section B.2.3 characterizes the conditions necessary for our main result. Fig. 3 (in the text) provides an illustration of this case. Specifically, we numerically solve a system of three equations (i.e., (119), (125), and (135)) to solve for the price signal coefficient b_1, and the fraction of informed investors at each date λ_1 and λ_2 for a given set of parameter values, with and without disclosure (i.e., for $d \in \{D, ND\}$), and then plot the date 2 fraction informed (i.e., λ_2) and the date 2 expected price (i.e., $\mathbb{E}[P_2]$ from equation (138) for different values of supply shock persistence ϕ.

B.2.1. Financial market equilibrium

For given disclosure and information choices, the derivation of the financial market equilibrium is standard. Fix the fraction λ_t of investors in generation t who acquire information about θ. We conjecture that prices are of the form:

$$P_1 = A_1 + B_1 s_{p1}, \quad \text{and} \quad P_2 = A_2 + B_2 s_{p2} + C_2 s_{p1} + x\eta, \tag{77}$$

where the $s_{pt} \equiv \theta + b_t(Z_t - \bar{Z})$ for $t \in \{1, 2\}$. In particular, the date t price provides a noisy, linear signal s_{pt} about θ to the uninformed investors of that generation. Moreover, the uninformed investors at date 2 can condition on the date 1 price to infer s_{p1}. This implies that the conditional beliefs of an uninformed investor at date $t = 1$ are given by:

$$\mathbb{E}_{U1}[\theta] = \frac{\tau_{p1} s_{p1}}{\tau_\theta + \tau_{p1}}, \quad \mathbb{V}_{U1}[\theta] = \frac{1}{\tau_\theta + \tau_{p1}} \equiv \frac{1}{\tau_{U1}}, \quad \text{where } \tau_{p1} \equiv \tau_{z1}/b_1^2. \tag{78}$$

The conditional beliefs of an uninformed investor at date $t = 2$ are more complex since the date 2 price signal is not conditionally independent of the date 1 signal. We have

$$\mathbb{E}_{U2}[\theta] = \frac{\frac{\tau_{z1}}{b_1^2} s_{p1} + \left(1 - \frac{b_2}{b_1}\phi\right) \frac{\tau_{z2}}{b_2^2} \left(s_{p2} - \frac{b_2}{b_1}\phi s_{p1}\right)}{\tau_\theta + \frac{\tau_{z1}}{b_1^2} + \left(1 - \frac{b_2}{b_1}\phi\right)^2 \frac{\tau_{z2}}{b_2^2}} \tag{79}$$

$$\mathbb{V}_{U2}^{-1}[\theta] = \tau_\theta + \frac{\tau_{z1}}{b_1^2} + \left(1 - \frac{b_2}{b_1}\phi\right)^2 \frac{\tau_{z2}}{b_2^2} \tag{80}$$

Note that if we further define the $t = 2$ 'incremental precision' τ_{p2} as

$$\tau_{p2} \equiv \left(1 - \frac{b_2}{b_1}\phi\right)^2 \frac{\tau_{z2}}{b_2^2} \tag{81}$$

then we can write these expressions concisely as

$$\mathbb{E}_{U2}[\theta] = \frac{\tau_{p1} s_{p1} + \tau_{p2} \frac{s_{p2} - \frac{b_2}{b_1}\phi s_{p1}}{\left(1 - \frac{b_2}{b_1}\phi\right)}}{\tau_\theta + \tau_{p1} + \tau_{p2}} \tag{82}$$

$$\mathbb{V}_{U2}^{-1}[\theta] = \tau_\theta + \tau_{p1} + \tau_{p2} \tag{83}$$

We now proceed to construct the financial market equilibrium by backward induction.

Date $t = 2$ trading round At $t = 2$ there are no future trading rounds left, so all investors optimally behave myopically. Investor i chooses optimal demand X_{it} to maximize CARA utility over next period wealth

$$X_{i2} \equiv \arg\max_x \mathbb{E}_{i2}[-e^{-\gamma\{W_2 + x(P_3 - P_2)\}}] \tag{84}$$

$$= \frac{\mathbb{E}_{i2}[V] - P_2}{\gamma \mathbb{V}_{i2}[P_3]} \tag{85}$$

This yields optimized expected utility

$$\mathbb{E}_{i2}\left[-e^{-\gamma W_2 + X_{i2}(V - P_2)}\right] = -e^{-\gamma W_2 - \frac{1}{2}\frac{\mathbb{E}_{i2}^2[V - P_2]}{\mathbb{V}_{i2}[V]}}. \tag{86}$$

By enforcing the market clearing condition and solving for P_2 we can easily pin down conditions that define the time 2 price coefficients. Specifically, note that

$$\lambda_2 \frac{\mathbb{E}_{I2}[V - P_2]}{\gamma \mathbb{V}_{I2}(V)} + (1 - \lambda_2) \frac{\mathbb{E}_{I2}[V - P_2]}{\gamma \mathbb{V}_{I2}(V)} = Z_2 \tag{87}$$

$$\Leftrightarrow P_2 = \overline{V} + x\eta + \frac{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} \mathbb{E}_{I2}[\theta] + \frac{1 - \lambda_2}{\mathbb{V}_{U2}(V)} \mathbb{E}_{U2}[\theta]}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1 - \lambda_2}{\mathbb{V}_{U2}(V)}} - \frac{\gamma}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1 - \lambda_2}{\mathbb{V}_{U2}(V)}} Z_2 \tag{88}$$

Equating coefficients with the initial conjecture yields

$$b_2 = -\frac{\gamma}{\lambda_2 \tau_u} \tag{89}$$

$$A_2 = \overline{V} - \frac{\gamma}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \overline{Z} \tag{90}$$

$$B_2 = \frac{\left(\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)} \mathbb{V}_{U2}[\theta] \frac{\tau_{p2}}{1-\frac{b_2}{b_1}\phi} \right)}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \tag{91}$$

$$C_2 = \frac{\frac{1-\lambda_2}{\mathbb{V}_{U2}(V)} \mathbb{V}_{U2}[\theta] \left(\tau_{p1} - \tau_{p2} \frac{\frac{b_2}{b_1}\phi}{1-\frac{b_2}{b_1}\phi} \right)}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \tag{92}$$

Date $t = 1$ trading round Now step back to $t = 1$.

An informed investor chooses demand to solve

$$\max_x \mathbb{E}_{I1} \left[-e^{-\gamma W_1 - \gamma x (P_2 - P_1) - \frac{1}{2} \frac{\mathbb{E}^2_{I2}[V-P_2]}{\mathbb{V}_{I2}[V]}} \right] \tag{93}$$

Using standard methods, it is tedious but straightforward to compute the expectation and show that her optimal demand is

$$X_{I1} = \frac{1}{\gamma} \frac{\mathbb{E}_{I1}[P_2 - P_1 - \beta_{I1}(V - P_2)]}{\mathbb{V}_{I1}(P_2 - P_1 - \beta_{I1}(V - P_2))} \tag{94}$$

where $\beta_{I1} = \frac{\mathbb{C}_{I1}(V-P_2, P_2-P_1)}{\mathbb{V}_{I1}(V-P_2)}$ is the conditional regression coefficient of $P_2 - P_1$ on $V - P_2$. Plugging the optimal demand back into the objective function and arranging terms yields optimized expected utility

$$\mathbb{E}_{I1} \left[-e^{-\gamma W_1 - \gamma X_{I1} (P_2 - P_1) - \frac{1}{2} \frac{\mathbb{E}^2_{I2}[V-P_2]}{\mathbb{V}_{I2}[V]}} \right] \tag{95}$$

$$= -\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{I1}(V - P_2)}} e^{-\gamma W_1 - \frac{1}{2} \frac{\mathbb{E}^2_{I1}[V-P_2]}{\mathbb{V}_{I1}(V-P_2)} - \frac{1}{2} \frac{\mathbb{E}^2_{I1}[P_2-P_1-\beta_{I1}(V-P_2)]}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}} \tag{96}$$

An uninformed investor who anticipates remaining uninformed at the second trading date chooses x to solve

$$\max_x \mathbb{E}_{U1} \left[-e^{-\gamma W_1 - \gamma x (P_2 - P_1) - \frac{1}{2} \frac{\mathbb{E}^2_{U2}[V-P_2]}{\mathbb{V}_{U2}[V]}} \right] \tag{97}$$

Similarly to the informed investor, we can also show that her optimal demand is

$$X_{U1} = \frac{1}{\gamma} \frac{\mathbb{E}_{U1}[P_2 - P_1 - \beta_{U1}(V - P_2)]}{\mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V - P_2))} \tag{98}$$

where $\beta_{U1} = \frac{\mathbb{C}_{U1}(V-P_2, P_2-P_1)}{\mathbb{V}_{U1}(V-P_2)}$ is the conditional regression coefficient of $P_2 - P_1$ on $V - P_2$. This demand leads to optimized expected utility

$$\mathbb{E}_{U1}\left[-e^{-\gamma W_1-\gamma X_{U1}(P_2-P_1)-\frac{1}{2}\frac{\mathbb{E}^2_{U2}[V-P_2]}{\mathbb{V}_{U2}[V]}}\right] \tag{99}$$

$$=-\sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{U1}(V-P_2)}}e^{-\gamma W_1-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)}-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}} \tag{100}$$

Finally, consider an uninformed investor who plans to acquire information before $t=2$. Her problem is to choose x to maximize

$$\mathbb{E}_{U1}\left[-e^{-\gamma(W_1-c_2)-\gamma x(P_2-P_1)-\frac{1}{2}\frac{\mathbb{E}^2_{I2}[V-P_2]}{\mathbb{V}_{I2}[V]}}\right] \tag{101}$$

$$=\mathbb{E}_{U1}\left[\mathbb{E}_{U2}\left[-e^{-\gamma(W_1-c_2)-\gamma x(P_2-P_1)-\frac{1}{2}\frac{\mathbb{E}^2_{I2}[V-P_2]}{\mathbb{V}_{I2}[V]}}\right]\right] \tag{102}$$

$$=\mathbb{E}_{U1}\left[-e^{-\gamma(W_1-c_2)-\gamma x(P_2-P_1)}\mathbb{E}_{U2}\left[e^{-\frac{1}{2}\frac{\mathbb{E}^2_{I2}[V-P_2]}{\mathbb{V}_{I2}[V]}}\right]\right] \tag{103}$$

$$=\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U2}(V)}}e^{\gamma c_2}\,\mathbb{E}_{U1}\left[-e^{-\gamma W_1-\gamma x(P_2-P_1)-\frac{1}{2}\frac{\mathbb{E}^2_{U2}[V-P_2]}{\mathbb{V}_{U2}[V]}}\right] \tag{104}$$

where the second line uses the law of iterated expectations, the second line pulls $\mathcal{F}_{U2}$ measurable things out of the inner expectation, and the final line computes the inner expectation, using the fact that $\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U2}(V)}}$ is a constant and can be pulled out of the expectation.

Because this objective function is a constant multiple of that for an uninformed investor who plans to remain uninformed, it leads to the same optimal demand

$$X_{U1}=\frac{1}{\gamma}\frac{\mathbb{E}_{U1}[P_2-P_1-\beta_{U1}(P_3-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))} \tag{105}$$

and to optimized expected utility

$$-\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U2}(V)}}e^{\gamma c_2}\sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{U1}(V-P_2)}}e^{-\gamma W_1-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)}-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}} \tag{106}$$

$$=-\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U1}(V-P_2)}}e^{\gamma c_2}e^{-\gamma W_1-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)}-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}} \tag{107}$$

With the $t=1$ optimal demands pinned down, we can now enforce the market clearing condition to pin down the coefficients on P_1. Specifically, note that

$$\frac{\lambda_1}{\gamma}\frac{\mathbb{E}_{I1}[P_2-P_1-\beta_{I1}(V-P_2)]}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}+\frac{1-\lambda_1}{\gamma}\frac{\mathbb{E}_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}=Z_1 \tag{108}$$

$$\Rightarrow P_1=\frac{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}\mathbb{E}_{I1}[P_2-\beta_{I1}(V-P_2)]}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}}$$
$$+\frac{\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}\mathbb{E}_{U1}[P_2-\beta_{U1}(V-P_2)]}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}}$$

$$-\frac{\gamma}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}}Z_1 \tag{109}$$

Note that

$$\mathbb{C}_{I1}(V-P_2,P_2)=\mathbb{C}_{I1}(-B_2s_{p2},B_2s_{p2})=-B_2^2b_2^2\frac{1}{\tau_{z2}} \tag{110}$$

$$\mathbb{V}_{I1}(P_2)=\mathbb{V}_{I1}(B_2s_{p2}+x\eta)=B_2^2b_2^2\frac{1}{\tau_{z2}}+\frac{x^2}{\tau_\eta} \tag{111}$$

$$\mathbb{V}_{I1}(V-P_2)=\mathbb{V}_{I1}(u+\theta-B_2s_{p2})=\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}} \tag{112}$$

Hence

$$\beta_{I1}=\frac{\mathbb{C}_{I1}(V-P_2,P_2)}{\mathbb{V}_{I1}(V-P_2)}=\frac{-B_2^2b_2^2\frac{1}{\tau_{z2}}}{\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}}}. \tag{113}$$

We can now compute

$$\mathbb{E}_{I1}[P_2-\beta_{I1}(V-P_2)] \tag{114}$$

$$=\mathbb{E}_{I1}\Bigg[A_2+B_2s_{p2}+C_2s_{p1}+x\eta$$

$$-\frac{-B_2^2b_2^2\frac{1}{\tau_{z2}}}{\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}}}\left(\overline{V}+x\eta+\theta+u-(A_2+B_2s_{p2}+C_2s_{p1}+x\eta)\right)\Bigg] \tag{115}$$

$$=\frac{\frac{1}{\tau_u}B_2\left(1-\frac{b_2}{b_1}\phi\right)+B_2^2b_2^2\frac{1}{\tau_z}}{\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}}}\theta+\text{other terms that do not depend explicitly on }\theta, \tag{116}$$

where we use $\mathbb{E}_{I1}[s_{p2}]=\mathbb{E}_{I1}[\theta+b_2\phi z_1]=\left(1-\frac{b_2}{b_1}\phi\right)\theta+\frac{b_2}{b_1}\phi s_{p1}$, and

$$\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))=B_2^2b_2^2\frac{1}{\tau_{z2}}+\frac{x^2}{\tau_\eta}-\frac{\left(B_2^2b_2^2\frac{1}{\tau_{z2}}\right)^2}{\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}}}=\frac{\frac{1}{\tau_u}B_2^2b_2^2\frac{1}{\tau_{z2}}}{\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}}}+\frac{x^2}{\tau_\eta} \tag{117}$$

Substituting these into the $t=1$ market clearing condition and grouping terms involving θ and z_1, we can pin down the linear statistic that P_1 must reveal

$$\theta-\frac{\gamma}{\lambda_1}\frac{\frac{\frac{1}{\tau_u}B_2^2b_2^2\frac{1}{\tau_{z2}}}{\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}}}+\frac{x^2}{\tau_\eta}}{\frac{\frac{1}{\tau_u}B_2\left(1-\frac{b_2}{b_1}\phi\right)+B_2^2b_2^2\frac{1}{\tau_z}}{\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}}}}(Z_1-\overline{Z}) \tag{118}$$

which gives the condition

$$b_1 = -\frac{\gamma}{\lambda_1} \frac{\frac{\frac{1}{\tau_u} B_2^2 b_2^2 \frac{1}{\tau_{z2}}}{\frac{1}{\tau_u} + B_2^2 b_2^2 \frac{1}{\tau_{z2}}} + \frac{x^2}{\tau_\eta}}{\frac{\frac{1}{\tau_u} B_2 \left(1 - \frac{b_2}{b_1}\phi\right) + B_2^2 b_2^2 \frac{1}{\tau_{z2}}}{\frac{1}{\tau_u} + B_2^2 b_2^2 \frac{1}{\tau_{z2}}}}. \tag{119}$$

Combined with the earlier condition for B_2, this gives us enough to pin down the financial market equilibrium, given fractions of informed traders λ_t. Returning to B_2, we have

$$B_2 = \frac{\left(\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)} \mathbb{V}_{U2}[\theta] \frac{\tau_{p2}}{1 - \frac{b_2}{b_1}\phi}\right)}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \tag{120}$$

which, after substituting in for all the variances and using the explicit expression for b_2 from earlier, is a complicated function of b_1 (and the λ's).

B.2.2. Information acquisition choices

Given the characterization of the financial market equilibrium in the previous section, one can characterize the optimal information acquisition choices each period.

Date $t = 2^-$ information acquisition Immediately after the $t = 1$ trading round, but strictly before the $t = 2$ trading round, investors who have remained uninformed must decide whether to purchase information. The forward-looking expected utilities from acquiring or not acquiring information at this stage (i.e., conditional on the information $\mathcal{F}_{U1} = \sigma(d, P_1)$ she observed in the first round) are

$$U_{I2^-} = \mathbb{E}_{U1}\left[e^{-\gamma(W_1 - c_2) - \gamma X_{U1}(P_2 - P_1) - \frac{1}{2}\frac{\mathbb{E}_{I2}^2[V-P_2]}{\mathbb{V}_{I2}[V]}} \right] \tag{121}$$

$$= -\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U1}(V - P_2)}} e^{\gamma c_2} e^{-\gamma W_1 - \frac{1}{2}\frac{\mathbb{E}_{U1}^2[V-P_2]}{\mathbb{V}_{U1}(V-P_2)} - \frac{1}{2}\frac{\mathbb{E}_{U1}^2[P_2 - P_1 - \beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V-P_2))}} \tag{122}$$

$$U_{U2^-} = \mathbb{E}_{U1}\left[e^{-\gamma W_1 - \gamma X_{U1}(P_2 - P_1) - \frac{1}{2}\frac{\mathbb{E}_{U2}^2[V-P_2]}{\mathbb{V}_{I2}[V]}} \right] \tag{123}$$

$$= -\sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{U1}(V - P_2)}} e^{-\gamma W_1 - \frac{1}{2}\frac{\mathbb{E}_{U1}^2[V-P_2]}{\mathbb{V}_{U1}(V-P_2)} - \frac{1}{2}\frac{\mathbb{E}_{U1}^2[P_2 - P_1 - \beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V-P_2))}} \tag{124}$$

The indifference condition for an interior equilibrium therefore requires[23]

$$\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U2}(V)}} e^{\gamma c_2} = 1. \tag{125}$$

[23] Note that here the 'interior' region in which the indifference condition characterizes the equilibrium, is the situation in which some positive mass of investors who were previously uninformed choose to acquire information at $t = 2$, but not the entire mass $1 - \lambda_1$ of such investors. The mass λ_1 from the first round do not 'forget' their information and so we necessarily have $\lambda_2 \geq \lambda_1$ in any equilibrium.

Date $t = 1^-$ ($t = 0$) information acquisition To establish the initial information equilibrium, we need to compute the ex-ante expected utilities of all types. Let $\mu_R = \begin{pmatrix} \mathbb{E}_0[V - p_2] \\ \mathbb{E}_0[p_2 - p_1] \end{pmatrix}$ be the vector of ex-ante expected returns and $\mathbb{V}_R = \mathbb{V}_0 \begin{pmatrix} V - p_2 \\ p_2 - p_1 \end{pmatrix}$ the ex-ante covariance matrix of returns.

The expected utility of a investor who remains uninformed at both stages is

$$U_{U0} = \mathbb{E}_0 \left[-\sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{U1}(V - P_2)}} e^{-\gamma W_1 - \frac{1}{2} \frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)} - \frac{1}{2} \frac{\mathbb{E}^2_{U1}[P_2 - P_1 - \beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}} \right] \tag{126}$$

$$= -\left(\frac{\left| \mathbb{V}_0 \begin{pmatrix} V - P_2 \\ P_2 - P_1 \end{pmatrix} \right|}{\mathbb{V}_{U2}(V) \mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V - P_2))} \right)^{-1/2} e^{-\gamma W_0 - \frac{1}{2} \mu_R' V_R^{-1} \mu_R'} \tag{127}$$

That of a investor who is uninformed at the first period and informed at the second period is

$$U_{UI0} = \mathbb{E}_0 \left[-\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U1}(V - P_2)}} e^{\gamma c_2} e^{-\gamma W_1 - \frac{1}{2} \frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)} - \frac{1}{2} \frac{\mathbb{E}^2_{U1}[P_2 - P_1 - \beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}} \right] \tag{128}$$

$$= -e^{\gamma c_2} \left(\frac{\left| \mathbb{V}_0 \begin{pmatrix} V - P_2 \\ P_2 - P_1 \end{pmatrix} \right|}{\mathbb{V}_{I2}(V) \mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V - P_2))} \right)^{-1/2} e^{-\gamma W_0 - \frac{1}{2} \mu_R' V_R^{-1} \mu_R'} \tag{129}$$

And that of a investor who is informed at both periods is

$$U_{I0} = \mathbb{E}_0 \left[-\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{I1}(V - P_2)}} e^{-\gamma W_1 - \frac{1}{2} \frac{\mathbb{E}^2_{I1}[V-P_2]}{\mathbb{V}_{I1}(V-P_2)} - \frac{1}{2} \frac{\mathbb{E}^2_{I1}[P_2 - P_1 - \beta_{I1}(V-P_2)]}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}} \right] \tag{130}$$

$$= -\left(\frac{\left| \mathbb{V}_0 \begin{pmatrix} V - P_2 \\ P_2 - P_1 \end{pmatrix} \right|}{\mathbb{V}_{I2}(V) \mathbb{V}_{I1}(P_2 - P_1 - \beta_{I1}(V - P_2))} \right)^{-1/2} e^{-\gamma (W_0 - c_1) - \frac{1}{2} \mu_R' V_R^{-1} \mu_R'} \tag{131}$$

If the $t = 2^-$ equilibrium is interior, we immediately have $U_{U0} = U_{UI0}$, which implies that a $t = 0$ interior equilibrium requires that investors be indifferent between being informed at both periods and being uninformed at both periods, $U_{U0} = U_{I0}$:

$$-\left(\frac{\left| \mathbb{V}_0 \begin{pmatrix} V - P_2 \\ P_2 - P_1 \end{pmatrix} \right|}{\mathbb{V}_{U2}(V) \mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V - P_2))} \right)^{-1/2} e^{-\gamma W_0 - \frac{1}{2} \mu_R' V_R^{-1} \mu_R'}$$
$$= -\left(\frac{\left| \mathbb{V}_0 \begin{pmatrix} V - P_2 \\ P_2 - P_1 \end{pmatrix} \right|}{\mathbb{V}_{I2}(V) \mathbb{V}_{I1}(P_2 - P_1 - \beta_{I1}(V - P_2))} \right)^{-1/2} e^{-\gamma (W_0 - c_1) - \frac{1}{2} \mu_R' V_R^{-1} \mu_R'} \tag{132}$$

$$\Leftrightarrow \sqrt{\mathbb{V}_{U2}(V)\mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V - P_2))} = \sqrt{\mathbb{V}_{I2}(V)\mathbb{V}_{I1}(P_2 - P_1 - \beta_{I1}(V - P_2))}e^{\gamma c_1} \tag{133}$$

$$\Leftrightarrow e^{\gamma c_1} = \sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{I2}(V)}}\sqrt{\frac{\mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V - P_2))}{\mathbb{V}_{I1}(P_2 - P_1 - \beta_{I1}(V - P_2))}} \tag{134}$$

Again using the $t = 2$ information condition, this can be simplified to

$$e^{\gamma(c_1 - c_2)} = \sqrt{\frac{\mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V - P_2))}{\mathbb{V}_{I1}(P_2 - P_1 - \beta_{I1}(V - P_2))}}. \tag{135}$$

B.2.3. Relation between x and $\mathbb{E}[P_2]$

The expected value of P_2 for the manager at the disclosure stage is

$$\mathbb{E}[P_2] = \overline{V} - \frac{\gamma}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{I2}(V)}} + x\bar{\eta}. \tag{136}$$

And in an interior equilibrium we have

$$\mathbb{V}_{U2}[V] = e^{2\gamma c_2}\mathbb{V}_{I2}[V] = \frac{e^{2\gamma c_2}}{\tau_u} \tag{137}$$

so that

$$\mathbb{E}[P_2] = \overline{V} - \frac{\gamma}{\tau_u(\lambda_2(1 - e^{-2\gamma c_2}) + e^{-2\gamma c_2})} \tag{138}$$

Hence, as in our benchmark analysis, it is sufficient to show that $\lambda_2(D) = \lambda_2(1) \geq \lambda_2(ND) = \lambda_2(0)$.

While analytically establishing this result is not tractable, we show numerically that the result obtains for a large region of the parameter space. Specifically, we have a system of three equations i.e., (119), (125), and (135), and three unknowns, i.e., b_1, λ_1 and λ_2, which we can solve numerically for a given set of parameter values and x.

B.3. Short-lived investors with persistent supply

In this section, we consider an extension to our benchmark analysis in which the asset supply shocks are persistent. Specifically, we assume that the aggregate supply of the risky security is Z_t, $t \in \{1, 2\}$ which follows $Z_t = \bar{Z} + \phi(Z_{t-1} - \overline{Z}) + z_t$ where $z_t \sim N(0, \tau_{zt})$ are normally distributed, independent of each other and other random variables, and we normalize $Z_0 \equiv \overline{Z}$. This implies that the investors' beliefs about fundamentals θ, and the resulting intermediate steps are as in Appendix B.2. However, we need to modify the equation defining b_1 since the $t = 1$ demand functions are myopic in this case. Similarly, the $t = 1$ information condition simplifies due to myopic behavior.

Because the $t = 2$ demand functions are identical (in functional form), the conditions defining B_2 and b_2 have the same functional forms

$$B_2 = \frac{\lambda_2\tau_u + (1-\lambda_2)\frac{e^{2\gamma c_2}-1}{e^{2\gamma c_2}}\left(1 - \frac{b_2}{b_1}\phi\right)\frac{\tau_{z2}}{b_2^2}}{\lambda_2\tau_u + (1-\lambda_2)\frac{\tau_u}{e^{2\gamma c_2}}} \tag{139}$$

$$b_2 = -\frac{\gamma}{\lambda_2\tau_u} \tag{140}$$

To pin down b_1, note that the $t = 1$ market clearing condition is

$$\frac{\lambda_1}{\gamma} \frac{\mathbb{E}_{I1}[P_2 - P_1]}{\mathbb{V}_{I1}(P_2 - P_1)} + \frac{1-\lambda_1}{\gamma} \frac{\mathbb{E}_{U1}[P_2 - P_1]}{\mathbb{V}_{U1}(P_2 - P_1)} = Z_1. \tag{141}$$

Hence, the price reveals

$$\theta - \frac{\gamma}{\lambda_1} \frac{B_2^2 b_2^2 \frac{1}{\tau_{z2}} + \frac{x^2}{\tau_\eta}}{B_2 \left(1 - \frac{b_2}{b_1}\phi\right)} (Z_1 - \overline{Z}) \tag{142}$$

and the equation defining b_1 is

$$b_1 = -\frac{\gamma}{\lambda_1} \frac{B_2^2 b_2^2 \frac{1}{\tau_{z2}} + \frac{x^2}{\tau_\eta}}{B_2 \left(1 - \frac{b_2}{b_1}\phi\right)}. \tag{143}$$

The $t = 2$ information condition is still

$$e^{\gamma c_2} = \sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{I2}(V)}} \tag{144}$$

$$\Rightarrow \tau_\theta + \frac{\tau_{z1}}{b_1^2} + \left(\frac{1}{b_2} - \frac{\phi}{b_1}\right)^2 \tau_{z2} = \frac{\tau_u}{e^{2\gamma c_2} - 1}. \tag{145}$$

The $t = 1$ information condition is

$$e^{\gamma c_1} = \sqrt{\frac{\mathbb{V}_{U1}(P_2 - P_1)}{\mathbb{V}_{I1}(P_2 - P_1)}} \tag{146}$$

$$\Rightarrow e^{2\gamma c_1} = \frac{B_2^2 \left(1 - \frac{b_2}{b_1}\phi\right)^2 \mathbb{V}_{U1}(\theta) + B_2^2 b_2^2 \frac{1}{\tau_{z2}} + \frac{x^2}{\tau_\eta}}{B_2^2 b_2^2 \frac{1}{\tau_{z2}} + \frac{x^2}{\tau_\eta}} \tag{147}$$

As in the benchmark case, it is sufficient for us to show that

$$\lambda_2 (D) = \lambda_2 (1) \geq \lambda_2 (ND) = \lambda_2 (0). \tag{148}$$

It is intractable to establish such a result analytically. However, we can demonstrate numerically the result obtains for a wide range of parameters. Specifically, we have a system of three equations i.e., (143), (145), and (147), and three unknowns, i.e., b_1, λ_1 and λ_2, which we can solve numerically for a given set of parameter values and x.

B.4. Short-lived investors with mean-variance preferences

The setup follows the benchmark described in Section 3, except that investors have mean-variance preferences over next period's wealth i.e., investor i at date t chooses $X_{i,t}$ to maximize:

$$X_{i,t} \equiv \arg\max_{x \in \mathbb{R}} \mathbb{E}_{it} \left[W_t + x\left(P_{t+1} - P_t\right)\right] - \frac{\gamma}{2} \mathbb{V}_{it} \left[W_t + x\left(P_{t+1} - P_t\right)\right]. \tag{149}$$

Moreover, we allow the manager to mix between disclosing and not. Specifically, she chooses a disclosure probability $r \in [0, 1]$, subject to disclosure cost $c_D > 0$ (i.e., if she ultimately discloses, whether under a pure or mixed strategy, she pays cost c_D, while if she does not disclose, she pays no cost). Let $d = D$ and $d = ND$ correspond to the events of disclosure and non-disclosure, respectively. Let q denote the (endogenous) probability that investors assign to the existence of the project, which is, in general, a function of disclosure d and the firm's type x, $q = q(d, x) = \mathbb{P}(x = 1|d)$ $d \in \{D, ND\}$ Specifically, we have $q(D, 1) = 1$, $q(D, 0) = 0$, and $q(ND, x) = \frac{p(1-r)}{1-p+p(1-r)}$ for either $x \in \{0, 1\}$ (i.e., in the event of non-disclosure, investors assign the same probability that $x = 1$ regardless of the firm's actual type since they do not observe the type in this case). The manager optimally chooses her disclosure strategy to maximize the expected date 2 price, taken the market belief function $q(\cdot)$ as given. Let

$$U_d(x; q) = \mathbb{E}\left[P_2 \middle| d, x\right]$$

denote the expected price conditional on the realized value of x and the disclosure decision d, given a conjecture for the market belief $q(\cdot)$ as a function of the disclosure. Formally, a type x manager's problem is to choose r to solve

$$U(x; q) \equiv \max_{r\in[0,1]} r \times (U_D(x; q) - c_D) + (1 - r) \times U_{ND}(x; q), \tag{150}$$

taking as given the market's belief $q(\cdot)$. An equilibrium disclosure probability is a probability r that simultaneously solves the manager's problem and is consistent with market beliefs. That is, if investors conjecture that the $x = 1$ manager discloses with probability r and assign belief $q(ND) = \frac{p(1-r)}{1-p+p(1-r)}$ in the event of non-disclosure, then the manager indeed finds it optimal to disclose with probability r.

B.4.1. Financial market equilibrium

For given disclosure and information choices, the derivation of the financial market equilibrium is standard and proceeds analogously to the baseline model. Fix the fractions λ_t of investors in generation t who acquire information about θ. We conjecture that prices depend on a sequence of price-signals that provide noisy signals of the private information of informed traders:

$$s_{p1} \equiv \theta + b_1(q)z_1; \quad \tau_{p1} \equiv \tau_z \frac{1}{b_1^2} \tag{151}$$

$$s_{p2} \equiv \theta + b_2(q)z_2 \quad \tau_{p2} \equiv \tau_z \frac{1}{b_2^2} \tag{152}$$

with the prices themselves conjectured to be

$$P_1 = A_1(q) + B_1(q)s_{p1} + q\overline{\eta}, \quad \text{and} \quad P_2 = A_2(q) + B_2(q)s_{p2} + C_2(q)s_{p1} + x\eta. \tag{153}$$

For clarity, we make explicit the fact that the coefficients depend, in general, on the market's $t = 1$ belief, q, about the manager's type. To eliminate notational clutter we mostly suppress this dependence going forward.

At $t = 2$, an arbitrary informed investor can condition on $\mathcal{F}_{I2} = \sigma(\theta, s_{p2}, s_{p1})$. We trivially have

$$\mathbb{E}_{I2}[\theta] = \theta \tag{154}$$

$$\frac{1}{\tau_{I2}} \equiv \mathbb{V}_{I2}(\theta) = 0. \tag{155}$$

A $t = 2$ uninformed investor can condition on $\mathcal{F}_{U2} = \sigma(s_{p2}, s_{p1})$ which yields

$$\mathbb{E}_{U2}[\theta] = \mathbb{V}_{U2}(\theta)\left(\tau_{p2}s_{p2} + \tau_{p1}s_{p1}\right) \tag{156}$$

$$\frac{1}{\tau_{U2}} \equiv \mathbb{V}_{U2}(\theta) = \frac{1}{\tau_\theta + \tau_{p1} + \tau_{p2}} \tag{157}$$

Similarly, at $t = 1$, an informed investor can condition on $\mathcal{F}_{I1} = \sigma(\theta, s_{p1})$, which yields

$$\mathbb{E}_{I1}[\theta] = \theta \tag{158}$$

$$\frac{1}{\tau_{I1}} \equiv \mathbb{V}_{I1}(\theta) = 0, \tag{159}$$

and an uninformed investor can condition on $\mathcal{F}_{U1} = \sigma(s_{p1})$, giving

$$\mathbb{E}_{U1}[\theta] = \mathbb{V}_{U1}(\theta)\tau_{p1}s_{p1} \tag{160}$$

$$\frac{1}{\tau_{U1}} \equiv \mathbb{V}_{U1}(\theta) = \frac{1}{\tau_\theta + \tau_{p1}} \tag{161}$$

We now proceed to construct the financial market equilibrium by backward induction.

$t = 2$ *trading round* At $t = 2$ investor i chooses optimal demand X_{it} to maximize her mean-variance preferences over next period wealth

$$X_{i2} \equiv \arg\max_{x \in \mathbb{R}} \mathbb{E}_{i2}[W_2 + x(P_3 - P_2)] - \frac{\gamma}{2}\mathbb{V}_{i2}\left(W_2 + x(P_3 - P_2)\right) \tag{162}$$

$$= \frac{\mathbb{E}_{i2}[P_3] - P_2}{\gamma \mathbb{V}_{i2}(P_3)} \tag{163}$$

This leads to mean-variance demand functions for both time-2 trader types

$$X_{I2} = \frac{\mathbb{E}_{I2}[P_3] - P_2}{\gamma \mathbb{V}_{I2}(P_3)} = \frac{1}{\gamma}\frac{\overline{V} + x\eta + \mathbb{E}_{I2}[\theta] - P_2}{\frac{1}{\tau_{I2}} + \frac{1}{\tau_u}} \tag{164}$$

and

$$X_{U2} = \frac{\mathbb{E}_{U2}[P_3] - P_2}{\gamma \mathbb{V}_{U2}(P_3)} = \frac{1}{\gamma}\frac{\overline{V} + x\eta + \mathbb{E}_{U2}[\theta] - P_2}{\frac{1}{\tau_{U2}} + \frac{1}{\tau_u}} \tag{165}$$

Enforcing market clearing yields

$$\lambda_2 X_{I2} + (1 - \lambda_2)X_{U2} = \overline{Z} + z_2 \tag{166}$$

$$\Rightarrow P_2 = \overline{V} + x\eta - \gamma \frac{1}{\frac{\lambda_2}{\frac{1}{\tau_u}} + \frac{1-\lambda_2}{\frac{1}{\tau_{U2}} + \frac{1}{\tau_u}}}\overline{Z} + \frac{\frac{\lambda_2}{\frac{1}{\tau_u}}s_{p2}}{\frac{\lambda_2}{\frac{1}{\tau_u}} + \frac{1-\lambda_2}{\frac{1}{\tau_{U2}} + \frac{1}{\tau_u}}} + \frac{\frac{1-\lambda_2}{\frac{1}{\tau_{U2}} + \frac{1}{\tau_u}}\frac{1}{\tau_{U2}}\left(\tau_{p2}s_{p2} + \tau_{p1}s_{p1}\right)}{\frac{\lambda_2}{\frac{1}{\tau_u}} + \frac{1-\lambda_2}{\frac{1}{\tau_{U2}} + \frac{1}{\tau_u}}} \tag{167}$$

and matching with the initial conjecture gives

$$b_2 = -\frac{\gamma}{\lambda_2 \tau_u} \tag{168}$$

$$A_2 = \overline{V} - \gamma \frac{1}{\frac{\lambda_2}{\frac{1}{\tau_u}} + \frac{1-\lambda_2}{\frac{1}{\tau_{U2}} + \frac{1}{\tau_u}}} \overline{Z} \tag{169}$$

$$B_2 = \frac{\frac{\lambda_2}{\frac{1}{\tau_u}} + \frac{1-\lambda_2}{\frac{1}{\tau_{U2}} + \frac{1}{\tau_u}} \frac{1}{\tau_{U2}} \tau_{p2}}{\frac{\lambda_2}{\frac{1}{\tau_u}} + \frac{1-\lambda_2}{\frac{1}{\tau_{U2}} + \frac{1}{\tau_u}}} \tag{170}$$

$$C_2 = \frac{\frac{1-\lambda_2}{\frac{1}{\tau_{U2}} + \frac{1}{\tau_u}} \frac{1}{\tau_{U2}} \tau_{p1}}{\frac{\lambda_2}{\frac{1}{\tau_u}} + \frac{1-\lambda_2}{\frac{1}{\tau_{U2}} + \frac{1}{\tau_u}}} \tag{171}$$

$t = 1$ *trading round* Now step back to $t = 1$. An arbitrary investor i chooses her demand to maximize mean-variance preferences over next-period wealth

$$X_{i1} \equiv \arg\max_{x \in \mathbb{R}} \mathbb{E}_{i1}\left[W_1 + x(P_2 - P_1)\right] - \frac{\gamma}{2} \mathbb{V}_{i1}\left(W_1 + x\left(P_2 - P_1\right)\right) \tag{172}$$

Consequently, the optimal demand is

$$X_{I1} = \frac{1}{\gamma} \frac{\mathbb{E}_{i1}[P_2] - P_1}{\mathbb{V}_{i1}(P_2)} \tag{173}$$

Recalling that $q = \mathbb{P}\left(x = 1|d\right)$ denotes the market's belief about the existence of the project, we have

$$\begin{aligned}
\mathbb{E}_{i1}[P_2] &= \mathbb{E}_{i1}[A_2 + B_2 s_{p2} + C_2 s_{p1} + x\eta] \\
&= A_2 + B_2 \mathbb{E}_{i1}[\theta] + C_2 s_{p1} + q\overline{\eta}
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{V}_{i1}(P_2) &= \mathbb{V}_{i1}\left(\mathbb{E}_{i1}\left[A_2 + B_2 s_{p2} + C_2 s_{p1} + x\eta \middle| x\right]\right) \\
&\quad + \mathbb{E}_{i1}\left[\mathbb{V}_{i1}\left(A_2 + B_2 s_{p2} + C_2 s_{p1} + x\eta \middle| x\right)\right] && (174) \\
&= \mathbb{V}_{i1}\left(A + B_2 \mathbb{E}_{i1}\left[s_{p2}\right] + C_2 s_{p1} + \mathbf{1}_{\{x=1\}}\overline{\eta}\right) + \mathbb{E}_{i1}\left[B_2^2 \mathbb{V}_{i1}\left[s_{p2}\right] + \mathbf{1}_{\{x=1\}} \frac{1}{\tau_\eta}\right] && (175) \\
&= q(1-q)\overline{\eta}^2 + B_2^2\left(\mathbb{V}_{i1}(\theta) + \tfrac{1}{\tau_{p2}}\right) + q\frac{1}{\tau_\eta} && (176) \\
&= B_2^2\left(\tfrac{1}{\tau_{i1}} + \tfrac{1}{\tau_{p2}}\right) + q\frac{1}{\tau_\eta} + q(1-q)\overline{\eta}^2 && (177)
\end{aligned}$$

The market clearing condition yields

$$\begin{aligned}
&\lambda_1 X_{I1} + (1-\lambda_1) X_{U1} = \overline{Z} + z_1 && (178) \\
&\Rightarrow P_1 = A_2 + q\overline{\eta} + C_2 s_{p1} - \frac{\gamma}{\frac{\lambda_1}{B_2^2 \frac{1}{\tau_{p2}} + q\frac{1}{\tau_\eta} + q(1-q)\overline{\eta}^2} + \frac{1-\lambda_1}{B_2^2\left(\frac{1}{\tau_{U1}} + \frac{1}{\tau_{p2}}\right) + q\frac{1}{\tau_\eta} + q(1-q)\overline{\eta}^2}} \bar{Z}
\end{aligned}$$

$$+\frac{\frac{\lambda_1}{B_2^2\frac{1}{\tau_{p2}}+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}B_2+\frac{1-\lambda_1}{B_2^2\left(\frac{1}{\tau_{U1}}+\frac{1}{\tau_{p2}}\right)+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}B_2\frac{1}{\tau_{U1}}\tau_{p1}}{\frac{\lambda_1}{B_2^2\frac{1}{\tau_{p2}}+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}+\frac{1-\lambda_1}{B_2^2\left(\frac{1}{\tau_{U1}}+\frac{1}{\tau_{p2}}\right)+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}}s_{p1} \tag{179}$$

Matching coefficients with the initial conjecture gives

$$b_1=-\frac{\gamma}{\lambda_1}\frac{1}{B_2}\left(B_2^2\frac{1}{\tau_{p2}}+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2\right) \tag{180}$$

$$A_1=A_2-\frac{\gamma}{\frac{\lambda_1}{B_2^2\frac{1}{\tau_{p2}}+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}+\frac{1-\lambda_1}{B_2^2\left(\frac{1}{\tau_{U1}}+\frac{1}{\tau_{p2}}\right)+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}}\bar{Z} \tag{181}$$

$$B_1=C_2+\frac{\frac{\lambda_1}{B_2^2\frac{1}{\tau_{p2}}+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}B_2+\frac{1-\lambda_1}{B_2^2\left(\frac{1}{\tau_{U1}}+\frac{1}{\tau_{p2}}\right)+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}B_2\frac{1}{\tau_{U1}}\tau_{p1}}{\frac{\lambda_1}{B_2^2\frac{1}{\tau_{p2}}+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}+\frac{1-\lambda_1}{B_2^2\left(\frac{1}{\tau_{U1}}+\frac{1}{\tau_{p2}}\right)+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}} \tag{182}$$

Solution for price coefficients Bringing together the coefficient equations from $t=1$ and $t=2$ gives the system

$$b_2=-\frac{\gamma}{\lambda_2}\frac{1}{\tau_u} \tag{183}$$

$$A_2=\overline{V}-\gamma\frac{1}{\frac{\lambda_2}{\frac{1}{\tau_u}}+\frac{1-\lambda_2}{\frac{1}{\tau_{U2}}+\frac{1}{\tau_u}}}\overline{Z} \tag{184}$$

$$B_2=\frac{\frac{\lambda_2}{\frac{1}{\tau_u}}+\frac{1-\lambda_2}{\frac{1}{\tau_{U2}}+\frac{1}{\tau_u}}\frac{1}{\tau_{U2}}\tau_{p2}}{\frac{\lambda_2}{\frac{1}{\tau_u}}+\frac{1-\lambda_2}{\frac{1}{\tau_{U2}}+\frac{1}{\tau_u}}} \tag{185}$$

$$C_2=\frac{\frac{1-\lambda_2}{\frac{1}{\tau_{U2}}+\frac{1}{\tau_u}}\frac{1}{\tau_{U2}}\tau_{p1}}{\frac{\lambda_2}{\frac{1}{\tau_u}}+\frac{1-\lambda_2}{\frac{1}{\tau_{U2}}+\frac{1}{\tau_u}}} \tag{186}$$

$$b_1=-\frac{\gamma}{\lambda_1}\frac{1}{B_2}\left(B_2^2\frac{1}{\tau_{p2}}+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2\right) \tag{187}$$

$$A_1=A_2-\frac{\gamma}{\frac{\lambda_1}{B_2^2\left(\frac{1}{\tau_{I1}}+\frac{1}{\tau_\xi}+\frac{1}{\tau_{p2}}\right)+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}+\frac{1-\lambda_1}{B_2^2\left(\frac{1}{\tau_{U1}}+\frac{1}{\tau_\xi}+\frac{1}{\tau_{p2}}\right)+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}}\bar{Z} \tag{188}$$

$$B_1=C_2+\frac{\frac{\lambda_1}{B_2^2\frac{1}{\tau_{p2}}+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}B_2+\frac{1-\lambda_1}{B_2^2\left(\frac{1}{\tau_{U1}}+\frac{1}{\tau_{p2}}\right)+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}B_2\frac{1}{\tau_{U1}}\tau_{p1}}{\frac{\lambda_1}{B_2^2\frac{1}{\tau_{p2}}+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}+\frac{1-\lambda_1}{B_2^2\left(\frac{1}{\tau_{U1}}+\frac{1}{\tau_{p2}}\right)+q\frac{1}{\tau_\eta}+q(1-q)\overline{\eta}^2}} \tag{189}$$

As in the baseline case, if we can characterize B_2 and b_1 then all of the other coefficients follow from direct substitution. Note that b_2 is already a function of exogenous parameters

$$b_2 = -\frac{\gamma}{\lambda_2}\frac{1}{\tau_u}$$

So, plugging in to the equation defining B_2 yields

$$\begin{aligned} B_2 &= \frac{\frac{\lambda_2}{\frac{1}{\tau_u}} + \frac{1-\lambda_2}{\frac{1}{\tau_{U2}}+\frac{1}{\tau_u}}\frac{1}{\tau_{U2}}\tau_{p2}}{\frac{\lambda_2}{\frac{1}{\tau_u}} + \frac{1-\lambda_2}{\frac{1}{\tau_{U2}}+\frac{1}{\tau_u}}} \\ &= \frac{\frac{\lambda_2}{\frac{1}{\tau_u}} + \frac{1-\lambda_2}{\frac{1}{\tau_u}+\frac{1}{\tau_\theta+\tau_z\frac{1}{b_1^2}+\tau_z\left(\frac{\lambda_2\tau_u}{\gamma}\right)^2}}\frac{\tau_z\left(\frac{\lambda_2\tau_u}{\gamma}\right)^2}{\tau_\theta+\tau_{p1}+\tau_z\left(\frac{\lambda_2\tau_u}{\gamma}\right)^2}}{\frac{\lambda_2}{\frac{1}{\tau_u}} + \frac{1-\lambda_2}{\frac{1}{\tau_\theta+\tau_z\frac{1}{b_1^2}+\tau_z\left(\frac{\lambda_2\tau_u}{\gamma}\right)^2}+\frac{1}{\tau_u}}} \end{aligned}$$

Hence, we have expressed B_2 purely in terms of b_1 and exogenous parameters. Now, substituting both b_2 and B_2 into the equation for b_1

$$\begin{aligned} b_1 &= -\frac{\gamma}{\lambda_1}\frac{1}{B_2}\left(B_2^2 b_2^2\frac{1}{\tau_z} + q\frac{1}{\tau_\eta} + q(1-q)\overline{\eta}^2\right) \\ &= -\frac{\gamma}{\lambda_1}\frac{1}{B_2}\left(B_2^2\left(\frac{\gamma}{\lambda_2\tau_u}\right)^2\frac{1}{\tau_z} + q\frac{1}{\tau_\eta} + q(1-q)\overline{\eta}^2\right) \end{aligned} \tag{190}$$

yields a single polynomial equation in b_1. With b_1 pinned down, b_2 and B_2 follow from substituting the equilibrium b_1 back into the previous two equations, and then all of the remaining coefficients follow from substituting in B_2, b_1 and b_2.

B.4.2. Information market equilibrium

Having characterized the financial market equilibrium, given a disclosure policy, we can now characterize investors' optimal information choices, given a disclosure policy. As in the baseline model, to keep the analysis transparent and eliminate cumbersome enumeration of corner cases, we focus on "interior" equilibria in the information market (i.e., equilibria in which $\lambda_1, \lambda_2 \in (0,1)$. Because investors have mean-variance preferences, the equilibrium conditions in the information market do not follow from the benchmark model, so we need to derive them directly. Plugging the optimal $t=2$ demand $X_{i2} = \frac{\mathbb{E}_{i2}[V-P_2]}{\gamma\mathbb{V}_{i2}[V]}$ into the objective function yields the optimized MV objective at the trading stage

$$\mathbb{E}_{i2}[W_2 + x(V-P_2)] - \frac{\gamma}{2}\mathbb{V}_{i2}(W_2 + X_{i2}\times(V-P_2)) = W_2 + \frac{1}{2\gamma}\frac{\mathbb{E}_{i2}^2[V-P_2]}{\mathbb{V}_{i2}(V)}.$$

Now, stepping back to time $t=2^-$ immediately before the trading round, recalling that $\mathcal{F}_{2^-} = \sigma\left(s_{p1}, d, x\right)$, the net value of choosing to become informed is

$$\mathbb{E}_{2^-}\left[W_2 + \frac{1}{2\gamma}\frac{\mathbb{E}_{I2}^2[V-P_2]}{\mathbb{V}_{I2}(V)} - c\right] = W_2 + \frac{1}{2\gamma}\frac{1}{\mathbb{V}_{I2}(V)}\mathbb{E}_{2^-}\left[\mathbb{E}_{I2}^2[V-P_2]\right] - c$$

while the value of remaining uninformed is similarly

$$\mathbb{E}_{2^-}\left[W_2+\frac{1}{2\gamma}\frac{\mathbb{E}^2_{U2}[V-P_2]}{\mathbb{V}_{U2}(V)}\right]=W_2+\frac{1}{2\gamma}\frac{1}{\mathbb{V}_{U2}(V)}\mathbb{E}_{2^-}\left[\mathbb{E}^2_{U2}[V-P_2]\right].$$

Equating these expressions yields the indifference condition that characterizes an interior equilibrium in the information market:

$$\frac{1}{2\gamma}\frac{1}{\mathbb{V}_{I2}(V)}\mathbb{E}_{2^-}\left[\mathbb{E}^2_{I2}[V-P_2]\right]-c=\frac{1}{2\gamma}\frac{1}{\mathbb{V}_{U2}(V)}\mathbb{E}_{2^-}\left[\mathbb{E}^2_{U2}[V-P_2]\right] \tag{191}$$

To simplify, note that for both $i\in\{I,U\}$, we can write

$$\begin{aligned}\mathbb{E}_{2^-}\left[\mathbb{E}^2_{i2}[V-P_2]\right]&=\mathbb{V}_{2^-}\left(\mathbb{E}_{i2}[V-P_2]\right)+\mathbb{E}^2_{2^-}[V-P_2]\\&=\mathbb{V}_{2^-}(V-P_2)-\mathbb{V}_{i2}(V-P_2)+\mathbb{E}^2_{2^-}[V-P_2]\\&=\mathbb{V}_{2^-}(V-P_2)+\mathbb{E}^2_{2^-}[V-P_2]-\mathbb{V}_{i2}(V).\end{aligned} \tag{192}$$

Furthermore, we can express

$$P_2=\omega_2\mathbb{E}_{I2}[V]+(1-\omega_2)\mathbb{E}_{U2}[V]-\gamma\frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)}+\frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}Z_2$$

with $\omega_2=\frac{\frac{\lambda_2}{\mathbb{V}_{I2}(V)}}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)}+\frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}$. Hence, we have

$$\begin{aligned}\mathbb{E}_{2^-}[V-P_2]&=\mathbb{E}_{2^-}\left[\omega_2(V-\mathbb{E}_{I2}[V])+(1-\omega_2)(V-\mathbb{E}_{U2}[V])+\gamma\frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)}+\frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}Z_2\right]\\&=\gamma\overline{Z}\frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)}+\frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}.\end{aligned}$$

Furthermore,

$$\begin{aligned}&\mathbb{V}_{2^-}(V-P_2)\\&=\mathbb{C}_{2^-}(V,V-P_2)-\mathbb{C}_{2^-}(P_2,V-P_2)\\&=\mathbb{C}_{2^-}(V,V-P_2)-\mathbb{C}_{2^-}(P_2,\mathbb{E}_{U2}[V-P_2])\\&=\frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)}+\frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}\\&\quad-\mathbb{C}_{2^-}\left(\mathbb{E}_{U2}[V]-\gamma\frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)}+\frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}\mathbb{E}_{U2}[Z_2],\gamma\frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)}+\frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}\mathbb{E}_{U2}[Z_2]\right)\end{aligned} \tag{193}$$

Using the law of total covariance, we have

$$\begin{aligned}\mathbb{C}_{2^-}(V,Z_2)&=\mathbb{C}_{2^-}(\mathbb{E}_{U2}[V],\mathbb{E}_{U2}[Z_2])+\mathbb{C}_{U2}(V,Z_2)\\\Rightarrow\mathbb{C}_{2^-}(\mathbb{E}_{U2}[V],\mathbb{E}_{U2}[Z_2])&=-\mathbb{C}_{U2}(V,Z_2)=-\frac{1}{b_2}\mathbb{C}_{U2}(V,b_2Z_2)=\frac{1}{b_2}\mathbb{V}_{U2}(\theta)\\\mathbb{C}_{2^-}(Z_2,Z_2)&=\mathbb{C}_{2^-}(\mathbb{E}_{U2}[Z_2],\mathbb{E}_{U2}[Z_2])+\mathbb{C}_{U2}(Z_2,Z_2)\\\Rightarrow\mathbb{C}_{2^-}(\mathbb{E}_{U2}[Z_2],\mathbb{E}_{U2}[Z_2])&=\mathbb{V}_{2^-}(Z_2)-\mathbb{V}_{U2}(Z_2)=\frac{1}{\tau_z}-\frac{1}{b^2}\mathbb{V}_{U2}(b_2Z_2)\end{aligned}$$

$$= \frac{1}{\tau_z} - \frac{1}{b_2^2} \mathbb{V}_{U2}(\theta)$$

And note that $b_2 = -\frac{\gamma}{\lambda_2} \frac{1}{\tau_u} = -\gamma \frac{\mathbb{V}_{I2}(V)}{\lambda_2}$. Hence, substituting into eq. (193) yields

$$\begin{aligned}
&\mathbb{V}_{2^-}(V - P_2) \\
&= \frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \\
&\quad - \mathbb{C}_{2^-}\left(\mathbb{E}_{U2}[V] - \gamma \frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \mathbb{E}_{U2}[Z_2], \gamma \frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \mathbb{E}_{U2}[Z_2]\right) \\
&= \left(\gamma \frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}\right)^2 \frac{1}{\tau_z} + \frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \\
&\quad + \lambda_2(1-\lambda_2)\left(\frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}\right)^2 \left(\frac{1}{\mathbb{V}_{I2}(V)} - \frac{1}{\mathbb{V}_{U2}(V)}\right)
\end{aligned}$$

Finally, substituting everything back into eq. (192) gives

$$\begin{aligned}
&\mathbb{E}_{2^-}\left[\mathbb{E}_{i2}^2[V - P_2]\right] \\
&= \mathbb{V}_{2^-}(V - P_2) + \mathbb{E}_{2^-}^2[V - P_2] - \mathbb{V}_{i2}(V) \\
&= \left(\gamma \frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}\right)^2 \left(\overline{Z}^2 + \frac{1}{\tau_z}\right) + \frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \\
&\quad + \lambda_2(1-\lambda_2)\left(\frac{1}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}\right)^2 \left(\frac{1}{\mathbb{V}_{I2}(V)} - \frac{1}{\mathbb{V}_{U2}(V)}\right) - \mathbb{V}_{i2}(V),
\end{aligned}$$

and plugging this back into the $t = 2$ information equilibrium condition, eq. (191), fully characterizes the information equilibrium at $t = 2$ as a function of λ_2 and the conditional variances of the two trader types, $\mathbb{V}_{i2}(V)$.

Consider now the $t = 1$ information equilibrium. Because traders are 1-period mean-variance optimizers, the net values of being informed or uninformed take similar forms to those at $t = 2$. The value of being informed is

$$\mathbb{E}_{1^-}\left[W_1 + \frac{1}{2\gamma} \frac{\mathbb{E}_{I1}^2[P_2 - P_1]}{\mathbb{V}_{I1}(P_2)} - c\right] = W_1 + \frac{1}{2\gamma} \frac{1}{\mathbb{V}_{I1}(P_2)} \mathbb{E}_{1^-}\left[\mathbb{E}_{I1}^2[P_2 - P_1]\right] - c$$

while the value of remaining uninformed is similarly

$$\mathbb{E}_{1^-}\left[W_1 + \frac{1}{2\gamma} \frac{\mathbb{E}_{U1}^2[P_2 - P_1]}{\mathbb{V}_{U1}(P_2)}\right] = W_1 + \frac{1}{2\gamma} \frac{1}{\mathbb{V}_{U1}(P_2)} \mathbb{E}_{1^-}\left[\mathbb{E}_{U1}^2[P_2 - P_1]\right].$$

Hence, the information equilibrium condition for an interior equilibrium is

$$\frac{1}{2\gamma} \frac{1}{\mathbb{V}_{I1}(P_2)} \mathbb{E}_{1^-}\left[\mathbb{E}_{I1}^2[P_2 - P_1]\right] - c = \frac{1}{2\gamma} \frac{1}{\mathbb{V}_{U1}(P_2)} \mathbb{E}_{1^-}\left[\mathbb{E}_{U1}^2[P_2 - P_1]\right] \tag{194}$$

For both $i \in \{I, U\}$, we can write

$$\begin{aligned}\mathbb{E}_{1^-}\left[\mathbb{E}_{i1}^2[P_2-P_1]\right] &= \mathbb{V}_{1^-}\left(\mathbb{E}_{i1}[P_2-P_1]\right)+\mathbb{E}_{1^-}^2\left[P_2-P_1\right] \\ &= \mathbb{V}_{1^-}\left(P_2-P_1\right)-\mathbb{V}_{i1}\left(P_2-P_1\right)+\mathbb{E}_{1^-}^2\left[P_2-P_1\right] \\ &= \mathbb{V}_{1^-}\left(P_2-P_1\right)+\mathbb{E}_{1^-}^2\left[P_2-P_1\right]-\mathbb{V}_{i1}\left(P_2\right). \end{aligned} \tag{195}$$

To simplify further, note that we can express

$$P_1=\omega_1\mathbb{E}_{I1}[P_2]+(1-\omega_1)\mathbb{E}_{U1}[P_2]-\gamma\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}Z_1$$

with $\omega_1=\frac{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)}}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}$. Hence, we have

$$\begin{aligned}\mathbb{E}_{1^-}\left[P_2-P_1\right] &= \mathbb{E}_{1^-}\Bigg[\omega_1\left(P_2-\mathbb{E}_{I1}[P_2]\right)+(1-\omega_1)\left(P_2-\mathbb{E}_{U1}[P_2]\right) \\ &\qquad +\gamma\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}Z_1\Bigg] \\ &= \gamma\overline{Z}\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}.\end{aligned}$$

Furthermore,

$$\begin{aligned}&\mathbb{V}_{1^-}\left(P_2-P_1\right) \\ &= \mathbb{C}_{1^-}\left(P_2,P_2-P_1\right)-\mathbb{C}_{1^-}\left(P_1,P_2-P_1\right) \\ &= \mathbb{C}_{1^-}\left(P_2,P_2-P_1\right)-\mathbb{C}_{1^-}\left(P_1,\mathbb{E}_{U1}\left[P_2-P_1\right]\right) \\ &= \frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}} \\ &\quad -\mathbb{C}_{1^-}\left(\mathbb{E}_{U1}[P_2]-\gamma\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}\mathbb{E}_{U1}\left[Z_1\right],\gamma\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}\mathbb{E}_{U1}\left[Z_1\right]\right).\end{aligned} \tag{196}$$

Using the law of total covariance, we have

$$\begin{aligned}\mathbb{C}_{1^-}\left(P_2,Z_1\right) &= \mathbb{C}_{1^-}\left(\mathbb{E}_{U1}\left[P_2\right],\mathbb{E}_{U1}\left[Z_1\right]\right)+\mathbb{C}_{U1}\left(P_2,Z_1\right) \\ \Rightarrow \mathbb{C}_{1^-}\left(\mathbb{E}_{U1}\left[P_2\right],\mathbb{E}_{U1}\left[Z_1\right]\right) &= -\mathbb{C}_{U1}\left(P_2,Z_2\right)=-\frac{1}{b_1}\mathbb{C}_{U1}\left(P_2,b_1Z_1\right)=\frac{1}{b_1}\mathbb{C}_{U1}\left(P_2,\theta\right) \\ \mathbb{C}_{1^-}\left(Z_1,Z_1\right) &= \mathbb{C}_{1^-}\left(\mathbb{E}_{U1}\left[Z_1\right],\mathbb{E}_{U1}\left[Z_1\right]\right)+\mathbb{C}_{U1}\left(Z_1,Z_1\right) \\ \Rightarrow \mathbb{C}_{1^-}\left(\mathbb{E}_{U1}\left[Z_1\right],\mathbb{E}_{U1}\left[Z_1\right]\right) &= \mathbb{V}_{1^-}\left(Z_1\right)-\mathbb{V}_{U1}\left(Z_1\right)=\frac{1}{\tau_z}-\frac{1}{b_1^2}\mathbb{V}_{U1}\left(b_1Z_2\right) \\ &= \frac{1}{\tau_z}-\frac{1}{b_1^2}\mathbb{V}_{U1}\left(\theta\right)\end{aligned}$$

And note that $\mathbb{C}_{U1}\left(P_2,\theta\right)=B_2\mathbb{V}_{U1}\left(\theta\right)$ and $b_1=-\frac{1}{B_2}\frac{\gamma}{\lambda_1}\mathbb{V}_{I1}\left(P_2\right)$, so we have

$$\frac{1}{b_1}\mathbb{C}_{U1}(P_2,\theta) = -B_2\frac{\lambda_1}{\gamma}\frac{1}{\mathbb{V}_{I1}(P_2)}B_2 = -\frac{\lambda_1}{\gamma}\frac{1}{\mathbb{V}_{I1}(P_2)}B_2^2\mathbb{V}_{U1}(\theta)$$

$$\frac{1}{\tau_z} - \frac{1}{b_1^2}\mathbb{V}_{U1}(\theta) = \frac{1}{\tau_z} - \left(\frac{\lambda_1}{\gamma}\frac{1}{\mathbb{V}_{I1}(P_2)}\right)^2 B_2^2\mathbb{V}_{U1}(\theta)$$

Hence, substituting into eq. (196) yields

$$\begin{aligned}
&\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)} + \frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}} \\
&\quad - \mathbb{C}_{1^-}\left(\mathbb{E}_{U1}[P_2] - \gamma\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)} + \frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}\mathbb{E}_{U1}[Z_1], \gamma\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)} + \frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}\mathbb{E}_{U1}[Z_1]\right) \\
&\quad = \left(\gamma\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)} + \frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}\right)^2\frac{1}{\tau_z} + \frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)} + \frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}} \\
&\quad + \lambda_1(1-\lambda_1)\left(\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)} + \frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}\right)^2\left(\frac{1}{\mathbb{V}_{I1}(P_2)} - \frac{1}{\mathbb{V}_{U1}(P_2)}\right)
\end{aligned}$$

Finally, substituting everything back into eq. (195) gives

$$\begin{aligned}
&\mathbb{E}_{1^-}\left[\mathbb{E}_{i1}^2[P_2 - P_1]\right] \\
&= \mathbb{V}_{1^-}(P_2 - P_1) + \mathbb{E}_{1^-}^2[P_2 - P_1] - \mathbb{V}_{i1}(P_2) \\
&= \left(\gamma\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)} + \frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}\right)^2\left(\overline{Z}^2 + \frac{1}{\tau_z}\right) + \frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)} + \frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}} \\
&\quad + \lambda_1(1-\lambda_1)\left(\frac{1}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2)} + \frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2)}}\right)^2\left(\frac{1}{\mathbb{V}_{I1}(P_2)} - \frac{1}{\mathbb{V}_{U1}(P_2)}\right) - \mathbb{V}_{i1}(P_2)
\end{aligned}$$

and plugging this back into the $t = 1$ information equilibrium condition, eq. (194), fully characterizes the information equilibrium at $t = 1$ as a function of λ_1 and the conditional variances of the two trader types, $\mathbb{V}_{i1}(P_2)$.

B.4.3. Optimal disclosure

With the equilibrium in the financial and information markets pinned down, the expected value of P_2 for the in the event of disclosure or non-disclosure is

$$U_d(x;q) = \mathbb{E}\left[P_2\middle|d,x\right] = \overline{V} - \frac{\gamma}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}\bar{Z} + x\bar{\eta}, \tag{197}$$

so that, as in the baseline model, the effect of disclosure operates through the risk premium $\frac{\gamma}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}\bar{Z}$ via its effect on the fraction informed λ_2 and the uninformed conditional variance $\mathbb{V}_{U2}(V)$ which depend endogenously on investors' trading and information acquisition decisions characterized above.

Note that the $x = 0$ firm always discloses with probability zero. While analytically characterizing the equilibrium is not tractable, we can solve for the equilibrium numerically. Note that for a given set of parameters, including a probability of mixing r, we can solve equation (190) and the date 1 and date 2 information conditions for $\{b_1, \lambda_1, \lambda_2\}$, which characterizes the financial market / information acquisition equilibrium. We then compare the expected utility from disclosing versus not for the $x = 1$ firm to determine whether there exists an equilibrium of each type. Specifically,

(i) if the value from disclosing (net of costs) is higher than the value from not disclosing in the case that investors conjecture that the $x = 1$ manager always discloses, $r = 1$, then a pure strategy equilibrium with disclosure by the $x = 1$ type exists;

(ii) if the value from not disclosing is higher than the value from disclosing (net of costs), in the case that investors conjecture that the $x = 1$ manager never discloses, $r = 0$, then a pure strategy equilibrium with no disclosure by the $x = 1$ type exists; and

(iii) if the $x = 1$ type is indifferent between disclosing (net of costs) and not disclosing, in the case that investors conjecture that the $x = 1$ manager discloses with probability $r \in (0, 1)$, then a mixed-strategy equilibrium exists in which the manager discloses with the conjectured probability r.

We illustrate the equilibrium disclosure strategies in the mean-variance setting in Fig. 5 under the same numerical parameter values that we used in the baseline model. When disclosure costs are sufficiently low, there only exists a pure strategy equilibrium with disclosure, while when disclosure costs are sufficiently high, there only exists a pure strategy equilibrium with no disclosure. For intermediate levels of costs, we find that all three types of equilibrium co-exist.

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